A Radio-Frequency-over-Fiber Link for Large-Array Radio Astronomy Applications

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Abstract

The Canadian Hydrogen Intensity Mapping Experiment (CHIME) is a specialized array of cylindrical radio telescopes that will study the nature of dark energy by mapping the three-dimensional distribution of neutral hydrogen gas in the universe detected from redshifted 21 cm radiation. CHIME will measure the characteristic Baryon Acoustic Oscillation (BAO) scale across the key redshift range $z \approx 0.8$ to $z \approx 2.5$ where the expansion history of the universe changes from one dominated by the attractive force of ordinary gravity to one dominated by dark energy.

While characterizing the CHIME analog receiver, the implementation of a Radio-Frequency-over-Fiber (RFoF) link to transport the signals from the antennas to the processing room has been investigated as an alternative to traditional coaxial cable. In this thesis, a prototype version of a custom-built RFoF link for CHIME is presented, as well as tests on its dynamic performance and gain and phase stability. We demonstrate that RFoF can be successfully used as a cost-effective solution for analog signal transport on the CHIME telescope and in other large-array radio astronomy applications.
Abrégé

L’expérience CHIME (Canadian Hydrogen Intensity Mapping Experiment) est un réseau de radiotélescopes cylindriques qui vont cartographier la distribution tridimensionnelle des atomes d’hydrogène neutres dans l’univers en détectant directement leurs émissions radio à 21 cm décalées vers le rouge afin d’étudier la nature de l’énergie sombre. CHIME mesurera l’échelle spatiale des oscillations acoustiques baryoniques (BAO) sur une plage de décalage vers le rouge allant de $z \approx 0.8$ à $z \approx 2.5$ dans les directions radiales and angulaires, là où se trouve la transition entre la décélération de l’expansion de l’univers dominée par les forces gravitationnelles, vers son accélération dominée par l’énergie sombre.

Nous étudions dans cette thèse la possibilité d’utiliser un lien optique pour transmettre les signaux radio du télescope au lieu des câbles coaxiaux traditionnels. Nous présentons une version prototype du lien radio optique de CHIME ainsi que les résultats des tests de performance dynamique et de stabilité de gain et de phase du lien. Ces tests démontrent que des liens radio optiques ont les performances nécessaires pour être utilisés avec succès dans le transport à faible coût des signaux astronomiques du télescope de CHIME et d’autres télescopes de radioastronomie.
Dedication

This thesis is dedicated to my family.
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Chapter 1

Introduction

CHIME, the Canadian Hydrogen Intensity Mapping Experiment\(^1\) is a novel radio telescope designed to study the expansion history of the universe and probe the nature of dark energy. When it is complete in 2017, CHIME will map the three-dimensional distribution of neutral hydrogen gas in the universe by directly detecting its redshifted 21-cm radiation. By measuring the scale of the Baryon Acoustic Oscillations across the redshift range \(z \approx 0.8\) to \(z \approx 2.5\) in both the angular and line-of-sight directions, CHIME will study the epoch when dark energy generated the transition from decelerated to accelerated expansion of the universe.

1.1 Dark Energy

Observations since 1998 \([60]\) \([68]\) have established that the expansion of the universe is accelerating. More recently, results from the Planck mission confirm that, based on the standard model of cosmology, the total mass-energy in the universe contains 4.9% ordinary matter, 26.8% dark matter and 68.3% the mysterious dark energy which, unlike any known form of matter or energy, counters the attractive force of gravity \([63]\) \([64]\). Understanding dark energy is one of the great drivers of cosmology today since it has deep implications in astronomy, high energy physics, general relativity and string theory.

One possibility is that the universe is permeated by an energy density which is constant in time and uniform in space. In this case, dark energy takes the form of Einstein’s cosmological constant \(\Lambda\), and the ratio of its pressure to its energy density (its equation of state) is \(w = P_{\Lambda}/\rho_{\Lambda} = -1\) at all times. Another possibility is that dark energy is a dynamical fluid. In this case the equation of state of the fluid would not likely be

\(^{1}\text{www.CHIMExperiment.ca}\)
constant, but it would be a function of the scale factor of the universe, \( a = (1 + z)^{-1} \), where \( z \) is redshift. Different theories have different predictions for the evolution of the equation of state.

Whether dynamical or a constant, one possible parametrization for the equation of state of dark energy is

\[
w(a) = \frac{P_\Lambda(a)}{\rho_\Lambda(a)} = w_0 + (1 - a)w_a
\]

where \( w_0 \) is the present value of \( w \), and \( w_a \) parametrizes the evolution of \( w(a) \). This parametrization is useful if dark energy is important at late times but insignificant at early times [4]. It also provides a framework to compare theory to experiments.

### 1.1.1 Cosmological Model

For a detailed review on the cosmological model, see [15] [27]. On large scales the universe is extremely isotropic and homogeneous. The space-time metric that describes a homogeneous, isotropic expanding universe is the Friedmann-Robertson-Walker (FRW) metric given by (using units with \( c = 1 \))

\[
ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]
\]

with \((r, \theta, \phi)\) as the comoving spatial coordinates, \( a \) is the scale factor (normalized to 1 today) and \( k \) is a constant representing the curvature of the space (\( k > 0 \) for a closed universe, \( k < 0 \) for an open universe, and \( k = 0 \) for a flat universe). Applying the FRW metric to Einstein’s field equations of general relativity gives the Friedmann equations

\[
\begin{align*}
\frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3} \quad \text{\quad (1.3)} \\
\left(\frac{\dot{a}}{a}\right)^2 &= H^2(t) = \frac{8\pi G \rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \quad \text{\quad (1.4)}
\end{align*}
\]

where \( P \) and \( \rho \) are the mean pressure and density of the contents of the universe and \( H(t) \) is the Hubble parameter. \( \Lambda \) is the cosmological constant, which can be interpreted as an additional component of the universe in the form of vacuum energy (dark energy) \( \rho_\Lambda = \Lambda/(8\pi G) = \text{constant} \). From the last two equations it follows that
\[ \dot{\rho} = -3H(\rho + P). \]  

Assuming that all the components of the energy density are perfect fluids that obey the equation of state

\[ P_i = w_i \rho_i \]  

where \( w_i \) is constant, it follows that

\[ \rho_i \propto a^{-3(1+w_i)} \]  

with \( w = 0, 1/3, -1 \) for radiation, matter and the cosmological constant respectively. We now introduce the critical density (defined as the density for which the spatial geometry is flat)

\[ \rho_c = \frac{3H^2}{8\pi G} \]  

so we can define the density parameters of the different components of the universe today as

\[ \Omega_r = \frac{\rho_{r0}}{\rho_{c0}} \quad \Omega_m = \frac{\rho_{m0}}{\rho_{c0}} \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{c0}} = \frac{\Lambda}{3H_0^2} \]  

where \( \rho_{c0} = 3H_0^2/(8\pi G) \) is the critical density today, \( H_0 \) is the value of the Hubble parameter today and \( \Omega_r, \Omega_m, \Omega_\Lambda \) are, respectively, the densities of radiation, matter and cosmological constant today relative to the critical density. Analogously, we define the parameter

\[ \Omega_k = -\frac{k}{H_0^2} \]  

so equation 1.4 can be written as
\[
\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda. 
\] (1.11)

\(\Omega_k\) is the “spatial curvature density” today. \(\Omega_k < 0\) for a closed universe, \(\Omega_k > 0\) for an open universe and \(\Omega_k = 0\) for a flat (euclidean) universe. The spatial geometry of the universe has been measured to be nearly flat. If the equation of state for dark energy has \(w \neq -1\), or if \(w\) changes with the scale factor, equation 1.11 can be generalized with \[4\]

\[
\Omega_\Lambda \rightarrow \Omega_\Lambda \exp \left\{ 3 \int_a^1 \frac{da'}{a'} [1 + w(a')] \right\}. 
\] (1.12)

### 1.2 Baryon Acoustic oscillations

The early universe consisted of a hot plasma in which photons and baryons were tightly coupled via Thomson scattering. Quantum mechanical fluctuations that were present at the beginning of the universe and that expanded to scales larger than the Hubble volume during inflation caused the development of overdense regions consisting of baryons, dark matter, and photons. These fluctuations excite sound waves in the relativistic plasma (acoustic oscillations) of the early universe due to the competing forces of radiation pressure and gravity in the overdensity regions: while an overdensity gravitationally attracts baryons inwards, the heat of photon-baryon interactions creates a large amount of outward pressure. After about 380000 years of expansion, the plasma cooled to below 3000 K and the electrons and protons in the plasma could combine to form neutral hydrogen atoms. This recombination happened at a redshift \(z \sim 1000\). Recombination abruptly decreases the sound speed and effectively ends the wave propagation. During the period between the generation of the perturbations and recombination, modes of different wavelength complete different number of oscillation periods [30]. These baryon and dark matter perturbations are the seeds of the formation of large scale structure in the universe at late times.

#### 1.2.1 Cosmic Sound

The acoustic oscillations in the primordial plasma are also imprinted in the late-time baryon power spectrum. A simple way to understand this is to consider a single spherical perturbation common to the dark matter and the baryons. The dark matter only interacts gravitationally and so it stays at the center of the sound wave, the origin of the
overdensity. The tightly coupled photon-baryon plasma will propagate outwards as an acoustic wave with a speed

\[ c_s = \frac{c}{\sqrt{3(1 + R)}} \]

\[ R = 3 \frac{\rho_b}{\rho_r} \propto \frac{\Omega_b}{1 + z} \]

(1.13)

where \( \rho_b \) is the baryon density [31]. At recombination, electrons and protons in the plasma combine to form neutral hydrogen atoms, photons interact to a much lesser degree with baryons and the pressure on the baryons is removed. This stops the baryon wave propagation while the photons propagate freely, forming what we now observe as the Cosmic Microwave Background (CMB). This leaves a shell of baryonic matter at a fixed characteristic radius which is imprinted on the distribution of the baryons as a density excess. The gravitational interaction between baryons and dark matter implies that dark matter also preferentially clumps on this scale [9]. This characteristic scale will appear as a bump in the correlation function of the baryon density field at the radius of the spherical shell (Figure 1.1). This feature will also appear as oscillations in the baryon power spectrum (Figure 1.2), since the correlation function and power spectrum form a Fourier pair (see Section 1.3 below).

The comoving distance a sound wave could have travelled in the photon-baryon fluid before recombination is [32]

\[ s = \int_{z_{rec}}^{\infty} \frac{c_s \, dz}{H(z)} = \frac{1}{\sqrt{\Omega_m H_0^2}} \frac{2c}{\sqrt{3z_{eq} R_{eq}}} \ln \left[ \frac{\sqrt{1 + R_{rec}} + \sqrt{R_{rec} + R_{eq}}}{1 + \sqrt{R_{rec}}} \right] \]

(1.14)

where "rec" and "eq" refer to the epochs of recombination and matter-radiation equality respectively and \( 1 + z_{eq} = \Omega_m / \Omega_r \). The value of \( z_{rec} \) is very insensitive to cosmology since it is mainly determined by atomic physics [32]. The additional parameters in \( s \) are accurately determined by measurements in the CMB power spectrum, fixing the comoving sound horizon at \( s \approx 147 \) Mpc [64].

### 1.2.2 Standard Ruler

Since the scale of the comoving sound horizon is known, it can be used as a standard ruler to learn about dark energy and the expansion history of the universe. We can observe the preferred clustering scale set by the Baryon Acoustic Oscillations (BAO) at different redshifts to constrain the Hubble parameter and the angular diameter distance.
Chapter 1. Introduction

Figure 1.1: The BAO peak in the correlation function of galaxies from the Sloan Digital Sky Survey (SDSS) of luminous red galaxies (LRG). Source: [30]

Figure 1.2: The ratio of the matter power spectrum with BAO to the power spectrum without BAO. The angular and frequency resolution of CHIME are set to detect the density enhancements at the 147 Mpc sound horizon and its second and third harmonics at 74 and 49 Mpc. Higher peaks are partially washed out by gravitational interactions. Error bars for a CHIME two year survey at a single effective redshift for a system temperature of 50 K are shown. Source: [24]
For a review of distance measures in cosmology refer to [27]. The fundamental distance measure, from which all other distances in the FRW metric are determined is the radial comoving distance

$$\chi(a) = \int_{t(a)}^{t_0} \frac{cdt'}{a(t')} = \int_a^1 \frac{cd\alpha'}{a'^2H(a')} \quad \rightarrow \quad \chi(z) = \int_0^z \frac{cdz'}{H(z')}$$

(1.15)

where we used $$dt = da/\dot{a}, \quad H = \dot{a}/a$$ and $$a = 1/(1 + z)$$. Now we introduce the angular diameter distance

$$d_A(z) = \frac{1}{1+z} \begin{cases} \frac{c}{H_0\sqrt{|\Omega_k|}} \sinh \left( \frac{H_0}{c} \sqrt{\Omega_k} \chi(z) \right) & \text{if } \Omega_k > 0 \\ \chi(z) & \text{if } \Omega_k = 0 \\ \frac{c}{H_0\sqrt{|\Omega_k|}} \sin \left( \frac{H_0}{c} \sqrt{-\Omega_k} \chi(z) \right) & \text{if } \Omega_k < 0. \end{cases}$$

(1.16)

In general, we need to know both $$\Omega_k$$ and $$\chi(z)$$ to determine cosmological distances. In a flat universe, cosmological distances are determined by $$\chi(z)$$ only and $$d_A(z) = \chi(z)/(1 + z)$$. In this case, the characteristic BAO scale, $$s_{\parallel}(z)$$, along the line-of-sight provides a measurement of the Hubble parameter through

$$H(z) = \frac{c\Delta z}{s_{\parallel}(z)}$$

(1.17)

while the tangential mode provides a measurement of the angular diameter distance

$$d_A(z) = \frac{s_{\perp}(z)}{\Delta \theta(1 + z)}.$$

(1.18)

This is shown in Figure 1.3. The BAO is a statistical standard ruler in the sense that the preferred length in the matter distribution cannot be directly observed. The early universe is permeated by many of the spherical acoustic waves described in Section 1.2.1 and hence the final density distribution is a linear superposition of the small-amplitude sound waves. This superposition of rings visually "hides" the characteristic scale. However, the acoustic signature is still detectable statistically through the two-point correlation function of the matter distribution, $$\xi(r)$$, which quantifies the excess clustering in the spatial matter distribution as function of relative distance. The BAO will appear as
a peak in the correlation function (Figure 1.1) and as oscillations in its power spectrum (Figure 1.2).

1.3 21 cm Radiation

In a neutral hydrogen atom (HI) the electron and the proton interact predominantly through the Coulomb interaction $V(r) \propto e^2/r$. The energy levels of the Hamiltonian with this potential are given by $E_n = -13.6 \text{ eV}/n^2$, where $n$ is a positive integer. In addition, there are relativistic corrections that lead to a fine structure of these energy levels. There is also another interaction that involves the intrinsic spins of both the electron and the proton. Since the proton has a magnetic moment, it is a source of a magnetic field. The electron interacts with this magnetic field, generating an interaction energy proportional to the intrinsic spin of the two particles.

When two spin 1/2 particles are combined in absence of orbital angular momentum (which is the case of the ground state of hydrogen), the net state has either spin 1 or spin 0. In fact, there are three spin 1 states, known as triplet states, and a single spin 0 state, known as the singlet state. From the analysis of the perturbation Hamiltonian, it follows that this spin-spin coupling breaks the degeneracy of the $1s_{1/2}$ states in hydrogen, lifting the energy of the triplet configuration, and lowering that of the singlet (figure 1.4). This splitting is known as hyperfine structure. Using time independent perturbation theory, the net energy difference between the singlet and the triplet states is
Figure 1.4: Hyperfine splitting of the ground state of hydrogen.

\[ \Delta E = \frac{8}{3} g_p m_e \alpha^2 E_0 = 5.88 \cdot 10^{-6} \text{ eV} \]  

where \( E_0 = 13.6 \text{ eV} \) is the (absolute value of the) ground-state energy, \( g_p \) is the proton gyromagnetic ratio and \( \alpha \) is the fine-structure constant. This energy corresponds to a wavelength \( \lambda_0 \approx 21.1 \text{ cm} \) and a frequency \( \nu_0 \approx 1420.4 \text{ MHz} \). For a detailed discussion on the quantum mechanics behind the hyperfine splitting of the ground state of hydrogen refer to [39] [81].

The transition from a triplet state to the singlet is highly forbidden. The spontaneous emission coefficient is \( A_{10} = 2.85 \cdot 10^{-15} \text{ s}^{-1} \). However, as the total number of atoms of neutral hydrogen in the Intergalactic Medium (IGM) is very large, this emission line is easily observed by radio telescopes. Also, the lifetime can be considerably shortened by other mechanisms like collisions with other hydrogen atoms and interaction with the CMB.

