Optical Whispering Gallery Modes in Chalcogenide $\text{As}_2\text{Se}_3$ Microspheres

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Résumé

Cette thèse présente une analyse théorique et expérimentale du couplage des microsphères anisotropes en verre de chalcogénure. Les modes de galerie résonants (WGMS) de microsphères isotrope sont aussi présentés et la relation de dispersion TE et TM des WGMS est dérivée à partir des équations vectorielles électromagnétiques en coordonnées sphériques. Les équations de Maxwell peuvent être résolues en 2D pour la résolution en 3D de microsphères axisymétriques ou a symétrie rotationnelle isotrope. Les quatre premiers WGMS TE et TM sont simulés dans le modèle 2D en utilisant la méthode des éléments finis. La capacité de liaison, le volume modal V et facteur de qualité Q dépendent de l'indice de réfraction et de la taille de la microsphère.

On décompose une onde lumineuse en multiples fronts d'onde plan à l'intérieur de la microsphère; le coefficient de couplage entre une microsphère et un microfil est déterminé par le nombre de WGMS et la distance entre la microsphère et microfil. Le facteur de qualité Q est analysé; le couplage TE & TM de microsphères non linéaire est introduit à partir de simulations Matlab.

Des expériences de couplage pour la transmission, la réflexion et le port à fonction «drop» sont conduites. Les ondes lumineuses pour le couplage proviennent d’une source de lumière à large bande incohérent et d’une source laser étroite accordable à bande étroite. La lumière à large bande a donné des résultats à haute sensibilité tandis que le laser cohérent facilite la mesure de couplage.

En dernier lieu, les microsphères de chalcogénure ont été utilisées comme élément de rétroaction pour un milieu amplificateur. En comparaison avec des microsphères de silice, les microsphères de chalcogénure génèrent une réponse qui est plus instable due à la perturbation par les porteurs libres et l'activité thermique.
Abstract

Anisotropic chalcogenide microsphere is introduced for coupling theoretical analyzing and coupling experiment. Whispering Gallery Modes (WGMs) of isotropic microsphere is introduced and the TE &TM WGMs dispersion relationship is derived from electromagnetic vector equations in the spherical coordinate. The Maxwell equations can be solved in 2D model for the 3D model of axisymmetric or Rotational symmetry isotropic microsphere. First 4 TE&TM WGMs are simulated in 2D model using finite-element weak method. The binding capability, mode volume $V$ and quality factor $Q$ depend on the refractive index and size of the microsphere.

Plane wavefront light wave is assumed to propagate inside the microsphere; coupling coefficient is determined by WGMs numbers and the distance between the microsphere and the micro-taper. Coupling related Q factor is analyzed; TE & TM nonlinear microsphere coupling is introduced with Matlab simulation.

Chalcogenide coupling experiments for transmission, reflection and drop-port function are conducted. The light waves for coupling are broadband incoherent light source and narrowband tunable laser. Broadband light gave sensitive results while the coherent laser gave easy coupling capability.

The chalcogenide microsphere was used as a feedback element of an amplifying medium. Comparing with silica microsphere, chalcogenide microsphere’s response is more unstable due to free carriers perturbation and thermal activity.
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Introduction

Amorphous arsenic triselenide, $\text{As}_2\text{Se}_3$, is a chalcogenide glass that finds applications in infrared optics and nonlinear optics; it can transmit light at wavelengths between 0.9 µm and 17.2 µm. The highly nonlinear properties also make $\text{As}_2\text{Se}_3$ a promising transmission material for the mid-infrared. The optical microsphere is a low cost, easily fabricated resonator; it has potential to be used on micro-laser, sensing, and as a dynamic optical memory [1]. A nonlinear microsphere is more sensitive to optical energy density; it has the potential to be used for optical switching and mid-infrared sensing [2].

In this project, chalcogenide microspheres are fabricated by heating a chalcogenide fiber taper to the softening temperature and forms naturally following a spherical shape with the interplay of electrostatic surface tension. A perfectly round microsphere made of isotropic material can strongly confine light with specific wavelengths and propagation direction by different radius. The coupling in and out to/from the microsphere is performed using a micro-taper made of silica. The near electromagnetic field, evanescent field and far electromagnetic field light are coexisting in the research of Whispering Gallery Modes (WGMs), which indicates the resonant modes inside the microsphere. In this thesis, the concepts of near electromagnetic field and evanescent electromagnetic field are of utmost importance to understand the dynamics of coupling.
An isotropic dielectric microsphere has the quality to retain the light that has been injected into it from a coupling waveguide. A silica micro-taper is used to deliver and couple coherent or non-coherent light to a microsphere. The electromagnetic field in resonance is reinforced to generate constructive phase and amplitude interference forming resonating modes. An around perfect chalcogenide microsphere shares some common characteristics as general dielectric microsphere, such as WGMs, the capability to couple with a micro-taper.

In order to inject light energy into a dielectric microsphere, a waveguide such as micro-taper with strong evanescent field is required because far field electromagnetic energy could not be captured by a dielectric microsphere. The modes properties of the microsphere can be evaluated by measuring and analyzing the coupling characteristics and resulting interference pattern from the micro-taper output.

The WGMs are excited either by evanescent field or by the modes coupling between the WGMs. On the contrary, the excited WGMs will affect the coupling capability (coupling coefficient) of the specific mode. Discrete frequencies longitudinal modes can be found in transmission interference pattern, which shows almost equally separated deep troughs identified by the Free Space Range (FSR). The FSR can be evaluated by the Fabry–Pérot interferometer formula [3].

\[
\Delta \nu = \frac{c}{2\pi r_0 n_s}
\]
where $\Delta \nu$ is the separation of the discrete frequencies related to FSR, $c$ is the velocity of light in free space, $r_0$ is the microsphere’s radius, and $n_e$ is the refractive index of the dielectric microsphere.

Chalcogenide $\text{As}_2\text{Se}_3$ waveguide has its unique nonlinear optical properties compared with silica waveguide. Chalcogenide is transparent to infrared light but opaque to visible light. Chalcogenide is sensitive to ambient photo and thermal effects. The photocurrent of $\text{As}_2\text{Se}_3$ is the bias current of free carriers excited by different illumination intensities; thermal activation such as temperature change can also contribute to the photocurrent increment. Without biasing voltage, the free carriers can significantly change the chalcogenide waveguide’s optical characteristics [3]. The thermo-optic coefficient and thermal dilatation coefficient also affect the stability of the WGMs.
Chapter 1 Whispering Gallery Modes

Whispering Gallery Modes (WGMs) are the electromagnetic resonances inside an axisymmetric dielectric waveguide such as microsphere, micro-disc, micro-cylinder and micro-bottle. In an isotropic optical microsphere, the dielectric wall of the resonator prevents the trapped light from being scattered easily; instead most photons are constrained in the optical potential well along the propagation path. When phase matching condition is satisfied at coupling point, the electromagnetic field of WGMs can be reinforced by coupling in light and forming spatial coherent modes. A dielectric microsphere can accommodate many different discrete frequency resonate WGMs with respect to its dimension and materials.

WGMs can accommodate two polarization modes: Transverse Electric (TE) modes and Transverse Magnetic (TM) modes. The three mode numbers $n$, $l$ and $m$ are radial, angular and azimuthal mode number which identifies the modes of the WGMs. Radial number $n$ is the number of field maxima in the direction along the light wave propagation direction on the surface of microsphere. Angular number $l$ is the number of wavelengths that can exist in the optical length of the resonator. The value $l-m+1$ is the number of field maxima in the polar direction. Any two of the three mode numbers can determine the modes’ characteristics as the third one can be retrieved from the given two mode numbers. The fundamental mode is defined by $n=1$ and $m=1$;
Light propagating around a microsphere can be thought of as “zig-zagging” along an equator shown on figure 1-1, where \( u_r \) is the radial vector, \( u_\phi \) is the angular vector and \( u_\theta \) is the azimuthal vector, \( \beta_l \) is the WGMs propagation constant and it has angular, azimuthal and radial vector branches, \( \beta_m \) is the equator project of \( \beta_l \).

![Figure 1-1 Whispering Gallery Modes propagation path [4]](image)

The analytic solutions of WGMs can be gotten by solving spherical vector Helmhotz equation. As an arbitrary divergenceless vector field \( \vec{E} \) can be represented in terms of two scalar fields (Debye potentials) \( A \) and \( F \) according to \( \vec{E} = \vec{\varphi} + \nabla \times \vec{\varphi} \), where \( \vec{\varphi} \) is the angular momentum operator [5]; the Debye potentials are closely related to the radial components of the electric and magnetic fields. The spherical vector Helmhotz equation can be derived by first establishing an electromagnetic field of the microsphere.

First, define two radial vector potentials:
\[
\begin{align*}
\vec{A} &= r A_l^m(r, \theta, \varphi) \hat{r} \\
\vec{F} &= r F_l^m(r, \theta, \varphi) \hat{r}
\end{align*}
\]

where \( r \) is the radius, \( \hat{r} \) is the radial vector, scalar potentials \( A_l^m(r, \theta, \varphi) \) and \( F_l^m(r, \theta, \varphi) \) are Debye potentials. The two Debye potentials \( A_{1l}^m(r, \theta, \varphi), F_{1l}^m(r, \theta, \varphi) \) inside and \( A_{2l}^m(r, \theta, \varphi), F_{2l}^m(r, \theta, \varphi) \) outside a microsphere can be expressed by [6]:

\begin{align*}
\text{inside} \quad \begin{cases}
A_{1l}^m(r, \theta, \varphi) &= A_l^m j_l(n_s k_0 r) P_l^m(\cos \theta) \sin(m \varphi) \\
F_{1l}^m(r, \theta, \varphi) &= F_l^m j_l(n_s k_0 r) P_l^m(\cos \theta) \sin(m \varphi)
\end{cases} \\
\text{outside} \quad \begin{cases}
A_{2l}^m(r, \theta, \varphi) &= A_l^m h_l^{(2)}(k_0 r) P_l^m(\cos \theta) \sin(m \varphi) \\
F_{2l}^m(r, \theta, \varphi) &= F_l^m h_l^{(2)}(k_0 r) P_l^m(\cos \theta) \sin(m \varphi)
\end{cases}
\end{align*}

where subscript \( l \) is radial mode number, subscript 1 indicates inside of the microsphere while subscript 2 indicates outside of the microsphere and \( m \) is azimuthal mode number, \( r, \theta, \varphi \) are the radial, azimuthal and angular parameters. The wave number for the dielectric microsphere is \( k = n_s k_0 \), \( n_s \) is the refractive index of the microsphere, while the wave number outside the microsphere is \( k_0 \), \( j_l(n_s k_0 r) \) is the spherical Bessel function and \( h_l^{(2)}(k_0 r) \) is the spherical Hankel function; \( A_l^m \) and \( F_l^m \) are normalization factors; and \( P_l^m(\cos \theta) \) is the corresponding associated Legendre polynomial.

Then, the vector function for electric and magnetic field can be express as [5]

\[
\begin{align*}
\vec{E} &= \frac{1}{\varepsilon} \nabla \times \vec{F} + \frac{1}{j_0 \mu \varepsilon} \nabla \times (\nabla \times \vec{A}) \\
\vec{H} &= \frac{1}{\varepsilon} \nabla \times \vec{A} - \frac{1}{j_0 \mu \varepsilon} \nabla \times (\nabla \times \vec{F})
\end{align*}
\]

For an isotropic dielectric, the scalar electric field equations can be expressed as [6]
\[
\begin{aligned}
E_r &= \frac{1}{j\omega \mu \varepsilon} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) A_r \\
E_\theta &= \frac{1}{er \sin \theta} \frac{\partial F_r}{\partial \phi} + \frac{1}{j \omega \mu \varepsilon r} \frac{\partial^2 A_\phi}{\partial r \partial \phi} \\
E_\phi &= -\frac{1}{er \sin \theta} \frac{\partial F_r}{\partial \phi} + \frac{1}{j \omega \mu \varepsilon r} \frac{\partial^2 A_\phi}{\partial r \partial \phi}
\end{aligned}
\] (1 - 6a)

\[
\begin{aligned}
H_r &= -\frac{1}{j \omega \mu \varepsilon} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) F_r \\
H_\theta &= \frac{1}{\mu r \sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{j \omega \mu \varepsilon} \frac{\partial^2 F_r}{\partial \phi \partial \theta} \\
H_\phi &= -\frac{1}{\mu r} \frac{\partial A_r}{\partial \phi} - \frac{1}{j \omega \mu \varepsilon} \frac{\partial^2 F_r}{\partial \phi \partial \theta}
\end{aligned}
\] (1 - 7a)

and the scalar magnetic field equations can be expressed as

\[ [6] \]

\[
\begin{aligned}
\bar{u}_r \times \bar{E}_1 &= \bar{u}_r \times \bar{E}_2 \\
\bar{u}_r \times \bar{H}_1 &= \bar{u}_r \times \bar{H}_2
\end{aligned}
\] (1 - 8a)

where the vector product can be expanded by:

\[
\begin{align*}
\bar{u}_r \times \bar{E}_1 &= \frac{1}{r^2 \sin \theta} \begin{vmatrix}
\bar{u}_r & r \bar{u}_\theta & r \sin \theta \bar{u}_\phi \\
E_{1r} & rE_{1\theta} & r \sin \theta E_{1\phi}
\end{vmatrix} = E_{1\theta} \bar{u}_\phi - E_{1\phi} \bar{u}_\theta \\
\bar{u}_r \times \bar{E}_2 &= \frac{1}{r^2 \sin \theta} \begin{vmatrix}
\bar{u}_r & r \bar{u}_\theta & r \sin \theta \bar{u}_\phi \\
E_{2r} & rE_{2\theta} & r \sin \theta E_{2\phi}
\end{vmatrix} = E_{2\theta} \bar{u}_\phi - E_{2\phi} \bar{u}_\theta \\
\bar{u}_r \times \bar{H}_1 &= \frac{1}{r^2 \sin \theta} \begin{vmatrix}
\bar{u}_r & r \bar{u}_\theta & r \sin \theta \bar{u}_\phi \\
H_{1r} & rH_{1\theta} & r \sin \theta H_{1\phi}
\end{vmatrix} = H_{1\theta} \bar{u}_\phi - H_{1\phi} \bar{u}_\theta \\
\bar{u}_r \times \bar{H}_2 &= \frac{1}{r^2 \sin \theta} \begin{vmatrix}
\bar{u}_r & r \bar{u}_\theta & r \sin \theta \bar{u}_\phi \\
H_{2r} & rH_{2\theta} & r \sin \theta H_{2\phi}
\end{vmatrix} = H_{2\theta} \bar{u}_\phi - H_{2\phi} \bar{u}_\theta
\end{align*}
\] (1 - 9a to 1 - 9d)
Substitute formula (1–9a), (1–9b), (1–9c), (1–9d) into (1–8a)(1–8b) accordingly, it gives:

\[
E_{1\theta} \bar{\mu}_\varphi - E_{1\theta} \bar{\mu}_\theta = E_{2\theta} \bar{\mu}_\varphi - E_{2\theta} \bar{\mu}_\theta \quad (1–10a)
\]

\[
H_{1\theta} \bar{\mu}_\varphi - H_{1\theta} \bar{\mu}_\theta = H_{2\theta} \bar{\mu}_\varphi - H_{2\theta} \bar{\mu}_\theta \quad (1–10b)
\]

Formula (1–10a) and (1–10b) can be used to derive the dispersion relationship for TE and TM modes as shown in Appendix A. The dispersion relationship can be solved to get WGMs mode number using numerical methods.
Introduction to TE WGMs

In Transverse Electric (TE) modes, electric field is parallel to the surface of the microsphere. The dispersion relationship of TE WGMs is given by (Appendix A):

\[
\frac{h_{l-1}^{(2)}(k_0 r)}{h_l^{(2)}(k_0 r)} = n_s \frac{j_{l-1}(n_s k_0 r)}{j_l(n_s k_0 r)}
\]  \hspace{1cm} (1 - 11a)

If the refractive index of the surrounding medium is \( n_{sm} > 1 \), \( (1 - 11a) \) can be revised by:

\[
n_{sm} \frac{h_{l-1}^{(2)}(k_0 r)}{h_l^{(2)}(k_0 r)} = n_s \frac{j_{l-1}(n_s k_0 r)}{j_l(n_s k_0 r)}
\]  \hspace{1cm} (1 - 11b)

From a ray optics’ point of view, the light propagating inside the micro-sphere experiences many total internal reflections (TIR) due to a higher refractive index than the air. For As\(_2\)Se\(_3\) glass \( n \approx 2.80 \), the critical angle is around 20.93° for air while the critical angle is 31.19° for silica waveguide with refractive index \( n \approx 1.45 \). Supposing that the surface of microsphere is smooth everywhere and has a perfect round shape, then the light wave can be coupled to a microsphere either by far field coupling or evanescent field coupling.

Once the light is coupled into a microsphere from evanescent field, it has less opportunity to be scattered out to far field radiation; while if the light is radiated into a dielectric microsphere, the microsphere cannot retain these photons in the WGMs; as the photons will be either scattered out directly or absorbed by free carriers inside the dielectric microsphere. For a nonlinear dielectric microsphere such as As\(_2\)Se\(_3\)
microsphere, the Two-Photon Absorption (TPA) loss and Free Carrier Absorption (FCA) loss are the main loss mechanisms to be considered except for the scattering.

From the quantum optics’ point of view, the quasi-plane light waves are confined by the electromagnetic wall of dielectric microsphere; the boundary of a microsphere and the air forms an asymmetric potential well for the transverse wave vector $k$ of propagating light, which can accommodate many modes of discrete frequencies. If the light with suitable wave vector is tunneled into the microsphere, the periodical waveguide will regulate and reinforce the phase-match photons to a coherent wavefront; forward and backward propagating wavefront forms standing wave inside the WGMs. The remaining light which cannot form constructive interference patterns will finally be dissipated.

