Compositional Strategies in Mensuration and Proportion Canons, ca. 1400 to ca. 1600

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Abstract.

This thesis presents a detailed list of mensuration and proportion canons written between 1400 and 1600 and an analysis of several of those canons. Proportion canons are a subset of mensuration canons, pieces in which one melody, under different rhythmic interpretations according to the rules of mensuration, appears simultaneously in more than one voice. The type of mensuration and proportion canons discussed in this thesis display simultaneous entry of the voices: all voices begin together and sing through the same music at different speeds. This type of composition is difficult to create, and places extreme demands on composers' ingenuity. Existing analysis of such pieces is extremely rare and not particularly revealing of compositional process. This thesis presents a method of analysis that explains proportion and mensuration canons in terms of initial conditions of pitch transposition and mensuration, and explores the contrapuntal options thus made available to the composer.

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Introduction.

How does one go about composing a piece in which all the voices sing the same music at different speeds? This question has intrigued me ever since I first heard, and saw, the four-from-one *Agnus dei II* mensuration canon from the *Missa L'homme armé* by Pierre de la Rue. Eventually I determined to find as many compositions like this one as possible, and to see what I could learn from them through analysis.

I was sure that someone before me would have made a collection of this type of pieces, or that I would encounter some research, especially analysis, on mensuration or proportion canons. While a few writers have considered mensuration and proportion canons as a group of pieces, I found much less than I expected within the scholarly literature on canons. Scattered references exist, beginning with Feininger’s 1937 work on canon and continuing up to the recent publication of the proceedings of the International Conference on Canons and Canonic Techniques in Leuven, Belgium, but there was no exhaustive, or even extensive, list of mensuration canons to be found.\(^1\) I also found very little analysis of this rather striking compositional problem and therefore I have had to adapt to my own ends the canon analyses that I have found.\(^2\) Therefore, this thesis fills a gap in studies on Renaissance music by presenting a list of mensuration and proportion canons culled from the more widely available sources, and by taking seriously the problem of their composition.\(^3\)

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This thesis has two primary goals: to gather in one place a set of compositions that have common traits, and to classify them according to the variations that they exhibit, and to analyze the pieces so as to describe their traits and the compositional strategies that they employ. I undertook the analysis with more than half an eye towards composition, but it could as easily be more abstract. For instance, the composition of mensuration canons strikes me as very much like a game of chess: initial conditions are established, rules for moving elements about are clearly stated and highly restrictive, and each move made affects all subsequent possibilities.

This thesis consists of one chapter on terminology, three analytical chapters, one historical chapter, and an appendix. The first analytical chapter is a discussion of two-voice canons, in which I describe the least complex kind of canon, in 2:1 proportion. This chapter includes a description of the parts of a proportion canon, the initial conditions of composition including placement of the faster voice, and transposition, and diagrams illustrating the crucial role that initial conditions play in determining the first melodic motion in the dux. The importance of dissonance treatment and the preparation of cadences is also discussed.

The second analytical chapter is about three-voice canons. It begins with 1:2:4 proportions of the type found in Senfl’s *Omne trinum perfectum* and then moves on to the triple proportions found in Josquin’s *Agnus dei II*, from the *Missa L’homme armé super voces musicales*. I discuss and illustrate options for melodic motion in the openings of these types of canon at length.

The analyses are focused on the *Agnus dei II* of the *Missa L’homme armé super voces musicales*, a piece that has intrigued music historians for centuries, as I explain in chapter five. I begin with analysis of canons in two-voices, and in the subsequent chapter I expand the same principles to address proportional canons in three-voices. My analysis incorporates two approaches simultaneously: it is an attempt to model the pieces using a mathematical formula as well as an exhaustive “all counterpoint options” exercise on two and three-voice canonic openings. Although formulas and counterpoint exercises have sometimes severe limits, it is my hope that these two approaches will lead to a better understanding of how one might write such a piece.

Chapter four is an analysis of the second *Agnus dei* from La Rue’s *Missa L’homme armé*, linked to Josquin’s by Glarean, and by the two pieces’ obvious similarity.
In this chapter I respond to the only previous analysis that I know of, by Giancarlo Bizzi, which I consider to be a starting point. This *Agnus* is quite resistant to analysis because of the complex relationships between the given voice and its derived voices. Its compositional parameters are quite severely restrictive, and do not lend themselves to formulaic description.

