ABSTRACT

An analytical method is described for computing the maximum field intensity at the surface of any smooth conductor of a multi-conductor system forming a regular polygon. In this method, each conductor in the system is replaced by a line charge at its center equal to the charge on its surface plus a system of line dipoles to account for the proximity effect. The line dipoles in the conductor under consideration are treated as individual line charges. Results obtained by this method for the two-conductor case are checked against the known exact solution. It is shown that this method is very accurate even if conductors are placed relatively close together (conductor spacing to diameter ratios less than 6).

This method is next applied to calculate the maximum voltage gradient on any conductor of a split bundle conductor configuration. The results show that for a constant cross-sectional area per phase, by "splitting" a conductor of an N-conductor bundle into p subconductors, thus forming an Nxp-conductor split bundle, the maximum voltage gradient can be reduced, but not as much as if the Nxp-conductor split bundle is converted into an (Nxp)-conductor bundle.
MAXIMUM VOLTAGE GRADIENTS ON SPLIT BUNDLE CONDUCTORS

by

Pericles Thanassoulis, B.Sc., B.Eng., Eng.
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A thesis submitted to the Faculty of Graduate Studies
and Research in partial fulfillment of the
requirements for the degree of
Master of Engineering

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July 1969.
ACKNOWLEDGEMENTS

The author wishes to thank Professor R. P. Comsa for his supervision and guidance. He also acknowledges the advice and suggestions offered by Dr. P. Silvester of the Electrical Engineering Department.

Special thanks are due to Miss Jean Glushik, who excellently typed the thesis at temperatures soaring around ninety degrees.

Grateful acknowledgement is made to the National Research Council of Canada under whose grant this research was conducted.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>i</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>iii</td>
</tr>
<tr>
<td><strong>CHAPTER I</strong> INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Trends towards Higher Voltage Levels of Power Transmission Systems</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Previous Work and Present Research on Bundle Conductors</td>
<td>1</td>
</tr>
<tr>
<td>1.3 Object of this Study</td>
<td>10</td>
</tr>
<tr>
<td>1.4 Terminology</td>
<td>12</td>
</tr>
<tr>
<td><strong>CHAPTER II</strong> THE TWIN-CONDUCTOR CASE</td>
<td>14</td>
</tr>
<tr>
<td>2.1 Single Line Charge Model</td>
<td>15</td>
</tr>
<tr>
<td>2.2 Uniform Field Model</td>
<td>17</td>
</tr>
<tr>
<td>2.3 King's Model</td>
<td>19</td>
</tr>
<tr>
<td>2.4 Successive Images Model</td>
<td>20</td>
</tr>
<tr>
<td>2.5 Electric Dipole Model</td>
<td>22</td>
</tr>
<tr>
<td>2.5.1 Basic Concepts</td>
<td>23</td>
</tr>
<tr>
<td>2.5.2 Field Arising from an Electric Dipole</td>
<td>29</td>
</tr>
<tr>
<td>2.5.3 Maximum Voltage Gradient of a Twin-Conductor Bundle Using the Dipole Model</td>
<td>32</td>
</tr>
<tr>
<td>2.6 Discussion</td>
<td>39</td>
</tr>
<tr>
<td>CHAPTER III</td>
<td>EQUATIONS FOR MAXIMUM VOLTAGE GRADIENTS OF BUNDLE CONDUCTORS FORMING A REGULAR POLYGON</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>3.1 Method</td>
<td>43</td>
</tr>
<tr>
<td>3.2 Unequal Charges on Conductors</td>
<td>46</td>
</tr>
<tr>
<td>3.2.1 Electric Field at Surface of Conductor under Consideration Due to its Own Charge</td>
<td>50</td>
</tr>
<tr>
<td>3.2.2 Electric Field at Surface of Conductor under Consideration Due to the Non-Uniform Charge Distribution on its Surface</td>
<td>51</td>
</tr>
<tr>
<td>3.2.3 Electric Field at Surface of Conductor under Consideration Due to Charges on the Other Conductors in the Bundle</td>
<td>51</td>
</tr>
<tr>
<td>3.2.4 Electric Field at Surface of Conductor under Consideration Due to Non-Uniform Charge Distribution on the Surfaces of the Other Conductors in the Bundle</td>
<td>52</td>
</tr>
<tr>
<td>3.3 Maximum Voltage Gradient at the Conductor Surface</td>
<td>56</td>
</tr>
<tr>
<td>3.4 Equal Charges on Conductors</td>
<td>57</td>
</tr>
<tr>
<td>3.5 Discussion</td>
<td>60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER IV</th>
<th>EQUATIONS FOR MAXIMUM VOLTAGE GRADIENTS OF SPLIT BUNDLE CONDUCTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Geometry of Conductors</td>
<td>65</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.2 Calculation of Charges on Conductors</td>
<td>69</td>
</tr>
<tr>
<td>4.3 Derivation of Equations for Determining the Maximum Voltage Gradi-</td>
<td>70</td>
</tr>
<tr>
<td>ents of Split Bundle Conductors</td>
<td></td>
</tr>
<tr>
<td>4.3.1 Method</td>
<td>70</td>
</tr>
<tr>
<td>4.3.2 Electric Field at Surface of Conductor under Consideration Due</td>
<td>73</td>
</tr>
<tr>
<td>to its Own Charge</td>
<td></td>
</tr>
<tr>
<td>4.3.3 Electric Field at Surface of Conductor under Consideration Due</td>
<td>73</td>
</tr>
<tr>
<td>to Charges on the Other Conductors of the Same Sub-Bundle</td>
<td></td>
</tr>
<tr>
<td>4.3.4 Electric Field at Surface of Conductor under Consideration Due</td>
<td>74</td>
</tr>
<tr>
<td>to Charges on the Conductors of the Other Sub-Bundles</td>
<td></td>
</tr>
<tr>
<td>4.3.5 Maximum Voltage Gradient at the Surface of the Conductor under</td>
<td>76</td>
</tr>
<tr>
<td>Consideration</td>
<td></td>
</tr>
<tr>
<td>CHAPTER V Application to Some Hypothetical Configurations</td>
<td>78</td>
</tr>
<tr>
<td>5.1 Configurations Studied</td>
<td>79</td>
</tr>
<tr>
<td>5.2 Results</td>
<td>81</td>
</tr>
<tr>
<td>CHAPTER VI Conclusions</td>
<td>97</td>
</tr>
<tr>
<td>APPENDIX A Derivation of Equations for Various Models of a Twin-Conduc-</td>
<td>100</td>
</tr>
<tr>
<td>tor Case</td>
<td></td>
</tr>
<tr>
<td>A.1 Exact Solution of the Twin-Conductor Case</td>
<td>100</td>
</tr>
</tbody>
</table>
### APPENDIX B
**Calculation of Charges on a Multi-Conductor System**
Page 106

### APPENDIX C
**Derivation of Cahen's Equation**
Page 112

### APPENDIX D
**FORTRAN Program Listings**
Page 118

### BIBLIOGRAPHY
Page 128
CHAPTER I
INTRODUCTION

1.1 Trends towards Higher Voltage Levels of Power Transmission Systems

Transmission of alternating-current power over several miles dates from 1886 when a line was built at Cerchi, Italy, to transmit 150 hp 17 miles at 2000 volts. Ever since the voltage of power transmission lines has progressively increased as shown in Figure 1-1. Up to about 1940, voltages have risen at a rate of roughly 6 KV per year. This rate, however, has increased for the later years and the slope has sharply steepened for the last decade. That is, for the last 10 years the voltage level of power transmission systems has increased from about 450 KV in the 1960's to about 750 KV in the 1970's, giving an average rate of voltage rise of the order of 30 KV per year. If this trend continues, then one can expect a voltage level of as high as 1500 KV by 1975.

1.2 Previous Work and Present Research on Bundle Conductors

A "bundle conductor" is a conductor made of two or more "subconductors", and is used as one phase conductor. A study of the literature in this field reveals the following terms used synonymously: grouped, multiple, split, dual
Figure 1-1. Trend in transmission voltage levels in 90 years.
conductors, or duplex, triplex, etc., when two, three, etc., conductors per phase are used.

As early as 1909, P. H. Thomas suggested "split conductors" for long, high tension lines\(^3,4\) for reducing the surge impedance of the line, thus increasing the power which may be transmitted at a given voltage drop. Reduced surge impedance and consequently higher power capabilities is indeed among the advantages of using bundle conductors. But other equally or more important advantages of bundle conductors are higher disruptive voltage with same cross-sectional area per phase and less rapid increase of corona loss and radio interference (RI) with increased voltage.\(^5,6\) Bundle conductors may also be advantageously used to increase the stability limit of long transmission lines by 25 to 50% depending upon the number of conductors in the bundle.\(^7,8\) Of course, these advantages must always be weighted against increased circuit cost,\(^2\) increased charging KVA if it cannot be utilized, etc.

With the increased use of Extra-High-Voltage (EHV) transmission lines and the prospect of going to Ultra-High-Voltage (UHV) range, the common aspects of
corona have become more important in the design of transmission lines. Corona may be regarded as a self-sustained partial breakdown of air in the non-uniform field around the conductor. It is caused by ionization of the air surrounding the conductor. Factors affecting corona other than the voltage of the conductor are: conductor diameter, line configuration, type of conductor, condition of the surface, weather, etc. Although in the early days of High-Voltage (HV) transmission, corona had to be avoided because of the energy loss associated with it, nowadays the RI aspect of corona has become more important so that in some areas this factor might dictate the limit of acceptable corona performance. Methods for reducing radio noise are through conductor-voltage-gradient reduction, i.e., the conductors must be designed and arranged in such a manner as to yield as low a surface potential gradient as possible.

For transmission lines with one conductor per phase, the assumption that the charge on the surface of the conductor is uniformly distributed is justifiable, provided the ratio of phase spacing to the diameter of the conductor is large. Normally, this is the case for one conductor per phase transmission lines and, therefore, the calculation of the
potential gradient around the surface of the conductor is a simple matter. However, when the phase of a transmission line is made up of bundle conductors, the assumption of uniform charge distribution on the surface of these conductors is not valid. The "proximity effect" of the conductors of the same phase will cause an appreciable field distortion around the surface of any conductor in that phase which may appreciably lower the disruptive critical voltage. The obvious method in calculating the potential gradient in the vicinity of any of these conductors is to solve Laplace's equation in the region outside these conductors. Since, however, the solution of Laplace's equation for such a problem is very difficult if not impossible, attempts have been concentrated towards models derived under various simplifying assumptions.

Dr. H. Poritsky, in 1932, published a paper\textsuperscript{10} in the A.I.E.E. Transactions where he describes the method by which he obtained the exact solution for the maximum potential gradient of a two-conductor bundle having charges of equal magnitude and of same polarity. At least two more papers\textsuperscript{11,12} have been published at later dates, presenting a rigorous analytical solution for an idealized model of a two-conductor
bundle. Although such solutions cover only the limited case of a two-conductor bundle, they provide an excellent means of determining the degree of accuracy of simplified models derived for the multi-conductor case.

S. B. Crary, in a paper\textsuperscript{13} published in 1932, describes the method by which he obtained general equations for determining the disruptive critical voltage on any conductor of a multi-conductor circuit. The model he used was to represent all the conductors in the system, except the one under consideration, by a line charge at their centers. The one under consideration, he replaced by a line charge passing through its center plus a conducting cylinder with the same radius as the conductor itself. Then, he made the following assumptions:

a. The field distortion on the surface of the conductor under consideration due to the conductors of the other phases and the ground is negligible.

b. The field produced by the other conductors of the same phase may be considered uniform
in the vicinity of the conductor under consideration.

This method has proved to be relatively simple in form and easy to use. For that reason, it has been widely used until today for calculating the voltage gradients on bundle conductors. The main drawback of this method is that it becomes very inaccurate as the ratio of bundle conductors spacing to conductor diameter approaches unity. Using as a criterion the two-conductor bundle case whose exact solution is known, S. B. Crary's equations are essentially correct for a ratio of spacing to conductor diameter as low as 6 to 1. However, as this ratio approaches unity, the percentage error for the two-conductor case increases rapidly and for the extreme case of a ratio of 1 to 1 this error reaches a maximum of about 27%.

Since a ratio equal to or greater than 6 covers the practical range of spacings of bundle-conductor phases, these approximate formulae developed by S. B. Crary have been quite frequently used by other researchers to study potential gradients on bundle conductors of practical transmission lines. M. Temoshok, in a paper published in 1948 used Crary's approximate equations to study different aspects of grouped
conductors consisting of one, two, three and four conductors per phase. So did C. J. Miller, Jr., in 1956 in his study\textsuperscript{14} of radio and corona characteristics of smooth bundle conductors. In 1958, E. T. B. Gross and L. R. Stensland carried out a study\textsuperscript{15} on the characteristics of twin-conductor arrangements from the viewpoint of voltage gradient and inductive reactance. They, also, have used the same approach to develop their equations.

Only in 1959, a paper\textsuperscript{16} by S. Y. King describes a new approach in determining the electric field near bundle conductors. In his attempt to keep all conductors equipotential surfaces, Dr. King used a model where he replaced each conductor in the bundle by a line charge slightly displaced from its center. As reported in his results, he has achieved a relatively small improvement over the method used by S. Crary, but his formulae are far more complex. The same model but a different mathematical approach - conformal transforms - has been used by A. S. Timascheff in his work\textsuperscript{17,18} published in 1961 and 1963. Although, the degree of accuracy is the same as that obtained by King's formulae, the method developed by Timascheff provides an excellent tool for obtaining accurate field patterns for multiple-conductor circuits.
Three more papers\textsuperscript{19,20,21} on bundle conductors were presented at the IEEE Winter Power Meeting in New York, in January 1969. Two of them dealt with the problem of determining the voltage gradient. One of them,\textsuperscript{20} by M. P. Sarma and W. Janischewskyj, describes a new method for determining the voltage gradient on bundle conductors. The model used by the authors was to replace the actual charge distribution on the conductor surfaces by a series of image line charges. To the authors' claim, this method can produce results to any desired degree of accuracy. In the writer's opinion, this method is undoubtedly unique in its content, and has a great theoretical value, but it seems to be limited in practical use because of the enormous complexity involved. Using numerical methods, a new technique called the "charge simulation technique" is described in the other paper\textsuperscript{21} presented at that meeting by M. S. Abou-Seada and E. Nasser. So far, however, this method covers only the unipolar twin-bundle line.