### 1.3.1 21 cm optical depth and brightness temperature

The intensity of the 21 cm transition is determined by the spin temperature, \( T_s \), defined through the equation

\[ \frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-T_*/T_s} \]  

where \( n_1 \) and \( n_0 \) are the number densities of electrons in the triplet and singlet states of the hyperfine level respectively, \( g_1 = 3 \) and \( g_0 = 1 \) are the respective statistical weights, and \( T_* = h\nu_0/k_b \approx 0.068 \text{ K} \) is the equivalent temperature of the hyperfine
transition energy. The spin temperature, whose name reflects the fact that the hyperfine splitting of the ground state of hydrogen arises from the coupling between the electron and proton spins, is just a shorthand for the ratio between the occupation number of the two hyperfine levels. This ratio establishes the intensity of the radiation emerging from a cloud of neutral hydrogen. Because all astrophysical processes have \( T_s \gg T_* \), approximately three of four atoms find themselves in the excited state \( [35] \).

Even though, at the redshifts relevant to CHIME, the 21 cm radiation is seen only in emission, it is instructive to go through the full derivation of the 21 cm differential brightness temperature including absorption. If we consider the 21 cm transition as a two level system, then the absorption coefficient of a cloud of hydrogen can be found from the Einstein coefficients (see \[70\] for details)

\[
\alpha(\nu) = \frac{c^2 A_{10} g_1}{8\pi v^2} \frac{g_0}{n_0} \left( 1 - \frac{n_1 g_0}{n_0 g_1} \right) \phi(\nu)
\]

where \( \phi(\nu) \) is the line profile; it is sharply peaked at \( \nu_0 \) and describes the relative effectiveness of frequencies in the neighbourhood of \( \nu_0 \) for causing transitions. The line profile is defined so that \( \int d\nu \phi(\nu) = 1 \). From equations 1.20 and 1.21 and using \( T_s \gg T_* \), \( \alpha(\nu) \) becomes

\[
\alpha(\nu) = \frac{c^2 A_{10} g_1}{8\pi v^2} \frac{g_0}{n_0} \left( 1 - e^{-T_*/T_s} \right) \phi(\nu) \approx \frac{c^2 A_{10} g_1}{8\pi v^2} \frac{T_*}{T_s} \phi(\nu) \approx \frac{3hc^2 A_{10} n_{HI}}{16k_B \nu T_s} \phi(\nu). \quad (1.22)
\]

In the last expression, \( n_{HI} \) is the neutral hydrogen density and there is an extra factor of 1/4 in equation 1.22 to account for the fraction of HI atoms in the hyperfine singlet state. In order to estimate the optical depth, \( \tau(\nu) \), as a function of observed frequency, due to absorption by the IGM along the line of sight to a given redshift, we must integrate \( \alpha(\nu) \) along the radiation path length, i.e. from the radiation source at \( z_{em} \) to \( z = 0 \).

If \( \alpha(\nu) \) is the absorption coefficient at frequency \( \nu \), then the increment of optical depth for photons of this frequency which are redshifted to frequency \( \nu_{obs} \) when they reach the earth is \( d\tau(\nu_{obs}) = \alpha(\nu) dl \), where \( dl = c dt \) is the element of proper distance at redshift \( z \) and \( dt \) is the time interval that radiation takes to travel the path length \( dl \). Then

\[
\tau(\nu_{obs}) = \int_0^l d\alpha(\nu) = \int_0^{z_{em}} \frac{dz \alpha(\nu)}{H(z)(1 + z)} \to \tau(z) = \int_{\nu_{obs}}^{\nu_{obs}(1+z_{em})} \frac{c \alpha(\nu)}{\nu H(z) \nu} d\nu \quad (1.23)
\]
where we used the fact that $\nu = \nu_{\text{obs}}(1 + z)$ so $dz = d\nu/\nu_{\text{obs}}$. As a first approximation, we ignore the broadening of the line profile and assume that it can be approximated by a delta function, $\phi(\nu) = \delta(\nu - \nu_0)$ so

$$\tau(z) = \frac{c\alpha(\nu_0)}{\nu_0 H(z)} = \frac{3hc^2A_{10}n_{HI}(z)}{16k_B\nu_0^2 T_s H(z)} \approx 5 \cdot 10^{-3}(1 + \delta_b)x_{HI} \left[ \frac{T_{\text{CMB}}(z)}{T_s} \right] \left( \frac{\Omega_b h^2}{0.02} \right) \left[ \frac{\Omega_m (1 + z)^3 + \Omega_{\Lambda}}{5.4} \right]^{-1/2} \left( \frac{1 + z}{2.5} \right)^2 \left( \frac{h}{0.7} \right)^{-1}$$

(1.24)

where $\nu_0 = \nu_{\text{obs}}(1 + z)$ and we have used $T_{\text{CMB}}(z) = 2.73(1 + z)$ K and $H(z) \approx H_0 [\Omega_m (1 + z)^3 + \Omega_{\Lambda}]^{-1/2}$. The neutral hydrogen density was written as [72] [91] $n_{HI} = x_{HI}n_b(1 + \delta_b)$, where $n_b = \bar{n}_b(1 + z)^3$ is the mean number density of cosmic baryons, with a local baryon overdensity $\delta_b$, and $x_{HI}$ is the fraction of neutral hydrogen. For a more exact calculation of $\tau(z)$ that considers both Hubble expansion and peculiar velocities refer to [35] [92].

Now, in the Rayleigh-Jeans limit, the equation of transfer along the line of sight for a background radiation field with brightness temperature $T_{\text{bkg}}$ and a cloud with uniform excitation temperature $T_s$ is (see [70])

$$T_b'(\nu) = T_{\text{bkg}}(\nu)e^{-\tau_{\nu}} + T_s(1 - e^{-\tau_{\nu}}).$$

(1.25)

Assuming that the radiation background includes only the CMB, then $T_{\text{bkg}} = T_{\text{CMB}}$ and the differential brightness temperature emergent from the cloud is

$$\Delta T_b'(\nu) = T_b' - T_{\text{CMB}} = (T_s - T_{\text{CMB}})(1 - e^{-\tau_{\nu}}) \approx (T_s - T_{\text{CMB}})\tau_{\nu}$$

(1.26)

so the observed differential brightness temperature is
\[ \Delta T_b(\nu) \approx \frac{[T_s - T_{CMB}(z)]}{1 + z} \tau_\nu(z) \]
\[ \approx 14(1 + \delta_b) x_{HI} \left[ 1 - \frac{T_{CMB}(z)}{T_s} \right] \left( \frac{\Omega_b h^2}{0.02} \right) \left[ \frac{\Omega_m (1 + z)^3 + \Omega_\Lambda}{5.4} \right]^{-1/2} \]
\[ \left( \frac{1 + z}{2.5} \right)^2 \left( \frac{h}{0.7} \right)^{-1} \text{mK} \]  

(1.27)

where in the second step we have used equation 1.24. Note that the 21 cm signal \( \Delta T_b \) traces the density fluctuations \( \delta_b \), making its measurement an excellent probe of cosmology.

Three processes determine the spin temperature [29] [35]: (1) absorption of CMB photons (as well as stimulated emissions); (2) collisions with other hydrogen atoms, electrons and protons; and (3) scattering of Lyman \( \alpha \) photons through excitation and de-excitation (Wouthuysen Field Effect). The rate of these processes is fast compared to the de-excitation time of the line, so that to a very good approximation the spin temperature is given by the equilibrium balance of these effects [65]. In this case \( T_s \) is a weighted average of the CMB temperature, \( T_{CMB} \), the gas kinetic temperature, \( T_K \), and the effective color temperature of the Ly\( \alpha \) radiation field, \( T_L \)

\[ T_s = \frac{T_{CMB} + y_c T_K + y_L T_L}{1 + y_c + y_L} \]  

(1.28)

where \( y_c, y_L \) are the efficiencies or coupling coefficients for collisions and Ly\( \alpha \) scattering. For a discussion and detailed calculation of the efficiencies see [29] [35] [65].

The evolution of \( T_s \) depends on how different mechanisms influence the efficiencies. When any of the efficiencies is very large \( T_s \) takes the corresponding temperature value. As explained in [92], after reionization \( T_s \) is coupled to \( T_K \), \( T_s \gg T_{CMB} \) so \( \Delta T_b \) is independent of \( T_s \) and the 21 cm radiation is seen in emission. In this case the differential brightness temperature simplifies to

\[ \Delta T_b \approx \Delta \bar{T}_b (1 + \delta_b)(1 + \delta_x) \]  

(1.29)

where \( \Delta \bar{T}_b \) is the average differential brightness temperature that depends on redshift,
\[ \Delta T_b \approx 14 \bar{x}_{HI}(z) \left( \frac{\Omega_b h^2}{0.02} \right) \left[ \frac{\Omega_m (1 + z)^3 + \Omega_\Lambda}{5.4} \right]^{-1/2} \left( \frac{1 + z}{2.5} \right)^2 \left( \frac{h}{0.7} \right)^{-1} \text{mK} \]  

(1.30)

and we have written \( x_{HI} = \bar{x}_{HI} (1 + \delta_x) \), with \( \bar{x}_{HI} \) as the average of the neutral fraction of hydrogen and \( \delta_x \) as the neutral fraction fluctuations.

### 1.3.2 Power spectrum

To first order in the fluctuations, we can write

\[
\Delta T_b(r) \approx \Delta T_b[1 + \delta_b(r) + \delta_x(r)] = \Delta T_b[1 + \delta_{21}(r)]
\]

(1.31)

where \( r = |r| \) is the radial distance corresponding to the observed frequency (redshift). In general, the variations in the brightness temperature depend on additional parameters, including variations in the Ly\(\alpha\) coupling coefficient, kinetic temperature and line-of-sight peculiar velocity gradient. However, if we ignore the velocity term then equation 1.31 is a good approximation for sufficiently low redshifts. See [35] for details.

\( \Delta T_b \) is the zeroth order approximation of the 21 cm signal, averaged over large angular scales at fixed radial distance. In this section we are interested in the 21 cm fluctuations, which are the ones that allow for the measurement of the BAO. The deviation from the mean 21 cm brightness temperature is given by

\[
\delta_{21}(r) = \frac{\Delta T_b - \Delta \bar{T}_b}{\Delta T_b}
\]

(1.32)

which is a zero mean random field. We will construct the angular power spectrum from the spherical harmonic expansion of \( \delta_{21}(r) \)

\[
\delta_{21}(r) = \delta_{21}(\hat{n}, \nu) = \sum_{l,m} a_{lm}(\nu) Y_{lm}(\hat{n}) \quad a_{lm}(\nu) = \int d^2\hat{n} \delta_{21}(\hat{n}, \nu) Y_{lm}^*(\hat{n})
\]

(1.33)

where each observed frequency corresponds to a different radial shell of the universe. Note that, except for \( a_{00}(\nu) \), the spherical harmonic expansion of \( \Delta T_b(r) \) is the same as
that of $\delta_{21}(r)$ with the additional prefactor $\Delta T_b(\nu)$. Now we write $\delta_{21}(r)$ in terms of its Fourier Transform

$$
\delta_{21}(r) = \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}_{21}(k)e^{i\mathbf{k}\cdot\mathbf{r}} \quad \tilde{\delta}_{21}(k) = \int d^3r \delta_{21}(r)e^{-i\mathbf{k}\cdot\mathbf{r}}.
$$

Inserting equation 1.34 into equation 1.33 for $a_{lm}(\nu)$ and using the Rayleigh expansion for the plane wave given by

$$
e^{ik\cdot r} = e^{ikr \cos \theta} = \sum_l i^l (2l + 1) j_l(kr) P_l(\cos \theta) = 4\pi \sum_{l,m} i^l j_l(kr) Y_{lm}^*(\hat{k}) Y_{lm}(\hat{n})
$$

we obtain

$$
a_{lm}(\nu) = \int d^2\hat{n} \left[ \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}_{21}(k)e^{i\mathbf{k}\cdot\mathbf{r}} \right] Y_{lm}^*(\hat{n})
$$

$$
= \int d^2\hat{n} \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}_{21}(k) \left[ 4\pi \sum_{l_1,m_1} i^{l_1} j_{l_1}(kr) Y_{l_1m_1}^*(\hat{k}) Y_{l_1m_1}(\hat{n}) \right] Y_{lm}^*(\hat{n})
$$

$$
= 4\pi \sum_{l_1,m_1} i^{l_1} \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}_{21}(k) j_{l_1}(kr) Y_{l_1m_1}^*(\hat{k}) \left[ \int d^2\hat{n} Y_{l_1m_1}(\hat{n}) Y_{lm}^*(\hat{n}) \right] \delta_{l l_1} \delta_{m m_1}
$$

$$
= 4\pi i^{l_1} \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}_{21}(k) j_{l_1}(kr) Y_{l_1m_1}^*(\hat{k}).
$$

In order to advance further and calculate $\langle a_{lm}(\nu)a_{l_1m_1}^*(\nu_1) \rangle$, we need to make some assumptions about the (real) random field $\delta_{21}(r)$.

We say that the random field $\delta_{21}(r)$ is homogeneous if its mean (zero in this case) and covariance (equal to its correlation function since it is a zero-mean random field) $\langle \delta_{21}(r_1)\delta_{21}(r_2) \rangle$ are invariant under translations. For the covariance this means

$$
\langle \delta_{21}(r_1)\delta_{21}(r_2) \rangle = \langle \delta_{21}(r_1 + \delta r)\delta_{21}(r_2 + \delta r) \rangle.
$$

If $\delta_{21}(r)$ is homogeneous it can be shown that its two-point correlation function depends on $r_{12} = r_1 - r_2$ only, that is, $\langle \delta_{21}(r_1)\delta_{21}(r_2) \rangle = \xi_{21}(r_{12})$. In this case the
correlation of \( \hat{\delta}_{21}(k) \) becomes

\[
\langle \hat{\delta}_{21}(k_1)\hat{\delta}_{21}^*(k_2) \rangle = \int d^3r_1 \int d^3r_2 \langle \delta_{21}(r_1)\delta_{21}(r_2) \rangle e^{-i(k_1\cdot r_1 - k_2\cdot r_2)} \\
= \int d^3r \int d^3r_2 \xi_{21}(r)e^{-i(k_1\cdot r + k_1\cdot r_2 - k_2\cdot r_2)} \\
= \int d^3r \xi_{21}(r)e^{-ik_1\cdot r} \int d^3r_2 e^{-i(k_1 - k_2)\cdot r_2} \\
= (2\pi)^3 P_{21}(k_1)\delta(k_1 - k_2)
\]

(1.38)

where \( P_{21}(k) = \int d^3r \xi_{21}(r)e^{-ik\cdot r} \) is the Fourier Transform of \( \xi_{21}(r) \) (the power spectrum of \( \delta_{21}(r) \)).

We say that \( \delta_{21}(r) \) is isotropic if its mean and covariance \( \langle \delta_{21}(r_1)\delta_{21}(r_2) \rangle \) are invariant under rotations. For the covariance this means

\[
\langle \delta_{21}(r_1)\delta_{21}(r_2) \rangle = \langle \delta_{21}(gr_1)\delta_{21}(gr_2) \rangle \quad g \in SO(3).
\]

(1.39)

If \( \delta_{21}(r) \) is both homogeneous and isotropic it can be shown that its two point correlation function depends on \( r_{12} = |r_1 - r_2| \) only, that is, \( \langle \delta_{21}(r_1)\delta_{21}(r_2) \rangle = \xi_{21}(r_{12}) \). In this case \( P_{21}(k) \) is a function of \( k = |k| \) only since

\[
P_{21}(k) = \int d^3r \xi_{21}(r)e^{-ik\cdot r} = \int k^2dr \xi_{21}(r) \int d^2\hat{n} \sum_l i^l(2l + 1)j_l(kr)P_l(\cos(\pi - \theta)) \\
= \sum_l (-i)^l(2l + 1) \int k^2dr \xi_{21}(r)j_l(kr) \int d^2\hat{n}P_l(\cos\theta) \\
= 4\pi \int k^2dr \xi_{21}(r)j_0(kr) = P_{21}(k)
\]

(1.40)

where in the second step we used the plane wave expansion. In this case, equation 1.38 becomes

\[
\langle \hat{\delta}_{21}(k_1)\hat{\delta}_{21}^*(k_2) \rangle = (2\pi)^3 P_{21}(k_1)\delta(k_1 - k_2)
\]

(1.41)

and we are ready to calculate the angular power spectrum. From equations 1.36 and 1.41 have
\[
\langle a_{lm}(\nu)a_{l1m1}^*(\nu_1) \rangle = (4\pi)^2 \delta^l(-i)^l \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k_1}{(2\pi)^3} \left\langle \tilde{\delta}_{21}(k) \tilde{\delta}_{21}^*(k_1) \right\rangle \\
j_l(kr) j_{l1}(k_1 r_1) Y_{lm}^*(\hat{k}) Y_{l1m1}(\hat{k}_1) \\
= (4\pi)^2 \delta^l(-i)^l \int \frac{d^3k}{(2\pi)^3} P_{21}(k) j_l(kr) j_{l1}(k_1 r_1) Y_{lm}^*(\hat{k}) Y_{l1m1}(\hat{k}_1) \\
= (4\pi)^2 \delta^l(-i)^l \int \frac{k^2 dk}{(2\pi)^3} P_{21}(k) j_l(kr) j_{l1}(k_1 r_1) \int d^2 \hat{k} Y_{lm}^*(\hat{k}) Y_{l1m1}(\hat{k}) \\
= \delta_{ll} \delta_{mm_1} C_l(\nu, \nu_1) (1.42)
\]

where we have defined (with \( r \) as the radial distance corresponding to the observed frequency \( \nu \))

\[
C_l(\nu, \nu_1) = 4\pi \int \frac{k^2 dk}{2\pi^2} P_{21}(k) j_l(kr) j_{l1}(k_1 r_1). (1.43)
\]

Throughout the last derivation we assumed that we can "see" the differential brightness temperature (equation 1.31) directly, which is equivalent to assume an experiment with an infinitely narrow bandwidth. As shown in [35] [72] [91], a more detailed calculation that takes into account the finite bandwidth of the experiment gives

\[
C_l(\nu, \nu_1) = 4\pi \int \frac{k^2 dk}{2\pi^2} P_{21}(k) \alpha_l(k, \nu) \alpha_l(k, \nu_1) (1.44)
\]

where \( \alpha_l(k, \nu) = \int dr R(r; r_0) j_l(kr) \). The integral is over the line of sight distance and \( R(r; r_0) \) describes the frequency response of the experiment; it is typically sharply peaked around \( r_0 \) corresponding to the redshift of interest. The \( C_l \)'s approach zero as \( \nu \) and \( \nu_1 \) depart from each other because the frequency-space window functions no longer overlap in this limit and because the fluctuations are uncorrelated on large scales [91].