The near electromagnetic field retains most of the WGMs’ energy, which is surrounded by the evanescent field. Part of the evanescent field is leaked out of the microsphere physical boundary. Without considering photons scattering loss, the evanescent field satisfies the energy conservation law and is bound by the electromagnetic wall. The boundary between the evanescent field and surrounding air, where the electromagnetic field’s amplitude approaches zero, is considered as the electromagnetic wall of the WGMs. In a nonlinear microsphere, some WGMs energy is radiated out of the electromagnetic wall to the far field radiation due to the scattering caused by anisotropic molecular structure in the mode volume and optically and thermally excited free carriers. WGMs energy can also be coupled out to a waveguide adjoined by evanescent field coupling.
The evanescent field’s phase shift is polarization dependent. Given a planer wavefront, the TE modes have less phase shift for TIR, but TM modes experience many more times of Goos-Hänchen shift in one cycle around the microsphere’s equator. The Goos–Hänchen effect is that linearly polarized light undergoes a small shift under totally internal reflection; the phase shift is perpendicular to the direction of propagation in the plane containing the incident and reflected beams. Due to different phase sensitivities to the evanescent field, the TE WGMs are more suitable for telecom application while TM WGMs are more suitable for sensing.

The wave number along the microsphere can be decomposed to have 3 vectors: $k_r, k_\theta, k_\varphi$. Wave numbers $k_r, k_\theta$ contribute to the transverse kinetic energy, which makes its propagation trace looking like “zig-zagging” along the equator. Wave number $k_\varphi$ is the propagating direction wave vector; the higher portion energy in $k_\varphi$, the lower order mode(higher quality factor) can be formed and vice versa.
Introduction to TM WGMs

In Transverse Magnetic (TM) modes, the magnetic field is parallel to the surface of dielectric microsphere. The dispersion relationship of TM Whispering Gallery Modes is given by (Appendix A):

\[-l + k_0 r \frac{h_{l-1}^{(2)}(k_0 r)}{h_l^{(2)}(k_0 r)} = - \frac{l}{n_s^2} + \frac{k_0 r}{n_s} \frac{j_{l-1}(n_s k_0 r)}{j_l(n_s k_0 r)} \]  \quad (1 - 12)

TM WGMs can coexist with TE WGMs along the same microsphere equator; besides, WGMs of the same frequency can also coexist in the same equator. If the spatial overlap of WGMs is small, no interference pattern will be generated; but if the spatial overlap is long enough, the interference cannot be ignored especially for nonlinear microsphere. The large spatial overlap wavefront of same order TM and TE WGMs tend to form TM-like WGMs profiles by the phase and amplitude superposition.

Chalcogenide \( \text{As}_2\text{Se}_3 \) microsphere has higher refractive index, higher free photo carriers density and higher linear loss than silica microsphere. It integrates the properties of partial semiconductor and partial optical resonator. It is more sensitive to ambient photo and thermal effects and it gives more unpredicted results comparing with a silica microsphere.
Simulation of WGMs with Comsol

Numerical simulation can give us a visual description for the WGMs. Maxwell’s equations in axisymmetric resonator can be solved through the finite element method. As 3D model for microsphere is hard for initializing the evanescent field and specifying WGMs’ configuration due to enormous amount of 3D grids and complicated boundary conditions convergence; it is necessary to convert the axisymmetric 3D model to a transverse 2D model before further numerical processing.

For the isotropic microsphere, TE or TM WGMs can be described with only 2 mode numbers without azimuthal dependence, and then the 3D problem can be solved and simulated by the angular and radial dependent 2D finite-element method. The finite-element weak method is chosen as the numerical method because it has been deeply studied and is known to give accurate and fast convergence solution.

As chalcogenide is an anisotropic material, the isotropic material simulation result could be quite different from the real experimental results of As$_2$Se$_3$ glass. Even a perfect round anisotropic microsphere cannot guarantee the total internal reflection (TIR) everywhere due to complicated scattering mechanism, which causes phase-mismatching or losing coherent. However, the simulation can give out general sense of the WGMs in the linear optics. The simulating platform is COMSOL version 3.5 which can create the geometry model and boundary relation with the help from CAD tools.
Converting 3D model to 2D model

The chalcogenide microsphere in WGMs is assumed to be an isotropic axisymmetric 3D resonator. The 3D Maxwell’s equations need converted to 2D equations and then are to be solved using the finite-element weak form method. The weak form method uses an integral form of dot multiply between Laplacian operator and a test function on each grid instead of using PDE function itself, which requires a weaker continuity on the field variables. It has been proven to give more accurate solution for complex geometry problems.

The 3D WGMs model is azimuthally dependent, which can be described by time-independent solution of the form [7]:

\[
H(r) = e^{iM\theta}\{H^x(x, y), iH^\theta(x, y), H^y(x, y)\}
\]

(1 – 13)

where \(M\) is the mode’s azimuthal mode order, and \(\theta\) is the azimuthal coordinate shown in figure 1-1. This model describes the ideal resonators which have low-loss dielectric mode waveguide space bounded by either perfect electric walls or perfect magnetic walls.

The ‘Laplacian’ weak term is derived to be [7]

\[
(\nabla \times \vec{H}^\ast) \cdot (\nabla \times H) = \frac{(\frac{A}{2} + B + xC)}{\varepsilon \varepsilon_\perp}
\]

(1 – 14)

where
\[ A = \left\{ \varepsilon_{\perp} \left( \vec{H}^\theta H^\theta - M(\vec{H}^\theta H^x + H^\theta \vec{H}^x) + M^2 \vec{H}^x H^x \right) + \varepsilon_{\parallel} M^2 \vec{H}^y H^y \right\} \quad (1 - 15a) \]

\[ B = \varepsilon_{\perp} \left[ (H^\theta - MH^x) \frac{\partial \vec{H}^\theta}{\partial x} + (\vec{H}^\theta - M\vec{H}^x) \frac{\partial H^x}{\partial x} \right] - \varepsilon_{\parallel} M \left( \vec{H}^y \frac{\partial H^\theta}{\partial y} + H^y \frac{\partial \vec{H}^\theta}{\partial y} \right) \quad (1 - 15b) \]

\[ C = \left\{ \varepsilon_{\perp} \frac{\partial \vec{H}^\theta}{\partial x} \frac{\partial H^x}{\partial x} + \varepsilon_{\parallel} \left[ \left( \frac{\partial \vec{H}^y}{\partial x} - \frac{\partial H^x}{\partial y} \right) \left( \frac{\partial H^y}{\partial x} - \frac{\partial H^x}{\partial y} \right) + \frac{\partial \vec{H}^\theta}{\partial y} \frac{\partial H^\theta}{\partial y} \right] \right\} \quad (1 - 15c) \]

\( \vec{H} \) is the Galerkin 'test function' of \( H \), \( \varepsilon_{\parallel} \) and \( \varepsilon_{\perp} \) are the material's relative permittivities in the axial direction and perpendicular plane.

The 'Penalty' weak term is shown to be [7]

\[ \alpha (\nabla \cdot \vec{H}^*) (\nabla \cdot H) = \alpha \left\{ \frac{\partial}{\partial x} + E + xF \right\} \quad (1 - 16) \]

where

\[ D = M^2 \vec{H}^\theta H^\theta - M(\vec{H}^\theta H^x + H^\theta \vec{H}^x) + \vec{H}^x H^x \quad (1 - 17a) \]

\[ E = (H^x - MH^\theta) \left( \frac{\partial H^x}{\partial x} + \frac{\partial \vec{H}^y}{\partial y} \right) + (\vec{H}^x - M\vec{H}^\theta) \left( \frac{\partial H^x}{\partial x} + \frac{\partial H^y}{\partial y} \right) \quad (1 - 17b) \]

\[ F = \left( \frac{\partial \vec{H}^x}{\partial x} + \frac{\partial \vec{H}^y}{\partial y} \right) \left( \frac{\partial H^x}{\partial x} + \frac{\partial H^y}{\partial y} \right) \quad (1 - 17c) \]

The temporal weak term is given by [7]

\[ \vec{H}^* \frac{\partial^2 H}{\partial x^2} = \frac{\vec{H}^x \frac{\partial^2 H^x}{\partial t^2} + \vec{H}^\theta \frac{\partial^2 H^\theta}{\partial t^2} + \vec{H}^y \frac{\partial^2 H^y}{\partial t^2}}{c^2} x = -f^2 \chi \frac{\vec{H}^x H^x + \vec{H}^\theta H^\theta + \vec{H}^y H^y}{c^2} \quad (1 - 18) \]

Since no term on above equations depends on azimuthal coordinate \( \theta \), the 3D problem can be solved with 2D coordinates only.
Boundary conditions for weak method

The continuity of the electric field gives the electric-wall boundary conditions [7]:

\[
\begin{align*}
H^x n_x + H^y n_y &= 0 \\
\frac{\partial H^x}{\partial y} - \frac{\partial H^y}{\partial x} &= 0 \\
\left( H^\theta - H^x M + x \frac{\partial H^\theta}{\partial x} \right) n_x - \left( H^y M - x \frac{\partial H^\theta}{\partial y} \right) n_y &= 0
\end{align*}
\] (1 - 19a) (1 - 19b) (1 - 19c)

Also, the continuity of the magnetic field gives the magnetic-wall boundary conditions [7]:

\[
\begin{align*}
H^y n_x - H^x n_y &= 0 \\
H^\theta &= 0 \\
\left( H^\theta - H^x M + x \frac{\partial H^\theta}{\partial x} \right) n_x + \left( H^y M - x \frac{\partial H^\theta}{\partial y} \right) n_y &= 0
\end{align*}
\] (1 - 20a) (1 - 20b) (1 - 20c)

No scattering loss is considered for this model. All the energy is assumed to fall in near electromagnetic field and evanescent electromagnetic field. The electromagnetic wall is the binding edge for WGMs.
COMSOL simulation parameters and setup

In the finite-element weak method for WGMs, neither the linear loss nor the scattering loss is considered; only the stable condition linear optical properties are to be investigated. The mode number is set to \( M = 90 \) to get the maximum coupling between the microsphere electric field and outside electric field and the working wavelength is set to 1550nm. All the parameters and configurations for simulation are list as below:

**Constants Table**

<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>299792458</td>
<td>2.997925e8</td>
<td>speed of light (m/s)</td>
</tr>
<tr>
<td>( k )</td>
<td>( 2\pi/c )</td>
<td>2.095845e-8</td>
<td>Wave number (s/m)</td>
</tr>
<tr>
<td>( fc )</td>
<td>( k^2 )</td>
<td>4.392566e-16</td>
<td>constant used internally --do not modify</td>
</tr>
<tr>
<td>( alpha )</td>
<td>1.0</td>
<td>1</td>
<td>penalty coefficient on Div H</td>
</tr>
<tr>
<td>( M )</td>
<td>90</td>
<td>90</td>
<td>azimuthal mode order</td>
</tr>
<tr>
<td>( delta_e )</td>
<td>0.0</td>
<td>0</td>
<td>fractional increment</td>
</tr>
<tr>
<td>( e1 )</td>
<td>( n_{as2}se^{3\delta_e(1+delta_e)} )</td>
<td>7.84</td>
<td>relative permittivity of ( u_{isotropic_dielectric_1} )</td>
</tr>
<tr>
<td>( e2 )</td>
<td>1.0</td>
<td>1</td>
<td>ditto for ( u_{isotropic_dielectric_2} )</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
<td>Notes</td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------</td>
<td>--------------------------------------</td>
<td></td>
</tr>
</tbody>
</table>
| $\Delta_{e\perp 1}$ | $0 \times 10^{-3}$ | 0  
| $e_{\perp 1}$   | $9.2725 \times (1 + \Delta_{e\perp 1})$ | relative permittivity of uniaxial_dielectric_1 perpendicular to cylindrical axis |
| $\Delta_{e\parallel 1}$ | $0 \times 10^{-3}$ | 0  
| $e_{\parallel 1}$ | $11.3486 \times (1 + \Delta_{e\parallel 1})$ | relative permittivity of uniaxial_dielectric_1 parallel to cylindrical axis |
| $e_{\perp 2}$   | 1.0           | relative permittivity of uniaxial_dielectric_2 perpendicular to cylindrical axis |
| $e_{\parallel 2}$ | 1.0           | ditto but parallel to cylindrical axis |
| $mf$            | c/wavelength  | 1.934145e14 match frequency |
| $ttgH$          | 1             | 1  
| $ttgE$          | 0             | 0  
| rectangle_mf    | c/wavelength  | 1.934145e14 |
| circle_mf       | c/wavelength  | 1.934145e14 |
**Scalar Expressions Table**

<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DivH</td>
<td>( (Hrad-Hazi*M+(Haxiz+Hradr)*r)/r )</td>
<td>divergence of magnetic field (should be zero!)</td>
</tr>
<tr>
<td>Drad</td>
<td>( (Haxi<em>M-Haziz</em>r)/r )</td>
<td>radial component of electric displacement</td>
</tr>
<tr>
<td>Dazi</td>
<td>( -Haxir+Hradz )</td>
<td>azimuthal component of electric</td>
</tr>
<tr>
<td>Symbol</td>
<td>Expression</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>-------------</td>
</tr>
<tr>
<td>$D_{axi}$</td>
<td>$(H_{azi} - H_{rad} * M + H_{zir} * r) / r$</td>
<td>axial component of electric displacement</td>
</tr>
<tr>
<td>$E_{rad}$</td>
<td>$Drad / erel$</td>
<td>radial component of electric field strength</td>
</tr>
<tr>
<td>$E_{azi}$</td>
<td>$Dazi / erel$</td>
<td>azimuthal component of electric field strength</td>
</tr>
<tr>
<td>$E_{axi}$</td>
<td>$Daxi / erel$</td>
<td>axial component of electric field strength</td>
</tr>
<tr>
<td>comment</td>
<td>$1$</td>
<td>elemental volume = $2 * pi * r * d_r * d_{phi}$</td>
</tr>
<tr>
<td>$Mag_{Azi}^2$</td>
<td>imag($H_{azi}$)$^2$</td>
<td>azimuthal component of magnetic field squared</td>
</tr>
<tr>
<td>$Mag_{Trans}^2$</td>
<td>real($H_{azi}$)$^2$ + real($H_{rad}$)$^2$</td>
<td>transverse component of magnetic field squared</td>
</tr>
<tr>
<td>$Elec_{Azi}^2$</td>
<td>real($E_{azi}$)$^2$</td>
<td>azimuthal component of electric field squared</td>
</tr>
<tr>
<td>$Elec_{Trans}^2$</td>
<td>imag($E_{axi}$)$^2$ + imag($E_{rad}$)$^2$</td>
<td>transverse component of electric field squared</td>
</tr>
</tbody>
</table>

Mesh for finite element weak method
Variables

- Dependent variables: $H_{rad}$, $H_{azi}$, $H_{axi}$, $H_{rad\_t}$, $H_{azi\_t}$, $H_{axi\_t}$
- Shape functions: $shlag(2,'H_{rad}')$, $shlag(2,'H_{azi}')$, $shlag(2,'H_{axi}')$
- Interior boundaries are not active.

Table of Boundary Settings

<table>
<thead>
<tr>
<th>Boundary</th>
<th>1-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>name</td>
<td>tangential_H</td>
</tr>
<tr>
<td>Constraint (constr = 0) (constr)</td>
<td>{H_{rad}_nr+H_{axi}_nz;0;0}</td>
</tr>
<tr>
<td>Constraint force (constrf)</td>
<td>{test(H_{rad}_nr+H_{axi}_nz);0;0}</td>
</tr>
</tbody>
</table>
Comsol simulation results for TE & TM WGMs

In total, 8 desired eigenvalues are set for this simulation, each eigenvalue corresponds to a unique WGMs from the fundamental mode to the fourth order mode with a polarization either in TE or TM. The microsphere radius is set to $r=60$ um.

From the TM modes definition, the magnetic fields are parallel to the surface; and figures 1-3, 1-7, 1-11, and 1-15 show from the fundamental TM mode to the fourth order TM mode. The lower TM mode has the shortest path along the equator, smaller transverse wave number, and easy to bind; and it is same for the TE modes, whose electric fields are parallel to the surface, figures 1-5, 1-9, 1-13, and 1-17 show from the fundamental TE mode to the fourth order TE mode.

For the first eigenvalue= $2.783 \times 10^{13}$, figures 1-3 and 1-4 show the electric field distribution of the fundamental TM mode; the profile of the fundamental TM mode is composed of the near electromagnetic field (red color on figure 1-3) with one maximum and decaying evanescent electromagnetic field around (cyan color on figure 1-3). The electric field inside the mode is perpendicular to the microsphere’s surface and has exponential decay from the near field center to evanescent field. Part of evanescent field lies outside the microsphere’s physical boundary.
For the second eigenvalue $= 2.809 \times 10^{13}$, figures 1-5 and 1-6 show the electric field distribution of the fundamental TE mode; the profile of the fundamental TE mode is composed of the near electromagnetic field (red on figure 1-5) with two opposite maxima and decaying evanescent electromagnetic field around (cyan on figure 1-5). The electric field inside the mode is parallel to the surface of the microsphere and has exponential decay from the two opposite peaks of near field to evanescent field. Also, part of evanescent field is outside the microsphere’s physical boundary. The profile of the fundamental TE mode looks same as that of the fundamental TM mode except that TE mode has two opposite peaks.
For the third eigenvalue $= 2.812 \times 10^{13}$, figures 1-7 and 1-8 show the electric field distribution of the second order TM mode; the profile of the mode is composed of the near electromagnetic field (red color on figure 1-7) with two opposite maxima and decaying evanescent electromagnetic field around (cyan color on figure 1-7) with clear envelop pattern for the two near field peaks. The electric field inside the mode is perpendicular to the surface of the microsphere and has exponential decay from the center of near field to evanescent field. Part of evanescent field lies outside the microsphere’s physical boundary, and evanescent field at the equator tightly wrapping around near fields gives less spatial field outside the microsphere than the fundamental TM mode.