Chapter five presents a short reception history of the Josquin *Agnus dei II*, in which I trace its curiously tenacious, if peripheral, hold on the musicological and theoretical imagination from the sixteenth century to the present day.

No discussion of mensural canons would be complete without a mention of the *Missa Prolationum* by Johannes Ockeghem. While there is no chapter on *Missa Prolationum*, the appendix contains a brief discussion of its structure and notation.

The appendix is a collection of short thumbnail sketches of each of the pieces that I collected. It includes information on the number of voices in the piece, the composer, the sources, editions, notation, and a short description of the music.

Much of the original material presented in this thesis is experimental. There is not a lot of analysis of individual pieces *per se*, although my conclusions about how to perform the experiments were arrived at by examining, and therefore analyzing, mensuration canons. The data that I examine are the series of initial *dux* motions under different sets of initial conditions in two and three voices. Only a few of the melodic motions that the *dux* might make will result in viable conditions for continuing the canon – consonances or legal dissonances. I explore these options in chapters two and three, and to a lesser extent in chapter four. These data are extremely important because they show that this body of canonic composition is not a series of one-offs or happy accidents, and that it is not divorced from non-imitative composition in the same period.

Clearly there are some limits to this study. The most important one in my view is that many more examples of the types of compositions that I have collected here probably exist. At the present time, however, an extensive search through source material is beyond my means. I suspect that unedited manuscripts probably hold examples of mensuration and proportion canons that remain unremarked because they are anonymous. Such pieces are easy to miss, and while they are certainly not very
numerous in the grand scheme of Medieval and Renaissance polyphony, I am reasonably sure that at least a few of them still lie undiscovered.
Chapter 1.

Terms and Definitions.

The term canon is a familiar one for modern musicians. Usually it evokes what is known as a round, a short piece in strict imitation that repeats, such as Row, row, row your boat. Generally, such pieces imitate at the unison and are not particularly complex in terms of harmony or rhythm, although they are highly entertaining to sing. Thus, what we now know as canon, from its most basic form in a round to its most complex form in the music of Ockeghem, Josquin, or J.S. Bach, among others, is grounded in the notion that given melodic material is imitated exactly between one voice and another. Thus, simply put, canons are polyphonic pieces that exhibit strict imitation between voices.

But canons are not necessarily imitative compositions. The word canon does not have to refer to music at all, of course. Even today the term has many meanings, as in literary canons or canon law, and they all derive from the Latin canon, which in turn comes from the Greek kanon, meaning rule, or law. For Renaissance musicians, a canon referred to a “rule or instruction for realizing a composition” that was often verbal, or perhaps written out in rhyme, in the manner of a “recipe” for singing the piece. The late fifteenth century composer and theorist Johannes Tinctoris defined canon in music as “a rule showing the composer’s intention behind a certain obscurity.” The obscurities involved varied in both degree and kind, but the “rules” did not necessarily

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1 R.E. Latham and D.R. Howlett, eds. The Dictionary of Medieval Latin from British Sources. (Oxford: Oxford University Press, 1997), 259-60. This dictionary gives six definitions for the term canon, including 1 Canon, rule (ecclesiastical), (basis of) canon law, 2 rule, table (math.), 3 officially recognized body of scriptures, 4 portion of the service including the words of consecration, 5 prescribed payment, 6 cleric attached to collegiate church. Not all of these definitions apply, but the notion of rule or prescription is clear in all but the last.

2 Peter Urquhart, Canon, Partial Signatures, and “Musica Ficta” in Works by Josquin DesPrez and his Contemporaries (PhD diss., Harvard University, 1988), iii.

direct the performers to sing imitative polyphony, what we now think of as a canon. These *canons* could instead indicate repetition of melodic material, transposition, or rhythmic modification, including the omission of rests; in short, *canon* in the Renaissance sense was a way of showing the performer what to do with the notation, not a description of a certain kind of piece.

To clarify this point, it is worth mentioning some aspects of music notation in the late Medieval and Renaissance periods, up to about 1600. For the present purpose, it is important that music was virtually never presented in score, but rather in parts, the format of which was somewhat variable in the late Middle Ages, and became mostly standardized by the sixteenth century, either in choirbook format (superius and tenor on the left page of each opening, altus and bassus on the right) or in separate partbooks. Compactness of notation was important in manuscripts, and parts were notated in whatever space accommodated them most efficiently.