To gain some intuition, we assume that the power spectrum can be approximated by a power law, \( k^3 P_{21}(k)/(2\pi^2) = \Delta_{21}^2(k) = (k/k_*)^n \) (see [9], [80]). Then it can be shown ([91] [35]) that in the limit in which the response function can be considered to be a delta function (valid on angular scales much larger than the radial response, or small \( l \), this is how we derived equation 1.43) the angular power spectrum becomes

\[
\frac{l^2 C_l(\nu, \nu)}{2\pi} \propto \Delta_{21}^2 \left( \frac{l}{r_0} \right)^n \frac{l\delta r}{r} << 1 (1.45)
\]
where $\delta r$ characterizes the bandwidth of the observation. Small scale modes, or large $l$, on the other hand, suffer a cancellation from the oscillatory Bessel functions, and

$$\frac{l^2 C_l(\nu, \nu)}{2\pi} \propto \Delta_{21}^2(l/r_0) \frac{r_0}{l\delta r} \gg 1 \quad (1.46)$$

In temperature units $C_l(\nu, \nu)$ takes a prefactor $\Delta T^2_b(\nu)$. In general, equation 1.44 (or 1.43) forms the basis for the analysis that makes it possible to relate the 21 cm fluctuations to cosmology. As explained in [56] [88], after reionization the neutral fraction fluctuations $\delta_x$ are proportional to the matter density fluctuations up to a weak (and simple) scale dependency. Since the 21 cm fluctuations allow the direct observation of the matter density field $\delta_b$, the power spectrum can be utilized to probe the geometry of the universe, derive cosmological constraints and test the equation of state of dark energy by measuring the BAO. In particular, redshifts in the range $z \approx 1 - 3$ are of great interest since they cover the regime in which dark energy begins to dominate the energy budget of the universe [65].

CHIME will be a novel radio telescope that will study the nature of dark energy by mapping the three-dimensional distribution of neutral hydrogen gas in the universe detected from redshifted 21 cm radiation. By measuring the scale of the Baryon Acoustic Oscillations across the redshift range $z \approx 0.8$ to $z \approx 2.5$ in both the angular and line-of-sight directions, CHIME will study the epoch when dark energy generated the transition from decelerated to accelerated expansion of the universe.

### 1.4 Outline

This thesis is structured as follows: A description of the CHIME instrument is given in Chapter 2. An introduction to Radio-Frequency-over-Fiber links and their use in radio astronomy applications is given in Chapter 3. Chapter 4 describes the current coax-based analog receiver for the two-element interferometer prototype for CHIME and the requirements on the dynamic performance of the RFoF link. Chapter 5 describes the analog optical link design and lab tests on its dynamic performance. In Chapter 6, the gain and phase stability of the optical link with changes in temperature of the RFoF transmitter and the optical fiber are presented. The first tests of the RFoF link in the two-element interferometer are shown in Chapter 7. A final discussion on the overall RFoF performance and improvements for the next version of the link are presented in Chapter 8.
Chapter 2

CHIME instrument

2.1 CHIME instrument design and characteristics

CHIME will consist of an array of five 100 m x 20 m cylindrical parabolic reflectors instrumented with more than 1200 dual-polarization feeds operating across the frequency range 400-800 MHz. The CHIME mechanical design is shown in Figure 2.1. The telescope will be built at the Dominion Radio Astrophysical Observatory (DRAO)\footnote{Official DRAO site.} near Penticton, Canada (Figure 2.2). CHIME has no moving parts or cryogenics, and employs a drift scan strategy, surveying the northern half of the sky each day as the earth rotates. Room temperature receivers with $\sim 50$ K noise performance are employed for the dual linearly polarized feeds that populate the focal length of each reflector. A digital channelizer/correlator will perform Fourier transform imaging, allowing a 90° north-south field of view while maintaining high angular resolution [24]. A summary of the CHIME design parameters is shown in table 2.1.

2.1.1 Frequency coverage

The ability of CHIME to constrain dark energy models can be quantified by the Dark Energy Task Force (DETF) Figure of Merit (FoM) which is formed from a combination of the uncertainties in the equation of state parameters $w_0$ and $w_a$ (equation 1.1) [4]. It depends on measuring the history of expansion both before and after dark energy emerges at a redshift slightly above $z = 1$, corresponding to an observed frequency of $\sim 700$ MHz. Figure 2.3 shows the improvement factor for the DETF FoM over our current knowledge from a BAO survey covering the redshift range from $z_{\text{min}}$ to $z_{\text{max}}$ as a function of $z_{\text{max}}$. If dark energy approximates a cosmological constant, then its effect
Figure 2.1: The CHIME design consists of an array of five 100 m x 20 m cylindrical parabolic reflectors instrumented with more than 1200 dual-polarization feeds operating across the frequency range 400-800 MHz.

Figure 2.2: The Dominion Radio Astrophysical Observatory (DRAO) is a facility for science and technology research related to radio astronomy. It currently operates three instruments: a 26-m fully steerable dish, an interferometric radio telescope and a solar flux monitor. It also features labs and equipment for the development of radio-frequency instrumentation. The observatory is located near Penticton, British Columbia, in a region regulated by the federal government to ensure a radio-quiet environment. Source: [24]
Table 2.1: CHIME experiment parameters [12] [24].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observing frequency</td>
<td>800 to 400 MHz</td>
</tr>
<tr>
<td>Observing wavelength</td>
<td>38 to 75 cm</td>
</tr>
<tr>
<td>Redshift</td>
<td>$z \approx 0.8$ to 2.5</td>
</tr>
<tr>
<td>System Noise Temperature</td>
<td>$\leq 50$ K</td>
</tr>
<tr>
<td>Beam Size</td>
<td>$\sim 0.26^\circ$ to $0.52^\circ$</td>
</tr>
<tr>
<td>Field of View N-S</td>
<td>$90^\circ$</td>
</tr>
<tr>
<td>Field of View E-W</td>
<td>$\sim 1.3^\circ$ to $2.5^\circ$</td>
</tr>
<tr>
<td>Cylinder Size</td>
<td>100 m x 20 m</td>
</tr>
<tr>
<td>Number of Cylinders</td>
<td>5</td>
</tr>
<tr>
<td>Collecting Area</td>
<td>10000 m$^2$</td>
</tr>
<tr>
<td>$f/d$</td>
<td>0.25</td>
</tr>
<tr>
<td>Dual-polarization antenna spacing</td>
<td>$\sim 31$ cm</td>
</tr>
<tr>
<td>Number of Antennas per Cylinder</td>
<td>256</td>
</tr>
<tr>
<td>Bandwidth of Channelized Outputs</td>
<td>$\sim 1$ MHz</td>
</tr>
</tbody>
</table>

is negligible at high redshifts. In this case, the improvement factor for the DETF FoM will be limited. However, high-redshift ($z > 1$) measurements are still useful for testing curvature and the standard model of cosmology (also known as the Λ Cold Dark Matter or ΛCDM model). If dark energy is more important at high redshifts than in the ΛCDM model, then high-redshift measurements become useful for dark energy constraints.

The improvement factor can be increased by reaching lower $z_{\min}$. This pushes CHIME to operate at as high a frequency as possible. The upper edge of the operating frequency range of CHIME is practically limited by interference caused by the cell phone band beginning above 800 MHz ($z_{\min} \approx 0.8$). The lower frequency (400 MHz) is chosen to give an upper redshift limit of $z_{\max} \approx 2.5$.

### 2.1.2 Resolution

To constrain cosmological models at useful levels of precision, maps should have sufficient spatial resolution to see the smallest features [12]. A 100 m-aperture telescope is sufficient to resolve a comoving separation of about 10 Mpc at $z \approx 1$, which is more than sufficient for resolving the third peak of the BAO, as seen in Figure 1.2 [17]. CHIME uses redshifts to map hydrogen in the third dimension, distance. For the comoving distance measurements to match the instrumental angular resolution, a frequency resolution in the range $\sim 1.6 – 4.1$ MHz is required in the redshift range of CHIME [12]. In practice, CHIME will have a frequency resolution below 1 MHz at all redshifts.
Chapter 2. CHIME instrument

2.1.3 Signal path

Sky signals are recorded using the following analog and digital processing chain: broadband antennas sensitive to both linear polarizations are placed at regular intervals along the focal line of each cylinder. The signals from the antennas are amplified using room temperature receivers with a noise performance below 50 K. After passing through the analog receiver, the signals from each polarization of each antenna are alias sampled and digitized with 8-bit analog-to-digital converters (ADCs) operating at 800 MHz. The data is Fourier transformed using a 2048-point Fast Fourier Transform (FFT), such that the data is channelized into 1024 frequency bins spanning the 400-800 MHz band of the instrument. This stage is implemented in Field-Programmable Gate Arrays (FPGAs) using special purpose firmware.

The data processing thereafter is separate for each polarization and each frequency channel (Figure 2.4). A custom high speed network allows the data to be shuffled amongst the FPGA motherboards and then sent to a large Graphics Processing Unit (GPU) cluster that will perform either a full $N^2$ correlation or an FFT based beam-former (we plan to implement a traditional $N^2$ correlator first, and then use it to benchmark an FFT based beam-former). Each GPU is programmed to process a fraction of the frequency channels grabbed from each antenna data stream. The transforms from each cylinder in CHIME will be correlated to form a two-dimensional image. Since each of the 1024 frequency bins encodes a two-dimensional shell of the universe in redshift, CHIME obtains the third dimension by observing different frequencies (redshifts).

Figure 2.3: Improvement factor for the DETF FoM over other experiments (Planck + DETF Stage II + BOSS) as a function of $z_{max}$ for different values of $z_{min}$. Source: [24]
The CHIME signal path: broadband antennas sensitive to both linear polarizations are placed at regular intervals along the focal line of each cylinder. The signals from the antennas are amplified using room temperature receivers and digitized. These broadband digital signals are separated by the digitizer into many narrow frequency channels which together span the 400-800 MHz band. The data processing thereafter is separate for each polarization and each frequency band. Data from every antenna on a cylinder are sent to a digital backend which correlates them all by performing a Fourier Transform. The transforms from each cylinder in CHIME will be correlated to form a two-dimensional image. The third dimension is obtained by observing different frequencies (redshifts).
2.1.4 Integration Time

Assuming $\Omega_{\text{HI}} \sim 10^{-3}$ at $z \sim 1.5$ (\cite{17}, \cite{65}) and using $\Omega_b \sim 0.04$ in equation 1.30 we find that the variations of the sky brightness due to 21 cm radiation presents a sky signal of about 350 $\mu$K. The noise per sky pixel in the measured sky temperature is given by

$$\Delta T = \frac{T_{\text{sys}}}{\sqrt{\Delta \nu t_{\text{int}}}}$$

(2.1)

where $T_{\text{sys}}$ is the system temperature of the radio telescope, $\Delta \nu$ is the channel bandwidth and $t_{\text{int}}$ the integration time. Assuming $T_{\text{sys}} \approx 50$ K and $\Delta \nu \approx 1$ MHz, the integration time required to bring the noise down to $\sim 100$ $\mu$K, below the level of the 21 cm signal, is about 69 hours per pixel. For a $\sim 0.21^\circ$-diameter beam surveying the northern half of the sky this corresponds to approximately $4.5 \cdot 10^3$ years of integration time for a single receiver. For 2560 single polarization feeds, this time reduces to approximately 2 years.

2.2 21 cm signal recovery using the spherical harmonic transit telescope formalism

In contrast to traditional radio astronomy applications that often involve observing point sources, mapping the universe with the 21 cm line requires rapidly probing large volumes with wide-field radio telescopes. This is because it involves mapping a faint and diffuse cosmic signal that covers all of the sky and needs to be separated from foreground contamination that is many orders of magnitude brighter. CHIME is a wide-field radio interferometer with fixed pointing that observes the sky as it transits through its field of view (drift scan strategy). Since CHIME maps large portions of the sky at once, the flat-sky approximation that is common in radio astronomy is not valid, and different approach that incorporates the curvature of the sky is required. In this section we will briefly describe the spherical harmonic transit telescope formalism \cite{73}, an all-sky approach that bypasses the curved sky complications and is particularly well suited to the analysis of wide-field radio interferometers. This formalism allows compact and computationally efficient representations of the data and its statistics that enable efficient and effective discrimination of the 21 cm signal from astrophysical foregrounds. The spherical harmonic transit telescope formalism is described in detail in \cite{73}.

In radio interferometry, the visibility $V_{ij}$ is the instantaneous correlation between two feeds $F_i$ and $F_j$. For simplicity, we assume that we are sensitive to the total intensity.
of the sky by taking a linear combination from a dual polarization antenna. For a fixed pair of feeds and a fixed frequency, the instantaneous visibility is given by

\[ V_{ij} = \langle F_i F_j^* \rangle = \frac{1}{\Omega_{ij}} \int d^2 \hat{n} A_i(\hat{n}) A_j^*(\hat{n}) e^{2\pi i \hat{n} \cdot u_{ij}} T(\hat{n}) \]  \quad (2.2)

where \( u_{ij} = (r_i - r_j)/\lambda \) is the spatial separations between the two feeds divided by the observed wavelength, \( \hat{n} \) is the position in the celestial sphere, and \( A_i(\hat{n}) \) gives the primary beam of feed \( i \). The visibilities have been normalized such that they are temperature like, and have been defined in terms of the sky brightness temperature, \( T(\hat{n}) \). \( \Omega_{ij} = \sqrt{\Omega_i \Omega_j} \) is the geometric mean of the individual beam solid angles.

\[ \Omega_i = \int d^2 \hat{n} |A_i(\hat{n})|^2. \]  \quad (2.3)

For details on the derivation of equation 2.2 refer to [79], [87]. Under the narrow-beam and flat-sky approximation, equation 2.2 becomes a Fourier transform mapping between the sky and the plane tangent to the phase reference position (the \( uv \)-plane). Instead, we realize that as the earth rotates both the primary beams and the baseline separations of a drift scan telescope rotate relative to the celestial sphere. This means that the visibilities are periodic with period equal to one sidereal day. In order to exploit this property, we first write the visibility in terms of the transfer function \( B_{ij} \)

\[ V_{ij}(t) = \int_{\Omega_T} d\Omega' B_{ij}(\theta', \phi', t) T(\theta', \phi') \]  \quad (2.4)

where the transfer function is

\[ B_{ij}(\theta', \phi', t) = \frac{1}{\Omega_{ij}} A_i(\theta', \phi', t) A_j^*(\theta', \phi', t) e^{2\pi i (\hat{n} \cdot u_{ij}) (\theta', \phi', t)}. \]  \quad (2.5)

Now, the rotation about the poles periodically over the course of a sidereal day creates a linear correspondence between \( t \) and the azimuthal angle \( \phi \). Also, note that the \( \phi \) dependence simply rotates the transfer function about the earth’s polar axis, so we can write
Chapter 2. CHIME instrument

\[ V_{ij}(\phi) = \int_{\Omega'} d\Omega' B_{ij}(\theta', \phi' - \phi) T(\theta', \phi'). \] (2.6)

Since \( V_{ij}(\phi) \) is periodic, it can be decomposed into its Fourier series, given by

\[ v_{ij}^k = \int_{2\pi} \frac{d\phi}{2\pi} V_{ij}(\phi) e^{-ik\phi}. \] (2.7)

The next step is to write \( B_{ij} \) and \( T \) in terms of their respective spherical harmonic expansions (note that these \( a_{lm} \) coefficients will not be the same as in Section 1.3 since \( T \) contains not only the 21 cm signal but also the foreground sources)

\[ T(\theta', \phi') = \sum_{l,m} a_{lm} Y_{lm}(\theta', \phi') \quad B_{ij}(\theta', \phi') = \sum_{l_1m_1} B_{l_1m_1}^{ij} Y_{l_1m_1}^*(\theta', \phi'). \] (2.8)

The reason why we expanded \( B_{ij} \) in terms of the conjugate spherical harmonics will become apparent below. Also notice that from the definition of the spherical harmonics, \( Y_{l_1m_1}(\theta', \phi') = Ne^{im_1\phi'} P_{l_1m_1}(\cos \theta') \), where \( N \) is a normalization constant, it is clear that

\[ B_{ij}(\theta', \phi' - \phi) = \sum_{l_1,m_1} B_{l_1m_1}^{ij} e^{im_1\phi} Y_{l_1m_1}^*(\theta', \phi'). \] (2.9)

Now insert equation 2.9 for \( B_{ij} \) and equation 2.8 for \( T \) into equation 2.7 to obtain
In practice, the visibilities are also corrupted by instrumental noise, so an additional term \( n_{ij}(\phi) \) must be added to equation 2.6. Assuming that the noise is stationary such that its statistics are independent of \( \phi \) and repeating the procedure above leads to

\[
v_{ij}^{m} = \sum_{l} a_{lm} B_{lm}^{ij} + n_{m}^{ij}.
\]  

Equation 2.11 describes the way the sky (contained in the \( a_{lm} \)'s) maps into the the observed data (the measured visibilities \( v_{ij}^{m} \)) given telescope design (determined by the beam transfer matrices \( B_{lm}^{ij} \) and the noise \( n_{m}^{ij} \)). For a given \( m \) and frequency \( \nu \), the measured visibilities are the projection of the \( l \)-modes on the sky for the measured \( m \). Note that different \( m \) modes do not mix (the stationarity assumption means that the noise \( m \) modes are uncorrelated) and that the finite size of the instrument ensures that the sums are finite.