For the fourth eigenvalue $= 2.838 \times 10^{13}$, figures 1-9 and 1-10 shows the electric field distribution of the second order TE mode; the profile of the mode is composed of the near electromagnetic field (red on figure 1-9) with two adjoined maxima and decaying evanescent electromagnetic field around (cyan on figure 1-9) with clear envelop pattern for the two near field peaks. The electric field of the near field is parallel to the surface.
of the microsphere and has exponential decay from the adjoined peaks of near field to evanescent field. Part of evanescent field is outside the microsphere’s physical boundary. The profile of the second order TE mode looks same as that of the second order TM mode except that the TE mode’s two near field maxima are on the same side of the profile.

![Figure 1-9 The 2nd order TE mode’s profile (2D)](image1)
![Figure 1-10 The 2nd order TE mode’s profile (3D)](image2)

For the fifth eigenvalue $= 2.841 \times 10^{13}$, figures 1-11 and 1-12 show the electric field distribution of the third order TM mode; the profile of the mode is composed of the near electromagnetic field (red color on figure 1-11) with three staggered maxima that two odd peaks are opposite to the even peak and decaying evanescent electromagnetic field around (cyan color on figure 1-11) with clear envelop pattern for the three near field peaks. The electric field of the near field is perpendicular to the surface of the microsphere and has exponential decay from the center of near field to evanescent field. Part of evanescent field lies outside the microsphere’s physical boundary, and evanescent field at the equator tightly wrapping around near fields occupies more spatial field outside the microsphere than the second TM mode.
For the sixth eigenvalue = $2.867 \times 10^{13}$, figures 1-13 and 1-14 show the electric field distribution of the third order TE mode; the profile of the mode is composed of the near electromagnetic field (red on figure 1-13) with two odd opposite maxima versus one relatively weak even peak and decaying evanescent electromagnetic field around (cyan on figure 1-13) with clear envelop pattern for the two near field peaks. The electric field of the near field is parallel to the surface of the microsphere and has exponential decay from the adjoined peaks of near field to evanescent field. Part of evanescent field is outside the microsphere’s physical boundary. The profile of the third order TE mode looks same as that of the third order TM mode except that the TE mode’s two odd near field maxima are on the same side of the profile.
For the seventh eigenvalue $= 2.871 \times 10^{13}$, figures 1-15 and 1-16 give out the electric field distribution of the fourth order TM mode; the profile of the mode is composed of the near electromagnetic field (red color on figure 1-15) with four staggered maxima that two odd peaks are opposite to the other even peaks and decaying evanescent electromagnetic field around (cyan color on figure 1-15) with clear envelop pattern for the four near field peaks. The electric field of the near field is perpendicular to the surface of the microsphere and has exponential decay from the center of near field to evanescent field. Part of evanescent field lies outside the microsphere’s physical boundary, and evanescent field at the equator tightly wrapping around near fields occupies less spatial field outside the microsphere than the third order TM mode.

![Figure 1-15](image1.png) The 4th order TM mode's profile (2D)  ![Figure 1-16](image2.png) The 4th order TM mode's profile (3D)

For the eighth eigenvalue $= 2.897 \times 10^{13}$, figures 1-17 and 1-18 give out the electric field distribution of the fourth order TE mode; the profile of the mode is composed of the near electromagnetic field (red on figure 1-17) with two strong side maxima opposite to the other two relatively weak inner peaks and decaying evanescent
electromagnetic field around (cyan on figure 1-17) with clear envelop pattern for the two near field peaks. The electric field of the near field is parallel to the surface of the microsphere and has exponential decay from the adjoined peaks of near field to evanescent field. Part of evanescent field is outside the microsphere’s physical boundary. The profile of the fourth order TE mode looks same as that of the fourth order TM mode, but that the TE mode’s two side near field maxima hold more energy than its inner near fields and its four peaks are symmetric.

![Figure 1-17 The 4\(^{th}\) order TE mode’s profile (2D)](image1)

![Figure 1-18 The 4\(^{th}\) order TE mode’s profile (3D)](image2)

The 8 simulation results demonstrate the some possible WGMs’ profiles for an isotropic microsphere without considering all kinds of losses. The real experiment results are more complicated than the simulation results due to fact of anisotropic material scattering, thermal noise and free carriers absorption.
Radial potential for binding WGMs

From the simulation results of the first four TE WGMs and TM WGMs, the energy distribution due to different transverse wave numbers $k_r, k_\theta$ is clearly to see. A higher vector portion $k_\varphi$ of the wave number $k$ is suitable for forming fundamental WGMs. TM WGMs avoid forming symmetric electric fields in the profile while TE WGMs’ profile is symmetric. WGMs with less evanescent field distribution at the equator line such as the second order WGMs have smaller coupling opportunity with external waveguide. Also, the larger effect area (most of red color in the simulating profiles) with high density side peaks’ modes have small probability to form due to the WGMs related coupling rate relationship, which will be introduce in Chapter 2. This simulation gives the generic WGMs’ profiles for the isotropic microsphere, but is doesn’t mean that these modes coexist for certain.

The reason why the WGMs have a higher probability to appear at the surface of microsphere can be explained by the one dimensional Helmholtz equation. In spherical coordinates, the radial part Helmholtz equation can be separated as

$$r^2 \frac{d^2 R}{dr^2} + 2r \frac{dR}{dr} + [k^2 r^2 - l(l + 1)] R = 0$$

(1 - 4)

where $k$ is wave number. If set $\frac{dR}{dr} = 0$ , $R$ will be confined in minima or maxima of the potential field. Without the first order part, the equation is evaluated as:

$$r^2 \frac{d^2 R}{dr^2} + [k^2 r^2 - l(l + 1)] R = 0$$

(1 - 5)

After equation (1-5) is divided by $r^2$, it gives:
\frac{d^2R}{dr^2} + \left[ k^2 - \frac{l(l+1)}{r^2} \right] R = 0 \quad (1-6)

Introduce \( ER = k^2 n(r)^2 R \) to both sides of equation, it gives potential equation for different wave number \( k \):

\frac{d^2R}{dr^2} + \left[ k^2 n(r)^2 + k^2 - \frac{l(l+1)}{r^2} \right] R = ER \quad (1-7)

Move the right of the equation to the left side and give:

\frac{d^2R}{dr^2} + \left[ -E - \left( \frac{l(l+1)}{r^2} - k^2 (1 + n(r)^2) \right) \right] R = 0 \quad (1-8a)

where

\[ V(r) = \left( \frac{l(l+1)}{r^2} - k^2 (1 + n(r)^2) \right) \quad (1-8b) \]

is the potential energy for radial field equation.

The radial field solution is given by

\[ R(r) = N_r \begin{cases} \alpha_j(kn_s r), & r \leq r_0 \\ b e^{-\alpha_s (r-r_0)}, & r > r_0 \end{cases} \quad (1-9a) \]

where

\[ N_r = \frac{2}{r_0^2 (1 + \frac{1}{\alpha_s r_0}) J_0^2(kn_s r_0) - \frac{1}{2} J_{l-1}(kn_s r_0) J_{l+1}(kn_s r_0)} \quad (1-10) \]

\[ \alpha_s = \sqrt{\beta_s^2 - k^2 n_0^2} \quad (1-11) \]
\[
\beta_l = \frac{\sqrt{l(l+1)}}{r_0} \tag{1-12}
\]

\(N_r\) is the normalization constant for radial function \(R(r)\), \(a, b\) are constants which to be determined by formulas (A-29) and (A-34) for TE mode and formulas (A-52) and (A-57) for TM mode. \(\beta_l\) is the propagation constant for Whispering Gallery Modes.

From the mathematical approximation above, the potential field along the radial direction of a dielectric microsphere for a given wave number \(k\) can be plotted with the formula (1-8a).

For a given chalcogenide microsphere with \(radius = 67.5\text{um} \ l=60\), figure 1-19 shows that an electromagnetic potential well is formed inside the microsphere near the surface when the microsphere’s refractive index is much higher than the air’s. It can
accommodate light with suitable transverse wave number. In this potential well, near field potential $R$ has the maximum value and the highest energy density with discrete frequencies as shown in the simulation results. The red color near field is constructed by balancing the energy coupled in and out through evanescent field to form WGMs resonating; and there is also big scattering loss in real experiment. The transverse wave number provides the tunneling capability from the evanescent electric field into the potential well.
Mode Volume V and effective mode area

The *mode volume* is used to describe energy density of optical spatial mode in three dimensions, and the definition of *mode volume* is given by [7]:

\[
V_{\text{mode}} = \frac{\iiint \epsilon |E|^2 \, dv}{\max[\epsilon |E|^2]}
\]  \hspace{1cm} (1-13)

where \( \epsilon \) is the permittivity and \( E \) is the electric field inside the mode.

For a dielectric microsphere, the *mode volume* consideration consists of both near electromagnetic field and evanescent field but not necessary far field. Plus, as the evanescent field is decaying exponentially away from the near field center, the evaluated integration can even ignore the evanescent field. The transversal 2 dimensional profile of the *mode volume* is called the *effective mode area*, which is an important parameter to evaluate the energy density in the nonlinear optical investigation, such as self phase modulation (SPM) and cross phase modulation (XPM). The microsphere WGMs’ *mode volume* had been studied to follow the quadratic dependence on the microsphere radius, \( V_{\text{mode}} \propto r_0^{(11/12)} \) [8], approximately and the *effective mode area* is linear dependent of the microsphere radius \( A_{\text{eff}} \propto r_0 \) [8].

Due to the un-perfect TIR (binding capability) on the surface of microsphere, part of the energy in near field of a dielectric microsphere will dissipate into far field sooner or later. The *mode volume* can only describe the steady state with stable light source, while transient state is better to be described by the *quality factor*. 
Quality Factor Q

The quality factor $Q$ is the ratio of the energy stored in the resonator to the energy supplied by the source. It describes the temporal characteristics of a resonator.

$$Q \equiv \frac{\omega}{P_{supplied}} = \frac{\omega}{\Delta \omega} = \frac{\lambda_0}{\Delta \lambda_{FWHM}} \quad (1 - 14)$$

For a chalcogenide microsphere, the Q factor is mainly determined by: chalcogenide material absorption, scattering losses (due to particles or irregular component), chalcogenide surface absorption losses (due to humidity or oxidization), WGMs loss and external coupling losses (tunneling loss when using another waveguide to inject energy into microsphere’s near field)

$$\frac{1}{Q_{total}} = \frac{1}{Q_{material}} + \frac{1}{Q_{scattering}} + \frac{1}{Q_{surface}} + \frac{1}{Q_{coupling}} + \frac{1}{Q_{WGM}} \quad (1 - 15)$$

The material of anisotropic dielectric has larger probability to scatter photons out of the mode volume to the far electromagnetic field. Chalcogenide glass is sensitive to humid environment and is easy to be oxidized in air. The surface refractive index change will reduce the mode binding capability of the dielectric-air surface. The potential change is simulated and shown on figures 1-20 and 1-21.
For a chalcogenide microsphere with \( \text{radius} = 67.5\, \text{um} \), \( l = 60 \), \( \text{oxidized surface depth} = 6.75\, \text{um} \), if the surface refractive index is lower than that of chalcogenide then the potential well has two levels, which can accommodate more than one WGMs for specific mode number \( l \) and cause mode coupling to reduce the quality factor for the desired mode. On the contrary, with the same dimension number but a higher refractive index on the surface, there will be deeper potential well at the surface; the potential well can also accommodate different level modes and makes it easy to couple the energy into the near electromagnetic field from evanescent field. At the end, this kind of mode coupling will lower the quality factor for the desired mode.

The Q factor of material loss can be calculated using the formula \( Q_{\text{material}} = \frac{2\pi n_{\text{eff}}}{\alpha} \); for chalcogenide microsphere, the linear absorption \( \alpha = 1.12/\text{m} \), \( Q_{\text{material}} = 10134170 \).

The WGMs loss is because of imperfect TIR for the curved dielectric surface of the microsphere. Not all the energies for a specific mode are reflected back to the mode volume, some of them are dissipated to the far field radiation. The quality factor due to the WGMs loss was first derived by Weinstein: [9]
where

\[ Q_{WGM} = \frac{1}{2} \left( l + \frac{1}{2} \right) \left( \frac{m}{2} \right)^{1/2} t_n^0 \left(\frac{m^2 - 1}{\sqrt{m^2 - 1}}\right) \frac{e^{i2T_n i \sqrt{m^2 - 1}}}{m^{1/2}} \]  \hspace{1cm} (1 - 16)

\[ T_n = \left( l + \frac{1}{2} \right) \left( \cosh^{-1}(n) - \frac{\sqrt{m^2 - 1}}{m} \right) + t_n^0 \left( l + \frac{1}{2} \right)^{1/2} \sqrt{m^2 - 1} \]  \hspace{1cm} (1 - 17)

and \( t_n^0 \) is the Airy function zero, \( m \) is the relative index of refraction.

The Q factor due to coupling, where the mode energy tunnels in and out the potential well, is to be discussed in Chapter 2.
Chapter 2 Coupling theory for micro cavities

Introduction to coupling theory

Micro-taper is a one dimensional waveguide with micron level diameter fabricated by heating a single mode optical fiber above its transition temperature and being stretched the desired dimension. The propagating modes of the micro-taper can leak out of the cladding at the taper’s waist due to weak binding capability, and partial mode volume and evanescent modes are outside of its cladding. There could be higher order HE modes existed along the micro-taper’s waist; TE and TM modes experience different phase-shifting along the micro-taper’s waist.

As the evanescent field is capable of extending away from the physical boundary of micro-taper, the evanescent modes can be used to tunnel energy to a desired optical potential well. Phase-matching condition between different waveguides further determines that only certain modes are suitable to be coupled into a microsphere; and only discrete frequency lights can resonate inside the microsphere.

Chalcogenide microsphere is a 10–100 micron radius glass sphere, which is fabricated by heating a chalcogenide taper’s tip to above its transition temperature while maintaining the temperature to allow the melted tip forming a sphere by the surface tension. The WGMs’ near field along the equator is close to surface of the dielectric microsphere and some of its mode volume and evanescent field extend outside of the
microsphere physical boundary, which makes it possible for the light in WGMs to tunnel into a micro-taper.

If two optical waveguides which have both external evanescent fields are approached close enough, the high energy photons with strong transverse wave vector branch can easily tunnel through the air gap barrier and propagate into the other waveguide; and if the phase matching condition is satisfied for the transverse wave vectors in the air gap, the penetrated wavefront will keep coherent along the WGMs’ path for a while.

For an anisotropic microsphere, the TIR capability around the surface is worse than that of an isotropic microsphere; some photons are scattered out of the microsphere, and some photons are reflected backward along the mode volume with different attenuation. The high density scattering can cause the light losing coherent and phase-mismatching at the coupling gap.
Phase Matching Condition at the Coupling Point

The phase matching condition in a uniform media can be expressed as:

\[ \phi_a = A_1 e^{ikx}, \phi_b = A_2 e^{i(k+G_n)x} \text{ where } G_n = 2n\pi \]  \hspace{1cm} (2 - 1)

The phase matching works effectively with the distance \( G_n = 2n\pi \) between tunneling points of different waveguides for a specific wave number. Within the evanescent fields of two different kinds of dielectric: silica and chalcogenide, the phase-matching condition needs the two waveguides’ evanescent field approached close enough so that the light with enough transverse wave vector branch can penetrate to the other waveguide. The distance \( G_n \) determines the wave numbers to be coupled.

As both the microsphere and the silica micro-taper are exposed in the same media such as the air, the phase-matching actually happens in the waveguide’s gap; so that nonlinear phenomena such as Four Wave Mixing (FWM) and Two Photon Absorption (TPA) are not supposed to happen if the gap is filled with linear optical media.

If the light satisfies the phase-matching condition at coupling point and WGMs resonating condition inside a microsphere then the wavefront will experience at least \( \pi \) phase shift at the coupling point; and it is called the shifted ray, while the uncoupled light is called bullet ray. The shifted ray around the microsphere destructively interferes with the bullet ray in the micro-taper. Given a broadband light source, the interference can generate regularly longitudinal transmission trough patterns of discrete wavelengths, which corresponds to resonating WGMs frequencies.
For a linear optical microsphere, the interference between shifted ray and bullet ray depends on not only the $\pi$ phase shift at the coupling point but also the polarization dependent \textit{Goos-Hänschen} shift along the propagation path. TM WGMs experience more phase shift with respect to TE WGMs. As the evanescent modes of a micro-taper are mainly HE$_{11}$ modes, then TM and TE modes have same opportunities to be coupled into TM and TE WGMs; and shifted ray from TM and TE WGMs will be coupled back to the micro-taper’s TM and TE modes with different phase shift.

Nonlinear microsphere may experience the phase shift due to self-phase-modulation (SPM) and cross-phase-modulation (XPM). Plus chemical unstable dielectric microsphere suffers from surface WGMs coupling due to surface oxidization and humid atmosphere.

Coherent broad band light source makes it easy to couple a microsphere and a micro-taper as long as the wavefront is coherent and the distance between the microsphere surface and the micro-taper’s core is close enough. For an anisotropic microsphere, using coherent broad band light source can easily get coupled but cannot guarantee high quality factor.
Planar wave presumption

The planar wavefront is hard to explain the WGMs’ scattering loss. However, it is easy to understand the coupling by planar wave assumption. Assuming that the guided light propagating along the micro-taper and WGMs are all planar wave, then light can be expressed by equations without considering linear losses:

\[
\begin{align*}
\frac{dE_T(z)}{dz} &= -i\beta_T E_T(z) \\
\frac{dE_S(z)}{dz} &= -i\beta_S E_S(z)
\end{align*}
\]

where $E_T(z)$ and $\beta_T$ are the electrical field, propagation constant in the micro-taper and $E_S(z)$ and $\beta_S$ are the corresponding parameters in the microsphere.