Although one, with this brief review, may conclude that a tremendous amount of work has been done on the subject of voltage gradients of bundle conductors, in the writer's opinion, this subject is by no means exhausted, especially
when one considers the new design challenges of the anticipated UHV lines which may evolve in the next few years. It is hoped, then, that people will maintain interest in this field and research will continue at an even faster pace.

1.3 Object of this Study

Radio noise on EHV lines is confined within acceptable limits by bundling the phase conductors. To the best knowledge of the author, the maximum number of conductors per phase used so far in any power transmission system in service is four. With the prospect of entering the UHV range this number will, undoubtedly, increase. Using a certain cross-sectional area per phase, based on economic current density, as a reference, various bundle configurations are investigated from the viewpoint of maximum voltage gradients. Such configurations comprise bundle conductors with their subconductors made up of bundles. In such a scheme the bundle conductors which will make up the subconductors will have to be placed relatively close together. Since the ratio of spacing of these conductors making up the subconductors to their diameter will be relatively small, perhaps much less than 6, standard analytical models for representing charged conductors, used so far for
determining voltage gradients cannot be employed. A new model, then, is desirable which will adequately represent the conductors when placed relatively close to other conductors (spacing to diameter ratio less than 6). Because of the increased number of conductors per phase, in order that the complexity of the problem may be kept at a minimum, this model should be as simple as possible without, of course, sacrificing the accuracy of the results beyond unacceptable limits.

The object of this study may, then, be summarized as follows:

1. Find a model which will adequately represent the conductor for purposes of voltage gradient calculations for relatively small spacing to diameter ratios (less than 6).

2. Develop a method which will enable one to calculate the maximum voltage gradient on bundle conductors with their subconductors made up of bundles.

3. Investigate various such bundle configurations from the viewpoint of maximum voltage gradients.
1.4 Terminology

The following terms used in the subsequent work will have the meaning as defined below:

a. A "bundle conductor" is defined as a group of conductors of equal diameters having their centers placed symmetrically in the periphery of a circle.

b. A "subconductor" is defined as any conductor of the group making up the bundle conductor.

c. If in a bundle-conductor system, the subconductors are made up of bundles, the term "split bundle conductor" is used to represent the phase conductor. The conductors making up the subconductor are also of equal diameters and have their centers placed symmetrically in the periphery of a circle.

d. The term "sub-bundle" is used instead of "sub-conductor" if the subconductor is made up of bundles.
e. The term "model" refers to the representation of a charged conductor by some other system for the purpose of making the calculations of the electric field near the conductor easier. Such a system may consist of line charges placed at or slightly off the center of the conductor, electric dipoles, etc.

f. "Radius of equivalent conductor" denotes the radius of a single conductor having an area of cross section equal to that of the total of all the subconductors constituting the phase.
CHAPTER II

THE TWIN-CONDUCTOR CASE

Throughout this work, the following assumptions are made about the conductors:

a. All conductors are smooth.

b. All conductors are of cylindrical shape.

c. All conductors are parallel to themselves and to an infinite plane.

The twin-conductor arrangement is studied first because of its simplicity and because exact solutions exist. Formulae developed by Dr. Poritsky for the exact calculation of the maximum voltage gradient for two cylindrical conductors of equal charge of the same sign, are given in Appendix A.

The aim here is to find a model which will represent the charged conductor with an accuracy within acceptable limits. The degree of accuracy of this model is checked against the exact solution. If a particular model is satisfactory it may be applied to the multi-conductor case. Several models are studied in this section and their degree of accuracy discussed.
2.1 **Single Line Charge Model**

In this model, each charged conductor is represented by a line charge of $Q$ coulombs/unit length, placed at the center of the conductor. The charge $Q$ has a magnitude equal to the total charge on the surface of the conductor. Equations for determining the maximum voltage gradient for the two-conductor case using this model are developed in Appendix A. For this model to perfectly represent the conductor, the line charges placed at the conductors' centers should produce equipotentials of the proper magnitudes and radii to coincide with the surfaces of the conductors. How truly this model represents the actual charged conductors is illustrated in Figure 2-1, which shows a plot of the normalized maximum unit voltage gradient, $G_o = G_{max}/G_{ave}$, versus the ratio of conductor spacing ($S$) to conductor diameter ($D$). Figure 2-1 also shows the percentage error in the solution obtained by using this model as compared to the exact one. A maximum error of about 15% occurs for an $S/D$ equal to 1. The drawback of this model is that its solution fails to approach the exact one even for relatively large $S/D$ ratios. Even for an $S/D$ equal to 15 a 3% error is introduced. Certainly this model represents
Figure 2-1. Comparison of maximum unit voltage gradient obtained by Single Line Charge Model and exact formula.
perfectly the charged conductors if S/D is very large. Several people\textsuperscript{14,24,25} have used this model to calculate voltage gradients for a three-phase transmission line with each phase made up of one conductor only.

2.2 Uniform Field Model\textsuperscript{13}

In this model the charged conductor, the maximum voltage gradient of which is to be found, is represented by a line charge of Q coulombs/unit length placed at its center plus a conducting cylinder of a diameter equal to that of the conductor. The other conductor is merely replaced by a line charge at its center. Each of these line charges has a magnitude equal to the charge on the surface of the conductor. The assumption made here is that the field produced by the line charge representing the other conductor may be considered uniform in the region of the conductor under consideration. Equations for determining the maximum voltage gradient for the two-conductor bundle, using this model, will be found in Appendix A. The correctness of this solution derived from this model as compared to the exact one is illustrated in Figure 2-2. For the extreme case where the two conductors are touching each other, an error of as high as 27\% occurs.
Figure 2-2. Comparison of maximum unit voltage gradient obtained by Uniform Field Model and exact formula.
However, this approximate solution agrees with the exact one to better than 1.2% for an S/D as low as 6. Since a ratio equal to or greater than 6 covers the practical range of spacings of bundle-conductor phases this model has been used almost exclusively to study\textsuperscript{5,6,14,15} voltage gradients on bundle conductors of practical transmission lines. The main disadvantage of this model results from the fact that it becomes highly inaccurate as the intragroup spacing to conductor diameter ratio approaches unity.

2.3 King's Model\textsuperscript{16}

This model resembles the "single line charge" one described in Section 2.1, but the line charges are slightly displaced from the center of each conductor. Since each of these line charges produces equipotential surfaces which, close to the source, are nearly cylindrical, the slight displacement from the center is made in an attempt to make the actual conductor take up the position of one of these equipotentials. With this model an almost negligible improvement is achieved over the "uniform field" one, but the complexity in the mathematics increases considerably. Formulae, as developed by Dr. King, are shown in Appendix A. A comparison between
the solution obtained by this model and the exact one is shown in Figure 2-3. Again here, as with the preceding models, the accuracy depends on the ratio of S/D, the greater the ratio the more accurate the model becomes.

2.4 Successive Images Model

This model, developed recently, consists mainly in replacing the actual charge distribution on the surface of the conductor by a series of "image line charges" slightly displaced from the center, of alternating polarity, so that the net charge is always equal to the charge on the surface of the conductor. The field intensity on the surface of any conductor in the system is then determined by finding the resultant of all the field components produced by each one of these "image line charges". This model is undoubtedly the most accurate one and in the author's words "... can be applied to calculate the electrostatic field, to any desired degree of accuracy,...". Although this method is of high theoretical value it seems to be limited in practical use because of the enormous complexity involved when applied to any practical problem. It is, however, the best method and to the best knowledge of the writer, the only one available for calculating voltage
Figure 2-3. Comparison of maximum unit voltage gradient obtained by King's Model and exact formula.
gradients of any multi-conductor system, if a very high degree of accuracy is desirable with no regard to complexity.

2.5 Electric Dipole Model

Thus far, four models have been examined, and with the exception of the last one, none of these seems to be suitable as far as accuracy is concerned, for small intragroup spacing to conductor diameter ratios (S/D). For an S/D less than 6, at least one and up to 27% error may result using any one of the first three models discussed in the preceding pages. Since the object here is to investigate maximum voltage gradients on split bundle conductors, the ratio of intragroup spacings of the conductors in the sub-bundle to their diameter may be relatively small. In any event, the investigation should be carried out with ratios as low as the accuracy of the model used permits. It is clear then that a relatively simple but more accurate model is needed.

A new, relatively simple, but much improved model as compared to the other ones is developed by the writer in the following pages. The main concept in establishing this model is to replace each charged conductor in an n-conductor
system by a line charge of \( Q \) coulombs/unit length placed at its center plus \( n-1 \) pairs of line charges of equal charge but opposite sign, separated by a small distance \( \delta \) so that they can form an electric dipole of a dipole moment \( \mathbf{P} = Q\delta \). The field intensity on the surface of the conductor under consideration will then be the resultant field of the vectorial sum of all the components of the fields produced by the single line charges and the dipoles. The single line charge of \( Q \) coulombs/unit length will account for the total charge on the surface of the conductor, and the dipoles will simulate the non-uniform charge distribution on the surface of the conductor due to the presence of the other conductors.

2.5.1 Basic Concepts: Let us assume that a conductor \( A \) of radius \( a \) carries a positive charge of \( Q \) coulombs/unit length on its surface, and it is placed at a height \( H \) above ground. The presence of the ground can be simulated, using the method of images, by another conductor of equal charge but opposite polarity. Thus, the problem of a positively charged conductor above ground is reduced to a new one consisting of two conductors of same charge but opposite polarity, separated by a distance \( 2H \). Using the well-known theorem of inverse
points, for the two conductors to take up the position of an equipotential surface, each of them must be replaced by a line charge placed at a distance \( \delta = \frac{a^2}{2H-\delta} \) from its center.

Let us next consider the case of two positively charged cylindrical conductors A and B placed at a height H above ground and separated by a distance S as shown in Figure 2-4a. If we assume that the conductor spacing to its diameter ratio is large enough so that the charge on each conductor may be replaced by a line charge placed at its center, the problem of two charged conductors above ground reduces to one consisting of four conducting cylinders and four line charges placed at their respective centers as shown in Figure 2-4b.

We wish now to replace conductor A by a system of charges so that when these charges and the charges on the other conductors are considered, conductor A takes up the position of an equipotential surface. Utilizing the well-known fact that an infinitely long line charge of \(+Q\) coulombs/unit length, placed parallel to an infinitely long conducting cylinder, will induce a charge distribution on the cylinder, which may be simulated by placing an image line charge of \(-Q\) coulombs/unit length in the cylinder at the inverse point, conductor A
Figure 2-4. Two positively charged conductors above ground.
Figure 2-5. System of charges representing two positively charged conductors above ground.

may be replaced by two line charges of opposite polarity in addition to a line charge at its center due to its own charge. The one of these two line charges due to the positive charge on conductor B, will be of negative polarity and placed at a distance \( \delta = \frac{a^2}{s} \) from the center of conductor A. The other one due to the image of conductor B, will be of positive polarity and placed at a distance \( \delta' = \frac{a^2}{\sqrt{(2H)^2 + s^2}} \) from the
center of conductor A. One can argue on the same grounds for conductor B so that the two-conductor bundle above ground can be replaced by a system of charges as shown in Figure 2-5.

The field at any point now outside these conductors will be due to all these line charges. Bearing in mind the fact that we are interested in determining the field intensity in the vicinity of these conductors, the following legitimate assumptions may be made:

1. Since in a real transmission system $H$ is much greater than the diameter of the conductor, the line charge in the conductor under consideration due to the image of any other conductor in the system may be assumed to be placed at its center.

2. The contribution to the field intensity at the surface of the conductor under consideration of the images of the other conductors in the system is negligibly small.

Thus, it is seen that for the case of a two-conductor bundle, each conductor is replaced by a line charge at its center plus a pair of line charges of opposite polarity. If
the distance from this pair of line charges to the point where the field is evaluated is large as compared to their separation.

![Figure 2-6](image)

Figure 2-6. (a) Two-conductor bundle. (b) Model.

this pair of line charges may be treated as an electric dipole of moment \( \mathbf{P} = Q \delta \). Thus each conductor in a two-conductor bundle can be simulated by a line charge and a dipole as shown in Figure 2-6. If the ratio of \( S/D \) is relatively small, for the conductor surfaces to coincide exactly with equipotentials of the proper magnitude and radii, more dipoles should be considered, but the model becomes more complex. It is considered justifiable here to sacrifice a little accuracy for the sake of simplicity.
This kind of argument may be extended to a conductor system made up of more than two conductors. For instance, for an n-conductor system, each conductor is replaced by a line charge equal to the charge on the surface of that conductor, placed at its center, plus n-1 line dipoles of moment \( \vec{P} = Q \delta \).

As stated earlier, for a pair of line charges of opposite polarity to be treated as a dipole, the distance from these line charges to the point where the field is evaluated, must be large compared to their separation. This condition, however, is not satisfied for the pairs of line charges in the conductor under consideration. It is necessary then, that the dipoles in that conductor be treated as individual line charges. This makes the model a little more complex, but such complexity cannot be avoided if a relatively high degree of accuracy is to be preserved.

2.5.2 Field Arising from an Electric Dipole: Let us consider two line charges of \( Q \) coulombs/unit length each, of equal magnitude and opposite sign separated by a small distance \( \delta \) as shown in Figure 2-7, forming a dipole of moment \( \vec{P} = Q \delta \). For convenience, as will be evident later, the origin is shifted by a small distance of \( \delta/2 \) and polar coordinates are
used. The field intensity at a point $p$ arising from these two line charges is to be evaluated. It is assumed that $r$ is large compared to length $\delta$ of the dipole.