For a given \( m \) mode, the visibility data can be written in a more compact form that includes all the frequencies \( \nu \) and feed pairs \( ij \) (the generalization of equation 2.11 to include the additional degrees of freedom). First, it can be shown that by the using properties of spherical harmonics and the fact that \( T \) is a real field, an index \( \alpha \) that specifies exactly the positive and negative values of \( m \) for all the feed pairs \( ij \) can be introduced, such that we can write (see [73] for details)
\[ v^\alpha_m = \sum_l a_{lm} B^\alpha_{lm} + n^\alpha_m \quad m \geq 0. \] (2.12)

We also introduce additional notation to write \( B^\alpha_{lm}, v^\alpha_m \) and \( n^\alpha_m \) in matrix form

\[
(B_m)_{(\alpha \nu)(l \nu')} = B^\alpha_{lm} \delta_{\nu \nu'} \quad (v_m)_{(\alpha \nu)} = v^\alpha_{m \nu} \quad (a_m)_{(l \nu)} = a^\nu_{lm}\] (2.13)

where in \((B_m)\) the row index labels all the combinations of baseline \(\alpha\) and frequency \(\nu\) and the column index labels all the combinations of multipole \(l\) and frequency \(\nu'\). With this notation equation 2.12 can be written as

\[ \mathbf{v} = \mathbf{aB} + \mathbf{n} \quad \forall m. \] (2.14)

The reduction of the measurement process to the simple linear mapping in equation 2.14, which uses a finite number of degrees of freedom, allows us to apply standard tools of signal processing in order to recover the 21 cm signal. The angular power spectrum of the sky signal can be recovered from equation 2.14 directly. The covariance of the visibilities is

\[
C_{(\alpha \nu m);(\alpha' \nu' m')} = \left\langle V^m_{\alpha \nu} V^{m' \ast}_{\alpha' \nu'} \right\rangle = \sum_{l \nu'} B^\alpha_{lm} \left\langle a_{lm \nu} a^*_{l \nu' \nu'} \right\rangle B^\alpha_{l \nu'} \left\langle n_{l \nu} n^*_{l \nu' \nu'} \right\rangle. \] (2.15)

Note that \( C_{(\alpha \nu m);(\alpha' \nu' m')} \) is a huge matrix (dimension of the order of \(10^8\) for an experiment like CHIME) since it is the covariance between all possible degrees of freedom: baselines, frequencies and \(m\) modes. However, under the assumption that the sky signal is a homogeneous and isotropic random field,

\[
\left\langle a_{lm \nu} a^*_{l \nu' \nu'} \right\rangle = C_l(\nu, \nu') \delta_{l \nu} \delta_{m \nu'}
\] (2.16)

then its two-point statistics become much simpler and importantly, they are uncorrelated in the \(m\) index. This means that both the amount of computation and data storage are significantly reduced (by factors of the order of \(10^6\) and \(10^3\) respectively) since \( C_{(\alpha \nu m);(\alpha' \nu' m')} \) is block diagonal and the statistics can be calculated in an \(m\) by \(m\) basis.
so we can write

\[
C_{(\alpha\nu);(\alpha'\nu')i} = \sum_l B_l^{\alpha\nu} B_l^{\alpha'\nu'} C_l(\nu, \nu') + N_{(\alpha\nu)(\alpha'\nu')} \quad \forall m \tag{2.17}
\]

where \(N_{(\alpha\nu)(\alpha'\nu')}\) is the power spectrum of the instrumental noise. In matrix notation

\[
C = B C_{\text{sky}} B^\dagger + N \tag{2.18}
\]

We have obtained a simple linear relationship between the covariance matrix of the data, \(C\), and that of the sky signal, \(C_{\text{sky}}\). As explained in chapter 1, the covariance of the 21 cm signal encodes the cosmological information that we are interested in (the BAO signal). In practice, \(C_{\text{sky}}\) contains both the 21 cm signal \((C_{21})\), which we are ultimately trying to recover, and foreground sources \((C_f)\) like diffuse synchrotron emission from the galaxy and emission from extragalactic point sources. To efficiently extract \(C_{21}\) we also need to understand the two-point statistics of the foregrounds. However, using the \(m\) mode formalism described above allows a powerful foreground removal technique in the form of the Karhunen-Loève transformation to be used, which allows us to identify a basis in which the astrophysical foregrounds and 21 cm signal are maximally separated. For details on foreground removal with the Karhunen-Loève transform refer to [73].
Chapter 3

R FoF links and their use in radio astronomy applications

3.1 Analog Optical links

Recently, there is a growing increase in the number of applications for analog fiber optic links as an alternative to traditional copper wire or coaxial cable to transport radio frequency (RF) signals. This is mainly because optical fiber transmission systems offer a wide range of benefits when compared to copper wire or coaxial cable. These include [1] [19] [34] [71]:

- The bandwidth available with optical fiber is enormous (more than 100 GHz).

- Optical fiber is considerably thinner and lighter than bulky coaxial cable or bundled twisted-pair cable. For underground conduits, high rise buildings, ships and aircrafts, the advantage in small size is considerable. The corresponding reduction in weight reduces the structural support requirements, reducing costs.

- For long-range applications (i.e. several kilometers), optical fiber is cheaper than copper. Also, the attenuation for optical fiber is significantly lower than even low attenuation coaxial cable and is constant over a wide frequency range\(^1\).

- Fiber optic cable is made up of dielectric materials and has high electrical resistance, so it does not act as an antenna to pick up radio frequency interference (RFI) and

\(^1\)For short-range applications (hundreds of meters) the difference in cost between fiber and coax is negligible. Also, for short-range links the attenuation for both fiber and coax is negligible but the RF to optical conversion inefficiencies of the modulation and photodetection devices dominate the fiber link loss and exceeds the coaxial loss.
Figure 3.1: Basic components of a RFoF link.

is immune to other electromagnetic interference (EMI) effects seen in copper wires, such as inductive pickup from other adjacent signal-carrying wires or coupling of electrical noise into the line from any type of nearby equipment. This is ideal when routing through noisy RF environments or running long lengths parallel to other fiber optic cables.

- Fiber optic cable does not have the problems of ground loops, sparks, and potentially high voltages inherent in copper wire and coax cable. It also allows separate grounds for the transmitter and receiver side, simplifying the grounding scheme.

- A longer life span of 20 to 30 years is predicted for optical fiber as compared to 12 to 15 years for conventional cables. The main reason for this is that glass does not corrode as metal does.

An analog optical link consists of all the components required to convey an electrical signal over an optical carrier [19]. Analog optical links used to transport RF signals over optical fiber are also referred to as Radio-over-Fiber (RoF) or Radio-Frequency-over-Fiber (RFoF) links. As shown in Figure 3.1, a general RFoF link contains three basic components: at the input, a modulation device converts the RF signal to the optical domain by varying one or more properties of the optical carrier. The optical fiber couples the modulation device to the photodetection device, which recovers the RF signal from the optical carrier.

Although the advantages of analog signal transport using RFoF are evident, the conversion from the electrical to optical domain and vice versa introduces not only signal losses, but also noise and nonlinear distortions. Unless the RFoF link performance is optimized, the use of optical fiber may result in additional signal degradation and increased noise compared to coax-based links. Thus, appropriate design and performance optimization are critical to the practical use of RFoF links.
3.2 Modulation and detection methods

Assuming, for simplicity, a perfectly monochromatic laser and neglecting noise, the electric field of the optical carrier at a fixed point can be written as $E(t) = E_0 \exp(\omega_0 t + \phi_0)$. In principle, the amplitude ($E_0$), intensity (proportional to $|E_0|^2$), frequency ($\omega_0$) or phase ($\phi_0$) of the optical carrier can be modulated to convey the RF signal over an optical link. As for the detection, two methods exist: direct detection, which can be applied for intensity modulation method, and coherent detection, which can be applied for amplitude, intensity, frequency or phase modulation [2]. While coherent detection may provide improved sensitivity with respect to direct detection, it is much more complex and expensive to implement than direct detection since it requires an optical local oscillator, an optical mixer and an optical filter for the demodulation device [19]. It also places stringent requirements on the linewidth of the source and local oscillator lasers [41]. Direct detection of an intensity modulated optical carrier is straightforward and all that is required is a photodiode, a device that produces a photocurrent that is directly proportional to the square of the electric field magnitude. This quantity is proportional to the intensity, which in turn is proportional to the optical power.

For these reasons, intensity modulation of the optical carrier followed by direct detection (IMDD) is by far the most popular and the most widely employed method today [19] [41], and will be the focus of the rest of this chapter (for the analysis of optical links using coherent detection methods see [46]). Two ways to implement the IMDD method are common: external modulation or direct modulation. In an externally modulated system the laser operates in continuous wave (CW) mode and the modulation is achieved externally with an optical modulator. With direct modulation, the electrical modulating signal is applied directly to the laser current to change the output of its optical intensity. Semiconductor diode lasers, which are described in next section, are the only ones with enough bandwidth to be used for direct modulation [19]. As we will see in Chapter 5, the RFoF link for CHIME is a directly modulated system. For a detailed analysis of RFoF links using IMDD refer to [2] [19] [21] [22] [41].

3.3 Link components

In this section we describe the basic components of a general RFoF link: optical fiber, modulation device and photodetector. As mentioned earlier, we will focus on the components for the implementation of the direct modulation IMDD method, which is the method used for the RFoF link for CHIME.
3.3.1 Optical fiber

An optical fiber is a cylindrical dielectric waveguide capable of conveying electromagnetic waves by confining them to a core region. The core has a slightly higher refractive index than the surrounding cladding thereby confining the light via total internal reflection. Practical fibers usually have additional layers (including protective outer coating and jacket layers) that protect and add strength to the fiber but do not contribute to its optical waveguide properties (Figure 3.2).

Two main types of optical fibers include multi-mode and single-mode optical fibers. In a single-mode fiber the core is so small (with typical diameters in the range 8-10 µm) that the only possible path is the one in which the light travels parallel to the length of the fiber, i.e., without any reflections off the core-cladding interface.

Multi-mode fiber has a larger core (typical diameters of 50 µm or larger), allowing not only the straight light path to propagate down the fiber but also multiple paths that involve one or more reflections off the core-cladding interface. Multi-mode fiber typically supports thousands of optical modes and the large physical size of the core improves the efficiency of coupling between the fibre and the other link components. However, in addition to chromatic dispersion (which is present in both single-mode and multi-mode fibers and is caused by the change of the propagation velocity with wavelength) multi-mode fibers suffer from modal dispersion, i.e., different propagation velocities for different modes. Since the various modes follow different paths through the core of the fiber, the modes will arrive at different times, according to the path taken. This causes
signal distortion [28]. Since modal dispersion (and dispersion in general) limits both the bandwidth and the transmission distance of the optical link [41], all high performance applications use single-mode fiber [19]. In particular, the RFoF link for CHIME uses single-mode fiber.

The combination of both constraints in the fiber (e.g. wavelength dependent attenuation and dispersion) and constraints in the devices (availability of diode lasers, external modulators and photodiodes at specific wavelength ranges) results in the common usage of just three primary wavelength bands for fiber optic links. The band around 850 nm was the first band used for fiber optic links because the first optical sources and detectors were available in this band [41]. The dominant bands in use at present are those located around 1300 nm and 1550 nm [19]. The band around 1300 nm has been widely used because, in this wavelength region, standard single-mode fiber exhibits a minimum of chromatic dispersion, which allows for higher frequency and/or longer length links. The current RFoF link for CHIME operates in this band. The band around 1550 nm offers the lowest attenuation but its use has also been motivated because of the availability of fiber optic amplifiers and because it is possible to modify the manufacturing process of the fiber to have optimum dispersion performance in this band [19] [41].

### 3.3.2 Modulation devices

**Direct modulation**

In direct modulation, the RF modulating signal is applied directly to the laser bias current to change the intensity of its output. This implies that the modulating signal must be within the modulation bandwidth of the laser. Only semiconductor diode lasers have sufficient bandwidth and efficiency to be used for direct modulation [19]. Several semiconductor laser types and their output spectra are shown in Figure 3.3. There are two basic requirements for laser operation: the ability to produce stimulated emission and a cavity that is resonant at the stimulated emission wavelength [86]. Forward biasing a semiconductor diode junction fabricated in an appropriate material (e.g. gallium arsenide, indium phosphide and mixtures of these compounds) results in spontaneous emission of radiation (emission that is generated by the random recombination of electrons and holes). In a laser we have light amplification by stimulated emission of radiation. Unlike the spontaneous process, in stimulated emission a photon triggers the emission of an identical photon (which has the same phase, frequency, polarization, and direction of travel as the incident photon). To produce laser light from a semiconductor requires a structure where an inverted population consisting of electrons and holes can be setup,
Figure 3.3: Top: some of the semiconductor laser types. Bottom: characteristic output spectrum of an FP laser (left) and of DFB, DBR and VCSEL (right).

from which the spontaneous emission can trigger stimulated emissions. In a semiconductor, the inverted population can be created and maintained by passing a dc current through the diode junction.

The second requirement for laser operation is that the stimulated emissions occur within an optical cavity that is resonant at the stimulated emission wavelength. The most common resonant cavity for diode lasers is one formed by two parallel plane mirrors. To get light out of the cavity, at least one of the mirrors is partially reflecting. Lasers with this cavity are called Fabry-Perot (FP) lasers. In a Fabry-Perot cavity we have resonance for any wavelength that results in addition in phase with the original wave after reflections off of both mirrors. Since there are multiple wavelengths that satisfy this condition, FP lasers emit several light wavelengths simultaneously. The current RFoF link for CHIME uses these type of lasers.

Further improvement in the output light coherence, minimizing dispersion and allowing transmission over longer distances, is achieved through additional structural elements that effectively select just one wavelength from the Fabry-Perot spectrum, and suppress the remaining wavelengths. This is done by inserting a selective grating element in the resonant cavity, as in the Distributed Feedback (DFB) lasers. It can also be done by inserting an outside Bragg grating, or gratings placed instead of the facet mirrors, as in the Distributed Bragg Reflector (DBR) lasers [23].

In the lasers mentioned above, the light propagates parallel to the semiconductor
junction and comes out of the sides. A different structure is the Vertical Cavity Surface Emitting Laser (VCSEL). In a VCSEL the light output is perpendicular to the surface of the semiconductor. This feature is desirable since it reduces cost and simplifies the laser-to-fiber coupling problem [41]. As DFB and DBR lasers, VCSELs are also single-mode lasers, so they have only one longitudinal mode in their spectrum.