To this end, music scribes, and in all likelihood the composers themselves, sometimes avoided writing out parts in their entirety, especially tenor parts in masses and motets. The kinds of shorthand notation that they used are a strong indication that conception and notational compactness are in fact impossible to separate. For example, it would seem that in order to create a piece on a tenor that repeats three times, each time at a different speed (using different note values) relative to its notated form (*integer valor*), a composer must plan the repetitions of the tenor and keep them firmly in mind when writing the other parts.

The connection between the format of written music and the processes involved in composing, imagining, or performing it is largely speculative on my part.

---


5 This statement applies to pieces that use pre-existing material in the tenor, which is the case in many isorhythmic motets and mass movements from the fifteenth century. Jessie Ann Owens, in *Composers at Work: The Craft of Musical Composition 1450-1600* (Oxford: Oxford University Press, 1997) mentions several times the directive to “begin with the tenor” given by writers of instructions in counterpoint in the sixteenth century. See chapter two, “Teaching Composition,” pp. 11-33.

6 Jessie Ann Owens’s work points in this direction of course, but here I am thinking of its ramifications. See also Margaret Bent, “Editing Early Music: the Dilemma of Translation,” *Early Music* 22.
My conjecture in this case is that repetition in a tenor part was the sort of thing that need not be written out, since score format was not a requirement for performance. A composer might conceivably be able to write an entire composition, with the help of a cartella, without ever once seeing in score the entirety of what he had written. Although I am not arguing for any particular view of compositional process in canonic writing, the notion of composing and performing without separate written-out voice parts forms a backdrop for my thinking about mensuration and proportion canons.

This thesis is not concerned with canons of what I call the “sequential” type, those that repeat different iterations of a melody in series within one voice. Nor am I exclusively concerned with canons that apply only to a tenor voice. Rather, the mensuration and proportion canons that I have collected here might be called “simultaneous” since the different interpretations of the given melody are presented simultaneously rather than one at a time, and the realization is not limited to a tenor part. I presume, of course, that the processes of composing and resolving both kinds of pieces were extremely similar. The same might be assumed for the realization of the canons thus composed: resolution of canons was a standard part of a musician’s job.

Of course, with the introduction of polyphonic printed music by Ottaviano Petrucci, the rationale of saving space and time that had been part of scribal practice began to change. As printed music reached a different, and perhaps less musically skilful audience than music manuscripts, works that had relied on the ability of a performer to realize a small amount of material according to a stated rule needed to be realized ahead of time by the printer.

The avoidance of fully written-out parts, and the need for canons (or canonic inscriptions) to elucidate them is as important for strictly imitative part-writing as it is for indicating performance procedures that apply to only one part. In general,


Renaissance theorists referred to voices that imitate exactly as *fuga*. *Fuga* might of course be indicated by a verbal *canon*, but was more likely to be present in the composition without any special announcements; imitative writing is after all a very widespread feature of European polyphony between 1400 and 1600.

The first real definition of the term *fuga* appears in the *Terminorum musicae diffinitorium* by Johannes Tinctoris,

Fuga est idemtitas partium cantus quo ad valorem nomen formam et interdum quo ad locum notarum et pausarum suarum.

Fugue is the likeness of the voice-parts in a composition as to the value, name, and shape of their notes and rests, and sometimes even to their degree on the staff.

This definition, however, says nothing about imitation specifically. “Likeness of the voice parts,” “value, name, and shape of notes and rests” and “position” (on the staff) are not confined to imitative textures. Those criteria can apply equally well to non-imitative contexts, such as “sequential” tenor canons.

I will not pursue this question of terminology exhaustively. What I wish to point out is that the terms we use today and the terms by which canonic procedures were known in the Renaissance are not entirely equivalent, and even the convenient substitution of *fuga* for *canon* is a little misleading. More importantly, the definitions of these terms in use in the Renaissance imply a great deal of continuity between different kinds of imitative procedures, which were not considered separately until much later. Although the term *fuga* has the advantage of being more historically appropriate than *canon* when speaking of strictly imitative pieces from the later Middle Ages and Renaissance, I use the two terms interchangeably when speaking of imitative

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10 Tinctoris, *Diffinitorium*, 32-33.

11 Tinctoris’s definition of *fuga* has implications for imitative writing that have been explored in the recent past. See Peter Urquhart, “Calculated to Please the Ear: Ockeghem’s Canonic Legacy.” *Tijdschrift van de Vereniging voor Nederlandse Musiekgeschiedenis* 47 (1997): 72-98. See also Schubert, “Counterpoint Pedagogy,” 511.
counterpoint, and I will refer to “rule” type canons as canonic inscriptions or canonic rubrics, even when the “rule” results in an imitative texture.