The general solutions to these equations are:

\[
\begin{align*}
E_T(z) &= E_T(0) e^{-i(\beta_T z + \varphi_T)} = E_T e^{-i(\beta_T z + \varphi_T)} \\
E_S(z) &= E_S(0) e^{-i(\beta_S z + \varphi_S)} = E_S e^{-i(\beta_S z + \varphi_S)}
\end{align*}
\]

where $E_T$ and $\varphi_T$ are the amplitude, phase of electric field amplitude in the taper while $E_S$ and $\varphi_S$ are counterparts in the microsphere.

The planar wavefront approximation makes the WGMs and the coupling mechanism easy to understand.
**Coupling coefficient**

The coupling coefficient is the capability to coupling the energy from a micro-taper to a microsphere, the value of coupling coefficient is always between 0 and 1. Figure 2-1 shows the profile of two coupled microsphere and micro-taper. The micro-taper’s radius is \( r_0 \) and the microsphere’s radius is \( R_0 \); and \( n_0, n_s, n_f \) are refractive indexes of air, microsphere and micro-taper correspondently. \( D \) is the distance from the micro-taper’s core to the microsphere’s surface.

![Figure 2-1 Profile of a microsphere coupling with a micro-taper](image)

The evanescent mode’s electric field of fiber taper can be written as [10]

\[
E = N_f e^{-\gamma_f (r-r_0)} \quad \text{when} \quad r > r_0 
\]  

(2 – 6)

\[
N_f = \frac{\alpha_f I_0(k_f r_0)}{V_f \sqrt{\pi} J_1(k_f r_0)} 
\]  

(2 – 7)
\[
\begin{aligned}
    k_f &= \sqrt{k^2 n_f^2 - \beta_f^2} \quad \text{(2-8)} \\
    \alpha_f &= \sqrt{\beta_f^2 - k^2 n_{cl}^2} \quad \text{(2-9)} \\
    \gamma_f &= \frac{\kappa_0(\alpha_f r_0)}{\kappa_1(\alpha_f r_0)} \quad \text{(2-10)} \\
    \psi_f &= kr_0 \sqrt{n_f^2 - n_{cl}^2} \quad \text{(2-11)}
\end{aligned}
\]

where \( n_f, n_{cl} \) are the refractive index of the fiber’s core and cladding, \( r_0 \) is the taper fiber’s radius, \( r \) is the radius of point whose evanescent field to be evaluated, \( k_f \) is the transverse wave vector, \( \alpha_f \) is the attenuation coefficient of micro taper’s evanescent field, \( \psi_f \) is the normalized frequency, \( \gamma_f \) is the decay constant at the core boundaries, \( N_f \) is the normalization constant selected for the integral of \( E^2 \) over the transverse area is unity, \( J_0, J_1 \) are the Bessel function of zero and first order and \( K_0, K_1 \) are the modified Hankel function of zero and first order. The propagation constant \( \beta_f \) can be determined with the micro-taper’s dispersion relationship by substituting formula (2-8) and (2-9) into it [10]:

\[
\frac{k_f J_0(k_f r_0)}{J_0(k_f r_0)} = \frac{\kappa_0(\alpha_f r_0)}{\kappa_0(\alpha_f r_0)} \quad \text{(2-12)}
\]

The coupling coefficient from a micro-taper to a microsphere is given by formula [10]

\[
\kappa(D) = \frac{k}{2\beta_r \sqrt{m}} (n_f^2 - n_0^2) N_f N_r R_0 \frac{2\pi}{\sqrt{q+1}} \left[ \frac{1}{\sqrt{q+1}} \right]^{N/2} H_N(0) e^{-\gamma_f(D-r_0)} \left[ \frac{1}{\gamma_f} - \frac{1}{\gamma_0} \right] j_0(k n_f R_0) + \frac{k n_f}{\gamma_f} j_1(k n_f R_0) \]
\[
(2 - 13a)
\]

\[
q = \frac{\gamma_f R_0}{m D} (D + R_0) \quad \text{(2 - 13b)}
\]
where $N_s$ is the normalization constant of the microsphere associated integral of WGMs’ propagation field intensity over the effective area [10],

$$N_s = \frac{1}{\sqrt{\int \frac{\pi m}{2(N-1)!N!R_0^2} \left(1 + \frac{1}{\alpha_s R_0}\right) J_l^2(k_n r_0) - J_{l-1}(k_n r_0) J_{l+1}(k_n r_0)}}$$  \hspace{1cm} (2 – 14a)

$$N = l - m$$  \hspace{1cm} (2 – 14b)

$$k = \frac{2\pi}{\lambda}$$  \hspace{1cm} (2 – 14c)

$$\alpha_s = \sqrt{\beta_l^2 - k^2 n_0^2}$$  \hspace{1cm} (2 – 14d)

$$\beta_l = \frac{\sqrt{\mu (l+1)}}{R_0}$$  \hspace{1cm} (2 – 14e)

and $m, l$ are the WGMs’ mode number, $R_0$ is the radius of microsphere, $D$ is the distance from the micro taper’s centre to the microsphere’s surface, $n_s, n_o$ are the refractive index of dielectric microsphere and micro taper’s cladding, $H_N(0)$ is the Hermite polynomial of order $N$ evaluated with argument 0, $\alpha_s$ is the attenuation coefficient of the microsphere’s evanescent field, $k$ is the wave vector, $\beta_l$ is the WGMs propagation constant which are discrete values.

The coupling coefficient is significantly determined by the refractive index of microsphere; the higher refractive index the higher coupling coefficient is. For a given coupling coefficient, the radius of the microsphere is inverse proportional to the coupled $k$ wave vector. Real coupling doesn’t always happen at one point only because there are many points around the nearest coupling point have coupling coefficient larger than 0.
Too many coupling points from one coupling zone can cause phase-mismatching. In order to evaluate the phase-mismatching, bent taper coupling radius $R_b$ and effective microsphere radius $R_e$ are introduced [10].

\[
R_e = \frac{R_b R_0}{R_b - R_0} \quad (2 - 15a)
\]

\[
\Delta \beta = \beta_f - \frac{m}{R_0} \quad (2 - 15b)
\]

\[
\kappa_{mismatching}^2 = \kappa^2(D) \frac{2 \pi R_e}{\gamma_f} e^{-\frac{(\Delta \beta)^2 R_e}{\gamma_f}} \quad (2 - 15c)
\]

The real coupling coefficient $\kappa_{mismatching}$ is determined by the curvature of the micro-taper at nearest coupling point, if the taper is kept straight then $1/R_b = 0$ and $R_e = R_0$, effective microsphere radius equals the physical radius; minimum phase mismatching has less effect on the coupling coefficient. If the bent micro-taper is used for coupling, the coupling length is increased and effective $R_e$ is larger than physical radius, which cause higher coupling rate for more WGMs; however, higher coupling coefficient is responsible for lower Q factor in the microsphere resonator.

The longitudinal interference pattern of the coupling result is used to evaluate the WGMs coupling and quality factor. From the indication of the interference pattern, the coupling capabilities are divided into three significant categories: critical coupling, optimized coupling and over coupling.

Critical coupling is the situation that the coupling coefficient between the micro-taper and the microsphere is so small that even input the highest energy from the micro-taper
can only invoke a weak interference pattern, which looks like longitudinal noises; optimized coupling is the situation that the interference pattern gives highest quality factor without changing the input energy; while over coupling means that even with a higher coupling coefficient and higher input energy the quality factor of the interference pattern is still lower than that of optimized coupling.
Q factor associated with coupling

As not all the photons can be kept inside the dielectric microsphere during the coupling between a microsphere and a micro-taper, the quantity evaluation of the loss in the exchange of energy can be shown by the well studied equation (2 - 16α). The transverse profile at the coupling point has two potential wells, one is for WGMs inside the microsphere and the other is for the HE_{11} fundamental mode around the micro-taper’s cladding mode. When the phase-matching condition for the wave numbers are satisfied between the two waveguides, photons in both potential wells can traverse back and forth through the media gap without losing coherent.

The Q factor due to energy tunneling can be express by [10]:

\[ Q_{coupling} = \frac{l^2 n_s}{k^3 c \varepsilon_0 N_s^2 n_0^2 R_0^6 Z_0 \left[ n_0 \left( \frac{\pi}{Y_1} \right)^2 j_1(kn_0R_0) j_{l+1}(kn_0R_0) - n_\phi \left( \frac{\pi}{Y_1} \right)^2 j_{l-1}(kn_0R_0) j_l(kn_0R_0) \right]^2} \quad (2 - 16a) \]

\[ \gamma_1 = l - \frac{1}{2} - kn_0R_0 \frac{j_1(kn_0R_0)}{j_{l+1}(kn_0R_0)} + \frac{(kn_0R_0)^2}{l} \quad (2 - 16b) \]

\[ \gamma_2 = l + \frac{1}{2} - kn_0R_0 \frac{j_{l+1}(kn_0R_0)}{j_l(kn_0R_0)} + \frac{(kn_0R_0)^2}{l} \quad (2 - 16c) \]

where \( c \) is the speed of light in air, \( \varepsilon_0 \) is the free space permittivity, \( Z_0 \) is the free space impedance, \( N_s \) is the normalization constant given in (2 - 14α), \( l \) is the whispering gallery modes’ number.
The $Q_{\text{coupling}}$ is mainly determined by the microsphere’s radius, refractive index and WGMs mode numbers. Without considering nonlinear properties, a chalcogenide microsphere with $n = 2.8$ is supposed to have a higher $Q_{\text{coupling}}$ factor than that of same dimension silica microsphere due to photons tunneling.

The tunneling quality factor $Q_{\text{coupling}}$ is the tunable contributor to the quality factor of a microsphere resonator. With a perfect round isotropic microsphere and a straight micro-taper optimized coupling, the effective energy density of the WGMs is supposed to be hundred times of the micro-taper’s effective energy density.
TE mode coupling between a micro-taper and a nonlinear micro-sphere

Now consider the scenarios that a straight micro-taper couples with a perfect round chalcogenide microsphere: the micro-taper is made from silica single mode fiber and its waist is narrow enough to extend evanescent field outside the cladding, and the chalcogenide microsphere has no surface oxidation. The shifted ray couples in the microsphere at point $a$ and couples out the microsphere from point $b$ as shown on the figure 2-2. The shifted ray without losing coherent will get interfered with the bullet ray around point $b$.

![Figure 2-2 Nonlinear microsphere coupling with a micro-taper](image)

Supposing there is no higher order coupling coefficient involved, then the TE modes coupling can be approximately described by orthogonal coupling equations [11]:
\[ E_{S}^{(a)} = i\kappa(D)E_{T}^{(in)} + \tau E_{S}^{(b)} e^{i\kappa L} \]  
\[ E_{T}^{(out)} = \tau E_{T}^{(in)} + i\kappa(D)E_{S}^{(in)} e^{i\kappa L} \]  
(2 - 17a)  
(2 - 17b)

where \( L \) is the propagation length around the micro-sphere, \( \kappa(D) \) is the coupling coefficient between the dielectric microsphere and the micro taper which is given by formula (2 - 13a), \( \tau \) is the transmission coefficient of the micro taper which satisfies:

\[ |\tau|^2 + |\kappa(D)|^2 = 1 \]  
(2 - 18)

For linear coupling system, orthogonal coupling condition and phase matching must be satisfied:

\[
\begin{bmatrix}
\langle \varphi_T | \varphi_T \rangle & \langle \varphi_T | \varphi_S \rangle \\
\langle \varphi_S | \varphi_T \rangle & \langle \varphi_S | \varphi_S \rangle
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]  
(2 - 19)

where \( \varphi_T \) is the phase of the bullet ray and \( \varphi_S \) is the phase of the shifted ray.

If take the chalcogenide micro-sphere’s nonlinear optical properties into consideration, then two-photon absorption (TPA) and free-carrier absorption (FCA) induced losses and self phase modulation (SPM) and cross phase modulation (XPM) effects must be included. For the scenarios with pump power, signals and converted signal conversion within the chalcogenide microsphere, the nonlinear equations can be written as [11]:

\[
\begin{aligned}
\frac{dE_{\text{pump}}(z)}{dz} &= -\frac{1}{2}(\alpha_{p \text{linear}} + \alpha_{p \text{TPA}} + \alpha_{p \text{FCA}})E_{\text{pump}}(z) + i\gamma_p |E_{\text{pump}}|^2 E_{\text{pump}}(z) \quad (2 - 20a) \\
\frac{dE_{\text{signal}}(z)}{dz} &= -\frac{1}{2}(\alpha_{s \text{linear}} + 2\alpha_{s \text{TPA}} + \alpha_{s \text{FCA}})E_{\text{signal}}(z) + i2\gamma_s |E_{\text{pump}}|^2 E_{\text{signal}}(z) \quad (2 - 20b) \\
\frac{dE_{\text{converted}}(z)}{dz} &= -\frac{1}{2}(\alpha_{c \text{linear}} + 2\alpha_{c \text{TPA}} + \alpha_{c \text{FCA}})E_{\text{converted}}(z) + i2\gamma_c |E_{\text{pump}}|^2 E_{\text{converted}}(z) \\
&\quad + i\gamma_c^2 \left( E_{\text{pump}}(z) \right)^2 E_{\text{signal}}^*(z)e^{-i\Delta k z} \quad (2 - 20c)
\end{aligned}
\]
where $\alpha_{p\,\text{Linear}}, \alpha_{s\,\text{Linear}}, \alpha_{c\,\text{Linear}}$ are linear losses for pump, signal, and converted signal in the microsphere, while $\alpha_{p\,\text{TPA}}, \alpha_{s\,\text{TPA}}, \alpha_{c\,\text{TPA}}$ are TPA nonlinear losses and $\alpha_{p\,\text{FCA}}, \alpha_{p\,\text{FCA}}, \alpha_{p\,\text{FCA}}$ are FCA nonlinear losses.

Chalcogenide nonlinear parameters $\gamma_p, \gamma_s,$ and $\gamma_c$ are given by [11]:

$$\gamma_p = \frac{n_2 \omega_{\text{pump}}}{c A_{\text{eff}}}$$  \hspace{1cm} (2 - 21)

$$\gamma_s = \frac{n_2 \omega_{\text{signal}}}{c A_{\text{eff}}}$$  \hspace{1cm} (2 - 22)

$$\gamma_c = \frac{n_2 \omega_{\text{converted}}}{c A_{\text{eff}}}$$  \hspace{1cm} (2 - 23)

where $n_2$ is nonlinear index coefficient of the chalcogenide micro-sphere, $A_{\text{eff}}$ is the effective mode area, $\omega_{\text{pump}}, \omega_{\text{signal}},$ and $\omega_{\text{converted}}$ are the frequencies for the pump, the signal and the converted signal.

The linear phase mismatch is defined as:

$$\Delta k = k_{\text{signal}} + k_{\text{converted}} - 2k_{\text{pump}}$$  \hspace{1cm} (2 - 24)

At the coupling point shown on the figure 2-2, the nonlinear relationship of electric fields for the wavefront at point $a$ and point $b$ which experience one cycle propagation around microsphere’s equator can be written with equations [11]:

$$E_{\text{pump}}^{(b)} = E_{\text{pump}}^{(a)} e^{\left(-\frac{(\alpha_{p\,\text{Linear}} + \alpha_{p\,\text{TPA}} + \alpha_{p\,\text{FCA}})}{2}\frac{\gamma_p}{\omega_{\text{pump}}}\left|E_{\text{pump}}^{(a)}\right|^2\right) L}$$  \hspace{1cm} (2 - 25)

$$E_{\text{signal}}^{(b)} = E_{\text{signal}}^{(a)} e^{\left(-\frac{(\alpha_{s\,\text{Linear}} + 2\alpha_{s\,\text{TPA}} + \alpha_{c\,\text{FCA}})}{2}\frac{\gamma_s}{\omega_{\text{signal}}}\left|E_{\text{pump}}^{(a)}\right|^2\right) L}$$  \hspace{1cm} (2 - 26)

$$E_{\text{converted}}^{(b)} = \left(E_{\text{converted}}^{(a)} + i\gamma_c \left|E_{\text{pump}}^{(a)}\right|^2 \frac{1-e^{-\left((\alpha_{c\,\text{Linear}} + \alpha_{c\,\text{TPA}} + \alpha_{c\,\text{FCA}} + \Delta \beta)\right) L}}{\alpha_{c\,\text{Linear}} + \alpha_{c\,\text{TPA}} + \alpha_{c\,\text{FCA}} + i\Delta \beta} \left|E_{\text{pump}}^{(a)}\right|^2 \right) L$$  \hspace{1cm} (2 - 27)
where the total phase mismatch is given by:

$$\Delta \beta = \Delta k + 2\gamma |E_{pump}^{(a)}|^2$$  \hspace{1cm} (2 – 28)$$

and \(\Delta k\) is linear phase mismatch given by (2 – 24).