The typical single-frequency optical power vs. laser current characteristic curve of a diode laser is shown in Figure 3.4 (left). For a laser current of the form \( i_L = I_L[1 + m \cos(\omega_m t + \phi_m)] \) where \( I_L \) is the bias current, \( m \) is the modulation index, and \( \omega_m \) is the RF modulating frequency, it can be shown [41] that the output optical power is of the form \( p_{OL} = P_{OL}[1 + m \cos(\omega_m t + \theta_m)] \), where \( P_{OL} \) is the average optical power. The small signal response is determined by the slope efficiency of the laser, \( s_l \), given by

\[
 s_l = \left| \frac{d p_{OL}}{d i_L} \right|_{i_L = I_L} .
\]  

As the \( p_{OL} \) vs. \( i_L \) curve shows, initially the output power increases slowly with current, since it is dominated by spontaneous emission and most of the photons are absorbed by the semiconductor. Above the threshold current level, stimulated emission dominates the optical output and the output increases at least 100 times more rapidly [19]. Above threshold and below saturation, the \( p_{OL} \) vs. \( i_L \) curve can be approximated very well by a straight line with a slope given by the slope efficiency \( s_l \). We must ensure that the laser is operating in the linear region. We also want to have a high slope efficiency, but this parameter is fundamentally limited by the quantum efficiency of the laser [19] [41].
The frequency dependence of the slope efficiency of the laser (its frequency response) is shown in Figure 3.4 (right). Directly modulated lasers have a low-pass second-order response which places a limit on the bandwidth that they support. The second-order response results from the interaction between carrier recombination and photon emission. The response can be derived by treating the semiconductor laser as a pair of coupled reservoirs, one for electrons (and holes) in the active region, and one for photons. The interactions between these two reservoirs can be described by a pair of coupled differential equations that are known as the laser rate equations. By solving the rate equations both the resonance frequency (relaxation frequency) and the damping factor of the frequency response are obtained. For a detailed description and solution of the rate equations refer to [19] [41]. The low frequency response is also degraded by bond wire and package parasitics. Finally, we mention that the range of the relaxation frequency is from about 1 GHz to about 40 GHz [19], so for CHIME it is safe to assume that we are within the modulation bandwidth of the laser (assuming that we are well above the threshold).

External modulation

For external modulation, the laser operates at a constant optical power (CW) and the intensity modulation of the optical carrier is introduced via a separate device. The increase in cost, complexity and size of the modulation device with respect to the direct modulation scheme is accepted in some cases because of their high performance, greater bandwidth and the greater control over which the modulation is carried out [21] [41]. There are also applications where external modulation is a requirement because effects such as frequency chirping (fluctuations in the frequency of the optical carrier caused by changes in the laser current) cannot be tolerated.

Although we will not go into much detail on external modulation devices, we will briefly describe the Mach-Zehnder structure implemented in lithium niobate, which is by far the most common external modulator [2] [19] [20] [22] [41]. The mechanism behind lithium niobate modulators is the linear electro-optic effect in which an electric field (originating from an external drive voltage) induces change in the refractive index, which in turn will lead to a change in phase. When an interferometric structure is used, the phase modulation is converted to intensity modulation [41]. The basic structure of the lithium niobate Mach-Zehnder modulator is shown in Figure 3.5. The idea is that the variation in the applied voltage can be used to change the phase between the two optical waveguide arms, such that the intensity of the output varies from a maximum (constructive interference) to a minimum level (destructive interference).
Figure 3.5: Simplified diagram of the Mach-Zehnder modulator. The variation in the applied voltage can be used to change the phase between the two optical waveguide arms, such that the intensity of the output varies from a maximum (constructive interference) to a minimum level (destructive interference).

### 3.3.3 Photodetectors

The principle behind photodetection using semiconductors is optical absorption. When the energy of the incident photons is greater than the band gap of the semiconductor, light incident on the semiconductor can be absorbed. This results in the creation of electron-hole pairs that are swept away when an electric field is applied across the semiconductor material, leading to a photocurrent in the external circuit.

By far the most common photodetector is the p-intrinsic-n (PIN) photodiode [19], which consists of an intrinsic (lightly doped) semiconductor sandwiched between the p-doped and n-doped regions. Since the intrinsic region has no free charges, its resistance is high and most of the voltage across the diode appears across this region, which is typically wide to increase the probability of absorption with respect to the p or n regions. Since the electric field is high in the intrinsic region, the carriers (electron-hole pairs) generated in this region are immediately conveyed to the external circuit as a photocurrent. From the electrical point of view the photodetector behaves as a diode, so forward biasing it results in a large electrical current that is in general much larger than the photogenerated current. Thus, PIN photodiodes are usually operated with a reverse bias. In this case the relation between the incident optical power, \( p_{OD} \), and the detected electrical current, \( i_D \), is linear to a good approximation [19], so

\[
i_D = r_d p_{OD}
\]
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where \( r_d \) is the responsivity of the photodiode. In general, \( r_d \) is a function of frequency. However, in a high-quality photodiode the responsivity will have a fairly flat frequency response [41].

3.4 Noise in RFoF links

In this section we describe the dominant noise sources in a RFoF link: thermal noise, shot noise and relative intensity noise [19]. In addition we describe the mode partition noise, which is relevant for the lasers that are used for the RFoF link prototype for CHIME. Stationarity and ergodicity of the random processes are assumed in what follows.

3.4.1 Thermal noise

Thermal noise arises from the thermally induced, random movement of electrical carriers in conductors. The mean-square value of the noise voltage across a resistor \( R \) at a temperature \( T \) in a small bandwidth \( \Delta \nu \) is given by

\[
\langle v_{tn}^2(t) \rangle = 4k_B T R \Delta \nu. \tag{3.3}
\]

Since the maximum noise power transfer from the noisy resistor to the rest of the circuit happens when the Thévenin equivalent resistance of the remaining circuit is equal to the noise generating resistance, then the available thermal noise power per bandwidth is

\[
P = \frac{\langle v_{tn}^2(t) \rangle}{4R} = k_B T \Delta \nu \tag{3.4}
\]

such that the one-sided power spectral density is just \( S(\nu) = k_B T \) in W/Hz. In particular, at room temperature, \( T = 290 \) K, the power spectral density is \( \sim 4 \cdot 10^{-21} \) W/Hz or \( \sim -174 \) dBm/Hz. For the purposes of analyzing the impact of thermal noise in the circuit, a physical resistor can be modelled as an ideal noise free resistor in series with a voltage source whose mean-square voltage is given by equation 3.3 or in parallel with a current source whose mean-square current is

\[
\langle i_{tn}^2(t) \rangle = 4k_B T \Delta \nu / R. \tag{3.5}
\]
Thermal noise has a flat or "white" spectrum for \(0 \leq \nu \ll k_B T / h\) where \(h\) is the Planck constant. At room temperature, the upper limit is \(\sim 6\) THz, so we can safely assume that thermal noise is white. For complex impedances the resistance in equation 3.3 is replaced by the real part of the complex impedance (an ideal reactive component generates no thermal noise).

### 3.4.2 Shot noise

When an average current \(\bar{I}\) is established via a series of independent, random charge carrier transits, a zero-mean, noise-like current, \(i_{sn}(t)\), is superimposed such that the total current is \(i(t) = \bar{I} + i_{sn}(t)\). Assuming that the statistics of the carrier (usually electrons) transits follow a Poisson distribution [13] then the one-sided spectral density for the shot current (in \(\text{A}^2/\text{Hz}\)) is given by

\[
S(\nu) = 2q\bar{I}
\]  

(3.6)

where \(q\) is the charge of the carriers. The corresponding mean-square value of the shot noise current over a bandwidth \(\Delta \nu\) is

\[
\langle i_{sn}^2(t) \rangle = 2q\bar{I}\Delta \nu.
\]  

(3.7)

The bandwidth over which the spectrum is flat depends on the device exhibiting the noise [19] (e.g. the photodiode). However, it is generally sufficient to assume that the shot noise spectrum is flat in the bandwidth of interest. The shot noise decreases as the average current decreases. From equations 3.5 and 3.7, the average photocurrent that produces a shot noise equal to the thermal noise is

\[
\bar{I}_{s=t} = \frac{2k_B T}{qR}
\]  

(3.8)

which is about 1 mA for \(R = 50\) \(\Omega\) and \(T = 290\) K. In a typical RFOF link for CHIME, the photodiode has a responsitivity \(r_d = 0.85\) mA/mW and the incident optical power is about 3 mW, corresponding to a photocurrent of approximately 2.6 mA. In this case, the shot noise power will be about 4 dB (or \(\approx -170\) dBm/Hz) above the thermal noise power.
3.4.3 Relative intensity noise

The laser’s output optical power exhibits random fluctuations caused by random emissions of spontaneous and stimulated photons in time, fluctuations in the laser pump current, and other factors. A common measure of the random fluctuations in the laser’s optical output power is the relative intensity noise (RIN). By writing the total optical power, $p_O(t) = \bar{P}_O + p_{rin}(t)$ where $p_{rin}(t)$ is a zero-mean random process, then the RIN is defined as

$$\frac{\langle p_{rin}^2(t)\rangle}{\bar{P}_O^2} = \text{RIN}(\nu)\Delta \nu$$

(3.9)

so the RIN is the one-sided power spectral density of $p_{rin}(t)/\bar{P}_O$ (units of Hz$^{-1}$) [18]. Here we have assumed that the spectral density is flat and that the measurement is limited to a bandwidth $\Delta \nu$. At the output of the photodiode, where the optical power is converted to an electric current, the photodiode responsivity cancels out so we can write the RIN in terms of the photocurrent (in dB/Hz) as

$$\text{RIN} = 10\log \left[ \frac{\langle i_{rin}^2(t) \rangle}{\bar{I}^2\Delta \nu} \right] \rightarrow \langle i_{rin}^2(t) \rangle = \bar{I}^210^{\text{RIN}/10}\Delta \nu$$

(3.10)

where $\bar{I}$ is the average photocurrent. The assumption that the RIN is constant holds for frequencies well below the relaxation resonant frequency of the laser, which is the case for the frequencies relevant to CHIME. For a discussion on the RIN spectrum refer to [19]. From equations 3.7 and 3.10 it is clear that the RIN decreases faster than the shot noise as the average photodetector current decreases. The minimum detectable RIN, below which the RIN is masked by the shot noise, is given by

$$\text{RIN}_{sn} = 10\log \left( \frac{2q}{\bar{I}} \right)$$

(3.11)

which is about -159 dB/Hz for the typical photocurrent in a RFOF link for CHIME. The manufacturer of the lasers for CHIME specifies RIN values up to -140 dB/Hz, so we expect that the RIN is the main contribution to the noise of the RFOF link.
3.4.4 Mode partition noise

For lasers that have multiple wavelengths in their spectra, Mode Partition Noise (MPN) describes how the intensities of all longitudinal optical modes (optical wavelengths) fluctuate. MPN is an important noise source in analog systems using lasers with multiple wavelengths in their spectra (e.g. Fabry-Perot) or with a single strong wavelength and weak side wavelengths (e.g. DFB). Mode partitioning refers to the distribution of the laser’s optical power between its different optical wavelengths in such a way that the power in individual wavelengths fluctuates but the total power in all wavelengths is relatively steady. Mode partitioning becomes a problem when dispersive propagation delays cause fluctuations in the powers of individual wavelengths to be separated in time, so that at a given time the fluctuations of various wavelengths no longer cancel one another [85].

3.5 Distortion and dynamic range in RFoF links

An RFoF link is closer to an active RF component (such as an amplifier) than to passive coax cable in the sense that the frequency response (or more generally, the frequency dependent scattering parameter matrix) of the RFoF link is not enough to completely characterize its performance. From the previous section, we know that there are several sources of noise in an RFoF link, so we need to define parameters that measure the effects of noise in the link. We will use parameters that are commonly used to characterize RF components like the Noise Factor, Noise Figure and equivalent noise temperature. This is not only convenient, but also necessary for the design of the link since the intrinsic RFoF link (modulation device, fiber and photodetector) needs to be properly coupled to additional RF amplification, filtering and matching stages in order to be of practical use in CHIME. A quick introduction to noise characterization is given in Appendix A.

In general, the transfer function (the relation between the input and the output) of modulation and photodetection devices, and the optical fiber (through effects like stimulated Brillouin scattering [28] [41]), is nonlinear. As for any active RF component, nonlinearities in the RFoF link limit its dynamic performance. The effect of nonlinear distortion on limiting the dynamic range of the RFoF link depends on how the transfer function of the whole link departs from a straight line, which is mainly determined by the modulation device [19]. Following the same method as with the noise performance, we will parametrize the nonlinear distortion of the RFoF link using figures of merit that are commonly used to characterize the nonlinear performance of RF components (e.g. amplifiers and mixers). Our figures of merit will be the 1 dB Compression Point (CP),
Intercept Points (IP) and Spurious-Free Dynamic Range (SFDR). In particular, we will focus on the Third Order Intercept Point (IP3), a figure of merit for the RFOF linearity that is appropriate for an octave-bandwidth system (the upper band frequency is twice the lower band frequency) like CHIME. The corresponding SFDR is used as a measure of the RFOF dynamic performance. An introduction to intermodulation distortion is given in Appendix B. We finally note that, unless we specify otherwise, when we characterize the dynamic performance of the RFOF link we will refer not just to the performance of the intrinsic link, but the performance of the link including additional amplification, filtering and matching stages included in the RFOF transmitter and receiver boards.

### 3.6 RFOF links in radio astronomy

RFOF links have found application in a variety of fields, including telecommunications (distribution of Cable Television [CATV] signals, cellular/Personal Communication System [PCS] up-link and down-link systems [19] [41] [67] [83]), military and aerospace (shipboard RF signal distribution, radar systems, military antenna remoting, electronic warfare systems [36] [43] [52] [58]) and medicine (Magnetic Resonance Imaging [MRI] signal transport [33] [53] [89] [90]).

The use of RFOF links in radio astronomy has also been investigated and successfully applied for local oscillator (LO) signal distribution and analog signal transport in large antenna arrays. Examples include the photonic local oscillator system for distribution of LO and timing references in the Atacama Large Millimeter Array (ALMA) [74] [75], the externally modulated, high bandwidth-distance product and high performance analog optical link for signal transport from the antennas to the processing facility in the Allen Telescope Array (ATA) [3] [84], and the directly modulated RFOF links for signal transport in the Australian Square Kilometer Array Pathfinder (ASKAP) [10] [26] and the Basic Element for SKA Training (BEST) [54] [55] [59].

As in the last three examples, the CHIME collaboration is currently investigating the implementation of a RFOF link to transport the signals from the antennas to the processing facility, an application that is traditionally implemented using coaxial cable. Optical fiber offers high bandwidth, low attenuation independent of frequency, small size, light weight and immunity to electromagnetic interference in contrast with coaxial cable as described in Section 3.1.

CHIME implements a receiver architecture where the RF signals are sent to the processing room via an analog optical link, where they are digitized. Analog transmission over a RFOF link reduces the likelihood of self-generated RFI pickup from the digitizing...
hardware compared to a receiver architecture with the digitizers located at the antennas and digital data transmission to the processing room over a digital optical link. From the reliability and maintainability point of view, the CHIME receiver architecture, with analog transmission over a RFOF link, has less electronics located at the antennas, which implies less power and cooling requirements and reduced maintenance at the antennas. It also implies more devices in a sheltered environment.

In general, RF signal transport in radio astronomy applications (including CHIME) is very demanding in terms of noise and $SFDR$ performance. In addition, gain and phase stability of the RFOF link are critical for its practical use in radio astronomy applications. The characterization of the prototype RFOF link for CHIME will be the main topic of the following chapters for this thesis.
Chapter 4

Analog receiver and RFoF performance requirements

As a prototype for testing the CHIME technology, a two-element radio interferometer was built at DRAO. While characterizing the CHIME analog receiver in the two-element interferometer, the implementation of a RFoF link to transport the analog signals from the antennas to the processing room has been investigated as an alternative to traditional coaxial cable. RFoF is an attractive option for the analog receiver in CHIME, and its advantages over coaxial cable include not only cost, but also small size and light weight, immunity to electromagnetic interference, low loss, and a separate grounding scheme for the front end RF chain and the digital back end.

Analog transmission over a RFoF link also reduces the likelihood of self-generated RFI pickup and the amount of electronics and maintenance at the antennas compared to a receiver architecture with the digitizers located at the antennas and digital data transmission to the processing room. However, the analog optical link also introduces some challenges that must be overcome, mainly related to linearity, noise, and stability. In this chapter we determine the dynamic performance requirements for the RFoF link. The $IP3$ will be used as the figure of merit for the RFoF linearity and is appropriate for an octave-bandwidth system like CHIME. The corresponding $SFDR$ will be used as measure of the RFoF dynamic performance.