As should be apparent from the forgoing discussion, there were many varieties of canonic procedure, some quite simple and direct, others extremely obscure. This is true of canonic inscriptions and of signs, and of the pieces that they represent. Many imitative pieces had no rubric at all, since in most cases signs are sufficient to show what is to be done. As Charles Turner points out, some rubrics “consist merely of telling the performer in words what could be communicated as easily by mensuration signs, clefs, repeat marks, signi and other aspects of conventional notation [...] Without doubt, the most common canonic inscription is that which simply informs the performer of the modus, tempus, and prolation employed.”

Among the types of imitative canonic procedures employed by composers from the late fourteenth to the end of the sixteenth century were imitative canons at various intervals, retrograde canons, inversion canons, double and triple canons, and canons that involve rhythmic transformation. These last could change notes into rests in one or more parts, omit specified notes or note values in any given part, or change the value of certain notes or rests. Among the canonic procedures that use rhythmic transformation are mensuration and proportion canons.

The terms “mensuration” and “proportion” canon are sometimes conflated, but are distinct categories. If they use mensuration signs to indicate proportion, then proportion canons are a subset of mensuration canons. Essentially, both terms indicate that a given melody should be performed at different rates of speed. Proportion canons are conceptually simpler, since all that is involved is the realization of a given series of

12 Obscurity and technical virtuosity were common to all art forms in the late fifteenth century, especially poetry and music. See John Beck, “Formalism and Virtuosity: Fanco-Burgundian Poetry, Music, and Visual Art, 1470-1520,” Critical Inquiry 10 (June 1984): 644-667 for a discussion of some other instances, especially poetry, in which procedure is as important as content. It is certainly true that virtuosic constructions were present in a variety of compositional techniques, including retrograde and augmentation procedures.


14 In German and French the terms seem to be quite distinct: “Proportionskanon,” “canon de proportion” and “canon mensurable” seem to be accepted terms, although none of them are really in widespread usage. See Newes, Fuge, 308 for a similar definition of mensuration and proportion canons.
pitches and rhythms by changing its relationship to a steady pulse. In a proportion
canon, all note values in one voice are related proportionally to one another and to the
pulse. The organization of note values within each voice remains the same. While it
could potentially be very complex, this procedure is commonly not as difficult as it
sounds, as I will show in chapter two.

Mensuration canons work by changing one or more note values from duple to
triple or vice versa. This could involve a proportional relationship (such as three
minims in the time of two, meaning that the semibreves are of equal duration) or a
mensural relationship (if the minims are equal and the semibreves in one voice are of
greater duration than in another voice.) A mensural relationship means that some note
shapes are re-interpreted under a different mensuration. In the above case, semibreves
have been re-interpreted as worth three minims rather than two.

The signs or the canonic inscriptions of the pieces under consideration here
indicate that the different realizations of the given melody should be realized
simultaneously. In these proportion canons, the voices maintain the same internal
rhythmic profile: all of the rhythms within each linear iteration stand in the same
relationship to one another no matter their particular relationship to the tactus. In the
following example, the first Benedictus from Josquin’s Missa L’homme armé super voce
musicales, the lower voice moves at half the speed of the upper voice. The pitches and
rhythms in both voices are notated in exactly the same way, however, and the only
difference is their relationship to one another; there is no difference in the organization
of durations within each voice.

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15 This need not involve the realization of a bit of notation into sound in several ways
simultaneously. As I mentioned earlier, some proportion canons are found in the tenors of masses or
motets and work by having the singers realize the tenor first in one way, then in another, and then
perhaps a third way while the tactus remains steady, and the other voices sing their parts as notated. This
idea can be expressed in a verbal canon, or by means of mensuration signs such as c, c, c, or o, all of
which can indicate proportional relationships. A well-known example is the tenor of Dufay’s motet
Nuper rosarum flores. Sequential repetition of a tenor under different mensurations is not uncommon. For
a discussion of De Orto’s Missa L’homme armé, see Rodin, Josquin, 183.
Mensuration canons, on the other hand, are slightly more complex, and they exist because of the particularly flexible nature of Medieval and Renaissance musical notation. Any given note value (long, breve, semibreve) could be divided into either two or three of the next smaller value. Mensuration canons work by changing at least one note value from duple to triple, so that, for example, a ternary breve in one voice is read as a binary breve in another voice. A change from ternary to binary or vice versa can also change the way that notes fit together rhythmically, so it is actually a less predictable procedure than using proportions. One of the more famous examples is Ockeghem’s Missa Prolationum, in which most of the movements begin as mensuration canons of various types.