The TE mode resonant field enhancement factor of pump in the chalcogenide microsphere is [11]

$$F_{pump} = \frac{|E_{pump}^{(a)}|}{|E_{pump}^{(in)}|} = \left| \frac{\kappa(D)}{1 - \left( \frac{\alpha_p \text{Linear} + \alpha_p \text{TPA} + \alpha_p \text{FCA}}{2} \right) \gamma_p |E_{pump}^{(a)}|^2 + k_{pump} \right|^L \right|$$  \hspace{1cm} (2 – 29)$$

and the TE mode transmission factor of pump for the micro-taper is

$$T_{pump} = \left| \frac{|E_{pump}^{(a)}|^2}{|E_{pump}^{(in)}|^2} \right|^2 = \left| \frac{\left( \frac{\alpha_p \text{Linear} + \alpha_p \text{TPA} + \alpha_p \text{FCA}}{2} \right) \gamma_p |E_{pump}^{(a)}|^2 + k_{pump} \right|_L^2$$  \hspace{1cm} (2 – 30)$$

The TE mode resonant field enhancement factor of signal in the chalcogenide microsphere is

$$F_{signal} = \frac{|E_{signal}^{(a)}|}{|E_{signal}^{(in)}|} = \left| \frac{\kappa(D)}{1 - \left( \frac{\alpha_s \text{Linear} + \alpha_s \text{TPA} + \alpha_s \text{FCA}}{2} \right) \gamma_s |E_{pump}^{(a)}|^2 + k_{signal} \right|^L \right|$$  \hspace{1cm} (2 – 31)$$

and the TE mode transmission factor of signal for the micro-taper is
The TE mode resonant field enhancement factor of signal in the chalcogenide microsphere is

\[ T_{\text{signal}} = \left( \frac{E_p^{(a)}}{E_{\text{pump}}^{(a)}} \right)^2 = \left( \frac{1 - e^{-\left[2\gamma_s \left| E_p^{(a)} \right|^2 + k_{\text{signal}} \right] L}}{1 - e^{-\left[2\gamma_s \left| E_p^{(a)} \right|^2 + k_{\text{signal}} \right] L}} \right)^2 \]  \hspace{1cm} (2 - 32)

\[ F_{\text{converted}} = \frac{\kappa(D)}{1 - \tau e^{-\left[2\gamma_s \left| E_p^{(a)} \right|^2 + k_{\text{converted}} \right] L}} \]  \hspace{1cm} (2 - 33)

A simulation for FWM is conducted based on nonlinear TE mode transmission coupling between a chalcogenide microsphere and micro-taper. All the material related parameters are listed in the Matlab code at Appendix B.

![Figure 2-3 TE mode resonant field enhancement factor for pump, signal and converted signal under FWM](image-url)
Figure 2-3 is the result of resonant field enhancement factor for pump and signal wavelength difference being 1nm, 1mW input power and 150um diameter chalcogenide microsphere. The result indicates that the WGMs power inside the microsphere can be reinforced to more than 100 times of the input power.
TM mode coupling between micro-taper and micro-sphere

The TM modes coupling needs to take Goos-Hänschen shift into consideration.

Supposing there is no higher order coupling coefficient involved, then the TM modes coupling can be approximated by orthogonal coupling equations:

\[
\begin{align*}
E^{(a)}_S & = i \kappa(D) E^{(in)}_T + \tau E^{(b)}_S e^{(ikL+\phi_{GH})} \\
E^{(out)}_T & = \tau E^{(in)}_T + i \kappa(D) E^{(in)}_S e^{(ikL+\phi_{GH})}
\end{align*}
\]  

(2 – 34)  

(2 – 35)

where \( L \) is the propagating length around the micro-sphere, \( \kappa(D) \) is the coupling coefficient between dielectric microsphere and the micro taper which is given by formula \((2 – 13a)\), \( \tau \) is the transmission coefficient of the micro taper.

The nonlinear relationship for the chalcogenide microsphere TM modes can be expressed by the equations from \((2 – 20a)\) to \((2 – 28)\).

The TM mode resonant field enhancement factor of pump in the chalcogenide microsphere is

\[
F_{pump} = \left| \frac{\kappa(D)}{1 - \tau e^{\left((\alpha_p \text{linear} + \alpha_p \text{TPA} + \alpha_p \text{FCA})/2 \right)|E^{(a)}_{pump}|^2}} \right|^l e^{\phi_{GH}}
\]

(2 – 36)

and the TM mode transmission factor of pump in the micro-taper is
The TM mode resonance field enhancement factor of signal in the chalcogenide microsphere is

\[
T_{\text{pump}} = \frac{\tau - \epsilon}{1 - \tau \epsilon} \left\{ \left( \frac{\alpha_p \text{Linear} + \alpha_p \text{TPA} + \alpha_p \text{FCA}}{2} \right) + i \left( \gamma_p \left| E_{\text{pump}}^{(a)} \right|^2 + k_{\text{pump}} \right) L + \phi_{\text{GH}} \right\}^2
\]  
(2 - 37)

The TM mode resonance field enhancement factor of signal in the chalcogenide microsphere is

\[
F_{\text{signal}} = \frac{\kappa(D)}{1 - \tau \epsilon} \left\{ \left( \frac{\alpha_s \text{Linear} + \alpha_s \text{TPA} + \alpha_s \text{FCA}}{2} \right) + i \left( 2 \gamma_s \left| E_{\text{pump}}^{(a)} \right|^2 + k_{\text{signal}} \right) L + \phi_{\text{GH}} \right\}
\]  
(2 - 38)

and the TM mode transmission factor of signal in the micro-taper is

\[
T_{\text{signal}} = \frac{\tau - \epsilon}{1 - \tau \epsilon} \left\{ \left( \frac{\alpha_s \text{Linear} + \alpha_s \text{TPA} + \alpha_s \text{FCA}}{2} \right) + i \left( 2 \gamma_s \left| E_{\text{pump}}^{(a)} \right|^2 + k_{\text{signal}} \right) L + \phi_{\text{GH}} \right\}^2
\]  
(2 - 39)

The TM mode resonance field enhancement factor of signal in the chalcogenide microsphere is

\[
F_{\text{converted}} = \frac{\kappa(D)}{1 - \tau \epsilon} \left\{ \left( \frac{\alpha_c \text{Linear} + 2 \alpha_c \text{TPA} + \alpha_c \text{FCA}}{2} \right) + i \left( 2 \gamma_c \left| E_{\text{pump}}^{(a)} \right|^2 + k_{\text{converted}} \right) L + \phi_{\text{GH}} \right\}
\]  
(2 - 40)

As TM and TE modes could coexist in the chalcogenide microsphere due to equal opportunity to couple the TM or TE polarization light wave from the HE_{11} mode of the micro-taper. For the same WGMs number and same frequency, degenerated TM WGMs can be formed when the
phase mismatching is small, and further research on the TM & TE WGMs dispersion of the microsphere is needed.
**Introduction to bent taper coupling**

Theoretically if the cone angle is small enough for everywhere along the deforming distance of a micro-taper, it is possible for the micro-taper to deliver energy with negligible loss in uncoupling state, which is called adiabatic.

A micro-taper has finite cladding at the waist if the micro-taper’s diameter is stretched to about 1-2 micron. From the original optical fiber to tapered waist, the effective index reduces monotonically and the binding capability of the waveguide keeps decreasing; as a result, more energy is propagating outside of the physical optical waveguide along the micro-taper as evanescent mode. However, without coupling with external waveguide, the evanescent mode keeps the energy propagating inside a virtual electromagnetic wall which binds the transverse wave vectors in.

If the micro-taper is straight and axisymmetric, the fundamental mode can couple only to higher order cladding local modes whose propagation constants are close to that of fundamental mode. However, for the non-axisymmetric micro-taper, the fundamental mode can couple to any modes which have closest propagation constants value. [12] By adjusting the curvature of the coupling taper, both TE and TM modes can be obtained, and it has significant effect on coupling experiment between a micro-taper and a microsphere.

If the silica micro-taper is adiabatic, then most energy is reserved in the fundamental mode $HE_{11}$. The fundamental mode $HE_{11}$ has the same opportunity to couple TE or TM modes to the fundamental WGMs with TE or TM polarization in discrete frequencies.
Some energy from the fundamental mode $HE_{11}$ can be coupled into higher order TE or TM WGMs; these higher order WGMs can interfere with the fundamental WGMs in the microsphere, and the interference pattern can be coupled back to the fundamental mode $HE_{11}$ mode in the micro-taper.

If the silica micro-taper is non-adiabatic and the taper is axisymmetric, then part of $HE_{11}$ mode energy can couple into $HE_{12}$ mode along the micro-taper’s waist. If the micro-taper is non-adiabatic and non-axisymmetric, the fundamental $HE_{11}$ mode energy can further couple into $TE_{01}$, $HE_{21}$ and $TM_{01}$ modes, which makes the coupling between the microsphere and micro-taper more complicated; more transverse wave vector from the evanescent modes of micro-taper can excite the more higher order WGM modes. There again exist interferences caused by different order WGMs inside the microsphere; when they are coupled back to the micro-taper, not all the energy will be coupled back into the fundamental $HE_{11}$ mode; and only $HE_{11}$ mode that can be monitored by experimental method.

The bent micro-taper changes not only the polarization and orders of the modes to be coupled but also the effective radius of the microsphere given by formula $(2 - 15a)$, which finally affect the Q factor and discrete frequencies to be coupled. In order to obtain a more stable research platform, it is recommended to fix the microsphere and the micro-taper with positioning equipment.
Deformed Sphere

When a dielectric microsphere is placed in the unstable temperature environment, the volume of microsphere increases due to thermal expansion, which is quite different for different materials and radii. All the WGMs accommodated inside the microsphere need to be adapted to new mode volume according to small change in the propagation constant along the microsphere equator.

The propagating wave vector shifting $\Delta k$ is evaluated by [10]

$$\Delta k = \left( l - m + \frac{1}{2} \right) \frac{\Delta r}{r^2}$$

(2 - 41)

And the wavelength shifting $\Delta \lambda$ is evaluated by

$$\Delta \lambda \approx \frac{\lambda^2 \Delta r}{2\pi r^2}$$

(2 - 42)

where $\Delta r$ is the change in radius. The shifting of propagation constant will cause the mode number changing accordingly, which appears in the longitudinal shifting of the coupling interference pattern.

Anisotropic Chalcogenide As$_2$Se$_3$ microsphere’s photon related free carriers inside the mode volume are easily to be invoked by coherent photons and the thermal effect. These free carriers without biasing can form particles along the mode volume of the WGMs and increase the local refractive index. The free carriers effect are unstable and usually have response time up to 20~30 second. The variance of refractive index change significantly affects the propagation constant in the chalcogenide microsphere. Plus, the irregular distribution of the free carriers will increase the coupling probability among
different order WGMs. However, the smaller size chalcogenide As$_2$Se$_3$ microsphere has the potential to be used on the optical & thermal sensing domain with a voltage biasing metal taper stem embedded into the microsphere.
Chapter 3 Coupling experimental results

Chalcogenide microsphere and silica micro-taper coupling experiment was designed to get the quantity result for fitting with established theories and experienced formulas. It also gives the comparison between the chalcogenide microsphere resonator and reported silica microsphere resonator. These coupling experiments include: a transmission coupling, a reflection coupling and an adding & dropping port characteristics of chalcogenide microsphere.

Coupling experiment for transmission

When a straight micro-taper is coupled to a dielectric microsphere, the shifted ray experiences at least $\pi$ shifting in phase for coupling in and out the microsphere and its amplitude is constructively reinforced by resonating inside the microsphere. The destructive interference between the two rays converts spatial resonating to discrete temporal coherent.

The transmission experiment was conducted in order to explore the longitudinal interference pattern between the bullet ray and the shifted ray for silica micro-taper and chalcogenide $\text{As}_2\text{Se}_3$ microsphere; as the interference is the simplest way to indicate the WGMs’ order and quality factor $Q$ of the resonator.
Experiment setup

This coupling experiment setup as shown on figure 3-1 is used to acquire the longitudinal transmission interference pattern. The candidate light sources are broadband source such as EDFA (C-BAND Erbium-Doped Fiber Amplifier INO) amplified spontaneous emissions (ASE) or a pulse laser (CALMAR Femtosecond pulsed laser) amplified by the EDFA which give a coherent broad band source. The light source is connected to the micro-taper, and the output of the micro-taper was connected to an OSA (Anritsu MS9710C Optical Spectrum Analyzer) through an isolator. As the broadband light passing through the micro-taper, a chalcogenide microsphere held by a stem is adjusted to couple with the silica micro-taper. The fine tuning equipment is capable of approaching the chalcogenide microsphere to the silica taper in micron level with 4 freedoms. All the experiment data was collected from the OSA to the Labview & Matlab environment through a GPIB after there is no vibration on the experimenting platform.

Figure 3-1 Experiment setup for transmission
Experiment procedure & results

When EDFA ASE was sent through the micro-taper without coupling, the OSA gave out the EDFA’s characteristic amplification curve. This initial data is collected to Matlab and used as the normalization data base. The wavelength sensitivity of the OSA was set to be 0.05\textit{nm}, and the number of sampling points was set to be 5001 points.

The chalcogenide microsphere is approached to the micro-taper’s waist while monitoring that if there is a critical coupling happening shown on the OSA. If the microsphere touched the micro-taper, then it should fine tune the angle between the chalcogenide microsphere and the micro-taper to capture the critical coupling result; And if there is no coupling, then slowly scan the microsphere from current position towards the micro-taper’s waist center while keeping close contact and optimized angle between the microsphere and the micro-taper. As the micro-taper is steepen in its axial direction, scanning the microsphere along the micro-taper’s cladding can give continuous propagation constant $\beta$ for the micro-taper; and if the micro-taper’s waist were thin enough, there would be large opportunity to acquire a critical. In order to get a transmission coupling, a micro-taper with 0.5$\sim$1.0\textit{$\mu$m} in diameter is recommended.

A normalized critical coupling result is shown on figure 3-2, which gives out noise like pattern at the range of -20$\sim$-24\textit{dB} and relatively high quality factor $Q$ for many troughs less than -24\textit{dB}. The trough is due to interference of bullet ray and shifted ray which experienced $\pi$ shifting in phase at this wavelength. However, these deep troughs are not stable enough to give a clearly longitudinal periodic interference patterns to identify the
microsphere resonator’s FSR. The sampled data is valid for around 30 seconds or less, after which the re-sampled data shows that the troughs keep shifting with different amplitudes.

![Normalized Spectrum](image)

Figure 3-2 Spectrum in the critical coupling of transmission experiment

The figure 3-2 shows that during the critical coupling happening, the average power spectrum has 20 dB insertion losses immediately. The high Q factor of a trough is due to the lower coupling coefficient, and instable interference pattern could be either due to modes coupling or the deformed mode volume.

After the critical coupling is explored, the next goal is to find the possible optimized coupling point and its interference pattern if there are over coupling situations. Without changing the input power, fine tuning the coupling position and angle between the microsphere equator and axial direction of the micro-taper; by comparing energy loss and the stability of the interference pattern of the longitudinal transmission, an
optimized coupling result was confirmed, which is normalized with respect to the initial spectrum and shown on figure 3-3.

![Normalized spectrum](image)

**Figure 3-3** Spectrum in the optimized coupling of transmission experiment

The optimized coupling result has -13dB loss with respect to the initial spectrum and clear and stable trough to differentiate the FSR of microsphere resonator and regularly side troughs. The quality factor Q for main trough at the 1550nm is given by:

$$Q = \frac{\lambda_0}{\Delta \lambda_{FWHM}} = \frac{1550.5\text{nm}}{0.46\text{nm}} = 3370$$

where $\lambda_0$ is the wavelength of trough around 1550nm to evaluate, $\Delta \lambda_{FWHM}$ is the width of the trough at full-width of half maximum. The FSR is measured to be around 3 nm. There also have more than 6 side troughs between every adjacent two main troughs or in each FSR. These side troughs have less amplitude value which means they have relatively small energy density inside the microsphere. Reviewing simulation results
from the chapter 1, the higher odd order WGMs have less energy density than the fundamental WGMs while have higher opportunity to couple the near field out to a micro-taper. So the stable side troughs are indications of higher order WGMs which have the same polarization as the fundamental modes of deep troughs.

If the microsphere is moved from current position back away from the waist, a weak coupling pattern will be acquired. Figure 3-4 shows a sample of weak coupling longitudinal interference pattern.

![Normalized Spectrum](image)

**Figure 3-4 spectrum in the weak coupling of transmission experiment**

The weak coupling has energy loss about 14dB in the spectrum with respect to initial measurement and weaker coupling curvature in the middle of the spectrum envelope. There are more clear and stable troughs to show the regular FSR and side troughs. Although the loss is larger than that of optimized coupling, the quality factor $Q =$
\( \lambda_0 / \Delta \lambda_{FWHM} = 1550.7 / 0.45 = 3446 \) around 1550nm is higher than that of optimized coupling.

If the microsphere is moved from the optimized coupling position towards the waist, the over coupling pattern will be acquired. Figure 3-5 shows a raw sampling inset around 1550 nm of the over coupling.

![Figure 3-5 Spectrum of over coupling of transmission experiment](image)

The over coupling result has -14dB loss with respect to the initial spectrum which has an -50dBm level around 1550nm and clear and stable FSR of microsphere resonator and 8 regularly side troughs. The interference pattern has almost no change except for the number of side troughs and the amplitude of the trough is less than that of optimized coupling for 1dB. The quality factor \( Q = \lambda_0 / \Delta \lambda_{FWHM} = 1550.27 / 0.46 = 3370 \) is given for the main trough at the 1550nm. Again, the side troughs are due to many high order WGMs with same polarization.
Return the chalcogenide microsphere back to the optimized coupling point. Use Pico-pulse laser with current set to 90 amplified by an EDFA as a coherent broad band source to the transmission coupling experiment. Figure 3-6 gives the sampling of the coupling spectrum for this light source.