4.1 Two-element interferometer

The two-element radio interferometer is shown in Figure 4.1. It is made up of two circular 8.53 m parabolic dish reflectors on an east-west line spaced 19 m apart. It was instrumented with custom amplifiers and feeds operating in the frequency range
Figure 4.1: The two-element interferometer is made up of two circular 8.53 m parabolic dish reflectors on an east-west line spaced 19 m apart. It was instrumented with custom amplifiers and orthogonal linear polarization feeds operating in the frequency range 425-850 MHz. The electronics hut (left) contains the digitizers and additional back end electronics.

425-850 MHz in orthogonal linear polarizations, and a back end consisting of ADCs and a custom five-channel correlator designed by the McGill Cosmology Instrumentation Laboratory. This prototype provides an excellent opportunity to test different feed and analog receiver designs, as well as different digitizer and correlator architectures. In particular, the two-element interferometer allowed testing the performance of a custom Radio-Frequency-over-Fiber (RFOF) link designed to transport the analog signals from the antennas to the processing room as an alternative to a coaxial cable-based receiver. All the field tests for the RFOF link were performed in this two-dish system.

4.2 Current analog receiver design and characteristics

A block diagram of the (coax-based) CHIME two-element interferometer receiver is shown in Figure 4.2. The feed for each dish is a four-square above a ground-plane based on the design presented in [49]. The focus receiver consists of a custom low noise amplifier (LNA) followed by a second-stage amplifier and filter. The feed analog stages are in individually shielded boxes. A 50 m coaxial cable is connected to the focus receiver to
Figure 4.2: Receiver block diagram for one channel of the two-element interferometer. The gain ($G$), noise figure ($NF$), input referred third order intercept point ($IIP_3$), and compression point ($CP$) shown for each block are referred to the block’s input.

transmit the signal to an electronics hut containing the digitizer and additional backend electronics. The 20 dB loss from the coax in Figure 4.2 includes the loss in both the cable and attenuators installed at each end of the cable. Inside the electronics hut, the signal passes through a second combined amplifier and filter box and is then fed into the digitizer and correlator. For a more detailed description of the CHIME two-element interferometer refer to [24].

The frequency response of the analog receiver is defined by a commercial band-pass filter with a 3 dB passband in the approximate range 475-800 MHz. The noise of the LNA is approximately 35 K ($NF = 0.5$ dB) across the CHIME band and this is the main contribution to the noise of the receiver electronics. The output signals from the receiver are directly sampled at 850 MSPS, recovering the second Nyquist zone from 425-850 MHz. There is no down-conversion stage. The ADC samples are processed by a custom FPGA-based correlator using an FX architecture: The signals are first put through a poly-phase filter bank (PFB), then Fourier transformed using a 2048-point FFT. The four signals (two dishes and two polarizations) are cross-correlated.

The RFoF link, having relatively high input noise ($NF \approx 27$ dB), is the last stage of the receiver chain with its output connected directly to the ADC. A gain of at least 50 dB upstream of the RFoF input is required to make its noise contribution negligible in comparison to the front-end LNA. To ensure the RFoF link has negligible impact on the entire analog chain, it is required to have an output referred dynamic performance that is substantially better than that of the ADC. The $SFDR$ of the RFoF is determined by its $IP_3$ and noise floor. This requirement translates to an Output $IP_3$ ($OIP_3$) much larger than the Input $IP_3$ ($IIP_3$) of the ADC and an output noise floor much lower than the input noise of the ADC.

\[\text{In general, the board where the ADC is mounted has some attenuation, mainly due to input matching}\]
4.3 ADC dynamic performance

For the RFoF link, the \textit{IP3} determines the level of the third-order Intermodulation Distortion products (\textit{IMD3}) as a function of the output signal level. In contrast, an ADC does not have a single, well defined \textit{IP3}. For a typical ADC, the \textit{IMD3} curve has three regions. For low level input signals, the \textit{IMD3} products remain relatively constant regardless of signal level. Near the ADC full-scale, the \textit{IMD3} products behave as expected (3 dB increase per every dB increase in the input power). The power level at which this begins depends on the specific ADC device. Above the full-range, the ADC acts as an ideal limiter and the \textit{IMD3} products become large due to clipping. For a detailed description of this subject see [47].

The approach followed here to determine the \textit{IP3} of the ADC is to measure the \textit{IMD3} products using two input tones such that the ADC is close to full range (which is typically the \textit{IMD3} value reported on ADC datasheets). A calculation of the ADC \textit{IP3} based on this measurement yields an optimistic value, and an overestimation of the ADC \textit{SFDR} as long as the ADC is below full range\(^2\). Thus, requiring that the \textit{OIP3} of the RFoF is much greater than this measure of the ADC \textit{IIP3} is enough to guarantee that the distortion generated by the RFoF is much smaller than that generated by the ADC.

Both the two-element interferometer and the CHIME pathfinder (a small version of CHIME consisting of two cylindrical telescopes with 256 total radio receivers that is currently under construction at DRAO) use the E2V\(^3\) EV8AQ160 quad-channel, 8-bit ADC implemented on a custom FPGA Mezzanine Card (FMC) compliant circuit board with two devices providing eight input channels. The insertion loss of the ADC board from its single ended input to the differential ADC is approximately 2 dB. Each ADC device can sample at rates up to 1.25 GSPS in four-channel mode with a full range of 0.5 V\(_{p-p}\) (-5 dBm) differential input. The measurements presented in this paper are sampled at 850 MSPS whereas CHIME will operate at 800 MSPS. A top view of the custom ADC board is shown in Figure 4.3.

The performance of the ADC is characterized by the \textit{SFDR} for our purposes. Measurements of the ADC \textit{IMD3} products in the CHIME band are shown in Figure 4.4.

\(^{1}\)In this document \textit{SFDR} always refers to the Spurious Free Dynamic Range as calculated from the two-tone third order intermodulation distortion products. This should not be confused with the \textit{SFDR} specified in ADC datasheets which is obtained from a single-tone test as the ratio of the signal power to the power of the strongest spurious signal.

\(^{2}\)http://www.e2v.com/

\(^{3}\)http://www.e2v.com/
Figure 4.3: The custom-built ADC board for CHIME includes two ADCs that can each digitize four analog channels with 8 bits at a maximum sampling rate of 1.25 GSPS, for a total of eight channels per board. Each ADC channel streams its data across eight differential signals to the FPGA. For CHIME, the ADCs will be operated at 800 MSPS to perform alias sampling of the 400-800 MHz signal. The ADC board can interface to any FPGA motherboard that implements the full FPGA Mezzanine Card (FMC) High Pin Count (HPC) standard. The two additional inputs are for the reference clock and the synchronization signal.

The $I_{IP3}$ of the ADC board was measured using two tones at 632.5 MHz and 652 MHz. We obtained $I_{IP3}=13$ dBm. The noise of the ADC board is determined from the Signal-to-Noise-and-Distortion ($SINAD$) measurement of the ADC (typically parametrized as Effective-Number-of-Bits, $ENOB$) and then combined with the $IP3$ measurement to obtain the $SFDR$. For a description of the $SINAD$, $ENOB$, and other specifications for quantifying the ADC dynamic performance refer to [47]. Figure 4.5 shows the $ENOB$ measurement at 632.5 MHz. A 1 dBFS (1 dB below full-scale) input tone was used. The total power from noise and distortion in the 425-850 MHz band is approximately $-48$ dBm, which corresponds to $SINAD = -5 - (-48) = 43$ dBFS. Note that the $SINAD$ already includes the 1 dB correction from the fact that we did not use a full-scale tone. The $ENOB$ of the ADC is

$$ENOB = \frac{SINAD - 1.76}{6.02} \approx 6.9 \text{ bits}$$

(4.1)

which is within the specification of the EV8AQ160 ADC. The (normalized) output noise floor of the ADC can be obtained from the $ENOB$. However, it is easier to use the
Figure 4.4: IMD3 measurements are shown for input tones close to the ADC full scale using the custom ADC board (left) and spectrum corresponding to the measurement at 7 dBFS (right). Two input tones at 632.5 MHz and 652 MHz were used. The ADC full range is 0.5 V_{p-p} or -5 dBm.

The fact that the -48 dBm power from noise and distortion is measured over a bandwidth of 425 MHz. Thus, the output noise floor of the ADC is

\[ P_{no} = -48 - 10\log(425 \cdot 10^6) \approx -134 \text{ dBm}. \]  

\[ (4.2) \]

The corresponding input noise floor (including the 2 dB insertion loss of the ADC board) is approximately \( P_{ni} = -132 \text{ dBm/Hz} \), resulting in \( SFDR = 97 \text{ dB-Hz}^{2/3} \) for the ADC board.
Figure 4.5: ADC ENOB measurement for the custom ADC board is shown. A 1 dBFS input tone at 632.5 MHz was used. The first 25 folded harmonics have been identified where clearly visible. The total power from noise and distortion in the 425-850 MHz band is approximately $-48 \text{ dBm}$ or $\text{SINAD} = 43 \text{ dBFS}$, which corresponds to $\text{ENOB} = 6.9$ bits. The normalized output noise power is $P_{no} = -134 \text{ dBm/Hz}$. 
Chapter 5

RFoF link design and dynamic performance

5.1 RFoF link design and characteristics

The selection of the components and modulation method for the RFoF design is mainly driven by two parameters: cost and performance. As already mentioned in Chapter 3, compared to other modulation and detection techniques (e.g. frequency and phase modulation of the optical carrier combined with coherent detection), intensity modulation combined with direct detection (IMDD) is the most widely employed method to convey an RF signal over an analog optical link, mainly due to its simplicity. In IMDD the RF signal modulates the intensity of an optical carrier, which then travels over the optical fiber and is detected by a photodetector. Two ways to implement the IMDD method are to use external modulation or direct modulation. In an externally modulated system the laser operates in continuous wave (CW) mode and the modulation is done externally with an optical modulator. These are high performance systems that provide high dynamic range and large bandwidth over long transmission distances (e.g. the Allen Telescope Array [3], [84]). However, the need for the additional external modulator and high performance, high power lasers increases the system costs dramatically. This, in the case of a large-array application like CHIME requiring thousands of RFoF links, is a major disadvantage.

The custom-built RFoF link for CHIME is a directly modulated system. The main advantage of direct modulation is simplicity and low cost as compared to externally modulated systems. For a short-range application like CHIME, where the deterioration of the optical signal in the fiber is small, the performance of the link is mainly determined
by the laser [19]. Distributed Feedback (DFB) lasers are generally preferred for analog applications due to their high linearity, high slope efficiency (the slope of the laser output power versus the laser current curve) and low noise characteristic compared to other lasers. However, these are the most expensive laser types due to their complicated fabrication [66]. Vertical-Cavity-Surface-Emitting Lasers (VCSELs) offer low cost and low power consumption due to the low threshold current. Their optical characteristics are similar to DFB lasers (a single wavelength peak is present in their spectrum) but their performance is, in general, below that of DFB lasers. Several VCSELs were tested in the lab as possible candidates for the RFoF prototype and SFDR values in the range 90-100 dB·Hz$^{2/3}$ were achieved. A similar RFoF link using VCSELs for antenna remoting in the Australian Square Kilometre Array Pathfinder (ASKAP) achieved 98 dB·Hz$^{2/3}$ [10] [26]. These values are below the requirements for CHIME. The current RFoF prototype uses a Fabry-Perot (FP) laser, which for short-range applications offers good performance and its costs are lower than the DFB laser costs. Due to its characteristic design, several wavelengths are present in the FP spectrum, which causes signal deterioration for transmission over distances exceeding a few kilometers due to dispersion in the fiber [14] [19] [48] [85]. For short-range applications like CHIME, this effect is very small and the constraints from the laser dominate. The performance of the link for distances larger than 100 m has not yet been investigated.

Figure 5.1 shows the prototype RFoF link design. The RF analog signal is placed on the optical carrier by directly modulating the laser current with the signal. The optical signal travels through the fiber and the photodiode converts the signal back to the electric domain for additional amplification and filtering. The RFoF transmitter features an AGX\(^1\) uncooled, linear multi-quantum well (MQW) FP laser emitting at 1310 nm, hermetically sealed in an industry-standard coaxial package with a single-mode fiber pigtail and subscriber connector/angled physical contact (SC/APC) connector. Integrated in the laser package are an optical isolator to prevent unwanted feedback into the laser and a monitor photodiode to control the laser optical output power. Each RFoF transmitter is electrically sealed in a dedicated aluminum box. The circuit schematic of the RFoF transmitter is shown in Figure 5.2 and the circuit schematic of the optical power control module is shown in Figure 5.3.

The RFoF receiver uses an AGX linear, low-capacitance photodiode sensitive to the wavelength range between 1100 nm and 1650 nm. This photodiode module is also a hermetically sealed coaxial package with a single-mode fiber pigtail and SC/APC connector. The first version of the RFoF receiver was constructed as a board with four receiver

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\(^1\)http://www.agxtech.com/
Figure 5.1: Basic diagram of the prototype CHIME RFoF link. The laser current is directly modulated by the RF signal. The optical signal travels through the fiber and the photodiode converts the signal back to the electric domain. The RFoF receiver includes an amplification chain and the same band pass filter (BPF) as the second stage amplifier of the two-element interferometer receiver.

channels, all enclosed in the same aluminum box. Each RFoF receiver channel includes an amplification chain and the same band defining filter as the second stage amplifier of the two-element interferometer receiver. The circuit schematic of the RFoF receiver is shown in Figure 5.4. A top view of the single-channel RFoF transmitter and four-channel receiver is shown in Figure 5.5. The approximate cost is US$ 200 per link in quantities of several hundred, not including the cost of fiber. The laser and the band-pass filter are the most expensive components.

5.2 Dynamic performance of the RFoF link

The performance of the RFoF was characterized with a suite of laboratory tests. All the measurements in this section are performed at 25 C using a 80 m fiber. The optical output power feedback control circuitry of the RFoF transmitter has been disabled for these tests as explained in Chapter 6. Without the RFoF receiver amplification chain, the gain of the link is approximately -20 dB, a value which is mainly determined by the slope efficiency of the laser and the responsivity of the photodiode [19] [22]. Amplification is needed in the receiver to boost the $OIP_3$ of the RFoF to a level well above that of the ADC while keeping the output noise floor below the input noise of the ADC. This is implemented in two stages. A commercial LNA is placed after the photodiode,
Figure 5.2: Circuit schematic of the RfoF transmitter. The block LaserBias (top) is the optical power control module, which is shown in detail in Figure 5.3.
Figure 5.3: Circuit schematic of the optical power control module.

The Photodiode inside the FP laser has a responsivity of $10 \rightarrow 200$ nA/mW.

Our system will need to be calibrated such that at both of these gains we can set the power level to 2 mW.

2 mW > 200 nA/mW > 2 kV/A = 0.8 V
2 mW > 10 nA/mW > 2 kV/A = 0.04 V

My reference is 2.5 V, therefore, I need G = 3.1 for 10 nA/mW and G = 62.5 for 200 nA/mW given 2 kV/A.

The LT3092 with 30 k and 10 Ohm bias resistors provides 30 mA. The laser typically needs 35 mA, the additional current being provided by the feedback circuit.

System is inefficient if current buffer sinks current, bias current source above to provide less current than expected.

Resistors limit total current from opamps (5 V - 3 V) 100 = 200 mA

Assuming at 28 mA bias current, Vbias approximately 3V = 2.1 V + 36 mA x 30 Ohms
Figure 5.4: Circuit schematic of the RFoF receiver.
Figure 5.5: The RFoF transmitter (top) features an uncooled, linear multi-quantum well (MQW) FP laser emitting at 1310 nm, hermetically sealed in an industry-standard coaxial package with a single-mode fiber pigtail and subscriber connector/angled physical contact (SC/APC) connector. Each RFoF transmitter is electrically sealed in a dedicated aluminum box. The RFoF receiver (bottom) uses a linear, low-capacitance photodiode sensitive to the wavelength range between 1100 nm and 1650 nm. This photodiode module is also a hermetically sealed coaxial package with a single-mode fiber pigtail and SC/APC connector. The first version of the RFoF receiver was constructed as a board with four receiver channels, all enclosed in the same aluminum box. Each RFoF receiver channel includes an amplification chain and the same band defining filter as the second stage amplifier of the two-element interferometer receiver.
followed by the band defining filter and a commercial high $IP_3$ amplifier as the final amplification stage. An amplification greater than the electric-optical-electric conversion loss is required to have margin to allow for attenuation pads at each stage, improving the impedance matching and reducing the in-band ripple from the filter. The end-to-end gain of the RFOF link in the CHIME band has been set to 4 dB with a corresponding $OIP_3$ of approximately 29 dBm. Measurements of the scattering parameters for the RFOF link are shown in Figure 5.6 and measurements of the output noise, 1 dB Compression Point ($CP$), $OIP_3$ and crosstalk between adjacent RFOF receiver channels are shown in Figure 5.7. A summary of the characteristics and dynamic performance of the RFOF link in the CHIME band is shown in Table 5.1.