Example 2, the opening of the Et resurrexit from Ockeghem’s Missa Prolationum, is a two-voice mensuration canon. The canon is realized from a single notated melody.

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17 Specific notes can be given values in a canonic rubric as well, as in the Old Hall Credo. See appendix 32, page 110, and appendix 34, page 113.

18 The principles of alteration and imperfection, whereby note values changed based on their position relative to other note values. For a full explanation see Carl Parrish, The Notation of Medieval Music, New York: Pendragon Press, 1978. It is worth mentioning that a proportion canon is in fact a kind of mensuration canon, since it can be expressed in terms of mensural changes, but a mensuration canon is not necessarily a proportion canon. Thus the Agnus dei of Ockeghem’s Missa Prolationum is a both a proportion and a mensuration canon, while the Kyrie of the same Mass is a mensuration canon, but not a proportion canon.
according to the two different mensuration signs shown. Those notes that are of
different values are boxed in the example.

Example 1.2. *Et resurrexit*, *Missa Prolationum*, Johannes Ockeghem. Note values that are
different in the two parts have been boxed.19

![Musical notation](image)

Within the categories of mensuration and proportion canon there are of course
many variations and sub-types. I have chosen to describe the examples that I have
collected on the basis of the following criteria: type: mensuration or proportion, the
presence or absence of non-canonic voices, the number of canonic voices, and use of
imitative entries.

A number of canons fall into more than one of these categories. For this
reason, the appendix is accompanied by a set of indices that is searchable by number of
voices, composer, type of opening, and mensural technique.

Certain terminology is not specific to mensuration and proportion canons. In
imitation canons, the term *dux* is used to denote the leading voice, and *comes* to denote
the following voice or voices. How is one to determine which voice is leading and
which is following if both voices enter together? The logic that I have applied is that
the faster or fastest moving voice is by default the *dux*, because it contains all the
information needed to realize any other voices that should be present. Any voice that

Musicological Society, 1947), 29. Note values that change from one part to another have been boxed.
moves more slowly in relation to the beat is therefore a *comes* “following” the *dux*, because its pitches sound later in real time.

All of the technical discussions that follow are based on the premise that the initial melodic motion in the *dux* is foundational for all of the motions that follow. The choice of the second *dux* pitch is naturally entirely dependant upon the first *dux* pitch as it appears in another, slower voice. Thus, the first *dux* pitch affects the second for harmonic as well as melodic reasons, and the chain of causation continues throughout the canon.

I have likened this to the game of chess, in which only a limited number of moves are available and each piece has its own rules, like the rules in counterpoint. The elements to be used (chess pieces or pitches) are arranged according to a set of initial conditions that are themselves limited (only one option in chess; a few more in counterpoint, provided by different mensural and pitch choices). At the beginning, the number of possible ways to manipulate the elements is extremely limited and tends to be highly formulaic, but it grows marginally greater as more choices are made, though each choice will limit the possibilities for future moves.

Renaissance canons are an extremely diverse set of compositions, partly linked together by the compositional procedures employed, partly by the nature of the instructions required to realize them. In the three chapters that follow I will mostly be concerned with enigmatic canons that do not use inscriptions, but rather derive two or more voices from one notated voice by using mensuration signs.

Each type of canon is really a different kind of challenge for the composer. Due to constraints on space and time I have not been able to explore the parameters of all the types of canon that I have collected in the appendix. They are extremely diverse, and grouping them according to very specific categories is not always successful. I hope that at the least it will be apparent that the techniques required for the writing of pieces like them are non-obvious.
Chapter 2.

Two-voice Proportion Canons

Two-voice proportion canons are the simplest kind of mensural canon. Though they could conceivably involve any proportional or mensural combination, all of the examples that I have collected are of the same type, 2:1 proportion canons.

In order to describe proportion canons thoroughly, several concepts need explanation. The most important thing about the analysis of these canons is that they have openings, middles, and endings. A canonic opening is the beginning of the piece, at which point all voices begin sounding simultaneously. In the canonic opening, each voice has a set of initial constraints that partly determine its behaviour. These constraints include the given speed of the voice and its distance in pitch from the other voices.