The broad band coherent source coupling result has -14dB loss with respect to its normalization spectrum; and the FSR is around 2.91 nm, which falls in the results of ASE light source coupling. The main troughs in spectrum shows deeper and narrower trend while side troughs are relatively shallow. The quality factor \( Q = \lambda_0 / \Delta \lambda_{FWHM} = 1549.85 / 0.05 = 30997 \) is given for the main trough at the 1550nm, which is limited by the resolution of the OSA. Figure 3-6 shows that chalcogenide microsphere remains high quality factor Q for wavelength from 1550nm to 1570nm. The fact that more side troughs along the FSR means that there are a lot of higher order WGMs had been invoked by the high-energy density coherent light source.
While moving the microsphere to the weak coupling point and over coupling point, it gave out almost the same spectrum as shown on figure 3-6. The average spectrum loss was -13dB at over coupling condition and was -14dB at weak coupling condition. The coupling kept generating same interference pattern after the chalcogenide microsphere was detached away from the silica micro-taper’s cladding with the distance up to $5\mu m$ except that the troughs’ amplitudes were getting weaker.
Summary

The coupling of the broad band bullet ray and its shifted ray gives out the longitudinal interference spectrum to indicate the invoked WGMs inside a chalcogenide microsphere. The coupling results depend on light sources, micro-taper’s dimension and thermal state of the microsphere.

The EDFA ASE provides all possible polarizations with continuous non-coherent light. The critical coupling condition requires phase-matching in the air gap between the chalcogenide microsphere and silica micro-taper. As the energy density inside the microsphere’s mode volume is not high enough to invoke more free carriers and any nonlinear properties, the coupling mainly generates fundamental and higher order WGMs. In this stage the coupling coefficient has more weight in the interference result. The coupling coefficient value between 0.1 and 0.2 gives clear and sharp transmission troughs in the longitudinal interference pattern.

The coherent light source gives more stable and consistent interference spectrum. The critical coupling result is almost same as the weak coupling result in which the quality factor Q and average loss of the coupling spectrum doesn’t change significantly when moving the chalcogenide microsphere around the optimized coupling point. The coherent light source is easy to satisfy the phase-matching condition in the evanescent field gap to construct the coupling conditions. The modes coupling and higher order modes are not as significant as that of ASE light source coupling, but the statistical variance of side troughs in the interference pattern indicates that the higher order modes coupling is related to the phase-matching condition. The formation of significant higher
order modes requires stable transverse wave numbers inside the chalcogenide microsphere. The amplified coherent source is a polarization sensitive light source, which cannot guarantee the micro-taper coupling out the desired polarization light to the chalcogenide microsphere due to the positioning. The possible reason could be that the coherent wave front makes its way by invoking free carriers along the equator and changes the refractive index of the mode volume, which makes the pulse light possessing a relatively higher wave number in the propagation direction, and the coherent source invoked evanescent field is almost same as that of ASE light source. The existence of higher order WGMs depends on the phase-matching condition for different refractive waveguide; even a small wave number difference in transverse direction can be reflected to a significant wave number difference in the microsphere. As the most wave vector is in the propagation direction, then most energy is left in the fundamental WGMs of the chalcogenide microsphere, which is shown by the high quality factor Q in the deep troughs of the spectrum.

The shape and positioning of the micro-taper determine the coupling feasibility and the coupling efficiency. As the straight micro-taper has less side effect on the coupling points overlapping, and it tends to give longitudinal transmission result with deep troughs and shallow side troughs. Curved micro-taper changes the effective radius of the microsphere; it not only changes the accommodated WGMs propagation constant as well as its sustaining condition but also causes more modes coupling among bullet rays. So a straight micro-taper is the necessary condition for acquiring a clear transmission longitudinal spectrum.
When chalcogenide microsphere being coupled, the energy density within the near field is much higher than that of the micro-taper. The high energy density and high linear loss of chalcogenide generate large thermal energy to raise the local temperature, which is indicated from microsphere surface melting and viscous chalcogenide material can be easily separated and attached to the micro-taper’s cladding.

As discussed in chapter 2 that thermal energy will deform the microsphere’s volume by thermal expansion; it changes the existing WGMs’ parameters and causes longitudinal shifting of the interference pattern. Beside, higher temperature invokes more free carriers, adding more probability for modes coupling and produces deep side troughs.

That the surface temperature is higher than chalcogenide transition temperature also helps the microsphere releasing residual molecular force towards forming the quasi-isotropic state controlled by the surface tension. The quasi-isotropic state makes the interference pattern statically stable.
Coupling experiment for reflection

When a broken micro-taper is coupled with a dielectric microsphere, the light goes through the micro-taper without experiencing coupling and being reflected directly from the micro-taper’s tip is called the bouncing ray, and the light experienced at least once coupling with the microsphere is the shifted ray. The light which doesn’t couple with the microsphere and is being scatter out of the micro-taper’s tip is called shooting ray. The shifted ray has more weight in coupling in and out the microsphere with same the propagating direction as the shooting ray. Part of the interfered wave front is reflected back, again some will experience the coupling again while the other part will go back straightly. As the reflection of micro-taper’s tip is so weak that the effect of second coupling can be ignored. So the reflection coupling result is considered as a simple way to evaluate the WGMs.

The reflection experiment uses the same dimensional chalcogenide microsphere as the transmission experiment and a silica taper with tip diameter less than 0.5 um. The reflection’s insertion loss of the micro-taper tip is around -30dB. This experiment is the complementary part of transmission coupling experiment. Single end micro-taper is easy for curving and positioning to get the coupling result quickly. It has the potential application of MID-IR optical sensing.

Experiment setup for coupling reflection

The light sources used for coupling reflection includes EDFA ASE source and the coherent source of EDFA amplified Pico-second pulse laser. Light source is connected
to the input port of a circulator, and the reflection port of the circulator is connected to the micro-taper; the micro-taper is clipped by a fiber holder used for rotating and positioning. Without any external stress or air flow, the silica micro-taper’s tip keeps straight for the part where the diameter is larger than 0.5 μm. The output port of the circulator is connected to an OSA. All data was collected from the OSA to the Labview & Matlab environment through GPIB. The resolution of the wavelength was set to be 0.05 nm and the number of sampling points was set to be 5001.

The chalcogenide microsphere can be moved freely to adjust its desired equator to tangentially align the axial direction of the silica micro-taper. In this experiment, the relatively position and angle between chalcogenide microsphere and the silica taper can be fine tuned. The setup is shown on figure 3-7.

Figure 3-7 Experiment setup for reflection
Experiment procedure & results

First, EDFA ASE source is sent to the port 1 of the circulator, the port 2 connected with micro-taper which decoupled with the chalcogenide microsphere, then the output of the circulator’s port 3 is the reflection from the micro-taper’s tip. This initial data was sampled to Matlab and is used as normalization data base for reflection.

Next, approach the chalcogenide microsphere to the single end micro-taper from thick side while watch out the interference spectrum on the OSA. Fine tune the angle between the axial direction of chalcogenide and the micro-taper axial direction and make them contacted to find the critical coupling. Scan the microsphere from current position to the micro-taper’s tip until critical coupling is found. Figure 3-8 gives the critical coupling normalized spectrum.

![Normalized Spectrum](image)

Figure 3-8 Critical coupling spectrum of the coupling reflection experiment
The critical coupling result has average spectrum loss -15dB with respect to decoupling state. The FSR equals 1.4 nm; the troughs in the interference pattern are easy to identify; no significant side trough can be found except the statically sampling noise, the quality factor Q can hardly be given due to shallow trough in interference spectrum.

Continually scanning the microsphere along the micro-taper towards its tip, as long as the taper is thin enough, the optimized coupling point can be found. When approaching the tip of micro-taper, the micro-taper is bent a little bit outward, which theoretically change the effective of the microsphere according the formula (2-15a) and the coupling coefficient. When the coupling loss is found to be minimal and the interference pattern’s envelope is clear to indentify, the optimized point for ASE is confirmed. Figure 3-9 shows the longitudinal spectrum. The average loss with respect to the initial reflection is about -12dB, and the quality factor at 1500nm is  

\[ Q = \frac{\lambda_0}{\Delta \lambda_{FWHM}} = \frac{1549.55}{0.75} = 2066. \]
After locating the optimized coupling point, it is possible to explore the over coupling and weak coupling state. Further scan the microsphere to the tip end of the micro-taper, avoid bending the taper and keep it as straight as possible. Usually the micro-taper is attracted and attached to the microsphere by static electric. An over coupling sample is shown by figure 3-10, where the average spectrum loss with respect to initial reflection is about -14dB, and the quality factor at 1550nm is given by $Q = \frac{\lambda_0}{\Delta \lambda_{FWHM}} = \frac{1550.25}{0.81} = 1914$.

![Normalized Spectrum](image)

**Figure 3-10** Over coupling spectrum of coupling reflection experiment

The weak coupling spectrum doesn’t give clear interference pattern and looks almost as critical coupling shown on figure 3-8, the average spectrum loss with respect to initial reflection is -15dB, and the quality factor at 1550nm is $Q = \frac{\lambda_0}{\Delta \lambda_{FWHM}} = \frac{1549.35}{1.25} = 1239$. 
After finishing the ASE light source experiment, moved the chalcogenide microsphere back to the optimized coupling point. Replace the ASE source with board band coherent light source. The amplified Pico-pulse laser source generated strong interference pattern from 1540 nm to 1640 nm for the micro-taper reflection without coupling, as it could not be used as normalization data base for the whole span, the initial data was collected just use as power reference as shown on figure 3-11.

Scanning the microsphere around the micro-taper’s ASE optimized coupling point gave out the interference pattern for critical, weak, optimized and over coupling. Figure 3-11 to 3-16 show the reflection spectrums for both decoupling state and all coupling states from 1540 nm to 1640 nm. As 1550 nm wavelength is of interest, and then all the coupling state is defined according to the interference patterns around 1550 nm.

![Figure 3-11](image1.png)  ![Figure 3-12](image2.png)

**Figure 3-11** Reflection of decoupling state  **Figure 3-12** Reflection of critical coupling

The critical coupling was got when the chalcogenide is more than 5 um away the micro-taper’s cladding. Figure 3-12 shows the critical coupling interference pattern. There is very weak interference around 1550 nm but relatively clear interference (weak coupling) around 1605nm~1625nm.
As the microsphere is approaching to the micro-taper and finally is attached to the micro-taper’s cladding, weak coupling was acquired for wavelength around 1550 nm. Figure 3-13 shows the weak coupling’s interference pattern around 1550 nm. There is clear interference around 1550nm and strong optimized coupling interference appeared from 1605nm-1620nm.

Scanning the microsphere along the micro-taper’s cladding down to tip’s direction. Strong optimized interference was caught at certain point. Figure 3-14 shows the interference result of optimized coupling for wavelength around 1550nm and also there is over coupling interference pattern for wavelength from 1605nm to 1635nm.
As the microsphere is moved from the optimized coupling point down to the tip coupled with thinner diameter micro-taper, the coupling interference pattern was getting weak but clear. Figure 3-15 shows the over coupling’s interference pattern around wavelength 1550nm. There is clear but not as strong as the interference pattern around 1550nm on figure 3-14 and there is also clear over coupling interference trend for the wavelength from 1615nm to 1630nm.

Continually move the microsphere along micro-taper to the place where the diameter is less than 0.5um, further over coupling result was shown around 1550nm. Figure 3-16 shows the result of clear further over coupling around wavelength 1550nm and there is critical coupling or weak coupling happened for the wavelength range from 1605nm to 1635nm.

The rest part of reflection experiment is realized using ANDO AQ4320B tunable laser source whose precision is set to 1pm. First using the ASE light source to locate the optimized coupling point, and then switched the light source to the tunable laser source. The output power was set to 1mW. The OSA was replaced by an ILX Lightwave FPM-8200 fiber optic power meter which is control by Labview program through GPIB; the power meter was programmed to synchronize with the tunable laser in setting the working wavelength; the power meter samples at least 3 times (3 channels) for each power output. All the data was collected by Labview program through GPIB, and the second sampling channel is used to plot the interference spectrum.

Figure 3-17 shows the power meter sampling result. This spectrum is quite different from the interference patterns of ASE or amplified pulse laser light source. It has
narrow and regular troughs. The quality factor around 1530nm is given by $Q = \frac{\lambda_0}{\Delta \lambda_{FWHM}} = \frac{1530.762}{0.001} = 1530762$. The trough width is less than 5 pm, which cannot be resolved by the OSA; on the contrary, the spectrum envelop of this tunable laser scanning is more like the interference pattern acquired by the OSA.

Figure 3-17 Tunable laser scanning for coupling reflection
Summary

The coupling reflection experiment takes advantage of the experience in the transmission coupling experiment. The single end taper is easy to fabricate, just stretch single mode fiber to broken with a desired shape. However, the insertion loss of a single end micro-taper is more than 30dB, which is the trade-off for the easy use.

Coupling reflection has high probability to acquire optimized coupling because there is no limitation in the fiber stretching stage. It is much easier to fabricate a micro-taper with local diameter less than 0.5um than a bi-taper with required waist less than 1um. Also, transferring a bi-taper whose diameter is less than 0.5um between different platforms requires lots of time and work. While single end micro-taper is portable and it can tolerate most of thermal and chemical environment.

For the ASE light source, coupling reflection result shows clearly the spatial dependent coupling coefficient. Thick cladding caused weak coupling interference, and thin cladding leads to optimized coupling or over coupling. The interference pattern of coupling reflection is statically stable but not clear, and the quality factor is low.

While using coherent light source, the coupling state is easier to establish. The coupling capability is related to both spatial position and wavelength. With coherent light source, the micro-taper is easy to establish phase-matching condition by strong spatial interference between the shooting ray and shifted ray. Compared with ASE light source, it is possible to get critical coupling from 5um away the micro-taper’s cladding. The
difference between 1550nm and 1620nm interference pattern shows that coupling coefficient depends on the excited frequencies of the WGMs.

Tunable laser light source gave more detail in the coupling reflection. With the reflection set-up, micro-taper can be rotated to acquire interference pattern with the desired polarization. Between each FSR of the longitudinal modes, there are numbers of deep troughs. These troughs are related to the higher order WGMs excited by the coherent light source.
Coupling experiment for drop-port

The goal of drop port experiment is to explore the difference between drop port spectrum and the transmission coupling interference pattern. Two micro-tapers, a chalcogenide microsphere and their positioning platform are required.

This experiment is to use two broken silica micro-tapers to couple with chalcogenide microsphere in the same equator; one micro-taper is connected to light source and used to couple light with chalcogenide microsphere to form WGMs, and the other micro-taper is connected to an OSA through a circulator and used to couple the WGMs out to be measured.

The coupling setup of drop port function is made up with two single end micro-tapers which both couple with the same microsphere almost in same equator in counter direction without touching each other. The coupling coefficient is preferred to be at weak coupling state, because there is limited space to extend the micro-tapers’ tip; relatively straight and thicker micro-tapers make the positioning feasible for three movable elements.
Experiment setup

The two micro-tapers are hold by two single-mode assemblies which are fixed on Thorlab surface-mount multimode fiber couplers by tip to tip. The chalcogenide microsphere is placed close and perpendicular to the two micro-tapers. One micro-taper is connected to EDFA ASE light source, the other one was connected to an OSA through an isolator. The desired micro-taper tip’s shape should not be affected by air flow in the laboratory. All data is collected from the OSA to the Labview & Matlab environment through GPIB. The resolution of the wavelength was set to be 0.05nm and the number of sampling points was set to be 5001. The setup is shown on figure 3-18.

Figure 3-18  Drop-port function setup and picture
Experiment procedure & results

Only amplified spontaneous emissions light source results is discussed for this experiment because amplified Pico-second laser causes directly coupling between the two micro-tapers.

First, overlap the two micro-tapers’ tip around 100um, inspect the coupling between the two micro-tapers; adjusted the output power of the ASE light so that there exists EDFA ASE spectrum when the two-tips are overlapping, and there is no power spectrum when the two tips are separated. As the micro-taper can be easy attracted by each other due to static electric, alcohol is needed to separate the connection. The initial spectrum was also collected as a power reference.

Next, separate the tip micro-taper apart along the axial direction, and move the microsphere to align the equator with the two micro-tapers tangentially. Push the two micro-tapers simultaneously at the microsphere’s equator so that both micro-tapers are bent and attached to the microsphere’s equator. Fine scan the microsphere along the two micro-taper and monitor the coupling results.

The coupling results are stable, which is shown on figure 3-19; fixed the microsphere and the add-port micro-taper, fine tune the axial position of the drop-port micro-taper while monitoring the coupling interference pattern change on the OSA. The figure 3-20, 3-21 and 3-22 show the change due to drop-port taper moving along the equator without touching the add-port micro-taper.
These figures show the small phase change due to moving the drop-port’s coupling point in spatial. As there is no light source from the drop-port micro-taper, the interference pattern isn’t generated by the micro-taper. The interference pattern is a duplication of WGMs’ spectrum at spatial dependent drop-port coupling point.
Summary

The drop-port works as WGMs spatial probe when working with low power ASE light source. The interference pattern is determined by the coupling coefficient between add-port micro-taper and the microsphere. As the add-port micro-taper doesn’t work at the optimized coupling point and relatively high coupling rate out of the drop-port, the microsphere’s capability of retaining energy is reduced; it is shown that the troughs in result spectrums are not deep and the quality factor Q is not high.

The drop-port doesn’t work with powerful light source and coherent broad band light source because the two micro-taper is so close that any power higher than critical power will totally change the interference pattern. The WGMs are standing waves, whose spatial phase can be measured by a drop-port probe with at least $\pi/2$ phase shifting. The noise is low for the drop-port spectrum due to no interfering inside the micro-taper.

The drop-port function of a micro-cavity has the potential to be applied on multi-port dynamic optical memory, signals routing with much lower coupling coefficient than add-port.
Chapter 4 As$_2$Se$_3$ Microsphere

Application Experiment

This chapter is to investigate different behaviors of chalcogenide microsphere invoked under the condition same as the silica microsphere. Erbium doped fiber cavity emissions is selected as the invoking light source, and As2Se3 microsphere coupling with a micro-taper works as output end feedback in cavity, the other feedback is a 99.9% broad band fiber brag grating whose center wavelength is 1550nm and its working band is around 10 nm. When the erbium doped fiber is excited by 980nm laser injected through the fiber brag gating, the continuous spectrum around 1550nm is being reflected back and forth in the cavity formed by the two feedbacks. As the microsphere’s feedback is only partial reflection on selected wavelengths, it supposed to send out coherent light at the minimal coupling transmission wavelength. The cavity’s light output is at the microsphere/micro-taper side.