Table 5.1: Characteristics and dynamic performance of the RFOF link in the 475-800 MHz band. All measurements are performed at 25 C using an 80 m fiber.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Wavelength</td>
<td>$\lambda$</td>
<td>1310 nm</td>
</tr>
<tr>
<td>Laser Operating Current Bias</td>
<td>$I_L$</td>
<td>30 mA</td>
</tr>
<tr>
<td>Laser Optical Output Power</td>
<td>$P_{OL}$</td>
<td>3.0 mW</td>
</tr>
<tr>
<td>1 dB (Input) Compression Point</td>
<td>$CP$</td>
<td>$&gt; 10$ dBm</td>
</tr>
<tr>
<td>Gain</td>
<td>$G$</td>
<td>4 dB</td>
</tr>
<tr>
<td>Input Return Loss</td>
<td>$</td>
<td>S_{11 dB}</td>
</tr>
<tr>
<td>Output Third Order Intercept</td>
<td>$OIP_3$</td>
<td>29 dBm</td>
</tr>
<tr>
<td>Output Noise Floor</td>
<td>$P_{no}$</td>
<td>$-143$ dBm/Hz</td>
</tr>
<tr>
<td>Noise Figure</td>
<td>$NF$</td>
<td>27 dB</td>
</tr>
<tr>
<td>Spurious-Free Dynamic Range</td>
<td>$SFDR$</td>
<td>115 dB $\cdot$ Hz$^{2/3}$</td>
</tr>
</tbody>
</table>

The main contribution to the noise of the RFOF link is the Relative Intensity Noise (RIN) of the laser. Since the dispersion effects introduced by the fiber are small near 1310 nm and for the transmission distances considered in CHIME (of the order of 100 m), the noise penalty introduced by the spectral impurity of the FP laser (e.g. Mode Partition Noise, see Chapter 3) is negligible [77] [85]. The RFOF link has a Noise Figure $NF = 27$ dB ($P_{ni} = -147$ dBm/Hz) and the corresponding Spurious-Free Dynamic Range is $SFDR = 115$ dB $\cdot$ Hz$^{2/3}$, or 57 dB in a 425 MHz bandwidth. These results are well matched to an 8 bit ADC, as they will have a negligible impact on the noise and linearity of the receiver chain. This performance over 80 m distances is comparable to that of more expensive high performance DFB laser-based RFOF links for radio astronomy applications (e.g. [54], [55], [59]).

The degradation of the ADC $SFDR$ introduced by the RFOF link can be estimated
by calculating the noise and $IIP^3$ of the cascaded RFoF+ADC system at the input of the RFoF using the methods described in appendices A and B. The input noise of the cascaded system is

$$P_{ni} = 10\log\left(10^{P_{no\, RFoF/10}} + 10^{P_{ni\, ADC/10}}\right) - G_{RFoF} \approx -135.7 \text{ dBm/Hz.} \quad (5.1)$$

The $IIP^3$ of the cascaded system is

$$IIP^3 = -10\log\left(10^{-OIP^3_{RFoF}/10} + 10^{-IIP^3_{ADC}/10}\right) - G_{RFoF} \approx 8.9 \text{ dBm.} \quad (5.2)$$

Thus, the $SFDR$ of the cascaded system is $SFDR \approx 96.4 \text{ dB} \cdot \text{Hz}^{2/3}$, a reduction of less than 1 dB with respect to the $SFDR$ of the ADC alone.

The crosstalk performance between adjacent RFoF receiver channels is better than 45 dB in the CHIME band. The crosstalk requirement for CHIME is set by the ADC board, which has a minimum crosstalk performance of 55 dB between adjacent channels in the CHIME band. In order to meet this requirement, the production version of the RFoF link for CHIME will also have each RFoF receiver individually sealed in its own aluminum box.
Figure 5.7: Output noise, Compression Point, $IP_3$ and crosstalk between adjacent receiver channels of the RFoF link at 25 °C using an 80 m fiber. The average normalized output noise power in the 475-800 MHz band (top left) is $P_{no} = -143$ dBm/Hz and the out-of-band noise floor is limited by the dynamic range of the Spectrum Analyzer. The $CP$ exceeds 10 dBm and the measurement (top right) is limited by the maximum output power of the Network Analyzer. The $OIP_3$ is approximately 29 dBm (bottom left) measured using two tones at 599.5 MHz and 600.5 MHz. The crosstalk performance between adjacent RFoF receiver channels (bottom right) is better than 45 dB in the CHIME band.
Chapter 6

Gain and phase stability of the RFoF link

The gain and phase stability of the analog receiver are critical in radio interferometry. Gain and phase fluctuations, resulting mainly from temperature effects in the components of the receiver, generate errors in the visibility data and loss in sensitivity [79]. The receiver stability determines how often the calibration procedure, such as the injection of a known signal across the array, needs to be performed.

In the case of the CHIME two-element interferometer, the RFoF receiver is located in a temperature controlled electronics hut with the back-end electronics. Thus it is expected that the gain and phase stability of the RFoF will be driven by temperature fluctuations of the transmitter and the fiber, which are in an outdoor environment. Measurements as a function of temperature will be presented separately for these two components below.

6.1 Fluctuations caused by changes in temperature of the RFoF transmitter

Measurements of the gain and phase variation of the RFoF link with changes in temperature of the RFoF transmitter box are shown in Figure 6.1. The RFoF transmitter was placed inside a climate controlled chamber and the temperature was varied from -25 C to 45 C in steps of 10 C. This encompasses the expected range of temperatures at DRAO. Both the RFoF receiver and the 80 m fiber were kept at 25 C during the measurements. The phase variation is shown as arg($S_{21}(T)$)−arg($S_{21}(T = 25 \text{ C})$), where arg($S_{21}(T = 25 \text{ C})$) is the phase of $S_{21}$ at 25 C and arg($S_{21}(T)$) is the phase of $S_{21}$ at
Figure 6.1: Gain and phase variation of the RFoF link with changes in temperature of the RFoF transmitter. An 80 m fiber was used. The temperature of the RFoF receiver and fiber was kept at 25 C. The phase variation is shown as the phase difference in degrees with respect to the phase at 25 C.

The temperature of the chamber. The in-band gain variation is approximately 3 dB for a 70 C temperature change, or approximately 0.04 dB/C, mainly caused by the change in the slope efficiency of the FP laser. The phase variation is below 6° in-band for a 70 C temperature change.

The RFoF transmitter includes an automatic optical power control module for the laser. When the module was enabled, higher gain variations were observed. As shown in Figure 6.2, this is due to a combination of a decrease of the slope efficiency of the laser as the temperature increases and the fact that the $p_{OL}$ vs. $i_L$ curve of the laser starts to compress as the current is increased. Since the main purpose of the optical power control system is to keep the laser bias current well above the threshold current of the laser (which increases with temperature), it is possible to disable the module as long as the laser bias current, which is now fixed, is well above the threshold current along the full range of expected temperatures for the RFoF transmitter. The power control module has been disabled for the measurements in this paper and it will be removed for the next version of the RFoF link using this laser.

In addition, the nonlinear and noise performance of the RFoF link as a function of the temperature of the RFoF transmitter were measured. Within the uncertainty of the measurements (about 1 dB), the $OIP3$, output noise, and consequently the $SFDR$ re-
Figure 6.2: Temperature dependence of the $p_{OL}$ vs. $i_L$ curve of a typical laser. As the temperature of the laser increases, the threshold current increases and the slope efficiency $s_l$ decreases. In general, the $p_{OL}$ vs. $i_L$ curve is not a straight line. The change in $s_l$ when moving along curves with constant optical power (e.g. enabling the automatic optical power control module for the laser) is different from the change in $s_l$ moving along curves with constant laser bias current (e.g. disabling the automatic optical power control module for the laser).
Figure 6.3: Gain and phase variation of the RFoF link with changes in temperature of the fiber. An 80 m fiber was used. The temperature of the RFoF transmitter and receiver was kept at 25 °C. The phase variation is shown as the phase difference in degrees with respect to the phase at 25 °C.

remained relatively constant during the test (at 29 dBm, −143 dBm/Hz, and 115 dB · Hz$^{2/3}$ respectively). However, the $IIP_3$ and the $NF$ rose with temperature: the $IIP_3$ changed from 23 dBm at -25 °C to 26 dBm at 45 °C while the $NF$ changed from 25 dB at -25 °C to 28 dB at 45 °C.

### 6.2 Fluctuations caused by changes in temperature of the fiber

Measurements of the gain and phase variation of the RFoF link with changes in temperature of the fiber are shown in Figure 6.3. The 80 m fiber was placed inside the chamber and the temperature was varied from -25 °C to 45 °C in steps of 10 °C. A standard 9/125 μm tight buffer patch cord was used. The gain variation is less than 0.1 dB for a 70 °C temperature change, very small compared to the gain variation due to the RFoF transmitter. However, the phase variation of the fiber is relatively high and will be the dominant source of phase variations for the interferometer. It is common to express the phase stability of the fiber in terms of the Thermal Coefficient of Delay ($TCD$) defined
Figure 6.4: Estimated $TCD$ of the fiber as a function of temperature. A velocity factor of 0.68 and a length of 80 m under standard conditions (25°C) was assumed for the fiber. Note that $\phi$ cannot be obtained directly from $S_{21}$ since the insertion phase of the other components of the RFoF link has not been measured.

\[
TCD = \frac{d\tau}{dT} \approx \frac{\Delta \phi}{\Delta T} \frac{10^6}{\tau} \tag{6.1}
\]

where $\tau$ is the insertion delay and $\phi$ is the respective insertion phase of the fiber. $d\tau/dT$ is determined by variations in the length and refractive index of the fiber [11]. Figure 6.4 shows the estimated $TCD$ obtained from the phase information in Figure 6.3 assuming that the fiber has a velocity factor of 0.68 and a length of 80 m under standard conditions (25°C). The $TCD$ is approximately 90 ppm/C at -25°C, relatively high compared to LMR-400 coax cable, for which cable providers typically specify a $TCD$ below 10 ppm/C. However, the phase stability of the optical link can be greatly improved by using loose tube cable, in which the manufacturing process ensures that the fiber is protected from stresses caused by temperature changes and mechanical forces [57]. For this cable design the $TCD$ reduces to values below 10 ppm/C [11] and, in general, has a better performance in outdoor applications. Since the change to loose tube causes no increase in the cost of the cable, this will be the fiber used for the production version of the CHIME RFoF link.
Figure 6.5: Two-element interferometer receiver block diagram using RFoF. The parameters of each block are referred to its respective input.

6.3 Gain and phase stability of the fiber-based receiver

Returning to the gain fluctuations, a method commonly used to reduce them in the optical link is to control the temperature of the laser by means of a thermistor and a Thermoelectric Cooler (TEC), with both components either in a separate module on the RFoF transmitter board or included in the laser package. The last option is discarded for CHIME because cooled lasers are about an order of magnitude more expensive than uncooled ones. Instead of trying to reduce the gain fluctuations by analog means, a much simpler and more cost-effective alternative is to perform a gain compensation of the analog receiver implemented in firmware or software after the digitization or during the calibration procedure. This is feasible as long as the signal degradation introduced by ADC quantization and noise is acceptable across the full range of temperature induced gain variations of the analog receiver.

In order to characterize this effect, the gain variation of the analog receiver with RFoF was also investigated. A diagram of the receiver chain with RFoF is shown in Figure 6.5. The current analog receiver design requires a 20 dB gain RFoF link in order to achieve the desired noise power at the input of the ADC. Optimally, the additional gain must be added at the input of the RFoF link in order to keep the RFoF output noise level below that of the ADC. For this test, the additional gain had to be added in the RFoF receiver so we could use the existing analog receiver electronics. The effect of this change in the overall system performance is negligible since the noise performance of the system is still dominated by the LNA and its nonlinear behavior is still dominated by the ADC. The new version of the RFoF link will have the additional gain in the transmitter. For this test, all the components of the receiver which are located at the focus (LNA, second-stage amplifier and RFoF transmitter) were placed in the chamber and the temperature inside
Figure 6.6: Gain and phase variation of the RFoF-based two-element interferometer receiver with changes in temperature of the elements at the focus. An 80 m fiber was used. The temperature of the RFoF receiver and fiber was kept at 25 °C. The phase variation is shown as the phase difference in degrees with respect to the phase at 25 °C.

was changed from -25 °C to 45 °C. The results are shown in Figure 6.6. The in-band gain variation is approximately 5 dB for a 70 °C temperature change, or approximately 0.07 dB/°C.

The signal degradation due to quantization and ADC noise in the case of an 8-bit ADC is shown in Figure 6.7. For a detailed discussion of the fractional increase in the noise of a Gaussian distributed input signal that results from quantization refer to [79]. For the two-element interferometer, the typical input power to each channel of the analog receiver is estimated as 100 K in a 425 MHz bandwidth, or about -92 dBm [24]. The average gain of the receiver in the 425-850 MHz band is approximately 72 dB at 25 °C which, including the 2 dB insertion loss of the ADC board results in -22 dBm power at the input to the ADC or a voltage standard deviation of about 3.7 bits. This level causes a negligible noise penalty due to quantization and ADC noise, while allowing headroom for receiver linearity and ADC saturation caused by external RFI. The total variation in the gain of the receiver (from -25 °C to 45 °C) results in an input signal variation between 3.4 bits (at 45 °C) and 4.2 bits (at -25 °C). As Figure 6.7 shows, the minimum of the quantization efficiency function is broad and the noise penalty at these levels is still negligible, so the gain calibration can be performed after digitization.
Figure 6.7: Fractional increase in the noise of a signal that results from the quantization with an 8-bit ADC. The input to the ADC is assumed to be Gaussian noise with standard deviation $\sigma_{\text{in}}$. The standard deviation of the signal after quantization is $\sigma_{\text{out}}$. 
Chapter 7

First tests on the CHIME two-element interferometer

The first tests of the RFoF link on the CHIME two-element interferometer at DRAO were performed in January 2013. The purpose of these tests was two-fold: (1) to compare the performance of the RFoF-based receiver to that of the coax-based receiver and (2) to test a technique that uses an injected reference signal to measure and calibrate the system gain.

7.1 Noise injection setup

The noise injection calibration procedure allows for the determination of the receiver channel relative gains in real time by correlating the receiver channels with an injected reference signal derived from a noise source. This is illustrated in Figure 7.1. Inside the electronics hut, a broadband noise source is switched on and off periodically. The output of this source is further amplified and shaped with the CHIME band-defining filters. The signal is then split, with one output sent directly to one of the ADC channels as the reference, while the other two signals are sent through coax cable to helical antennas located in the bottom of each dish. The radiated noise is sensed by the feed at the focus of each dish and sent through the analog receiver to the ADC. By comparing the output correlation matrix with the noise source on and off, the complex gain of each receiver channel can be measured with respect to the reference channel.

At the focus of each dish, the output of the feed is sent through the LNA, second-stage amplifier, and filter box. It is then split with one output connected to the ADC through the RFoF system (Figure 6.5) while the other output is connected to the ADC through coax cable and a second combined amplifier and filter box located inside the electronics.
7.2 System temperature

With this setup, we proceeded to compare the noise performance of the two systems. The system temperature was calculated from 24 hours of raw data recorded in drift-scan mode and comparing it to the 408 MHz Haslam map [40], excluding the bright radio source Cas A. The Haslam map was convolved with a Gaussian kernel determined by the beam parameters of the dish, and scaled using an estimated spectral index $\alpha_{\text{est}} = -2.5$. As Figure 7.2 shows, the noise performance of the coax-based and fiber-based receiver are consistent to within about 5 K across the band, which falls within the systematic uncertainty of this measurement. The system temperature of both systems is roughly $T_{\text{sys}} \approx 100$ K midband, of which about 40 K is caused by ground spill and loss in the feed. The spectral ripple in the $T_{\text{sys}}$ measurement is caused by standing waves between the ground plane and the reflector [24].
Figure 7.2: Estimated system temperature, $T_{sys}$, for the coax-based and fiber-based receiver across the CHIME band from raw data measured by comparing to the Haslam map excluding Cas A. The Haslam map was convolved with a Gaussian kernel determined by the beam parameters of the dish, and scaled using an estimated spectral index $\alpha_{est} = -2.5$. The scattering of the data points in some regions of the spectrum is due to RFI. A particular fit at 632 MHz is shown in the right plot.
Chapter 8

Conclusions and future work

The Canadian Hydrogen Intensity Mapping Experiment (CHIME) is currently under development at the Dominion Radio Astrophysical Observatory (DRAO) in Penticton, Canada. It is an array of cylindrical telescopes with more than 2500 radio receivers and has no moving parts or cryogenics. When it is complete in 2017, CHIME will map the three-dimensional distribution of neutral hydrogen gas in the universe by directly detecting the redshifted 21-cm radiation. By measuring the scale of the Baryon Acoustic Oscillations (BAO) across the redshift range $z \approx 0.8$ to $z \approx 2.5$ in both the angular and line-of-sight directions, CHIME will study the epoch when dark energy generated the transition from decelerated to accelerated expansion of the universe.