In a simultaneous proportion canon, the dux is the voice that moves faster than the other voices. The comes therefore moves more slowly. In a 2:1 proportion canon, the dux moves twice as fast as the comes in relation to a steady pulse. Since the dux pitches are projected forward in time in the comes, half of the dux consists of free counterpoint against an already composed comes.

The second initial condition is transposition between dux and comes. The common transpositions are at the unison, the fifth, and the octave. Canons at the fifth will have different initial conditions from canons at the unison or octave. The importance of initial conditions is that they pose constraints on the first melodic motion in the dux. These constraints turn out to be very severe, excluding many possible motions in the dux.

Under some circumstances there may be only one or two acceptable options for the dux. The reason for this lies partly in the basic concept of species counterpoint. Counterpoint rules from the fourteenth century onwards only allow dissonances on minims, semiminims, and fusas. Breves and semibreves in first species were necessarily consonant. This very simple fact places an even greater constraint on harmonic content in a mensuration or proportion canon. When written out, a note value that can accept a dissonance in one voice may become a note value that demands a consonance in another voice. For example, any passing minims in the
dux of a 2:1 proportion canon will become semibreves in the comes, and then care must be taken to ensure that the dux makes only acceptable dissonances.

Both the forgoing and the following discussions make some important assumptions. The first is that the beat falls on the semibreve. While this is not an immutable rule in the musical practice of the fifteenth and sixteenth centuries, I will use it for the sake of clarity. When we are confronted with mensuration signs such as c and ε, the latter is a sign of diminution, or it shows that the breve, rather than the semibreve, is meant to fill one beat. In other words, it shows that the music is twice as fast as notated. The signs c and ε simultaneously, as in the canons in this chapter, indicates that one of the voices is being sung as though its semibreves were actually breves, and the steady pulse referred to breves, thus, twice as fast as the other voice. The distinction between diminution and augmentation is important, since by diminishing the note values, the composer enables dissonance to take place in the faster moving voice. For the moment, I am only considering music that is in duple mensuration.

One other term that I will use is rhythmic position (RP), which refers to the numbered sequence of semibreve beats. The numbering starts with the downbeat and continues until the end of the piece, with each rhythmic position numbered successively. The RPs fall in different places in real time, so a given number appears in two different places in the score. In proportion canons, rhythmic positions share a pitch and rhythm identity in different parts. In other words, a given RP looks just like the identically numbered RP in another voice. In non-proportional mensuration canons, this does not hold true: a given RP will not necessarily share an identity with the same number in another voice.

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Example 2.1. Score with rhythmic positions labeled in the *Benedictus, Missa Sine nomine*, Josquin des Prez.

The following musical figures show the possibilities for first melodic motion given the initial constraints listed above each figure. The acceptable motions are placed in a box, those that yield unacceptable dissonances are marked with an “X.”

**Figure 2.1. Duple mensuration, two-voices, 2:1 proportion, unison, fastest voice in semibreves.**

There are clearly not many options for the composer: only a motion of a third up or down will satisfy the initial conditions. Even if one were to use semibreves and minims rather than breves and semibreves, the result would still be the same given these initial conditions, because even in minims one cannot leap to a dissonance, and ordinarily a dissonance arrived at by step must be left by step in the same direction.

The following figures show the different possibilities for initial *dux* motion when the pitch interval of imitation changes. I considered only the unison, fifth, and octave for historical reasons; those are the acceptable opening sonorities for a piece of fifteenth or sixteenth century music. I have not included the repeat of the opening note as an option in the following figures, although that is always an option.
Figure 2.2. Duple mensuration, two-voices, 2:1 proportion, *comes* at the fifth above.

Figure 2.3. Duple mensuration, two-voices, 2:1 proportion, *comes* at the fifth below. The *Benedictus* of Josquin’s *Missa Sine nomine* uses this option, with a descending third.

Figure 2.4. Duple mensuration, two-voices, 2:1 proportion, *comes* at the octave above.

Figure 2.5. Duple mensuration, two-voices, 2:1 proportion, *comes* at the octave below.
Table 2.1. *Dux* options for two-voice 2:1 proportion canons.