![Diagram of a line dipole](Image)

Figure 2-7. Line dipole.

Potential at $p$ due to these line charges is:

$$V_p = \frac{Q}{2\pi \varepsilon_0} \ln \frac{r_1}{r}$$  \hspace{1cm} (2-1)

where

$$r_1 = \sqrt{\delta^2 + r^2 + 2\delta r \cos \theta}$$

$$= r \sqrt{1 + \left(\frac{\delta}{r}\right)^2 + \frac{2\delta}{r} \cos \theta}$$

Since $r \gg \delta$, the term $(\frac{\delta}{r})^2$ is very much smaller than unity and
therefore it may be neglected without introducing an appreciable error.

Hence \[ r_1 \approx r \sqrt{1 + \frac{2\delta \cos \theta}{r}} \]

Therefore \[ V_p \approx \frac{Q}{4\pi \varepsilon_0} \ln(1 + \frac{2\delta \cos \theta}{r}) \] (2-2)

The radial and tangential components of the field at \( p \) are:

\[ E_r = -\frac{3V_p}{3r} = \frac{Q\delta}{2\pi \varepsilon_0} \frac{\cos \theta}{r^2} \frac{1}{1 + \frac{2\delta \cos \theta}{r}} \]

\[ E_\theta = -\frac{1}{r} \frac{3V_p}{3\theta} = \frac{Q\delta}{2\pi \varepsilon_0} \frac{\sin \theta}{r^2} \frac{1}{1 + \frac{2\delta \cos \theta}{r}} \]

Denoting the dipole moment by \( P = Q\delta \) above equations become:

\[ E_r = \frac{P}{2\pi \varepsilon_0} \frac{\cos \theta}{r^2} \frac{1}{1 + \frac{2\delta \cos \theta}{r}} \] (2-3)

\[ E_\theta = \frac{P}{2\pi \varepsilon_0} \frac{\sin \theta}{r^2} \frac{1}{1 + \frac{2\delta \cos \theta}{r}} \] (2-4)

The radial and tangential field components arising from a dipole may be further simplified by using Taylor-series expansion for logarithms. In general:

\[ \ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots \ (-1)^{n-1} \frac{x^n}{n} \]
Using the first term only of the above series, equation (2-2) becomes:

\[ V_p \approx \frac{Q \delta}{2\pi \varepsilon_0} \frac{\cos \theta}{r} \]

The radial and tangential components of the field at p now become:

\[ E_r \approx \frac{P}{2\pi \varepsilon_0} \frac{\cos \theta}{r^2} \quad (2-5) \]

\[ E_\theta \approx \frac{P}{2\pi \varepsilon_0} \frac{\sin \theta}{r^2} \quad (2-6) \]

Equations (2-5) and (2-6) are of a much simpler form than (2-3) and (2-4). The validity of this last approximation will be examined in the next section where the maximum voltage gradient of a two-conductor bundle is calculated.

### 2.5.3 Maximum Voltage Gradient of a Two-Conductor Bundle

Using the Dipole Model: The maximum voltage gradient on the surface of one of a two-conductor bundle of radius \(a\) will now be calculated using the dipole model, and the result will be compared with the known exact solution. The conductor configuration is shown in Figure 2-8 and consists of two conductors of the same potential with respect to ground which
is assumed to be at an infinitely large distance. The electric field at point \( p \) on the surface of conductor \( B \) will be due to:

Figure 2-8. The two-conductor bundle and the model representing the charged conductors.

a. A line charge of +2\( Q \) coulombs/unit length placed at the center of conductor \( B \).

b. A line charge of -\( Q \) coulombs/unit length placed at a distance \( \delta \) from the center of conductor \( B \).

c. A line charge of +\( Q \) coulombs/unit length placed at the center of conductor \( A \), and
d. A line dipole of moment $\mathbf{p} = Q\delta$ in conductor A.

The normal component of the total field intensity on the surface of conductor B at p will be:

$$E_n = \frac{2Q}{2\pi\varepsilon_0 a} - \frac{Q\cos\delta}{2\pi\varepsilon_0 b} + \frac{Q\cos\alpha}{2\pi\varepsilon_0 r} + \frac{P}{2\pi\varepsilon_0} \left( \frac{\cos\theta}{r^2} \cos\alpha \right)$$

$$+ \frac{\sin\theta}{r^2} \sin\alpha \left( 1 + \frac{1}{r} \right)$$

The maximum field intensity will be at $\phi = 0$. At this point:

$$r = S + a$$
$$b = \delta + a$$
$$\delta = \frac{a^2}{S}$$
$$\alpha = 0$$
$$\beta = 0$$
$$\theta = 180^\circ$$

Therefore

$$E_{\text{max}} = \frac{Q}{2\pi\varepsilon_0} \left[ \frac{2}{a} - \frac{1}{a+a^2} + \frac{1}{S+a} - \frac{a^2}{S(S+a)^2} \frac{1}{1 - \frac{2a^2}{S(S+a)}} \right]$$

Rearranging above equation and letting $\zeta = a/S$ we obtain:
\[ E_{\text{max}} = \frac{Q}{2\pi \varepsilon_0 a} \left[ 2 - \frac{1}{1+\zeta} - \frac{\zeta^3}{(1+\zeta)^2} - \frac{1}{1-\frac{2\zeta^2}{1+\zeta}} \right] \]  \hspace{1cm} (2-7)

The last term in Equation (2-7) is the contribution of the dipole.

If we let \( E_{\text{ave}} = \frac{Q}{2\pi \varepsilon_0 a} \) and define the unit surface gradient (G) to represent the magnitude of the electric field on the surface of the conductor when a potential of unity is applied to this conductor, we obtain:

\[ G_{\text{max}} = G_{\text{ave}} \left[ 2 - \frac{1}{1+\zeta} - \frac{\zeta^3}{(1+\zeta)^2} - \frac{1}{1-\frac{2\zeta^2}{1+\zeta}} \right] \]  \hspace{1cm} (2-8)

How well this model represents the actual charged conductors from the viewpoint of maximum voltage gradient, as compared to the exact solution, is illustrated in Figure 2-9. Numerical values for the maximum voltage gradient obtained by using the exact solution and the dipole model are also listed in Table 5-1. Even for the extreme case of an S/D equal to 1 the solution for the maximum voltage gradient obtained from the dipole model, agrees with the exact one to better than 0.8%. For an S/D equal to 2, the error is just a little more
Figure 2-9. Comparison of maximum unit voltage gradient obtained by Dipole Model and exact formula.
than 0.6%. It is thus seen that the dipole model can produce a solution for the maximum voltage gradient which almost matches the exact one.

It should be pointed out, however, that this model, at its best, can produce a solution as accurate as the first order imaging process. The two-conductor bundle problem was also solved using this latter method. The maximum voltage gradient calculated by this method agrees to within 1.85% of the exact solution. It seems, then, that the approximations made in deriving the field equations arising from a dipole have tended to improve the accuracy of the solution obtained by using this model.

If the simplified but slightly less accurate Equations (2-5) and (2-6) derived in Section 2.5.2 are used, Equation (2-8) becomes:

\[ G_{\text{max}} = G_{\text{ave}} \left[ 2 - \frac{1-\xi}{1+\xi} - \frac{\xi^3}{(1+\xi)^2} \right] \]  

(2-9)

The effect of using these simplified equations for the dipole field is shown in Figure 2-9 (dotted line). Numerical values are also listed in Table 5-1. The maximum error for the
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extreme case of an S/D equal to 1 is about 2.5%, whereas this error drops to about 0.7% for an S/D equal to 2. It is clear then from these results that the error introduced by using these simplified equations for the dipole field is negligibly small, at least for an S/D equal to or greater than 2, and therefore Equations (2-5) and (2-6) will be used in all subsequent calculations.

2.6 Discussion

The solution for the maximum voltage gradient of the two-conductor arrangement has been obtained in this chapter by using four different models. The exact solution and the various approximate solutions obtained from these models are shown, for purposes of comparison, in Figure 2-10. Undoubtedly, the dipole model, although a little more complex, gives the most accurate solution. The advantage of this model is that it is relatively simple, very accurate for calculating the maximum voltage gradient for S/D ratios less than 6 - a desirable feature for the study of the maximum voltage gradients on split bundle conductors - and it can be directly extended to multi-conductor systems. It has therefore been decided to use this model as a building block in developing
Figure 2-10. Comparison of maximum unit voltage gradient obtained by various approximate methods and exact formula.
equations for the study of the more complex cases of multi-conductor systems.
Throughout this work, only single phase lines are considered, with the effect of the image charge lines, due to earth, on the voltage gradient of the conductors, neglected. Although the error by neglecting the other phases is negligibly small, if a three-phase line with ground wires is to be considered, the following remarks must be borne in mind:

a. Because of the great distances between phases as compared to the conductor separation in the bundle, the distribution of the charge on a conductor's surface is assumed to be unaffected by the presence of the charges in the other phases.

b. The charge on each conductor is to be determined by considering all three phases and the ground wires.

c. The resultant field on any conductor is, by the principle of superposition, the vectorial sum of the fields due to all charges in the three phases
and ground wires.

The concept of using multiple dipoles in a conductor to simulate the non-uniform charge distribution on its surface is extended in this chapter to more than two conductors in a bundle. These conductors are assumed to be placed symmetrically in the periphery of a circle.

3.1 Method

In order to make the case general, a p-conductor bundle is considered, where p is the total number of conductors in the bundle. Each charged conductor in the system is represented by a line charge at its center equal to the total charge on its surface plus p-1 line dipoles. The dipoles in the conductor under consideration are treated as individual line charges.

The maximum field intensity on the surface of any conductor in the system will be, by the principle of superposition, the vectorial sum of the fields due to the following system of charges:

a. A line charge of magnitude equal to the charge
on the surface of the conductor under consideration placed at its center.

b. p-l pairs of line charges placed inside the conductor under consideration.

c. p-l line charges, each one placed at the center of each of the other conductors.

d. p-l line dipoles placed inside each of the other conductors in the system.

If one assumes that the charges on all conductors are equal, a considerable simplification in the equations results. For a p-conductor bundle forming a regular polygon, this assumption is practically correct. For purposes of illustration, the charges in picocoulombs of a four-conductor bundle with an intragroup spacing of 20 in., placed at 460 in. above ground with a unipotential applied, are shown in Figure 3-1a. This assumption, however, cannot be extended to split bundle conductors because charges on each conductor differ quite widely. To illustrate the point, each conductor of the previous example, is replaced by a sub-bundle of four sub-conductors with same total cross-sectional area and an
(a) Four-conductor bundle.

(b) Four-conductor split bundle.

Figure 3-1. Charges on conductors of a multi-conductor system with applied voltage of +1.0 and height above ground 460 in. Diameter of equivalent conductor, 1.5 in.
intrasubgroup separation of twice the diameter of the sub-conductor. The charge on each conductor, with this configuration, is shown in Figure 3-1b.

3.2 Unequal Charges on Conductors

The general case of unequal charges on conductors is treated first so that the result may be applied later to the split bundle conductor arrangement. The case of equal charges on conductors is discussed next and may be treated as a special case of the general one. General equations for calculating the charges on a multi-conductor system are derived in Appendix B.

The general case of a p-conductor bundle with all the important geometric dimensions is shown in Figure 3-2. Let us assume that the maximum voltage gradient on the surface of conductor one is to be determined. Note that conductor one may be any conductor in the bundle. Because of symmetry, a coordinate system can be chosen such that all the sine terms of the field components, when added, cancel out, resulting in a considerable simplification. Such a coordinate system must have its X-axis passing through the center of the
Figure 3-2. A p-conductor bundle and the charge system representing it.
bundle and the center of the conductor under consideration as shown in Figure 3-2. Also, because of symmetry, the maximum field intensity on the surface of conductor one will be at point 0.

With reference to Figure 3-2, the following equations can be derived from the geometry of the conductors:

\[ \theta_i = \frac{\pi (i-1)}{p} \]  
(3-1)

\[ s_i = \frac{s}{\sin \frac{\pi}{p}} \sin \left( \frac{\pi (i-1)}{p} \right) \]  
(3-2)

\[ s_{ij} = \left| \frac{s}{\sin \frac{\pi}{p}} \sin \left( \frac{\pi (j-i)}{p} \right) \right| \]  
(3-3)

\[ r_i = \sqrt{(s_i + a \sin \theta_i)^2 + (a \cos \theta_i)^2} \]  
(3-4)

\[ \beta_i = \tan^{-1} \left( \frac{a \cos \theta_i}{s_i + a \sin \theta_i} \right) \]  
(3-5)

\[ \delta_i = \frac{a^2}{s_i} \]  
(3-6)

\[ \alpha_i = \frac{\pi}{2} - (\theta_i + \beta_i) \]  
(3-7)

\[ \alpha_{oi} = \frac{\pi}{2} + \alpha_i \]  
(3-8)
\[ r_i' = \sqrt{(a + \delta_i \sin \theta_i)^2 + (\delta_i \cos \theta_i)^2} \] (3-9)

\[ a_i' = \tan^{-1} \left( \frac{\delta_i \cos \theta_i}{a + \delta_i \sin \theta_i} \right) \] (3-10)

\[ \gamma_i = \frac{\pi}{2} + \theta_i \] (3-11)

\[ \phi_i = \gamma_i - \beta_i \] (3-12)

\[ \delta_{ij} = \frac{a^2}{S_{ij}} \] (3-13)

\[ P_{ij} = Q_j \delta_{ij} \] (3-14)

where \( i \) is an integer with values 2, 3, \ldots p

\( j \) is an integer with values 1, 2, \ldots p

\( \theta_i \) is the angle between the Y-axis and a line joining the centers of conductor one and the \( i \)th conductor

\( S_i \) is the center to center distance between conductor one and the \( i \)th conductor

\( S_{ij} \) is the center to center distance between the \( i \)th and \( j \)th conductors

\( r_i \) is the distance from the point on the surface of conductor one where the field
is evaluated to the center of the ith conductor.