The similar experiment on silica microsphere had been reported; also, in this experiment, a silica microsphere is investigated as the feedback with the same setup as a comparison.
Experiment setup

For this experiment, components needed are listed as following: one 980nm diode laser and controller, 25 meter long erbium doped fiber, 10nm broad band centered at 1550nm fiber brag grating, silica micro-taper, silica microsphere, chalcogenide microsphere, isolator, and the OSA sample suite.

As shown in figure 4-1, the 980nm laser is connected to one port of fiber brag grating, and the other port of the fiber brag grating is connected to the 25 meter long erbium doped fiber; the other end of the erbium doped fiber is connected to a low loss micro-taper (less than 10dB), the micro-taper’s output is connected to the OSA through a circulator. The up limitation of the working current for the 980nm diode laser is 200mA. During this experiment, only the chalcogenide or the silica microsphere is allowed to be moved to acquire an optimized coupling.

![Figure 4-1 Experiment setup for microsphere as a cavity feedback](image)

All the data sampled by the OSA is transferred to Labview & Matlab environment through GPIB; the resolution of the wavelength on the OSA was set to be 0.05nm and the number of sampling points was set to be 5001.
Experiment Results

When there’s no coupling between the microsphere and the micro-taper, the 980nm diode laser with the current 200mA invokes the erbium doped fiber cavity to generate one high peak on the OSA at the longest wavelength edge 1554.7 nm of the fiber bragg grating. Adjust the 980nm laser diode working current to remove the peak on the spectrum caused by the micro-taper’s feedback, and the decoupled state without any peak is set to the initial state.

From the initial state, use the silica microsphere as the feedback coupling microsphere; when there is a coupling happened between the silica microsphere and the micro-taper, there is one stable peak appeared around 1550nm on the spectrum. Scanning the microsphere along the micro-taper changes the peak to a wavelength closed to original one.

![Figure 4-2 Spectrum of erbium doped fiber cavity using silica microsphere as feedback](image)
Figure 4-2 shows the spectrum result using silica microsphere coupling as the feedback. There is only one peak generated for the silica microsphere coupling as the feedback. It is very stable during the experiment. The quality factor is given by $Q = \frac{\lambda_0}{\Delta \lambda_{FWHM}} = \frac{1553.12}{0.02} = 76656$, which is limited by the OSA’s precision.

Keeping the same setup configuration and exciting lasers current, decouple the microsphere and the micro-taper, take place of the silica microsphere with a chalcogenide microsphere, and repeat the coupling procedure as for the silica microsphere. When there is a coupling happened, many peaks appeared inside the fiber brag grating’s wavelength range.

The stability of the multi-peak is different for weak coupling, over coupling and optimized coupling. At the weak or critical coupling state, the multi-peaks are not located at fixed wavelength, the peaks wavelength kept changing without any regular orders and the peaks’ amplitudes are different from each other. When it is in the optimized coupling and over coupling state, the variation of the peaks’ wavelength is slow or just partial peaks changed the wavelength. In the optimized and over coupling the amplitudes of some peaks still kept changing up to 30% of its highest peak. The stable wavelength peaks could change their amplitude from a few seconds to 60 seconds. Figure 4-3 shows a multi-peaks cavity output taking chalcogenide microsphere as a feedback.
Figure 4-3 Spectrum of erbium doped fiber cavity using chalcogenide microsphere as feedback

The quality factor around 1550nm is given by $Q = \frac{\lambda_0}{\Delta \lambda_{FWHM}} = 1550.35/0.07 = 22147$. The peaks separation is about 0.75 nm. The gain is larger than 35dB. In the figure 4-2, there are also unstable side peaks along with most of the main peaks. As the fiber brag grating does not reflect different frequencies with exactly same reflectance, the relatively lower frequencies have the higher reflectance; and more peaks tend to appear on the right part of the fiber brag grating’s wavelength range.

At over coupling state, peaks changing is slow and new peaks always appeared around certain wavelengths. Figure 4-5 shows the multi-peaks’ wavelengths 120 second after the spectrum sampling of figure 4-4. While the peaks with low fiber brag grating reflectance tend to disappear or generate more frequently than that with relatively higher cavity reflectance. In any case, there is side peak coexisted with main peaks.
For stable feedback peaks, there are tiny changes at the main peaks due to cavity frequency selection. Figure 4-6 and 4-7 show the side peak change with 30 seconds spectrum sampling interval.

Decouple the current chalcogenide microsphere and changed a different radius chalcogenide microsphere to repeat above procedure for chalcogenide microsphere. The experiment results are shown on figure 4-8, 4-9. The difference in radius doesn’t significantly change the feedback peaks’ wavelength. The peaks appearance is still unforeseeable. And the peaks’ stability is same as previous experiment of chalcogenide microsphere as feedback.
Summary

The gain cavity’s micro lasers are related to the feedback’s transmission. The fiber brag grating has more than 99% reflection for the ±5nm wavelength around 1550nm; and the cavity is 25 meters long, which is able to accommodate more density longitudinal modes; the mode whose frequency meets the high reflectance of the transmission coupling will be reinforced by the erbium doped fiber cavity. The coupled microsphere is working as the feedback for the cavity. Coupled silica microsphere provides high reflectance for a single wavelength, while coupled chalcogenide microsphere provides high reflectance for unforeseeable multi-wavelength.

The frequency with high transmission rate on the interference pattern cannot invoke the resonating modes in the erbium doped fiber cavity. The light whose frequency falls in the interference troughs has the opportunity to be reflected back from the microsphere along the reverse path of the shifted ray. The reflected light is reinforced in the erbium doped fiber cavity and forms micro-laser.
The high reflectance is determined by the particles inside microspheres near field and coupling coefficient of in the direction of the reverse shifted ray; these particles are formed due to photo and thermal effects. Photons in the shifted ray are scattered by free carriers and particles. Counter scattered photons with same phase form a wavefront. In a closed loop cavity such as the equator of a microsphere, counter propagating wavefront constructively interferes with forth propagating wavefront to form WGMs.

Perfect silica microsphere has quasi-isotropic lattice distribution along its equator. The wavefront being scattered back has less phase-mismatch, so it is easy to for the highest reflectance wavelength to form the strong resonating mode in the gain cavity.

Chalcogenide microsphere is an anisotropic one even if its viscous stress is released during fabrication. The anisotropic structure cannot keep the reflected wave front phase matching all the time. Plus, the chalcogenide As2Se3 are sensitive to environment light and temperature change; many photo carriers and thermal carriers can be formed around the affected surface, and they have life time up to 30 second. When photons are scattered by these free carriers, noises are generated. The uncertainty multi-peak laser is caused by the unforeseeable reflectance inside the chalcogenide microsphere.
Conclusion

Anisotropic chalcogenide microsphere was investigated from both theoretical simulation and coupling experiments. It shows some common properties as reported silica microsphere while it has some unique response for higher energies coupling experiments. Chalcogenide microsphere’s quality factor Q is much lower than silica microsphere’s, and its WGMs’ stability is affected by input power of the light source, coherent state of the light source, environment light radiation and thermal state. Chalcogenide microsphere has stronger modes binding capability than silica microsphere.

The anisotropic structure scatters photons of shifted ray into phase mismatching state. Chalcogenide microsphere coupling with micro-taper is sensitive when using ASE light source. On the contrary, coherent broad band light source reduces the requirement for the isotropic scattering to form the phase-matching wave front. All the transmission coupling results consist with the coupling reflection experiment results, while transmission coupling works with high power light source and coupling reflection is suitable to portable experiment environment. Drop-port function is investigated and shows the spatial-phase relationship of the WGMs.

Using the coupled microsphere as reflection feedback of a gain cavity gives the clear view of the difference between silica microsphere and chalcogenide microsphere. Chalcogenide microsphere has uncertain reflectance inside the WGMs’ near field under high energy density.
To fully realize the potential of the chalcogenide microsphere resonator, further investigation which isolates its anisotropic properties from environment light and thermal effects is needed. It should be aimed at the following goals:

- Fabrication of the chalcogenide microsphere with an embedded metal pin as the voltage bias stem;
- Coupling with bias voltage to eliminate the photocurrent effect.

The chalcogenide microsphere has the potential to be used on dynamic optical memory and MID-IR sensing domain.
Appendix A  TE & TM WGMs derivation and Matlab Code

**TE-modes** [13]:

The electric field is given by

\[
\begin{align*}
E_r &= 0 \\
E_\theta &= \frac{1}{\epsilon r \sin \theta} \frac{\partial F_r}{\partial \phi} \\
E_\varphi &= -\frac{1}{\epsilon r} \frac{\partial F_r}{\partial \theta}
\end{align*}
\]  
(A-1)  
(A-2)  
(A-3)

and the magnetic field is given by

\[
\begin{align*}
H_r &= -\frac{1}{j \omega \mu \epsilon} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) F_r \\
H_\theta &= -\frac{1}{j \omega \mu \epsilon r} \frac{\partial^2 F_r}{\partial r \partial \phi} \\
H_\varphi &= -\frac{1}{j \omega \mu \epsilon r \sin \theta} \frac{\partial^2 F_r}{\partial \phi}
\end{align*}
\]  
(A-4)  
(A-5)  
(A-6)

The boundary conditions at the surface of microsphere of TE modes satisfy (1-8a) and (1-8b).

Inside the microsphere:

\[
F_{1r} = r F_{i m}^m(r, \theta, \varphi) = r a_j(kr) P_{i m}^m(\cos \theta) \sin(m\varphi)
\]  
(A-7)

\[
E_{1\theta} = \frac{1}{\epsilon r \sin \theta} \frac{\partial F_{1r}}{\partial \varphi} = \frac{1}{\epsilon r \sin \theta} \frac{\partial \left( r F_{i m}^m(r, \theta, \varphi) \right)}{\partial \varphi}
\]

\[
= \frac{1}{\epsilon r \sin \theta} \frac{\partial \left( r a_j(kr) P_{i m}^m(\cos \theta) \sin(m\varphi) \right)}{\partial \varphi}
\]
Outside the microsphere:

\[ F_{2r} = r F_{1m}^m(r, \theta, \varphi) = r b h_i(2)(kr) p_i^m(\cos \theta) \sin(m\varphi) \]  
(A - 10)

\[ E_{2\theta} = \frac{1}{\varepsilon_0 r \sin \theta} \frac{\partial F_{2r}}{\partial \varphi} \]

\[ = \frac{1}{\varepsilon_0 r \sin \theta} \frac{\partial (r F_{1m}^m(r, \theta, \varphi))}{\partial \varphi} \]

\[ = \frac{1}{\varepsilon_0 r \sin \theta} \frac{\partial (r b h_i(2)(kr) p_i^m(\cos \theta) \sin(m\varphi))}{\partial \varphi} \]

\[ = \frac{1}{\varepsilon_0 r \sin \theta} r b h_i(2)(k_0 r) p_i^m(\cos \theta) \cos(m\varphi) m \]  
(A - 11)

\[ E_{2\varphi} = -\frac{1}{\varepsilon_0 r} \frac{\partial F_{2r}}{\partial \theta} \]

\[ = -\frac{1}{\varepsilon_0 r} \frac{\partial (r b h_i(2)(kr) p_i^m(\cos \theta) \sin(m\varphi))}{\partial \theta} \]

\[ = -\frac{1}{\varepsilon_0 r} r b h_i(2)(k_0 r) p_i^m(\cos \theta)(- \sin \theta) \sin(m\varphi) \]  
(A - 12)

Put (A-8) (A-9) into (1-9a) and put (A-11) (A-12) into (1-9b) and substitute them into (1-8a) and got one relationship for coefficients \( a \) and \( b \).

\[ \frac{a j_i(kr)}{\varepsilon} = \frac{b h_i(2)(k_0 r)}{\varepsilon_0} \]  
(A - 13)
\[ H_{1\theta} = -\frac{1}{j\omega \varepsilon} \frac{\partial^2 F_{1r}}{\partial r \partial \theta} = \frac{a P_l^m(\cos \theta)(\sin \theta) \sin(m \varphi)}{j\omega \varepsilon} \left[j_l(k_r) + k r j_l'(k_r)\right] \quad (A - 14) \]

\[ H_{1\varphi} = -\frac{1}{j\omega \varepsilon \sin \theta} \frac{\partial^2 F_{2r}}{\partial r \partial \varphi} = -\frac{a m P_l^m(\cos \theta) \cos(m \varphi)}{j\omega \varepsilon \sin \theta} \left[j_l(k_r) + k r j_l'(k_r)\right] \quad (A - 15) \]

\[ H_{2\theta} = -\frac{1}{j\omega \varepsilon_0 \varepsilon_r} \frac{\partial^2 F_{1r}}{\partial r \partial \theta} = \frac{b P_l^m(\cos \theta)(\sin \theta) \sin(m \varphi)}{j\omega \varepsilon_0 \varepsilon_r} \left[h_l^{(2)}(k_0 r) + k r h_l^{(2)'}(k_0 r)\right] \quad (A - 16) \]

\[ H_{2\varphi} = -\frac{1}{j\omega \varepsilon_0 \varepsilon_r \sin \theta} \frac{\partial^2 F_{2r}}{\partial r \partial \varphi} = -\frac{b m P_l^m(\cos \theta) \cos(m \varphi)}{j\omega \varepsilon_0 \varepsilon_r \sin \theta} \left[h_l^{(2)}(k_0 r) + k r h_l^{(2)'}(k_0 r)\right] \quad (A - 17) \]

Put (A-14) (A-15) into (1-9c) and put (A-16) (A-17) into (1-9d) and substitute them into (1-8b) and got another relationship for coefficients \(a\) and \(b\).

\[ \frac{a}{\varepsilon} [r j_l(k_r)]' = \frac{b}{\varepsilon_0} \left[r h_l^{(2)}(k_0 r)\right]' \quad (A - 18) \]

Merge (A-13) and (A-18) the dispersion relationship for TE WGMs is:

\[
\frac{h_l^{(2)}(k_0 r)}{h_l^{(2)}(k_0 r)} = n_s \frac{l_{l-1}(n_s k_0 r)}{l_l(n_s k_0 r)}
\]  

(A - 19)
Matlab code for TE mode whispering gallery mode

function [wavelength] = TE_WGM(r,ns,l)
    nr=ns*r;
    relation=@(k0) besselh(l-1,2,k0*r)/besselh(l-1,2,k0*r)-ns*besselj(l-1,k0*nr)/besselj(l-1,k0*nr);
    wavelength=linspace(1500e-9,1560e-9,5000);
    k=2*pi./wavelength;
    counter=1;
    for i=1:50000
        try
            result=fzero(relation, k(i));
        catch exception
            result=0;
        end
        if(result~=0 && ~isnan(result))
            k0(counter)=abs(result);
            counter=counter+1;
        end
    end
    K0=sort(k0);
    counter=1;
    for i=1:length(K0)
        lastone=K0(i);
        if(i==1)
            k0(counter)=lastone;
            counter=counter+1;
        else
            if(abs(lastone-k0(counter-1))> abs(0.01*lastone))
                k0(counter)=lastone;
                counter=counter+1;
            end
        end
    end
    wavelength=sort(2*pi./k0*1e9);
    end
**TM modes** [13]:

The electric field is given by

\[
\begin{align*}
E_r &= \frac{1}{j\omega \mu \varepsilon} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) A_r \\
E_\theta &= \frac{1}{j\omega \mu \varepsilon} \frac{\partial^2 A_r}{\partial r \partial \theta} \\
E_\varphi &= \frac{1}{j\omega \mu \varepsilon \sin \theta} \frac{\partial^2 A_r}{\partial \varphi \partial \theta}
\end{align*}
\] (A – 20, A – 21, A – 22)

and the magnetic field is given by

\[
\begin{align*}
H_r &= 0 \\
H_\theta &= \frac{1}{\mu \varepsilon \sin \theta} \frac{\partial A_r}{\partial \varphi} \\
H_\varphi &= -\frac{1}{\mu \varepsilon} \frac{\partial A_r}{\partial \theta}
\end{align*}
\] (A – 23, A – 24, A – 25)

The boundary conditions at the surface of microsphere of TM modes satisfy (1-8a) and (1-8b).