<table>
<thead>
<tr>
<th>Initial conditions (fastest voice in semibreves.) Duple mensuration, two-voices, 2:1 proportion</th>
<th>First melodic motion in the <em>dux</em>.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>unison</strong></td>
<td>[+3] [-3]</td>
</tr>
<tr>
<td><em>comes at the fifth above</em></td>
<td>[-2] [+3] [-4]</td>
</tr>
<tr>
<td><em>comes at the fifth below</em></td>
<td>[+2] [-3] [+4]</td>
</tr>
<tr>
<td><em>comes at the octave above</em></td>
<td>[+3] [-3] [+4]</td>
</tr>
<tr>
<td><em>comes at the octave below</em></td>
<td>[+3] [-3] [-4]</td>
</tr>
</tbody>
</table>

By way of comparison, table 2.2 shows the first melodic motion in the canons that I collected. Clearly these composers do not make use of anywhere near the full range of options potentially available to them.

Why is there not a greater variety in the choice of initial *dux* motion on the part of composers? It is difficult to know how to answer that question. Certainly aesthetics may have played a part. Given imitation at the lower fifth, a leap of a fourth upwards to an octave, while breaking no rules, may simply not have looked as attractive as a leap down a third. A preference for alternating perfect and imperfect sonorities could be a cause, but there is no way to be sure. In any case, there is no example like this among the canons that I collected. All of the historical examples use either a second or a third as their first melodic motion.
<table>
<thead>
<tr>
<th>Appendix number; Page number</th>
<th>Piece</th>
<th>Composer</th>
<th>Interval of imitation</th>
<th>First motion in dux (RP 2)</th>
<th>Second motion in dux (RP 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9, page 101</td>
<td>Benedictus*</td>
<td>Josquin</td>
<td>Unison</td>
<td>[0]</td>
<td>[+3]</td>
</tr>
<tr>
<td>11, page 101</td>
<td>In nomine*</td>
<td>Josquin</td>
<td>Unison</td>
<td>[0]</td>
<td>[-3]</td>
</tr>
<tr>
<td>35, page 115</td>
<td>Christe, Missa Petrus Apostolus</td>
<td>Obrecht</td>
<td>5th above</td>
<td>[0]</td>
<td>[+3]</td>
</tr>
<tr>
<td>36, page 116</td>
<td>Puzzle-canon VI</td>
<td>Anon.</td>
<td>5th below</td>
<td>[+2]</td>
<td>[+2]</td>
</tr>
<tr>
<td>38, page 118</td>
<td>Benedictus, Missa Sine Nomine (see ex. 2.1)</td>
<td>Josquin</td>
<td>5th below</td>
<td>[0]</td>
<td>[-3]</td>
</tr>
<tr>
<td>25, page 106</td>
<td>Agnus, Missa Prolationum</td>
<td>Ockeghem</td>
<td>5th below</td>
<td>[+2]</td>
<td>[+3]</td>
</tr>
<tr>
<td>39, page 119</td>
<td>Benedictus, Missa Iste Confessor Domini</td>
<td>La Rue</td>
<td>5th below</td>
<td>[0]</td>
<td>[0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2) 2</td>
</tr>
<tr>
<td>33, page 111</td>
<td>Eslongies Suy</td>
<td>Anon. (Tr. 87)</td>
<td>8th below</td>
<td>[0]</td>
<td>[0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-3) 3</td>
</tr>
</tbody>
</table>

*Missa L’homme armé super voce musicales.

To show how the choice of first melodic motion impacts composition, I will illustrate two-voice canon composition with a step-by-step example. First, we must establish the initial conditions. These are the number of voices (two), the mensural or

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2 First melodic motion is [+2].

3 First melodic motion is [-3].
proportional relationship (2:1), the transposition level (the octave), and which voice is faster-moving (the upper voice). Here is the beginning:

Figure 2.6. Initial conditions for a sample two-voice canon.

Having established this, we know from the examples above that the only possible motions in the dux are [+3], [-3], and [-4]. Let’s go down a third, shall we?

Figure 2.7. First melodic motion in the dux, rhythmic positions labeled.

The next task is to project this note choice forward into the comes.

Figure 2.8. First melodic motion in both dux and comes.

And then to choose the next dux note, consonant with the second comes note.

Figure 2.9. Second melodic motion in the dux. Comes rhythmic positions labeled.