$r_i'$ is the distance between the point on the surface of conductor one where the field is evaluated and the $-Q_i$ line charge in conductor one.

$a$ is the radius of the conductors.

$Q_j$ is the charge on the surface of the jth conductor.

$P_{ij}$ is the magnitude of the dipole moment in the ith conductor due to the charge on the jth conductor.

3.2.1 Electric Field at Surface of Conductor under Consideration Due to its own Charge: Let $E_1$ be the magnitude of the electric field on the surface of the conductor due to its own charge. Then

$$E_1 = \frac{1}{2\pi \varepsilon_0} \frac{Q_1}{a}$$  \hspace{1cm} (3-15)$$

where $Q_1$ is a line charge (coulombs/unit length) placed at the center of the conductor under consideration and representing the total charge on the surface of the
3.2.2 Electric Field at Surface of Conductor under Consideration

Due to the Non-Uniform Charge Distribution on its Surface: Let $E_2$ be the magnitude of the maximum field intensity on the surface of the conductor due to non-uniform charge distribution on its surface. Then

$$E_2 = \frac{1}{2\pi\varepsilon_0} \left[ \sum_{i=2}^{P} Q_i \left( \frac{1}{a} - \frac{\cos \alpha_i}{r_i} \right) \right]$$

(3-16)

where $Q_i$ is a line charge (coulombs/unit length) placed at a distance $\delta_i = \frac{a^2}{S_i}$ from the center of the conductor under consideration with a magnitude equal to the charge on the surface of the $i^{th}$ conductor.

3.2.3 Electric Field at Surface of Conductor under Consideration

Due to Charges on the Other Conductors in the Bundle: If $E_3$ denotes the magnitude of the maximum field intensity on the surface of the conductor due to the charges on the other conductors in the bundle, then
3.2.4 Electric Field at Surface of Conductor under Consideration Due to Non-Uniform Charge Distribution on the Surfaces of the Other Conductors in the Bundle: As already discussed in previous sections, the non-uniform charge distribution on the surface of a conductor is simulated by p-1 line dipoles. The field intensity on the surface of the conductor under consideration, arising from these dipoles, is determined by finding the radial and tangential components at the point of interest of a dipole with a moment equal to the vectorial sum of all individual dipole moments in the conductor.

If \( P_{ij} \) is the magnitude of the dipole moment in the \( i^{th} \) conductor due to the charge on the \( j^{th} \) conductor, the magnitude of the total dipole moment \( (P_{ti}) \) in the \( i^{th} \) conductor will be:

\[
P_{ti} = \sum_{\substack{j=1 \atop j \neq i}}^{P} P_{ij} \cos \theta_{oij} \tag{3-18}
\]

where \( P_{ij} = Q_j \delta_{ij} \)
Yoij is the angle between the line of action of the total dipole moment and the line of action of the Pij moment as shown in Figure 3-2.

A very simple expression for the total dipole moment in each conductor may be obtained, if, for each conductor, an X - Y coordinate system is chosen in the same manner as for conductor one in Figure 3-2, so that when adding vectorially the individual dipole moments, the components along the Y-axis cancel out.

Let us define:

\[
\theta_{ij} = \left| \frac{\pi}{P}(j-i) \right| \quad (3-19)
\]

\[
S_{ij} = \left| \frac{S}{\sin \frac{\pi}{P}} \sin \left[ \frac{\pi}{P}(j-i) \right] \right| \quad (3-20)
\]

\[
\delta_{ij} = \frac{a^2}{S_{ij}} \quad (3-21)
\]

where \( \theta_{ij} \) is the angle between the Y-axis in the \( i \)th conductor and a line joining that conductor with the \( j \)th conductor.

\( S_{ij} \) is the center to center distance between

...
the \( i \)th and \( j \)th conductors

\[ i \text{ is an integer with values } 2, 3, \ldots p \]

\[ j \text{ is an integer with values } 1, 2, \ldots p, j \neq i. \]

Since

\[ \gamma_{ij} = \frac{\pi}{2} - \theta_{ij} \]

the magnitude of the total dipole moment becomes:

\[ P_{ti} = \sum_{j=1}^{p} P_{ij} \sin \theta_{ij} = \sum_{j=1}^{p} Q_{ji} \delta_{ij} \sin \theta_{ij} \]

Substituting Equations (3-19), (3-20) and (3-21) in the above expression, we obtain:

\[ P_{ti} = \sum_{j=1}^{p} Q_{j} \left( \frac{a^2 \sin \frac{\pi}{p}}{S \sin \frac{\pi}{p} (j-i)} \right) \sin \left( \frac{\pi}{p} (j-i) \right) \]

\[ = \frac{a^2}{S} \sin \frac{\pi}{p} \sum_{j=1}^{p} Q_{j} \quad (3-22) \]

Equation (3-22) expresses the magnitude of the total dipole
moment in the $i$th conductor.

The radial and tangential components of the field at the point of interest, arising from the "multipole" in the $i$th conductor with total moment $P_{ti}$ are:

$$E_{r_i} = \frac{P_{ti}}{2\pi \varepsilon_0} \frac{\cos \phi_i}{r_i^2} \quad (3-23)$$

$$E_{\phi_i} = \frac{P_{ti}}{2\pi \varepsilon_0} \frac{\sin \phi_i}{r_i^2} \quad (3-24)$$

The normal components to the surface of the conductor at the point of interest are:

$$E_{r_{in}} = \frac{P_{ti}}{2\pi \varepsilon_0} \frac{\cos \phi_i \cos \alpha_i}{r_i^2}$$

$$E_{\phi_{in}} = \frac{P_{ti}}{2\pi \varepsilon_0} \frac{\sin \phi_i \cos \alpha_{oi}}{r_i^2}$$

since $\cos \alpha_{oi} = \cos \left(\frac{\pi}{2} + \alpha_i\right) = -\sin \alpha_i$

above equation becomes:

$$E_{\phi_{in}} = - \frac{P_{ti}}{2\pi \varepsilon_0} \frac{\sin \phi_i}{r_i^2} \sin \alpha_i$$

Let $E_4$ be the magnitude of the maximum field intensity on the surface of the conductor due to all dipoles in all
conductors in the bundle, except the one under consideration. Then

\[ E_4 = \sum_{i=2}^{p} (E_{r_i} + E_{\phi_i}) \]

\[ = \sum_{i=2}^{p} \frac{P_t_i}{2\pi \varepsilon_0} \left( \frac{\cos \phi_i \cos \alpha_i - \sin \phi_i \sin \alpha_i}{r_i^2} \right) \]

\[ = \sum_{i=2}^{p} \frac{P_t_i}{2\pi \varepsilon_0} \frac{\cos (\phi_i + \alpha_i)}{r_i^2} \quad (3-25) \]

If we substitute Equation (3-22) to (3-25), we obtain:

\[ E_4 = \frac{1}{2\pi \varepsilon_0} a^2 \sin \frac{\pi}{S} \sum_{i=2}^{p} \left[ \frac{\cos (\phi_i + \alpha_i)}{r_i^2} \sum_{j=1}^{p} Q_j \right] \quad (3-26) \]

3.3 Maximum Voltage Gradient at the Conductor Surface

The magnitude of the maximum electric field at the surface of conductor one is obtained by adding algebraically Equations (3-15), (3-16), (3-17) and (3-26). The direction of this field is along the X-axis shown in Figure 3-2.

If we denote the maximum voltage gradient at the
surface of conductor one by $E_{\text{max}}$, the following expression is obtained:

$$E_{\text{max}} = E_1 + E_2 + E_3 + E_4$$

$$= \frac{1}{2\pi\varepsilon_0} \left[ \frac{Q_1}{a} + \sum_{i=2}^{P} Q_i \left( \frac{1}{a} - \frac{\cos a_i}{r_i} \right) + \sum_{i=2}^{P} \frac{Q_i}{r_i} \right]$$

$$+ \frac{a^2}{S} \sin^2 \frac{\pi}{P} \sum_{i=2}^{P} \left( \frac{\cos(\phi_i + \alpha_i)}{r_i^2} \sum_{j=1}^{P} \frac{Q_j}{r_j} \right) \right]$$

By rearranging we obtain:

$$E_{\text{max}} = \frac{1}{2\pi\varepsilon_0} \left[ \frac{Q_1}{a} + \sum_{i=2}^{P} \left( \frac{Q_i}{a} - \left( \frac{\cos a_i}{r_i} - \frac{\cos a_i}{r_i^2} \right) \right)$$

$$- \frac{a^2}{S} \sin^2 \frac{\pi}{P} \frac{\cos(\phi_i + \alpha_i)}{r_i^2} \sum_{j=1}^{P} \left[ Q_j \right] \right]$$

Equation (3-27) may be used to calculate the maximum voltage gradient at the surface of any conductor in a bundle of $p$ conductors unequally charged.

3.4 Equal Charges on Conductors

Again a $p$-conductor bundle is considered. Assuming that the charges on all conductors are equal, Equation (3-27),
derived in Section 3.3, may be reduced to a relatively simpler form. Two terms in Equation (3-27) will be simplified under this assumption. These are:

a. Summation of charges

Because \( Q_1 = Q_2 = \ldots = Q_i = \ldots = Q_j = \ldots = Q_p = Q \)

\[
\sum_{i=2}^{p} Q_i = (p-1)Q \tag{3-28}
\]

b. Total dipole moment in each conductor

Again because

\( Q_1 = Q_2 = \ldots = Q_i = \ldots = Q_j = \ldots = Q_p = Q \)

\[
\sum_{j=1, j \neq i}^{p} Q_j = (p-1)Q \tag{3-29}
\]

Using Equation (3-29), the total dipole moment expressed by Equation (3-22) becomes:

\[
Pt = Q \frac{a^2}{S} (p-1) \sin \frac{\pi}{p} \tag{3-30}
\]

Equation (3-30) expresses the magnitude of the total dipole moment in the \( i \)th conductor. It may be noted here that the \( i \)th conductor can be any conductor in the bundle. In other words,
once the geometry of the conductors in the bundle is fixed, the magnitude of the total dipole moment in each conductor can be readily calculated by using Equation (3-30) with no reference to any particular conductor. This is the reason for dropping the subscript $i$ from Equation (3-30). The line of action of the total dipole moment in each conductor is along the bundle radius passing through the center of that conductor as shown in Figure 3-2.

Using Equations (3-28) and (3-30), the general Equation (3-27), developed in Section 3.3, may be rewritten as follows for the case of equal charges on all conductors:

$$E_{\text{max}} = \frac{Q}{2\pi \varepsilon_0} \left[ \frac{1}{a} + \frac{p-1}{a} \right] - \left( \sum_{i=2}^{P} \left( \frac{\cos \alpha_i}{r_i} - \frac{\cos \alpha_i}{r_i} \right) \right)$$

$$- \frac{a^2}{8} (p-1) \sin \frac{\pi}{P} \left( \frac{\cos (\phi_i + \alpha_i)}{r_i^2} \right)$$

or
Equation (3-31) may be used to calculate the maximum voltage gradient at the surface of any conductor in a bundle of \( p \) conductors equally charged.

3.5 Discussion

Two general equations have been developed in this chapter. Either one may be used to determine the maximum voltage gradients of bundle conductors, forming a regular polygon, depending on whether the charges on these conductors are assumed to be equal or not. Although these equations are of a much more complex form than Cahen's equation, derived in Appendix C, they permit an accurate evaluation of the maximum voltage gradient for conductor spacing to diameter ratios less than 6, where Cahen's equation becomes very inaccurate. For purposes of illustration, Figures 3-3 and 3-4 show a plot of the normalized maximum unit voltage gradient versus the
Figure 3-3. Comparison of maximum unit voltage gradient obtained by Cahen's equation and Dipole Model.
Figure 3-4. Comparison of maximum unit voltage gradient obtained by Cahen's equation and Dipole Model.
conductor spacing to diameter ratio for the cases of a four-conductor and a six-conductor bundle. One can clearly see from these figures that the solution obtained from the general equations developed in this chapter deviates considerably from the one obtained from Cahen's equation. The maximum deviation occurs for a ratio of S/D equal to 1 and is about 21% and 17% respectively.
CHAPTER IV

EQUATIONS FOR MAXIMUM VOLTAGE GRADIENTS
OF SPLIT BUNDLE CONDUCTORS

The evolution of power transmission above 765 KV and the prospective need for a still higher voltage (1000 to 1500 KV) indicate a need for a fresher examination of the design problems associated with lines of this UHV class. The problem of radio noise is becoming more pronounced at these voltage levels and, although other methods may be invented in the future, the practice so far for reducing radio noise is by bundling the phase conductors. The standard way of achieving this is by splitting the phase conductor into a number of subconductors placed symmetrically in the periphery of a circle.

A new bundle conductor configuration may be obtained by splitting not only the phase conductor but even the subconductor in the bundle, so the term "split bundle conductor" is introduced to describe this configuration, as stated in Chapter I, Section 1.4. It is the purpose of this chapter to present the development of a mathematical expression for determining the maximum voltage gradient on any conductor of a split bundle conductor system.
4.1 Geometry of Conductors

In order to make the problem general, a pxN-conductor system is considered, where \( p \) is the number of conductors in the sub-bundle and \( N \) is the number of sub-bundles in the phase. The cross section of a single-phase line with such a configuration is shown in Figure 4-1. The numbering sequence of the conductors in the sub-bundle, the numbering sequence of the sub-bundles in the phase along with all important geometric dimensions are also shown in that figure. Because of the rather complex geometry of this configuration, general equations are developed in this section expressing all geometric distances and angles of interest. Bearing in mind the fact that there are so many parameters to be varied in this case, it is hoped that these general equations and the digital computer will prove to be very useful tools for studying the maximum voltage gradients of a split bundle conductor system.