As a prototype for testing the CHIME technology, a two-element radio interferometer was built at DRAO. While characterizing the CHIME analog receiver in the two-element interferometer, the implementation of a Radio-Frequency-over-Fiber (RFoF) link to transport the analog signals from the antennas to the processing room has been investigated as an alternative to traditional coaxial cable. RFoF is an attractive option for the analog receiver in CHIME, and its advantages over coaxial cable include not only cost, but also small size and light weight, immunity to electromagnetic interference, low loss, and a separate grounding scheme for the front end RF chain and the digital back end.

Analog transmission over a RFoF link also reduces the likelihood of self-generated radio frequency interference (RFI) pickup and the amount of electronics and maintenance at the antennas compared to a receiver architecture with the digitizers located at the antennas and digital data transmission to the processing room. However, the analog optical link also introduces some challenges that must be overcome, mainly related to linearity, noise, and stability. In this thesis, a prototype version of a low cost RFoF link for CHIME has been presented. The design is based on a directly modulated Fabry-
Perot (FP) laser, operating at ambient temperature, and a single-mode fiber. The RFoF was characterized with a suite of laboratory tests and very good dynamic performance was achieved, comparable to more expensive high performance Distributed Feedback (DFB) laser-based RFoF links used elsewhere for radio astronomy applications. The link achieves a Noise Figure ($NF$) of 27 dB and a Spurious-Free Dynamic Range ($SFDR$) of $115 \text{ dB} \cdot \text{Hz}^{2/3}$. With this performance, it was demonstrated that, for the CHIME receiver and digitizer design, the noise and linearity are dominated by the 8-bit analog to digital converter (ADC) that follows the RFoF link, meaning that these challenges have been addressed at level that makes them negligible for the overall RFoF+ADC system.

In addition, the gain and phase stability of the RFoF link were investigated. The temperature induced gain fluctuations of the RFoF link are approximately 0.04 dB/C, mainly caused by the change in the slope efficiency of the Fabry-Perot (FP) laser. It was shown that the gain fluctuations for the fiber-based CHIME receiver cause a negligible noise penalty due to quantization and ADC noise of an 8-bit ADC, so the gain calibration can be performed after digitization. The temperature induced phase fluctuations are dominated by the change in the insertion delay of the fiber, which is determined by variations in its length and refractive index. For the particular tight buffer fiber used in the tests the measured Thermal Coefficient of Delay ($TCD$) is in the range 5-90 ppm/C over the range of expected temperatures for the fiber. In comparison, the TCD for the coaxial cable used in the coax-based CHIME receiver is expected to be below 10 ppm/C.

Finally, it was shown during the RFoF characterization on the CHIME two-element interferometer that there is no measurable noise degradation in the fiber-based CHIME receiver as compared to the coax-based receiver. Near-term improvements to this RFoF system will include separating the RFoF receivers into individually shielded boxes to improve the crosstalk performance and using loose tube fiber to improve the phase stability in an outdoor environment.

Valuable experience was obtained during the design and characterization of the RFoF link for CHIME, demonstrating that RFoF can be successfully applied for analog signal transport in large-array radio astronomy applications at low cost.
Appendix A

Noise characterization

A.1 Equivalent noise temperature

At RF frequencies noise characterization involves the measurement of noise power. The noise generated by a linear device can be considered as additive noise concentrated at its input as shown in Figure A.1. Although the figure shows an amplifier, this model can be generalized to other linear devices. If the noisy device with power gain $g$ generates noise with power spectral density $p_{ni}$ (in W/Hz) with respect to its input, it can be modelled as a noiseless device with power gain $g$ preceded by additive noise with power spectral density $p_{ni}$.

The additive noise can be thermal noise or can arise from other processes (see Section 3.4 for a description of thermal noise and other sources of noise in a RFoF link). In many cases, the noise processes involved have a white spectrum (i.e. a flat power spectral density), at least over the bandwidth of interest, identical to that of thermal noise. Since they are indistinguishable in this case, it is convenient to regard the total noise power from the contribution of all noise sources as a level of thermal noise. If the noise added by device at its input has power spectral density $p_{ni}$, then the equivalent noise temperature of the device is

$$T_e = \frac{p_{ni}}{k_B}.$$  \hspace{1cm} (A.1)

If a matched noise source with equivalent noise temperature $T_s$ is connected to the device’s input and a matched impedance is connected to the device’s output, the measured output noise power spectral density is
Appendix A. Noise characterization

Figure A.1: A noisy amplifier with power gain $g$ that generates noise with power density $p_{ni} = k_B T_e$ (in W/Hz) with respect to its input. $T_e$ is the equivalent noise temperature of the amplifier. The amplifier can be modelled as a noiseless amplifier with power gain $g$ preceded by additive noise with power density $p_{ni}$. If a matched noise source with equivalent noise temperature $T_s$ (usually 290 K) is connected to the amplifier’s input and a matched impedance is connected to the amplifier’s output, the measured output noise power spectral density is $p_o = g k_B (T_s + T_e)$. This model can be generalized to other linear devices.

$$p_o = g k_B (T_s + T_e).$$ (A.2)

It is common to express the noise power spectral density in dBm/Hz, given by

$$P_n \text{ (in dBm/Hz)} = 10 \log \left[ 10^3 p_n \text{ (in W/Hz)} \right].$$ (A.3)

One way to understand these units is that if a device has an input noise power spectral density $P_{ni}$ in dBm/Hz, then the total input noise (in dBm) added over a bandwidth $B$ is $P_{ni} + 10 \log [B \text{ (in Hz)}]$. Also, since the device is linear, it is possible to refer its input noise with respect to the output simply as

$$p_{no} = g p_{ni} \quad \rightarrow \quad P_{no} = P_{ni} + G$$ (A.4)

where $G = 10 \log(g)$ is the power gain of the device in dB.

Finally, a RF system usually consists of multiple stages or devices connected in cascade
where each adds noise to the system. When there are multiple devices in cascade, the equivalent noise temperature (or equivalently, the input noise power spectral density) of the multistage system can be calculated from [6]

\[ T_e = T_1 + \frac{T_2}{g_1} + \frac{T_3}{g_1 g_2} + \cdots \]  

(A.5)

where \( T_e \) is the equivalent noise temperature of the multistage system, \( T_1 \) and \( g_1 \) are the noise temperature and (linear) power gain of the first component in the cascade, \( T_2 \) and \( g_2 \) are the noise temperature and power gain of the second component in the cascade, and so on. In the usual case where the gains of the devices in the cascade are much greater than one, it can be seen that the noise the equivalent noise temperature of the multistage system is mainly determined by the first stages (typically the first stage dominates the overall noise temperature).

A.2 Noise Factor and Noise Figure

Consider a noisy linear device, such as an amplifier or a passive component with equivalent noise temperature \( T_e \). Suppose that the device is driven by a noise source with equivalent noise temperature \( T_s \) (e.g. a matched noisy resistor). The Noise Factor \( (nf) \) of the device is defined as the ratio of the total available output noise power spectral density (due to both the source and device) and the contribution to that from the source alone, in the later case supposing that the device is noise free [82]. From the previous section and assuming linearity and flat noise power spectral densities for both the noise source and the noisy device it is easy to show that

\[ nf = 1 + \frac{T_e}{T_s} = 1 + \frac{p_{ni}}{p_{ns}}. \]  

(A.6)

Note that the assumption of flat noise power spectral densities implies that the \( nf \) can be obtained as the ratio of the respective total noise powers in a given bandwidth (the \( nf \) is independent of the bandwidth used for the measurement). Some textbooks (e.g. [6] [19] [41]) define the \( nf \) of a linear device as the ratio of the input signal-to-noise-ratio (SNR) to the output SNR (for a given bandwidth). These definitions are equivalent. The Institute of Electrical and Electronics Engineers (IEEE) has standardized the value of the equivalent noise temperature of the noise source to room temperature, \( T_s = 290 \) K.
The Noise Figure ($NF$) is simply

$$NF = 10\log(nf).$$  \hfill (A.7)

Finally, when a system consists of multiple stages or devices connected in cascade where the $nf$ of each stage is known, then the $nf$ of the multistage system can be obtained from

$$nf = nf_1 + \frac{nf_2 - 1}{g_1} + \frac{nf_3 - 1}{g_1 g_2} + \cdots$$  \hfill (A.8)

where $nf$ is the noise figure of the multistage system, $nf_1$ and $g_1$ are the noise figure and (linear) power gain of the first component in the cascade, $nf_2$ and $g_2$ are the noise figure and power gain of the second component in the cascade, and so on. In a well designed receiver chain, only the noise factor of the first amplifier should be significant.
Appendix B

Review of Intermodulation Distortion

Intermodulation distortion (IMD) is a popular measure of the linearity of amplifiers, mixers, and other RF components. The Compression Point (CP) and the third-order Intercept Point (IP3) are figures of merit for these specifications and allow for the determination of the Spurious-Free Dynamic Range (SFDR) of RF components. These figures of merit will be used to describe the performance of the RFoF link and a quick review on these concepts is given in this section.

B.1 Compression Point

Generally, a nonlinear two-port network (an amplifier, for example) has a linear region of operation at which, for a single frequency input, the output has the same frequency (carrier) and the gain is independent of input power (amplitude) level. This is shown in the single frequency transfer function in Figure B.1. The gain in this region is called small-signal gain, and is determined by the slope of the transfer function. In general, the small signal gain is a function of frequency. As the input power increases, the gain of the device starts to decrease and the compression region begins. For a single frequency input, the output is no longer sinusoidal in this region, and some of the output power appears in the harmonics rather than in the carrier frequency (Harmonic Distortion).

A common measurement of compression is the 1-dB Compression Point (CP). It is defined as the input power (or sometimes the corresponding output power) that results in a 1-dB decrease in the gain of the device under test with respect to its small-signal gain.
Appendix B. Review of Intermodulation Distortion

Figure B.1: Single frequency transfer function of the device under test (left). Compression Point (right).

B.2 Intermodulation Distortion and Intercept Points

Intermodulation Distortion \((IMD)\) occurs when a signal with two or more frequency components passes through a two-port network with a nonlinear transfer function. The spectrum of the output is comprised of the original frequencies and additional spurious frequency components. To understand \(IMD\), the single frequency transfer function of the device shown in Figure B.1 is expanded as a power series around the operation point (assumed zero for simplicity).

\[
 v_{\text{out}}(v_{\text{in}}) = \sum_{n=0}^{\infty} c_n v_{\text{in}}^n = c_0 + c_1 v_{\text{in}} + c_2 v_{\text{in}}^2 + c_3 v_{\text{in}}^3 + \cdots . \tag{B.1}
\]

For an input signal of the form \(v_{\text{in}}(t) = a\cos(\omega_1 t) + a\cos(\omega_2 t)\), where \(a\) is in volts, the expansion of the second and third order terms in equation B.1 gives

\[
 c_2 v_{\text{in}}(t)^2 = c_2 a^2 \left\{ \cos((\omega_1 + \omega_2) t) + \cos((\omega_1 - \omega_2) t) + \frac{1}{2} [\cos(2\omega_1 t) + \cos(2\omega_2 t)] + 1 \right\}
\]

\[
 c_3 v_{\text{in}}(t)^3 = \frac{3c_3a^3}{4} \left\{ \cos((2\omega_1 - \omega_2) t) + \cos((2\omega_2 - \omega_1) t) + \cos((2\omega_1 + \omega_2) t) + \cos(2\omega_2 + \omega_1) + 3 [\cos(\omega_1 t) + \cos(\omega_2 t)] + \frac{1}{3} [\cos(3\omega_1 t) + \cos(3\omega_2 t)] \right\} \tag{B.2}
\]
The power spectrum of the output is shown in Figure B.2. The last two equations show that the output contains frequency components that were not present in the input. The second-order term produces a DC offset, harmonic distortion of the form $2\omega_1$ and $2\omega_2$, and mixing terms of the form $\omega_1 + \omega_2$, $|\omega_1 - \omega_2|$. The second order mixing and harmonic distortion terms are important in broadband systems where they may interfere with desired signals at those other frequencies. However, for an octave bandwidth system like CHIME all the second order distortion products lie outside the passband and can be removed by filtering.

The third-order term produces amplitude distortion (a scaled copy of the input), harmonic distortion and mixing terms. As before, the harmonic distortion and most of the mixing products can be removed by filtering. However, the third-order Intermodulation Distortion products ($IMD3$) of the form $|2\omega_1 - \omega_2|$ and $|2\omega_2 - \omega_1|$ are close to the original input frequencies and filtering them is difficult. They may fall within the passband of the system and cause interference. These two intermodulation products are used to calculate the Third Order Intercept Point. The amplitude each $IMD3$ term is $A_3 = 3c_3a^3/4$ while the amplitude of each carrier tone is $A_1 = c_1a$. The output power in dBm of carrier ($P_{o1}$) and $IMD3$ products ($P_{o3}$) as function of the input power (per tone), for a reference resistance $R$, is given by
Appendix B. Review of Intermodulation Distortion

\[
P_{o1} = P_i + 10 \log (|c_1|^2)
\]
\[
P_{o3} = 3P_i + 10 \log \left[ (2R)^2 \left( \frac{3|c_3|}{4} \right)^2 \right. 10^{-6} \right] \tag{B.3}
\]

where \( P_i = 10 \log (a^2/2R) \) is the input power (per tone) in dBm and \( G = 10 \log (|c_1|^2) \) is the small signal gain in dB. The power of \( IMD3 \) products increases 3 dB per every dB increase in the power of each input tone as long as the input power is well below \( CP \). This is shown in Figure B.3. The intersection of the third-order line with the line produced by the linear term is known as the Third-Order Intercept Point (\( IP3 \)). It can be defined with respect to the input (\( IIP3 \)) or the output (\( OIP3 \)) and these two values are related by \( OIP3 = IIP3 + G \). In reality, compression in the device prevents the power from the carrier and \( IMD3 \) products from growing without bound. The blue solid line in Figure B.3 shows the real response of the device. The \( IP3 \) point most often will never be reached in practice, because the respective curves (linear and third-order) will saturate before reaching it [93]. However, the \( IP3 \) is defined as if the linear and third-order terms increased without bound and is used as figure of merit for the linearity of the system.

By making \( P_{o1} = P_{o3} = OIP3 \) when \( P_i = IIP3 \) in equation B.3, the \( IIP3 \) (in dBm) can be written in terms of the coefficients \( c_1 \) and \( c_3 \) as

\[
IIP3 = 10 \log \left( \frac{10^3 4}{2R 3} \left| \frac{c_1}{c_3} \right| \right) \tag{B.4}
\]

However, for purposes of measuring the \( IP3 \), it is more convenient to write the \( OIP3 \) (in dBm) in terms of \( P_{o1} \) and \( P_{o3} \). From Figure B.3 it is easy to show that

\[
OIP3 = P_{o1} + \frac{P_{o1} - P_{o3}}{2} \tag{B.5}
\]

When a system consists of multiple stages or devices connected in cascade where the \( IIP3 \) of each stage is known then, assuming that all the intercept points are independent and uncorrelated, the \( IIP3 \) (in dBm) of the multistage system can be obtained from [6]

\[
IIP3 = -10 \log \left[ 10^{-IIP3_1/10} + 10^{-IIP3_2-G_1/10} + 10^{-IIP3_3-G_1-G_2/10} + \ldots \right] \tag{B.6}
\]
where $IIP_3$ is the input $IP_3$ (in dBm) of the multistage system, $IIP_3_1$ and $G_1$ are the $IIP_3$ (in dBm) and power gain (in dB) of the first component in the cascade, $IIP_3_2$ and $G_2$ are the $IIP_3$ (in dBm) and power gain (in dB) of the second component in the cascade, and so on.

### B.3 Spurious-Free Dynamic Range

The $SFDR$ is defined as the signal-to-noise ratio when the power in each $IMD_3$ product equals the noise power at the output [25]. From Figure B.3, this is just $SFDR = P_{\text{max}} - P_{ni}$, where $P_{ni}$ and $P_{no}$ are the input and output referred noise power of the device respectively and $P_{no} = P_{ni} + G$. It can be easily shown that $P_{\text{max}} = (2IIP_3 + P_{ni})/3$ so

$$SFDR = \frac{2}{3}(IIP_3 - P_{ni}) = \frac{2}{3}(OIP_3 - P_{no})$$

(B.7)

If the noise of the system is normalized to 1 Hz, then the $SFDR$ has units of dB-Hz$^{2/3}$. Finally, it should be noted that that $CP$, $IP_3$ and $SFDR$ are functions of frequency, and in general, the distortion is worse at higher frequencies [45].
Bibliography