Inside of microsphere:

\[
A_{1r} = r A_l^m(r, \theta, \varphi) = r a j_l(kr) P_l^m(\cos \theta) \sin(m\varphi)
\] (A – 26)

\[
\begin{align*}
H_{1\theta} &= \frac{1}{\mu \varepsilon \sin \theta} \frac{\partial A_{1r}}{\partial \varphi} = \frac{1}{\mu \varepsilon \sin \theta} \frac{\partial (r a j_l(kr) P_l^m(\cos \theta) \sin(m\varphi))}{\partial \varphi} \\
&= \frac{1}{\mu \varepsilon \sin \theta} \frac{\partial (r a j_l(kr) P_l^m(\cos \theta) \sin(m\varphi))}{\partial \varphi} \\
&= \frac{1}{\mu \varepsilon \sin \theta} r a j_l(kr) P_l^m(\cos \theta) \cos(m\varphi) m
\end{align*}
\] (A – 27)

\[
\begin{align*}
H_{1\varphi} &= -\frac{1}{\mu \varepsilon} \frac{\partial A_r}{\partial \theta} = -\frac{1}{\mu \varepsilon} \frac{\partial (r a j_l(kr) P_l^m(\cos \theta) \sin(m\varphi))}{\partial \theta} \\
&= -\frac{1}{\mu \varepsilon} r a j_l(kr) P_l^m(\cos \theta)(-\sin \theta) \sin(m\varphi)
\end{align*}
\] (A – 28)
Outside of microsphere:

\[ A_{2r} = rA_{l}^{m}(r, \theta, \varphi) = rbh_{l}^{(2)}(kr)P_{l}^{m}(\cos \theta) \sin(m\varphi) \]  \hspace{1cm} (A - 29)

\[ H_{2\theta} = \frac{1}{\mu r \sin \theta} \frac{\partial A_{2r}}{\partial \varphi} = \frac{1}{\mu r \sin \theta} \frac{\partial(rA_{l}^{m}(r, \theta, \varphi))}{\partial \varphi} \]

\[ = \frac{1}{\mu r \sin \theta} \frac{\partial \left(rbh_{l}^{(2)}(kr)P_{l}^{m}(\cos \theta) \sin(m\varphi)\right)}{\partial \varphi} \]

\[ = \frac{1}{\mu r \sin \theta} rbh_{l}^{(2)}(k_{0}r)P_{l}^{m}(\cos \theta) \cos(m\varphi)m \]  \hspace{1cm} (A - 30)

\[ H_{2\varphi} = - \frac{1}{\mu r \sin \theta} \frac{\partial A_{2r}}{\partial \theta} = - \frac{1}{\mu r} \frac{\partial \left(rbh_{l}^{(2)}(kr)P_{l}^{m}(\cos \theta) \sin(m\varphi)\right)}{\partial \theta} \]

\[ = - \frac{1}{\mu r} rbh_{l}^{(2)}(k_{0}r)P_{l}^{m'}(\cos \theta)(- \sin \theta) \sin(m\varphi) \]  \hspace{1cm} (A - 31)

Put (A-27) (A-28) into (1-9c) and (A-30) (A-31) into (1-9d) and then substitute them into (1-8b) and got one relationship for coefficients \( a \) and \( b \).

\[ a_{j_{l}}(kr) = bh_{l}^{(2)}(k_{0}r) \]  \hspace{1cm} (A - 32)

\[ E_{1\theta} = \frac{1}{j\omega \mu r} \frac{\partial^{2}A_{1r}}{\partial \theta \partial \varphi} = - \frac{aP_{l}^{m'}(\cos \theta)(\sin \theta) \sin(m\varphi)}{j\omega \mu r} [j_{l}(kr) + krj_{l}'(kr)] \]  \hspace{1cm} (A - 33)

\[ E_{1\varphi} = \frac{1}{j\omega \mu r \sin \theta} \frac{\partial^{2}A_{1r}}{\partial \theta \partial \varphi} = - \frac{aP_{l}^{m'}(\cos \theta) \cos(m\varphi)}{j\omega \mu r \sin \theta} [j_{l}(kr) + krj_{l}'(kr)] \]  \hspace{1cm} (A - 34)

\[ E_{2\theta} = \frac{1}{j\omega \mu e_{r}} \frac{\partial^{2}A_{2r}}{\partial \theta \partial \varphi} = - \frac{bP_{l}^{m'}(\cos \theta)(\sin \theta) \sin(m\varphi)}{j\omega \mu e_{r}} \left[h_{l}^{(2)}(k_{0}r) + krh_{l}^{(2)'}(k_{0}r)\right] \]  \hspace{1cm} (A - 35)

\[ E_{2\varphi} = \frac{1}{j\omega \mu e_{r} \sin \theta} \frac{\partial^{2}A_{2r}}{\partial \theta \partial \varphi} = \frac{bP_{l}^{m'}(\cos \theta) \cos(m\varphi)}{j\omega \mu e_{r} \sin \theta} \left[h_{l}^{(2)}(k_{0}r) + krh_{l}^{(2)'}(k_{0}r)\right] \]  \hspace{1cm} (A - 36)

Put (A-33) (A-34) into (1-9a) and (A-35) (A-36) into (1-9b) and then substitute them into (1-8a) and got one relationship for coefficients \( a \) and \( b \).
\[
\frac{\alpha}{\varepsilon} [j_1(kr) + kr j'_1(kr)] = \frac{b}{\varepsilon_0} \left[ h^{(2)}_l(k_0 r) + kr h^{(2)'}_l(k_0 r) \right]
\] (A - 37)

Substitute (A-32) into (A-37), eliminate \( a \) and \( b \), the dispersion relationship is:

\[
-l + k_0 r \frac{h^{(2)}_{l-1}(k_0 r)}{h^{(2)}_l(k_0 r)} = -\frac{l}{n_s^2} + \frac{k_0 r j_{l-1}(n_s k_0 r)}{n_s j_l(n_s k_0 r)}
\] (A - 38)
Matlab code for TM mode whispering gallery mode

```matlab
function [wavelength] = TM_WGM(r,ns,l)

nr=ns*r;

relation=@(k0)-l+k0*r*besselh(l-1,2,k0*r)/besselh(l-1,2,k0*r)+1/ns/ns-k0*r/ns*besselj(l-1,k0*nr)/besselj(l,k0*nr);

wavelength=linspace(1500e-9,1560e-9,5000);
k=2*pi./wavelength;

counter=1;
for i=1:50000
    try
        result=fzero(relation, k(i));
    catch exception
        result=0;
    end
    if(result~=-0 && ~isnan(result) )
        K0(counter)=abs(result);
        counter=counter+1;
    end
end

K0=sort(K0);
for i=1:length(K0)
    lastone=K0(i);
    if(i==1)
        k0(counter)=lastone;
        counter=counter+1;
    else
        if(abs(lastone-k0(counter-1))> abs(0.01*lastone) )
            k0(counter)=lastone;
            counter=counter+1;
        end
    end
end

wavelength=sort(2*pi./k0*1e9);
end
```
Appendix B  Matlab code for the simulation of nonlinear coupling between microsphere and micro taper

function Cavity(Pp_i,delta_W,Diameter)
% pump power, in Watt
% wavelength difference between pump and signal in nm,
% microsphere diameter in meter
NUM=8000;
Wp=linspace(1500e-9,1600e-9,NUM);
Ws=Wp-1e-9*delta_W;
Wc=Wp+1e-9*delta_W;

c=299792458;  % m/s
h=6.62606876e-34;  % Plank constant
coupling_a=7;  % virtual coupling angle for designate a coupling coefficient
L=pi*Diameter;  % Cavity length
sigma=sin(coupling_a/180*pi);  % coupling coefficient
tau=cos(coupling_a/180*pi);  % transmission coefficient
np=2.80;  % pump refractive index for chalcogenide
ns=2.80;  % signal refractive index for chalcogenide
nc=2.80;  % converted refractive index for chalcogenide
omega_p=2*pi*c./Wp;
omega_s=2*pi*c./Ws;
omega_c=2*pi*c./Wc;
kp=2*pi./Wp;  % wave number for pump
ks=2*pi./Ws;  % wave number for signal
kc=2*pi./Wc;  % wave number for converted
n2=1.1e-13*1e-4;  % nonlinear refractive index for chalcogenide
% Aeff=37e-12;  % reference value from As2Se3 Fiber 37um^2
Aeff=0.0946e-12;
gama_p=n2.*omega_p/c/Aeff;  % nonlinear parameter
Gama=gama_p;

delta_k=(ns.*omega_s+nc.*omega_c-2*np.*omega_p)/c;  % linear phase mismatch

% Ep_i=Pp_i^0.5;
% Ep_a=j*sigma*Ep_i+tau*Ep_b*exp(j*kp*L);
% Ep_o=tau*Ep_i+j*sigma*Ep_b*exp(j*kp*L);
% Es_a=j*sigma*Es_i+tau*Es_b*exp(j*kp*L);
% Es_o=tau*Es_i+j*sigma*Es_b*exp(j*kp*L);
% Ec_a=j*sigma*Ec_i+tau*Ec_b*exp(j*kp*L);
% Ec_o=tau*Ec_i+j*sigma*Ec_b*exp(j*kp*L);

% Ep_a=j*sigma*Ep_i+tau*Ep_b*exp(j*kp*L);
% Ep_o=tau*Ep_i+j*sigma*Ep_b*exp(j*kp*L);
% Es_a=j*sigma*Es_i+tau*Es_b*exp(j*kp*L);
% Es_o=tau*Es_i+j*sigma*Es_b*exp(j*kp*L);
% Ec_a=j*sigma*Ec_i+tau*Ec_b*exp(j*kp*L);
% Ec_o=tau*Ec_i+j*sigma*Ec_b*exp(j*kp*L);

delta_k=(ns.*omega_s+nc.*omega_c-2*np.*omega_p)/c;  % linear phase mismatch
and the refractive index is supposed to be improved by a curving array

\[ \beta_{TPA} = 0.25 \times 10^{-11}; \]

for As₂Se₃, the TPA \( \beta = 0.25 \text{ cm/GW} = 0.25 \times 10^{-11} \text{ m/W} \)

\[ \sigma_F = 1.45 \times 10^{-21}; \]

\( \sigma \) cross section area of FCA \( 1.45 \times 10^{-17} \text{ cm}^2 \)

\[ \tau_{eff} = 0.5 \times 10^{-9}; \]

% Carrier lifetime 0.5 ns

% The linear absorption loss for As₂Se₃ = 1.12 × 10⁻² 1/cm

for \( k = 1: \text{NUM} \)

\[ \text{Pump} = \sigma^2 \times \text{Pp}_a \times \text{abs} \left( \tau \times \exp \left( i(k_p(k) + \Gamma(k)) \times L \right) \right) - a_L \times \text{Pp}_a \times \beta_{TPA}/A_{eff} + \sigma_F \times \beta_{TPA} \times \tau_{eff} / 2 / h / c / A_{eff} / A_{eff} \times W_p(k) \times \text{Pp}_a^2 \times L / 2 - 1)^2; \]

try

\[ \text{Pp}_a(k) = \text{fzero}(\text{Pump}, \text{Pp}_i); \]

catch exception

end

end

\[ \text{aT} = \text{Pp}_a \times \beta_{TPA}/A_{eff}; \]

\[ \text{aF} = \sigma_F \times \beta_{TPA} \times \tau_{eff} / 2 / h / c / A_{eff} / A_{eff} \times W_p \times \text{Pp}_a \times \text{Pp}_a^2; \]

\[ \text{aL} = a_L + a_T + a_F; \]

\[ a_2 = a_L + 2 \times a_T + a_F; \]

% \( \text{Pp}_a = \text{Ep}_a^2; \)

\[ \delta_{beta} = \delta_k + 2 \times \Gamma(k) \times \text{Pp}_a; \]

\[ \text{Fp} = \exp \left( -a_1 \times L / 2 + i \times (\Gamma(k) \times \text{Pp}_a + k_p) \times L \right); \]

\[ \text{Tp} = \exp \left( -a_1 \times L / 2 + i \times (\Gamma(k) \times \text{Pp}_a + k_p) \times L \right) \times \left( 1 - \exp \left( -a_1 \times L / 2 + i \times (\Gamma(k) \times \text{Pp}_a + k_p) \times L \right) \right) / 2; \]

\[ Tp_{dB} = 10 \times \log_{10}(\text{Tp}); \]

\[ \text{peak}(1) = 0; \]

\[ \text{peaks} = 0; \]

\[ \text{last_wavelength} = 0; \]

\[ \text{fsr} = 0; \]

\[ \text{first_peak} = 0; \]

for \( k = 2: \text{NUM} - 1 \)

\[ \text{if}(\text{Tp}(k-1) > \text{Tp}(k) \text{ and } \text{Tp}(k+1) > \text{Tp}(k)) \]

\[ \text{peak}(k) = 1; \]

\[ \text{peaks} = \text{peaks} + 1; \]

\[ \text{if}(\text{last_wavelength} = 0) \]

\[ \text{fsr} = W_p(k) - \text{last_wavelength} + \text{fsr}; \]

\[ \text{else} \]

\[ \text{first_peak} = W_p(k); \]

\[ \text{fsr} = 0; \]

\[ \text{end} \]

\[ \text{last_wavelength} = W_p(k); \]

\[ \text{else} \]

\[ \text{peak}(k) = 0; \]
peak(NUM)=0;

fsr=fsr/(peaks-1)

Fs=abs(sigma./(1-tau.*exp(-a2*L/2+1i*(2*Gama.*Pp_a+ks)*L)));

Ts=abs((tau-exp(-a2*L/2+1i*(2*Gama.*Pp_a+ks)*L))/(1-tau*exp(-a2*L/2+1i*(2*Gama.*Pp_a+ks)*L)).^2);

Ts_DB=10*log10(Ts);

Fc=abs(sigma./(1-tau.*exp(-a2*L/2+1i*(2*Gama.*Pp_a+kc)*L)));

Leff=abs((1-exp(-a1+1i*delta_beta)*L))./(a1+1i*delta_beta));

conversion_efficiency=(Gama.*Pp_i.*Leff.*Fp.^2.*Fs.*Fc).^2.*exp(-a2*L);

conversion_efficiency_DB=10*log10(conversion_efficiency);

[C,I]=max(conversion_efficiency_DB);

PPI=linspace(0,1,NUM);

omega_power=2*pi*c/1550e-9;

Gama_power=n2*omega_power/c/Aeff;

W=Wp(I);

Kp=2*pi/W;                  % wave number for pump

Ks=2*pi/(W-1e-9*delta_W);  % wave number for signal

Kc=2*pi/(W+1e-9*delta_W);  % wave number for converted

for k=1:NUM

    Pump_Nonlinear=@(Pp_a_)sigma^2*PPI(k)-Pp_a_*abs(tau*exp(1i*(Kp+Gama_power*Pp_a_)*L-(aL+Pp_a_*Beta_TPA/Aeff*sigma_F*Beta_TPA*Tau_eff/2/h/c/Aeff/Aeff*W*Pp_a_^2)*L/2)));

    try
        PPA_Nonlinear(k)=fzero(Pump_Nonlinear,PPI(k));
        catch exception
    end

    Pump_Linear=@(Pp_a_)sigma^2*PPI(k)-Pp_a_*abs(tau*exp(1i*(Kp)*L-(aL)*L/2)));

    try
        PPA_Linear(k)=fzero(Pump_Linear,PPI(k));
        catch exception
    end

end

aT_power=PPA_Nonlinear*Beta_TPA/Aeff;
aF_power=sigma_F*Beta_TPA*Tau_eff/2/h/c/Aeff/Aeff*W*PPA_Nonlinear.^2;

for time=1:2

    if(time==1)
        a1_power=aL+aT_power+aF_power;
        a2_power=aL+2*aT_power+aF_power;
        PPA=PPA_Nonlinear;
        delta_k=delta_k+2*Gama_power*PPA;
        Fp_power=abs(sigma./(1-tau*exp(-a1_power*L/2+1i*(Gama_power*PPA+Kp)*L)));
        Fs_power=abs(sigma./(1-tau*exp(-a2_power*L/2+1i*(2*Gama_power*PPA+Ks)*L)));
        Fc_power=abs(sigma./(1-tau*exp(-a2_power*L/2+1i*(2*Gama_power*PPA+Kc)*L)));
    end

end
Leff_power = abs((1-exp(-
(a1_power+1i*delta_beta_power)*L))/(a1_power+1i*delta_beta_power));

conversion_efficiency_power=(Gama_power*PPI.*Leff_power.*Fp_power.^2.*Fs_power.*Fc_power
).^2.*exp(-a2_power*L);

conversion_efficiency_db_power=10*log10(conversion_efficiency_power);
conversion_efficiency_db_power_nonlinear_phase=conversion_efficiency_db_power;
else
   a1_power=aL;
   a2_power=aL;
   PPA=PFA_Linear;

   delta_beta_power=delta_k+2*Gama_power*PPA;
   Fp_power=abs(sigma/(1-tau*exp(-a1_power*L/2+1i*(Kp)*L)));
   Fs_power=abs(sigma/(1-tau*exp(-a2_power*L/2+1i*(Ks)*L)));
   Fc_power=abs(sigma/(1-tau*exp(-a2_power*L/2+1i*(Kc)*L)));
   Leff_power=abs((1-exp(-
(a1_power+1i*delta_beta_power)*L))/(a1_power+1i*delta_beta_power));

   conversion_efficiency_power=(Gama_power*PPI.*Leff_power.*Fp_power.*Fs_power.*Fc_power
).^2.*exp(-a2_power*L);

   conversion_efficiency_db_power=10*log10(conversion_efficiency_power);
   conversion_efficiency_db_power_linear_phase=conversion_efficiency_db_power;
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Kfsr=2*pi/first_peak;
Kfsr_shift=2*pi/(first_peak+0.01*fsr);

for k=1:NUM
   Pump_fsr_shift=@(Pp_a_)sigma^2*(PPI(k)+Pp_i)-Pp_a_*abs(tau*exp(1i*(Kfsr_shift+Gama_power*Pp_a_)*L-
(aL+Pp_a_*Beta_TPA/Aeff+sigma_F*Beta_TPA*Tau_eff/2/h/c/Aeff/Aeff*W*Pp_a_^2)*L/2)-1)^2;
   try
      PPA_first_peak_shift(k)=fzero(Pump_fsr_shift,PPI(k)+Pp_i);
   catch exception
      end
end

Tp_shift=abs((tau-exp(-a1*L/2+1i*(Gama.*PPA_first_peak_shift+Kfsr_shift)*L))../(1-
tau*exp(-a1*L/2+1i*(Gama.*PPA_first_peak_shift+Kfsr_shift)*L))).^2;

for k=2:NUM-1
   if(Tp_shift(k-1)>Tp_shift(k) && Tp_shift(k+1)>Tp_shift(k))
required_power=Pp_i+PPI(k)
break;
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure
plot(Wp*1e9,Fp,Wp*1e9,Fs,Wp*1e9,Fc);
legend('Pump','Signal','Converted');
title('Resonat field enhancement');
xlabel('Wavelength(mm)');
ylabel('Resonat field enhancement factor');

figure
plot(Wp*1e9,Tp_dB);
title('Transmission');
ylabel('Transmission(dB)');
xlabel('Wavelength(mm)');
References