Before these geometric equations may be written down, some basic quantities must be defined:

\[ p \] is the total number of conductors in the sub-bundle

\[ N \] is the total number of sub-bundles in the phase

\[ m \] is an integer representing the \( m^{th} \) sub-bundle in the phase
Figure 4-1. Cross section of a single-phase line with a split bundle conductor configuration.
n is an integer representing the n\textsuperscript{th} sub-bundle in the phase

i is an integer representing the i\textsuperscript{th} conductor in any sub-bundle

j is an integer representing the j\textsuperscript{th} conductor in any sub-bundle

a is the radius of the conductor

d is the diameter of the conductor

A is the conductor spacing in the sub-bundle

S is the sub-bundle spacing in the phase

H is the height of the center of the split bundle above ground

\Psi is the angle between the radius of the split bundle passing through the center of the n\textsuperscript{th} sub-bundle and the radius of the sub-bundle passing through the center of conductor one.

Note that the radius of a sub-bundle refers to the radius of a circle formed by the centers of the subconductors in that sub-bundle, whereas the radius of the split bundle refers to the radius of a circle formed by the centers of the sub-bundles
in the phase.

Referring to Figure 4-1, the following equations can be derived directly from the geometry of the conductor configuration:

\[ R_A = \frac{A}{2\sin^2 \frac{\pi}{P}} \]  \hspace{1cm} (4-1)

\[ R_S = \frac{S}{2\sin^2 \frac{\pi}{N}} \]  \hspace{1cm} (4-2)

\[ \gamma_{ni} = \frac{2\pi}{P}(i-1) + \gamma \]  \hspace{1cm} (4-3)

\[ \theta_{nm} = \frac{2\pi}{N}(n-m) \]  \hspace{1cm} (4-4)

\[ U_{ni} = \tan^{-1}\frac{R_A \sin \gamma_{ni}}{R_S + R_A \cos \gamma_{ni}} \]  \hspace{1cm} (4-5)

\[ \phi_{nimj} = \theta_{nm} + U_{ni} - U_{mj} \]  \hspace{1cm} (4-6)

\[ \beta_{ni} = \gamma_{ni} - U_{ni} \]  \hspace{1cm} (4-7)

\[ E_{ni} = \sqrt{R_S^2 + R_A^2 + 2R_S R_A \cos \gamma_{ni}} \]  \hspace{1cm} (4-8)
Note that $\alpha_n$ is the angle between a reference axis parallel to the ground plane and the split bundle radius passing through the center of the $n$th sub-bundle, as shown in Figure 4-1.

4.2 Calculation of Charges on Conductors

A method for determining the charge on each conductor of any multi-conductor system is shown in Appendix B. Equation (B-14) adapted to the notation of this section becomes:

$$[V_{ni}] = [P_{nimj}] [Q_{nimj}]$$
Solving this equation for the charges we have:

\[ \mathcal{Q}_{ni} = [C_{nimj}] [V_{mj}] \]  \hspace{1cm} (4-14)

where \( [C_{nimj}] = [P_{nimj}]^{-1} \)

\[ P_{nimj} = \frac{1}{2\pi\varepsilon_0} \ln \frac{S_{nimj}}{S_{nimj}} \]

\( S_{nimj} \) is the distance between the centers of the \( i \)th conductor in the \( n \)th sub-bundle and the \( j \)th conductor in the \( m \)th sub-bundle.

\( S_{nimj} \) is the distance between the centers of the \( i \)th conductor in the \( n \)th sub-bundle and the image of the \( j \)th conductor in the \( m \)th sub-bundle.

4.3 Derivation of Equations for Determining the Maximum Voltage Gradients of Split Bundle Conductors

4.3.1 Method: In deriving equations for determining the maximum voltage gradient on any conductor in a split bundle conductor system, the following assumption is made:

The sub-bundles in the phase are far enough apart so that the field produced by the conductors in the other sub-bundles may be considered uniform in the region of
the sub-bundle which contains the conductor under consideration.

For this assumption to be valid, the ratio of the minimum distance between any two conductors in any two sub-bundles to the conductor diameter, must not be less than 6.

Thus, two models are employed here for the calculation of the maximum field intensity on a conductor in a split bundle conductor system. The contribution to the field intensity of the conductors in all other sub-bundles is calculated by using the Uniform Field Model. The contribution to the field intensity of the conductors in the sub-bundle with the conductor under consideration is calculated by using the Dipole Model. The maximum field intensity at the surface of any conductor in the phase will be, by the principle of superposition, the vectorial sum of these two main field components plus the field intensity due to the charge on the surface of the conductor under consideration.

To ease the mathematics, two coordinate systems are employed, the $X_{Ai} - Y_{Ai}$ referring to the sub-bundle with the conductor under consideration and the $X_{Si} - Y_{Si}$ referring to the other sub-bundles in the phase, as shown in Figure 4-2. The angle between the X-axes of these two coordinate systems is $\beta_{ni}$ and is defined by Equation (4-7).
Figure 4-2. Coordinate systems for calculating the maximum voltage gradient on any conductor of a split bundle conductor system.
4.3.2 Electric Field at Surface of Conductor under Consideration Due to its Own Charge: An equation for the electric field at the surface of a conductor due to its own charge has been derived in Chapter III, Section 3.2. Its magnitude is given here again for the sake of completeness.

\[ E_1 = \frac{1}{2\pi \varepsilon_0} \frac{Q_1}{a} \quad (4-15) \]

4.3.3 Electric Field at Surface of Conductor under Consideration Due to Charges on the Other Conductors of the Same Sub-Bundle: This field component has also been calculated in Chapter III, Section 3.2. If we let \( E_{XA} \) be its magnitude, we may write directly from Equation (3-27) by excluding the first term:

\[ E_{XA} = \frac{1}{2\pi \varepsilon_0} \left[ \sum_{i=2}^{P} \left( \frac{Q_i}{a} - \left< \frac{\cos \alpha_i'}{r_i} - \frac{\cos \alpha_i}{r_i} \right> \right) \
- \frac{a^2}{S} \sin \pi \frac{\cos(\phi_i + \alpha_i)}{r_i} \sum_{j=1 \atop j \neq i}^{P} Q_j \right] \quad (4-16) \]

Note that the symbols here are as defined in Chapter III.
Equation (4-16) can be rewritten in the following form:

\[ E_{XA} = \frac{1}{2\pi \varepsilon_0} W \]

where \( W = \sum_{i=2}^{p} \left( \frac{Q_i}{a} - \left( Q_i \left( \frac{\cos \alpha_i'}{r_i} - \frac{\cos \alpha_i}{r_i} \right) \right. \right. \]

\[ - \frac{a^2 \sin \pi}{S^2} \left( \frac{\cos(\phi_i + \alpha_i)}{r_i^2} \sum_{j=1}^{p} Q_j \right) \]

The direction of this field component is along the \( X_{Ai} \)-axis. Its magnitude in the \( X_{Si} - Y_{Si} \) coordinate system becomes:

\[ E_{XA} = \frac{1}{2\pi \varepsilon_0} \sqrt{(W \cos \beta_{ni})^2 + (W \sin \beta_{ni})^2} \]  

(4-18)

4.3.4 Electric Field at Surface of Conductor under Consideration Due to Charges on the Conductors of the Other Sub-Bundles:

Let \( E_S \) be the magnitude of the maximum electric field at the surface of the conductor under consideration due to the charges on the conductors of the other sub-bundles. Using general
equations developed by S. B. Crary\textsuperscript{13} and referring to Figure 4-2, we obtain:

\[ E_S = \frac{2}{2\pi \varepsilon_0} \sqrt{\left(\frac{Q_{mj}}{S_{nimj}} \cos \alpha_{nimj}\right)^2 + \left(\frac{Q_{mj}}{S_{nimj}} \sin \alpha_{nimj}\right)^2} \]

(4-19)

where \( Q_{mj} \) is the charge on the surface of the \( j \)th conductor in the \( m \)th sub-bundle.

\( S_{nimj} \) is the distance between the centers of the \( i \)th conductor in the \( n \)th sub-bundle and the \( j \)th conductor in the \( m \)th sub-bundle.

\( \alpha_{nimj} \) is the angle between the \( X_{Si} \)-axis and a line joining the centers of the \( j \)th conductor in the \( m \)th sub-bundle and the \( i \)th conductor in the \( n \)th sub-bundle.

Let \( Z_c = \frac{Q_{mj}}{S_{nimj}} \cos \alpha_{nimj} \) \hspace{1cm} (4-20)

\[ Z_s = \frac{Q_{mj}}{S_{nimj}} \sin \alpha_{nimj} \]

(4-21)

where \( Z_c \) represents the field component along the \( X_{Si} \)-axis.
$Z_s$ represents the field component along the $Y_{S_i}$-axis.

Then

$$E_S = \frac{1}{2\pi \varepsilon_0} \sqrt{(2Z_C)^2 + (2Z_S)^2}$$

(4-22)

4.3.5 **Maximum Voltage Gradient at the Surface of the Conductor under Consideration**: Let $E_{\text{max}}$ be the maximum field intensity at the surface of the conductor under consideration due to its own charge and the charges on the surfaces of all the other conductors in the system. Then Equations (4-15), (4-18) and (4-22), if added properly, will yield:

$$E_{\text{max}} = \frac{1}{2\pi \varepsilon_0} \left[ \frac{Q_1}{a} + \sqrt{E_{X_S}^2 + E_{Y_S}^2} \right]$$

(4-23)

where

$$E_{X_S} = W\cos \theta_{ni} + 2Z_C$$

(4-24)

$$E_{Y_S} = W\sin \theta_{ni} + 2Z_S$$

(4-25)

$$\xi = \tan^{-1} \left( \frac{E_{Y_S}}{E_{X_S}} \right)$$

(4-26)

The maximum voltage gradient at the surface of the conductor under consideration is:

$$E_{\text{max}} = \frac{1}{2\pi \varepsilon_0} \left[ \frac{Q_1}{a} + \sqrt{E_{X_S}^2 + E_{Y_S}^2} \right]$$

(4-27)
Equation (4-27) along with Equations (4-17), (4-20), (4-21), (4-24) and (4-25) may be used to calculate the maximum voltage gradient at the surface of any conductor in a split bundle conductor system. Equation (4-26) may be used to calculate the angle $\xi$. This angle defines the point on the surface of the conductor where the maximum voltage gradient is with respect to the $X_{Si}-Y_{Si}$ coordinate system.
CHAPTER V
APPLICATION TO SOME HYPOTHETICAL CONFIGURATIONS

The method of analysis proposed in the preceding chapters is applicable to any split bundle conductor configuration, provided all conductors in each sub-bundle and all sub-bundles in the phase are placed symmetrically in peripheries of circles.

As expected, because of the asymmetry involved, all conductors will not be subjected to the same maximum voltage gradient. For clarity, two more terms are introduced here and defined as follows:

a. The term "relative maximum voltage gradient" refers to the maximum voltage gradient at a point on the surface of a conductor with respect to the voltage gradient at any other point on the same conductor.

b. The term "absolute maximum voltage gradient" refers to the greatest maximum voltage gradient at the surface of a conductor with respect to the maximum surface gradient on any other conductor in the phase.

Also, three more symbols are defined below, in addition to those
of Chapter IV:

D is the diameter of an equivalent conductor placed at the center of the sub-bundle and having a cross-sectional area equal to that of the total of all conductors constituting the sub-bundle.

'D' is the diameter of an equivalent conductor with cross-sectional area equal to that of the total of all conductors constituting the phase.

G\text{max} is the absolute maximum unit voltage gradient at the surface of a conductor in the phase.

5.1 **Configurations Studied**

The study of practically any configuration can be carried out with the digital computer programs listed in Appendix D. One may, by changing only a few cards (of the order of 10 to 15), vary any of such parameters as H, S, A, N, p, 'D' or 'v' and study the effect on the maximum voltage gradient on any conductor in the configuration.

The results presented in this chapter were obtained from the study of the following hypothetical configurations:

1. \( N=2 \) \[ p=2 \]
In all these cases the maximum unit voltage gradient was computed as a function of \( S \) with the A/d ratio as the running parameter. Parameters \( H \) and 'D' were kept constant at 900 in. and 3.5 in. respectively.

For purposes of comparison, each of the above split bundle conductor configurations was converted into two different bundle conductor configurations. One of them was formed by replacing each sub-bundle by an equivalent conductor of diameter \( D \), placed at its center, thus having an \( N \)-conductor bundle, while the other was formed by taking all subconductors in all
sub-bundles and placing them symmetrically in the periphery of a circle, thus having an \((Nxp)\)-conductor bundle.

### 5.2 Results

For purposes of illustration, the configuration for the case of \(N=3\), \(p=3\) is shown in Figure 5-1. The numbering sequence of each subconductor in the sub-bundle as well as each sub-bundle in the phase is also shown in that figure. The numbers in brackets give the maximum unit voltage gradient on the corresponding conductor for \(S\) equal to 15 in. and \(A/d\) equal to 4. The arrow in each conductor shows the direction in which the surface gradient is maximum.

For corona breakdown studies, the absolute maximum voltage gradient occurring anywhere on the surface of any conductor in the system is of vital concern. Figures 5-2 through 5-10 depict the absolute maximum unit voltage gradient as a function of the sub-bundles separation for various \(A/d\) ratios. These figures also depict the maximum unit voltage gradient (dashed line) which would occur if each \(Nxp\)-conductor split bundle were converted into an \(N\)-conductor bundle of an equivalent conductor diameter \(D\), while Figure 5-11 depicts the maximum unit voltage gradient which would occur if each \(Nxp\)-conductor split bundle were converted into an \((Nxp)\)-conductor bundle of subconductor diameter \(d\).
Figure 5-1. Maximum field intensity of a 3x3-conductor split bundle.
Figure 5-2. Absolute maximum unit voltage gradient of a 2x2-conductor split bundle versus sub-bundles separation for various A/d ratios. Dashed line indicates the maximum voltage gradient, if each sub-bundle is replaced by an equivalent conductor at its center.
Figure 5-3. Absolute maximum unit voltage gradient of a 2x3-conductor split bundle versus sub-bundles separation for various A/d ratios. Dashed line indicates the maximum voltage gradient, if each sub-bundle is replaced by an equivalent conductor at its center.
Figure 5-4. Absolute maximum unit voltage gradient of a 2x4-conductor split bundle versus sub-bundles separation for various $A/d$ ratios. Dashed line indicates the maximum voltage gradient, if each sub-bundle is replaced by an equivalent conductor at its center.
Figure 5-5. Absolute maximum unit voltage gradient of a 3x2-conductor split bundle versus sub-bundles separation for various A/d ratios. Dashed line indicates the maximum voltage gradient, if each sub-bundle is replaced by an equivalent conductor at its center.
Figure 5-6. Absolute maximum unit voltage gradient of a 3x3-conductor split bundle versus sub-bundles separation for various A/d ratios. Dashed line indicates the maximum voltage gradient, if each sub-bundle is replaced by an equivalent conductor at its center.
Figure 5-7. Absolute maximum unit voltage gradient of a 3x4-conductor split bundle versus sub-bundles separation for various A/d ratios. Dashed line indicates the maximum voltage gradient, if each sub-bundle is replaced by an equivalent conductor at its center.
Figure 5-8. Absolute maximum unit voltage gradient of a 4x2-conductor split bundle versus sub-bundles separation for various $A/d$ ratios. Dashed line indicates the maximum voltage gradient, if each sub-bundle is replaced by an equivalent conductor at its center.
Figure 5-9. Absolute maximum unit voltage gradient of a 4x3-conductor split bundle versus sub-bundles separation for various A/d ratios. Dashed line indicates the maximum voltage gradient, if each sub-bundle is replaced by an equivalent conductor at its center.
Figure 5-10. Absolute maximum unit voltage gradient of a 4x4-conductor split bundle versus sub-bundles separation for various A/d ratios. Dashed line indicates the maximum voltage gradient, if each sub-bundle is replaced by an equivalent conductor at its center.
Figure 5-11. Maximum unit voltage gradient of an (Nxp)-conductor bundle versus subconductors separation. Each of these bundle conductor configurations was obtained by taking all subconductors in all sub-bundles and placing them symmetrically in the periphery of a circle.
The effect of the subconductors geometry, in each sub-bundle, on the absolute maximum voltage gradient was also examined, and for the case of N=3, p=3, it is illustrated in Figure 5-12. If the subconductors in a sub-bundle are assumed to rotate about an axis through its center, the maximum voltage gradient varies between a minimum and a maximum value. The subconductors are said to have an optimum geometry when the absolute maximum voltage gradient on any subconductor in the sub-bundle has a minimum value. The optimum geometry occurs when \( y \) is equal to \( \pi/p \). The results presented in this chapter were obtained with the subconductors in each sub-bundle in the optimum position.

It is interesting to note by examining these results that by "splitting" a conductor in a bundle, one does not necessarily succeed in reducing the absolute maximum voltage gradient. For a ratio of A/d equal to 2, for all the configurations examined, \( G_{\text{max}} \) increases for \( p \) greater than 2 but it decreases for \( p \) equal to 2. This is attributed to the fact that by "splitting" a conductor into more than two subconductors, the total charge does not divide equally on each subconductor, with those closest to the center of the split bundle carrying the least charge. Whereas, if a conductor is "split" into two subconductors, with an optimum geometry, the charge divides almost equally between them. As the sub-bundles separation
Figure 5-12. Variation of maximum unit voltage gradient on subconductors of sub-bundle three if it rotates 120° about an axis through its center.
increases, although the inequality of the charges on the subconductors becomes more pronounced, the proximity effect in the vicinity of the subconductors constituting the sub-bundle, is reduced. Reduction of the proximity effect makes the charges distribute more uniformly on the surfaces of the subconductors, and hence lessens the absolute maximum voltage gradient. For instance, for the case of $N=3$, $p=3$, shown in Figure 5-6, $G_{\text{max}}$ keeps dropping until the ratio of $A/d$ reaches about 10 for a sub-bundle spacing equal to or greater than 30 in. For $A/d$ ratios greater than 10, $G_{\text{max}}$ starts to increase because now the proximity effect of the subconductors in the neighboring sub-bundles becomes more pronounced.

It is thus seen that by "splitting" a conductor into several subconductors, the absolute maximum voltage gradient may be reduced considerably. If, for instance, each of the conductors of a three-conductor bundle, with a conductor spacing of 36 in., is "split" into three subconductors, $G_{\text{max}}$ may be reduced by as much as 15% (see Figure 5-6). This reduction, however, may be almost doubled (see Figure 5-11) if all the subconductors in the sub-bundles are spread out symmetrically in the periphery of a circle, forming a nine-conductor bundle.

By examining carefully all the results presented in
this chapter, one can conclude that no real benefit, as far as voltage gradient is concerned, may be derived from the split bundle conductor arrangement, unless by some means each sub-bundle in a split bundle conductor can be treated as a unit, i.e. each sub-bundle can be regarded as an integral part of the phase. With this idea in mind, one may be tempted to suggest the "spread out" type of conductor, or the "flat" type of conductor. The former may be thought of as having a construction similar to that of the "expanded" conductor, with the aluminum strands possibly replaced by a single layer of aluminum stranded subconductors, but the ratio of subconductor spacing to its diameter must be carefully chosen. The latter type may be thought of as being constructed of two aluminum stranded conductors, with a core of stranded steel between them to take up the mechanical stresses. However, mechanical, aerodynamical, electrodynamical and other problems associated with conductors of overhead transmission lines, must be examined first before such types of conductors may be given any serious thought.
An accurate analytical method has been presented whereby the maximum voltage gradient of a p-conductor bundle can be calculated. This method, based on the Dipole Model and thus termed "dipole method", permits an accurate calculation of the maximum voltage gradient on any conductor of a p-conductor bundle, even for conductor spacing to diameter ratios relatively small (less than 6), where Cahen's equation or other analytical methods fail to produce accurate results. For the two-conductor case, the solution obtained by this method agrees with the exact one to better than 0.7% for a conductor spacing to diameter ratio of as low as 2. Even for the extreme case of a ratio of unity, the error does not exceed 2.5%.

Although this method can be carried out by desk calculations, particularly if conductors geometry is relatively simple and charges on conductors are assumed to be equal, it is best suited for a digital computer. The computing time is very small, since this method does not include any iterative or other computer time consuming schemes.

For the practical range of spacings of bundle-conductor phases at present, this method can hardly be of any advantage, because of the existing less accurate but much simpler analytical
methods, although it may be applied to accurately solve analogous field problems of suitable configurations in other areas. However, for voltage levels of the UHV class, the conductor configuration in the phases may have to be changed. For the study of split bundle conductor configurations, this method is of paramount importance.

While the split bundle conductor configuration may serve as a means for reducing voltage gradients, it has not proved to be the best arrangement, for, if the same number of subconductors is to be erected in the split bundle configuration, why not erect them in the bundle configuration and achieve a greater voltage gradient reduction. But the advantage of split bundle conductors stands out if each sub-bundle is to be treated as an integral part of the phase. It is reported that for transmission lines of the UHV class, as many as 14 subconductors per phase may be necessary. The stringing of 42 subconductors, excluding the ground wires, in a tower is by no means an easy job. It seems very likely that the situation will improve, if one has to string only 1/3 of them or even less, even if they are heavier.

Probably the conductor of the UHV line will have to depart considerably from the conventional one, as far as construction is concerned. From the voltage gradient point
of view only, the types of conductors suggested at the end of Chapter V seem to be promising, but unless an intensive investigation is conducted to examine all electrical and mechanical problems of any such type of conductor, it would be absurd and premature to make any definite suggestions.
APPENDIX A

DERIVATION OF EQUATIONS FOR VARIOUS MODELS
OF THE TWIN-CONDUCTOR CASE

A.1 Exact Solution of the Twin-Cylindrical Conductor Case

Formulae,\textsuperscript{10} as developed by Dr. H. Poritsky, for calculating the maximum voltage gradient near a twin-cylindrical conductor arrangement of equal charges of the same sign, are as follows*:

\[ E_{\text{max}} = E_{\text{ave}} \frac{\pi}{c} \frac{\sqrt{s-d}}{\sqrt{s+d}} \frac{1+3q+5q^2+7q^3+9q^4+5\ldots}{1-q+q^2+q^3+q^4+\ldots} \]

\[ (A-1) \]

where \( c = \ln \frac{\sqrt{s+d} + \sqrt{s-d}}{\sqrt{s+d} - \sqrt{s-d}} \)

\( q = e^{-\frac{\pi^2}{2c}} \)

\( s = \) center to center spacing between conductors

\( d = \) diameter of conductor

\[ E_{\text{ave}} = \frac{Q}{2\pi \varepsilon_0 a} \]

* These formulae are converted to the MKS system for uniformity.
\[ \varepsilon_0 = \text{permittivity of free space} \]
\[ Q = \text{charge per unit length of conductor} \]
\[ a = \text{radius of conductor} \]

A.2 **Single Line Charge Model**

Assuming the ground to be at infinity, Figure A-1a shows the actual two-conductor bundle, while Figure A-1b shows the representation of the two-conductor bundle by its model, i.e. a line charge \( Q \) per unit length equal to the charge on the surface of each conductor placed at the center of the conductor.

The potential at a point \( p \) in the region outside the conductors is:

\[
V_p = -\frac{Q}{2\pi\varepsilon_0} \ln r_1 - \frac{Q}{2\pi\varepsilon_0} \ln r + c
\]

where
\[
r_1 = \sqrt{s^2 + r^2 + 2rs\cos\theta}
\]

\( c \) = a constant of integration.

Therefore
\[
V_p = -\frac{Q}{2\pi\varepsilon_0} \left[ \frac{1}{2} \ln(s^2 + r^2 + 2rs\cos\theta) + \ln r \right] + c
\]

The radial component of the electric field at \( p \) is:

\[
E_r = -\frac{\partial V_p}{\partial r} = \frac{Q}{2\pi\varepsilon_0} \left[ \frac{r + s\cos\theta}{s^2 + r^2 + 2rs\cos\theta} \right] + \frac{1}{r}
The maximum electric field, normal to the surface of the conductor, will be at \( r = a, \theta = 0 \).

![Diagram of charged conductors](image)

Figure A-1. (a) Actual charged conductors. (b) Model.

Hence:

\[
E_{\text{max}} = E_r \left|_{r=a} \right. = \frac{Q}{2\pi \varepsilon_0} \left[ \frac{a+s}{(a+s)^2} + \frac{1}{a} \right]
\]

Let \( \zeta = \frac{a}{s} \)
\[ E_{\text{ave}} = \frac{Q}{2\pi \varepsilon_0 a} \]

Then \[ E_{\text{max}} = E_{\text{ave}} \left[1 + \frac{a}{s(1 + \frac{a}{s})}\right] = E_{\text{ave}} \left[1 + \frac{\zeta}{1 + \zeta}\right] \] (A-2)

If we define the unit surface gradient (G) to represent the magnitude of the electric field on the surface of the conductor when a potential of unity is applied to it, we obtain:

\[ G_{\text{max}} = G_{\text{ave}} \left[1 + \frac{\zeta}{1 + \zeta}\right] \] (A-3)

A.3 **Uniform Field Model**

Consider a two-conductor bundle made up of two conductors A and B. The maximum surface voltage gradient on conductor B is to be found. Figure A-2a shows the actual conductors, while Figure A-2b shows the model which represents the conductors for the purpose of calculating the surface gradient on conductor B. The assumption here is that the field produced by the charge on conductor A in the region of conductor B is uniform. Then the uniform field in the vicinity of conductor B will be:

\[ E_u = \frac{Q}{2\pi \varepsilon_0 s} \]
At the surface of a conducting cylinder in a uniform field, the maximum field intensity is $2E_u$. The maximum field on conductor B will be at point p and it will consist of twice this uniform field plus the field produced by the line charge at its center.

\[ E_{\text{max}} = \frac{Q}{2\pi \varepsilon_0 a} + 2E_u = \frac{Q}{2\pi \varepsilon_0} \left[ \frac{1}{a} + \frac{2}{s} \right] \]

If \( \zeta = \frac{a}{s} \)
\[ E_{\text{ave}} = \frac{Q}{2\pi \varepsilon_0 a} \]

Above equation becomes:

\[ E_{\text{max}} = E_{\text{ave}}[1 + 2\xi] \quad \text{(A-4)} \]

Therefore, the maximum unit surface gradient is:

\[ G_{\text{max}} = G_{\text{ave}}[1 + 2\xi] \quad \text{(A-5)} \]

### A.4 King's Model

Formulae,\(^{16}\) as developed by Dr. King, for determining the maximum voltage gradient on the surface of a two-conductor bundle, are given below:

\[ E_{\text{max}} = E_{\text{ave}} \left[\frac{(s+d)a}{sa + a^2 - s^2 - d^2}\right] \]

where \( s = \) center to center spacing of conductors

\[ d = 2a = \text{diameter of conductor} \]

\[ \delta = \frac{d}{2} \sqrt{\frac{1}{2} \left[ \frac{s}{d} + 1 \right]^2 + \left( \frac{s}{d} - 1 \right)^2} - \frac{s}{2} \]
APPENDIX B

CALCULATION OF CHARGES ON A
MULTI-CONDUCTOR SYSTEM

Consider for simplicity three cylindrical conductors in a flat configuration, parallel to an infinite plane, as shown in Figure B-1.

Assume that \( d \ll 2H \), i.e. the image of conductor one is so far away that the charge on conductor one is not affected by its image. Assume next that conductor one carries a charge of \(+Q\) coulombs/unit length, the other conductors being uncharged. Then its image will carry a charge of \(-Q\) coulombs/unit length. The potential at point \( p \) is due to the positive charge on conductor one and the negative charge on its image. Then

\[
V_p = \frac{Q_1}{2\pi \varepsilon_0} \ln \frac{r_2}{r_1}
\]

Let us define:

\( V_{11} = \) potential on surface of conductor one due to its own charge

\( V_{21} = \) potential on surface of conductor two due to charge on conductor one

\( V_{31} = \) potential on surface of conductor three due to the charge on conductor one.
Potential on surface of conductor one is obtained if we let:

\[ r_1 = \frac{d}{2} \]

\[ r_2 = 2H \]

Then

\[ V_{11} = \frac{Q \ln 4H}{2 \pi \varepsilon_0 d} \]

(B-1)

Figure B-1. Three-conductor system above ground.
Potential on surface of conductor two is obtained if we let:

\[ r_1 = s \]

\[ r_2 = \sqrt{(2H)^2 + s^2} \]

Then

\[ V_{21} = \frac{Q_1}{2\pi \varepsilon_0} \ln \frac{\sqrt{(2H)^2 + s^2}}{s} = \frac{Q_1}{2\pi \varepsilon_0} \ln \sqrt{\left(\frac{2H}{s}\right)^2 + 1} \]

If we assume that \( \left(\frac{2H}{s}\right)^2 \gg 1 \) we have:

\[ V_{21} = \frac{Q_1}{2\pi \varepsilon_0} \ln \left(\frac{2H}{s}\right) \quad (B-2) \]

Similarly, if we let:

\[ r_1 = 2s \]

\[ r_2 = 2H \]

we obtain:

\[ V_{31} = \frac{Q_1}{2\pi \varepsilon_0} \ln \frac{2H}{2s} \quad (B-3) \]

If we assume next that conductor two is charged with \( +Q_2 \) coulombs/unit length and conductors one and three are uncharged, we obtain:

\[ V_{22} = \frac{Q_2}{2\pi \varepsilon_0} \ln \frac{4H}{d} \quad (B-4) \]

\[ V_{12} = \frac{Q_2}{2\pi \varepsilon_0} \ln \frac{2H}{s} \quad (B-5) \]

\[ V_{32} = \frac{Q_2}{2\pi \varepsilon_0} \ln \frac{2H}{s} \quad (B-6) \]
Assuming finally that conductor three is charged with \( +Q_3 \) coulombs/unit length with the others remaining uncharged, we have:

\[
V_{33} = \frac{Q_3}{2\pi\varepsilon_0} \frac{\ln 4H}{d} \tag{B-7}
\]

\[
V_{13} = \frac{Q_3}{2\pi\varepsilon_0} \frac{\ln 2H}{2s} \tag{B-8}
\]

\[
V_{23} = \frac{Q_3}{2\pi\varepsilon_0} \frac{\ln 2H}{s} \tag{B-9}
\]

If we define \( V_1 \) to be the total potential of conductor one due to its own charge and charges on other conductors, assume all conductors are charged, and use the principle of superposition, we obtain:

\[
V_1 = V_{11} + V_{12} + V_{13}
\]

Substituting Equations (B-1), (B-2) and (B-3) to above, we have:

\[
V_1 = \frac{Q_1}{2\pi\varepsilon_0} \frac{\ln 4H}{d} + \frac{Q_2}{2\pi\varepsilon_0} \frac{\ln 2H}{s} + \frac{Q_3}{2\pi\varepsilon_0} \frac{\ln 2H}{2s} \tag{B-10}
\]

Let

\[
P_{11} = \frac{1}{2\pi\varepsilon_0} \frac{\ln 4H}{d}
\]

\[
P_{12} = \frac{1}{2\pi\varepsilon_0} \frac{\ln 2H}{s}
\]

\[
P_{13} = \frac{1}{2\pi\varepsilon_0} \frac{\ln 2H}{2s}
\]

Then (B-10) becomes:
Similarly, we obtain for conductors two and three:

\[ V_1 = P_{11}Q_1 + P_{12}Q_2 + P_{13}Q_3 \quad (B-11) \]

\[ V_2 = P_{21}Q_1 + P_{22}Q_2 + P_{23}Q_3 \quad (B-12) \]

\[ V_3 = P_{31}Q_1 + P_{32}Q_2 + P_{33}Q_3 \quad (B-13) \]

The coefficients \( P_{11}, P_{12}, \) etc. are constants and depend entirely on the geometry of the conductors, if conductors are placed in free space. Then, Equations (B-11), (B-12) and (B-13) can be simultaneously solved for the charges.

For the general case of an \( n \)-conductor system, we have:

\[ V_1 = P_{11}Q_1 + P_{12}Q_2 + \ldots + P_{1n}Q_n \]
\[ V_2 = P_{21}Q_1 + P_{22}Q_2 + \ldots + P_{2n}Q_n \]
\[ \vdots \]
\[ V_m = P_{m1}Q_1 + P_{m2}Q_2 + \ldots + P_{mn}Q_n \]

In matrix form above equations become:

\[ [V_i] = [P_{ij}] [Q_j] \quad (B-14) \]

Solving (B-14) for the charges, we have:

\[ [Q_i] = [C_{ij}] [V_j] \quad (B-15) \]
where \([C_{ij}] = [P_{ij}]^{-1}\)

If the potential to ground is known and the geometry of the conductors fixed, Equation (B-15) will give the charges on all conductors in the system considered.
APPENDIX C

DERIVATION OF CAHEN'S EQUATION

Assume $n$ subconductors per phase placed symmetrically in the periphery of a circle as shown in Figure C-1.

![Diagram of a bundle of $n$ conductors placed symmetrically in the periphery of a circle.](image)

Figure C-1. A bundle of $n$ conductors placed symmetrically in the periphery of a circle.
Referring to this figure, the following geometric equations can be written:

\[ \theta_i = \frac{\pi}{n} \]  
\[ R = \frac{s}{2 \sin \frac{\pi}{n}} \]  
\[ s_i = \frac{s \sin \frac{\pi}{n}}{\sin \frac{\pi}{n}} \]

where \( \theta_i \) is the angle between the Y-axis and the \( i \)th conductor
\( s_i \) is the distance between conductor one and the \( i \)th conductor
\( R \) is the bundle radius
\( i \) is an integer with values 1, 2, 3 ... \( n-1 \).

Let \( Q_1 \) be the charge on conductor one, \( Q_2 \) the charge on conductor two and so on. Also let \( H \) be the height of the bundle center above ground. Assume that:

\[ Q_1 = Q_2 = \ldots Q_n = Q \]

\[ \frac{2H}{s_i} \gg 1 \]
If $V_1$ is the potential of conductor one with respect to ground, we can write:

$$V_1 = \frac{Q}{2\pi \varepsilon_0} \left[ \ln \frac{2H}{r} \cdot \frac{2H}{s_1} \cdot \frac{2H}{s_2} \cdots \frac{2H}{s_i} \cdots \frac{2H}{s_{n-1}} \right]$$

$$= \frac{Q}{2\pi \varepsilon_0} \left[ \ln \left( \frac{2H}{r \cdot s_1 \cdot s_2 \cdots s_i \cdots s_{n-1}} \right)^n \right]$$

or

$$V_1 = \frac{nQ}{2\pi \varepsilon_0} \ln \frac{2H}{R_{eq}} \quad \text{(C-4)}$$

where $r$ is the radius of the conductor.

$$R_{eq} = \sqrt[2n-1]{r \cdot s_1 \cdot s_2 \cdots s_i \cdots s_{n-1}}$$

Using Equation (C-3), we obtain:

$$R_{eq} = \sqrt[2n-1]{r \cdot \frac{s}{\sin \frac{\pi}{n}} \cdot \frac{s}{\sin ^2 \frac{\pi}{n}} \cdots \frac{s}{\sin ^{n} \frac{\pi}{n}} \cdots \frac{s}{\sin ^{(n-1)} \frac{\pi}{n}} \frac{\sin ^{(n)} \frac{\pi}{n}}{\sin ^{n} \frac{\pi}{n}} \cdots \frac{s}{\sin ^{(n-1)} \frac{\pi}{n}}}$$

Let $$\lambda = \frac{\sin \frac{\pi}{n} \cdot \sin ^2 \frac{\pi}{n} \cdots \sin ^{n} \frac{\pi}{n} \cdots \sin ^{(n-1)} \frac{\pi}{n}}{(\sin ^{n} \frac{\pi}{n})^{n-1}}$$

$$= \frac{n}{2^{n-1}(\sin ^{n} \frac{\pi}{n})^{n-1}}$$
Hence \[ \lambda = \frac{n}{(2\sin \frac{\pi}{n})^{n-1}} \] (C-5)

Then \[ R_{eq} = \sqrt[n]{rs^{n-1} \lambda} \] (C-6)

Solving (C-4) for the charge, we obtain:

\[ Q = \frac{2\pi \varepsilon_0}{n \ln \left( \frac{2H}{R_{eq}} \right)} \] (C-7)

If \[ E_{ave} = \frac{Q}{2\pi \varepsilon_0 r} \] we obtain by using Equation (C-7):

\[ E_{ave} = \frac{V_1}{nr \ln \left( \frac{2H}{R_{eq}} \right)} \] (C-8)

Starting with S. B. Crary's general equations, the maximum voltage gradient on conductor one will be:

\[ E_{1max} = \frac{2Q}{2\pi \varepsilon_0 r} \left[ \sqrt{\left( \sum_{i=1}^{n-1} \frac{\cos \theta_i}{s_i} \right)^2 + \left( \sum_{i=1}^{n-1} \frac{\sin \theta_i}{s_i} \right)^2} + \frac{Q}{2\pi \varepsilon_0 r} \right] \]

Substituting Equations (C-1) and (C-3) to the above and rearranging, we have:

\[ E_{1max} = \frac{Q}{2\pi \varepsilon_0 r} \left[ 1 + \frac{d}{s} \sqrt{\left( \sum_{i=1}^{n-1} \frac{\cos \frac{\pi}{n} \theta_i}{\sin \frac{\pi}{n} s_i} \right)^2 + \left( \sum_{i=1}^{n-1} \frac{\sin \frac{\pi}{n} \theta_i}{\sin \frac{\pi}{n} s_i} \right)^2} \right] \]
Simplifying, we obtain:

\[ E_{\text{max}} = \frac{0}{2\pi \varepsilon_0 r} \left[ 1 + \frac{d}{s} \sin \frac{\pi}{n} \sqrt{\left( \sum_{i=1}^{n-1} \frac{1}{\tan \frac{\pi}{n}} \right)^2 + (n-1)^2} \right] \]

where \( d \) = diameter of conductor.

Because of symmetry:

\[ \sum_{i=1}^{n-1} \frac{1}{\tan \frac{\pi}{n}} = 0 \]

Therefore

\[ E_{\text{max}} = E_{\text{ave}} [1 + \frac{d}{s} (n-1) \sin \frac{\pi}{n}] \quad (C-9) \]

Since potential gradient varies cosinusoidally around the conductor, it can be expressed as follows:

\[ E_1 = E_{\text{ave}} [1 + A \cos \psi] \quad (C-10) \]

where \( A = \frac{d}{s} (n-1) \sin \frac{\pi}{n} \)

\( \psi \) is the angle formed by the bundle radius passing through the center of conductor one and a line joining the center of conductor one to a point on the surface of the conductor where the potential gradient is evaluated.
Equation (C-10) is known as Cahen's equation, and because of its simplicity, it is almost exclusively used for determining potential gradients on bundle conductors if the ratio of intragroup spacing to conductor diameter is greater than 6. For ratios less than 6, at least 1% and up to 27% error may result.

In terms of the unit voltage gradient \( G \), Equation (C-10) becomes:

\[
G_1 = \frac{E_1}{V_1} = \frac{1}{nr \ln \left( \frac{2H}{R_{eq}} \right)} (1 + A\cos \phi) \tag{C-11}
\]
APPENDIX D
FORTRAN PROGRAM LISTINGS

For reference, three typical programs used to obtain the results presented in Chapter V are listed in this section. Also, at the end of this section, sample results are shown for purposes of illustration. The programs are in FORTRAN IV language, G Level, suitable for use on IBM 360 series computers.
MAX VOLTAGE GRADIENT ON BUNDLE CONDUCTORS AT PERIPHERY OF A CIRCLE
CONDUCTORS IN SUBGROUP ARE REPLACED BY ONE CONDUCTOR OF EQUAL
CROSSSECTIONAL AREA. %UNIFORM FIELD MODEL
H=900.e0
PI=3.14159265
N0=4
DT=3.5
S0=39.e0
N=2
65 WRITE(6,1101)N
1101 FORMAT(1H1,5X,2HNO OF BUNDLE COND. #,I3)
R=N
BL=R/((2.0*SIN(PI/R))**(N-1))
DP=2.5
55 WRITE(6,1102)DP
1102 FORMAT(1H1,5X,2HDIAMETER OF BASIC COND #,F4.2)
C=DP/SCRT(R)
WRITE(6,1103)D
1103 FORMAT(1H1,5X,2HDIAMETER OF BUNDLE COND. #,F6.4)
SA=12.e0
15 RAT=SA/D
REQ =((0/2.0)*BL*(SA**(N-1))
AA= (D/SA)*((R-1.0G)*SIN(PI/R)
BCCNS= ((2.0)*H)**N/REQ
GM =((2.0)/(D*ALCN(BCCNS)))*(1.0+AA)
WRITE(6,1104)SA,RAT,GM
1104 FORMAT(1H1,5X,3HS #,F6.2,5X,5HS/D #,F6.2,5X,4HS/GM #,F12.8)
IF(SA.GE.SJ) GO TO 10
SA=SA+3.e0
GO TO 15
10 IF(DP.GE.DT) GO TO 50
DP=DP+1.e0
GO TO 55
50 IF(N.EC.N0) GO TO 60
N=N+1
GO TO 65
60 STOP
END

TOTAL MEMORY REQUIREMENTS 200400 Bytes
C MAX. VOLTAGE GRADIENT CA BUNDLE CONDUCTORS PLACED AT THE PERIPHERY
C OF A CIRCLE. CONDUCTORS IN SUBGROUPS ARE PLACED AT PERIPHERY
C OF A CIRCLE %UNIFORM FIELD MODEL<
H=900.0
PI=3.1415926
NO=16
DT=3.5
SO=39.0
N=4

65 WRITE(6,1101)
1101 FORMAT(1H1,5X,20HNO OF BUNDLE COND. #,I3)
R=N
BL=R/((2.0*SIN(PI/R))**(N-1))
D=2.5
55 WRITE(6,1102)
1102 FORMAT(1H0,5X,25HDIAMETER CF BASIC COND. #,F4.2)
DS=D/SCRT(R)
WRITE(6,1103)
1103 FORMAT(1H0,5X,26HDIAMETER CF BUNDLE COND. #,F6.4)
SA=12.0
15 RAT=SA/DS
REQ=(DS/2.7)*BL*(SA**(N-1))
AA=(DS/SA)*(R-1.0)*SIN(PI/R)
BCONS=((2.0*H)**A)/PEC
GM=(2.0/(DS*ALOG(BCONS)))*(1.0+AA)
WRITE(6,1104)
1104 FORMAT(1H2,5X,25HSA,RAT,GM #,F6.2,5X,5HS/C #,F6.2,5X,4HGM #,F12.8)
IF(SA. GE. SO) GC TO 1C
SA=SA+3.0
GO TO 15
10 IF(D. GE. DT) GO TO 6C
D=D+1.0
GO TO 55
50 IF(N. GE. NO) GO TO 6C
N=N+2
GO TO 65
69 STOP
END

TOTAL MEMORY REQUIREMENTS 0.04E2 BYTES
C MAX. VOLTAGE GRADIENT CN SPLIT BUNDLE CONDUCTORS *DIPOLE MOMENT
C METHOD < 
C CHARGES CN SPLIT BUNDLE CONDUCTORS 
DIMENSION SP(20,20), S(20,20), EV(40), TAU(40), SEG(40), UV(40), GAMA(40,1), THETA(6,6), PHI(20,20)
DIMENSION BETA(2C), ALPHA(2C,2C), QC(20), ATHETA(2C), AS(2C), ADELA(201), AR(2C), ABETA(2C), AALPHA(2C), AALPHAP(2C), ASP(20,20), APP(20,20), AGAMMA(2C), APPC(2C), GMAGAMMA(2C,2C), ALPHAO(2C)
DIMENSION P(9,9), L1(9), M1(9), V(9), Q(9)
H=9.0, 0, C
PI=3.14159265
E=8.85
WRITE(6,2222)
2222 FORMAT(1H1)
AC=3.5
AC=3.5
3C8 S=3.0, C
N=3
ANP=(R+2.0)/2.0
R=K
K=3
MC=NN*KC
P0=K
ADS=AC*SQRT((1.0/(PC*R)))
SA=3.0, 0, C
3C6 AC=SA-6.0, 0, ADS*SIN(PI/PC)
WRITE(6,2222)
2222 FORMAT(1H1)
AC=3.5
AC=3.5
3C8 S=3.0, C
N=3
ANP=(R+2.0)/2.0
R=K
K=3
MC=NN*KC
P0=K
ADS=AC*SQRT((1.0/(PC*R)))
SA=3.0, 0, C
3C6 AC=SA-6.0, 0, ADS*SIN(PI/PC)
WRITE(6,2222)
2222 FORMAT(1H1)
AC=3.5
AC=3.5
3C8 S=3.0, C
N=3
ANP=(R+2.0)/2.0
R=K
K=3
MC=NN*KC
P0=K
ADS=AC*SQRT((1.0/(PC*R)))
SA=3.0, 0, C
3C6 AC=SA-6.0, 0, ADS*SIN(PI/PC)
WRITE(6,2222)
2222 FORMAT(1H1)
AC=3.5
AC=3.5
3C8 S=3.0, C
N=3
ANP=(R+2.0)/2.0
R=K
K=3
MC=NN*KC
P0=K
ADS=AC*SQRT((1.0/(PC*R)))
SA=3.0, 0, C
3C6 AC=SA-6.0, 0, ADS*SIN(PI/PC)
WRITE(6,2222)
2222 FORMAT(1H1)
AC=3.5
AC=3.5
3C8 S=3.0, C
N=3
ANP=(R+2.0)/2.0
R=K
K=3
MC=NN*KC
P0=K
ADS=AC*SQRT((1.0/(PC*R)))
SA=3.0, 0, C
3C6 AC=SA-6.0, 0, ADS*SIN(PI/PC)
WRITE(6,2222)
2222 FORMAT(1H1)
AC=3.5
AC=3.5
3C8 S=3.0, C
N=3
ANP=(R+2.0)/2.0
R=K
K=3
MC=NN*KC
P0=K
ADS=AC*SQRT((1.0/(PC*R)))
SA=3.0, 0, C
3C6 AC=SA-6.0, 0, ADS*SIN(PI/PC)
WRITE(6,2222)
2222 FORMAT(1H1)
AC=3.5
AC=3.5
3C8 S=3.0, C
N=3
ANP=(R+2.0)/2.0
R=K
K=3
MC=NN*KC
P0=K
ADS=AC*SQRT((1.0/(PC*R)))
SA=3.0, 0, C
3C6 AC=SA-6.0, 0, ADS*SIN(PI/PC)
WRITE(6,2222)
2222 FORMAT(1H1)
AC=3.5
AC=3.5
3C8 S=3.0, C
N=3
ANP=(R+2.0)/2.0
R=K
K=3
MC=NN*KC
P0=K
ADS=AC*SQRT((1.0/(PC*R)))
SA=3.0, 0, C
3C6 AC=SA-6.0, 0, ADS*SIN(PI/PC)
UV(I) = ATAN((RA*SIN(GAMA(I)))/(RS+RA*COS(GAMA(I))))
UV(J) = ATAN((RA*SIN(GAMA(J)))/(RS+RA*COS(GAMA(J))))
BETA(I) = GAMA(I) - UV(I)
PHI(NN, MM) = THETA(NN, M) + UV(J) - UV(I)
EV(I) = SQRT(RS**2+RA**2+2.*RS*RA*COS(GAMA(I)))
EV(J) = SQRT(RS**2+RA**2+2.*RS*RA*COS(GAMA(J)))
IF(NN-MM) = 12, 15, 12
15 SIN(NN, MM) = ADS/2.0
GO TO 19
12 SIN(NN, MM) = SQRT(EV(I)**2+EV(J)**2-2.0*EV(I)*EV(J)*COS(PHI(NN, MM)))
AC1 = EV(J)*SIN(PHI(NN, MM))
AC2 = SIN(NN, MM)
ALPHA(NN, MM) = ATAN(AC1/SQRT(AC2**2-AC1**2))
19 TAU(NN) = SEG(N)+UV(I)
TAU(MM) = SEG(M)+UV(J)
SP(NN, MM) = SQRT((2.0*EV(I)**2+EV(J)**2+2.*EV(I)*EV(J)*COS(PHI(NN, MM))))
EV(J)*COS(TAU(MM)) - EV(I)*COS(TAU(NN)))**2
10 P(NN, MM) = B*ALOG(SP(NN, MM)/S(NN, MM))
WRITE(6, 2032)
2032 FORMAT(1HC, 5X, PHP-MATRIX)
DO 6*NN=1, MMQ
6* WRITE(6, 1G17)(P(NN, MM), NN=1, MMQ)
1010 FORMAT(1HC, 6F10.6)
CALL MINV(P, 9, D, L1, M1)
WRITE(6, 2033)
2033 FORMAT(1HC, 5X, 8HC-MATRIX)
DO 77NN=1, MMQ
77 WRITE(6, 2034)(P(NN, MM), NN=1, MMQ)
2034 FORMAT(1HC, 6F10.6)
DO 20MNN=1, MMQ, 1
20 VI(MM) = 1.0
CALL GPRED(P, V, Q, 9, S, 1)
WRITE(6, 1035)
1005 FORMAT(1HC, 5X, 8HC-MATRIX)
DO 55NN=1, MMQ, 1
55 WRITE(6, 1036)C(NN)
1036 FORMAT(1HC, 6F10.6)
WRITE(6, 2034)
2034 FORMAT(1HC, 5X, 9HCUND. NC, 5X, 16=MAX. VOLT. GRAC., 5X, 5HANGLE)
DO 99K1=1, K1
99 WRITE(6, 2034)
C CHARGE CONVERSION
M1=M1+I-1
DO 1 M1=1, M1
IF(NN-KI)*101, 101, 102
101 L=NN+I-1
K=L
ANL=NN+K*(N-1)
QC(L) = C(ANL)
LA=L
GO TO 104
104 NN=NN-K1
L=LA+ANL
K=L
C MAX. VOLTAGE GRADIENT DUE TO CHARGES ON CONDUCTORS IN SUB-BUNDLE
C WITH CONDUCTOR UNDER CONSIDERATION

AA = ADS/2.0
TSUM1 = CC
TSUM2 = CC
QCT = CC
DO 20 CC K = 2, K0
AK = K

DPM = CC
ATHETA(K) = (PI/PL0)*(AK-1.0)
AS(K) = (AA/SIN(PI/PL0))*SIN((PI/PL0)*(AK-1.0))
ADELTA(K) = AA**2/AS(K)
AR(K) = SQRT((AS(K) + AA*SIN(ATHETA(K)))**2 + (AA*CCS(ATHETA(K)))**2)
ABETA(K) = ATAN((AA*COS(ATHETA(K)))/(AS(K) + AA*SIN(ATHETA(K))))
AALPHA(K) = PI/2.0 - (ATHETA(K) + ABETA(K))
ALPHAP(K) = PI/2.0 + AALPHA(K)

IF(AK.GT.ANP) GG TG 66
AGAMA(K) = PI/2.0 + ATHETA(K)
GO TO 66

AGAMA(K) = 2.0*C*PI - (PI/2.0*C + ATHETA(K))

PHIC(K) = AGAMA(K) - ABS(ABETA(K))
TSUM1 = TSUM1 + CC(K)*(CCS(ALPHAP(K)) / AR(K) - COS(AALPHA(K)) / AR(K))
DO 33 L = 1, K0
AL = L

IF(L-K)11, 33, 11

ASP(K, L) = ABS((AA/SIN(PI/PL0))*SIN((PI/PL0)*(AL- AK))
APP(K, L) = (QC(L)*AA**2)/ASP(K, L)

IF(L-K)3, 3, 4

3 KE = 1
GO TO 33

4 KE = 2

33 GAMA(K, L) = -(1.0)*KE*((PI*(PO-2.0)/(2.0*PO) + PI*(AL-1.0)/PO) - PI*(1AK-2.0)/PO)
DPM = DPM + APP(K, L)*CCS(GAMA(K, L))

350 CONTINUE
PROD = DPM*(CCS(PHIC(K))*CCS(AALPHA(K)) + SIN(PHIC(K))*COS(ALPHA(K))

1)/AR(K)**2)
TSUM2 = TSUM2 + FRCC

250 CONTINUE
K = CCT - (TSUM1 - TSUM2)
BETA0 = (BETA1*18.0/PI)

C VOLTAGE GRADIENT DUE TO CHARGE ON SURFACE OF CONDUCTOR UNDER
C CONSIDERATION
AVG = QC(1)/AA
C CONTRIBUTION OF OTHER SLE-BUNDLES IN THE PHASE
TSUMC = C*G
TSUMS = G*G
DO 500 K = 1, N0
IF(M-N) EQ 1, 500, 501
501 SUMC = n*G
SUMS = n*G
DO 550 J = 1, K0
MN = K0*(N-1)+1
MM = K0*(M-1)+J
SLMC = SUMC+(Q(MM)*CCS(ALPHA(NN,MM)))/S(NN,MM)
550 SUMS = SUMS+(Q(MM)*SIN(ALPHA(NN,MM)))/S(NN,MM)
TSUMC = TSUMC + SUMC
TSUMS = TSLMS + SUMS
50C CONTINUE
ZC = TSUMC
ZS = TSUMS
C TOTAL MAX. VOLTAGE GRADIENT CN CONDUCTOR UNDER CONSIDERATION
IF(W LT 0.0) GO TO 60C
EXS = W * CCS(BETA(I)) + 2.0*ZC
EYS = W * SIN(BETA(I)) + 2.0*ZS
AAA = SQRT(EXS**2 + EYS**2)
GM = B*(AVG + AAA)
DEL = ABS(ATAN(EYS/EXS))
IF(EXS LT 0.0) GC TO 70C
IF(EYS LT 0.0) GC TO 71C
DELP = DEL
GO TO 77C
71C DELP = -DEL
GO TO 770
70C IF(EYS LT 0.0) GC TO 72C
DELP = PI - DEL
GO TO 77C
72C DELP = PI + DEL
77C DELPO = (DELP*180.0)/PI
WRITE(6, 2005) NN, GM, DELPO
2005 FORMAT(1HC, 5X, I5, 10X, F12.8, 10X, F8.4)
GO TO 900
60C WRITE(6, 1200)
1200 FORMAT(1HC, 18HW NEGATIVE, CHECK)
90C CONTINUE
IF(A GE AP) GC TO 301
A = A + 4.0*ADS
GO TO 302
301 IF(SA GE SP) GC TO 305
SA = SA + 3.0
GO TO 306
305 IF(AC GE ADO) GC TO 307
AD = AD + 1.0
GO TO 308
307 STOP
END

TOTAL MEMORY REQUIREMENTS 47576 BYTES
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BIBLIOGRAPHY