Now, since this canon will be very short, we must choose where and how the cadence will occur. If we want to have a standard cadence, then there is no flexibility in the choice of pitches and rhythms. The cantizans, or upper part must make a 7-6 suspension against the tenorizans, or lower part, and then resolve upwards, making the progression 6-8 with the tenorizans, which must always descend by step. Knowing this, we can write the cadence as follows, with cantizans in the top voice and tenorizans in the lower voice.
Before we can fill in the middle of the canon, we must put the 2-1 cadence motion into the *dux*. At the end of measure two the cadence forms a dissonant eleventh, which is corrected in Figure 2.12. If there is no way to make the cadence fit, the canon’s length could be adjusted to make the cadence fall in a different part of the *dux*.

Now we have to fill in the missing notes, the first one on RP 4 in the *dux*. This note must make sense in RP 4 in the *comes* as well. Since the second half of RP4 is a P11 above the bass, it will function best as a passing tone, so there is only one option available for RP4: a P12, which gives a strong beat consonance and enables descending passing motion.

Now there remains only RP 6 and 7 in the *dux*, both of which are only limited by the pitches in the *comes*. Any legal counterpoint is permissible.
Figure 2.13. Filling in the final dux notes.

Now the canon is finished. While in this short example there is not much space for free material in the dux, in a longer canon there would be a much greater number of rhythmic positions available for elaborating the dux melody. In the artificial canon above, only RP 6 and 7 can be considered free, since they do not appear in the comes and therefore have no direct impact on the choice of later notes in the dux. To put this another way, all mensuration canons that begin simultaneously will have a section of counterpoint that is not repeated in the comes, somewhere after the beginning of the dux but before the final cadence. It is this material that I am calling “free.” The longer the canon is, the longer the “free” section can be.

To put the preceding sample canon into perspective, I want to point out that the cadences in historical examples of this type of composition are not always standard. Sometimes the pieces do not use either the 6-8 cadence or the 3-1 cadence. Example 2.2 is from Josquin, and there is no standard cadence. Admittedly this example comes from the middle of a Benedictus, where the cadence is a pause in the text, not the end of a text, but the In nomine that follows does not continue the canon.

In Example 2.3 the In nomine has a final cadence that is really just 3-1, with a delay in the upper voice resolution.

Example 2.2. Qui venit, Missa L’homme armé s.v.m., Josquin. Adapted from Smijers, ed., Opera Omnia, 27.
Example 2.3. *In nomine, Missa L'homme armé s.v.m.*, Josquin. Adapted from Smijers, ed., *Opera Omnia*, 27.

![Musical example](image)

The following example, by Heinrich Finck, is the longest two-voice canon that I found, and it shows some of the possibilities that shorter canons cannot use, especially large-scale structural use of sequence. Each half of the canon has several cadential sixth-to-octave motions without suspensions, each of which is followed by a rest in one part, creating the impression of a new phrase beginning. Since the *dux* is the lower voice, it has at its midpoint the *cantizans* part taken from the final cadence. Every note after the signum in the bass is free counterpoint.

Example 2.4. *Agnus dei (Missa a 3 voce)* Heinrich Finck.⁴

![Musical example](image)

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⁴ From Heinrich Finck, *Ausgewählte Werke*, Bd. 6, ed. Lothar Hoffmann-Erbrecht (Frankfurt: C.F. Peters, 1962), 15. Note values are original. Cadential motions are boxed, and some unusual dissonances are shown with arrows.
Example 2.5. Continuation of the previous example.

The method described above is only one possibility in terms of compositional technique. There may well be others, and there is no way to know how any composer actually set about producing canons like those above. Nevertheless, it does illustrate a compositional method that works for composing two-voice canons in 2:1 proportion.
This method works as follows. The crucial steps are the opening and the cadence which, because it is a canon, may be relieved of formulaic requirements. The necessity of preparing the cadence forces the composer to insert some material into the middle of the *dux*. In the case of a 2:1 canon, this will be at exactly the halfway point in the *dux*, at which time there will be a version (possibly decorated) of the *tenorizans* or *cantizans*, depending upon which one is uppermost at the final cadence, and whether the *dux* or the *comes* is the higher voice. Once this is done, the composer has two points of arrival within the canon. Depending upon the length of the canon, the composer may choose to insert more cadence points or simply to fill in the spaces that are left, in the first half with imitative material, and in the second with “free” material above or below the *comes*. 
Chapter 3.

The Josquin Agnus dei II and Other Three-Voice Proportion Canons

I have found three examples of unaccompanied three-voice canons: Omne trinum perfectum (Senfl), Agnus dei II (Josquin), and Le ray au soleyl (Ciconia). Each of these canons has different initial conditions than the others. For convenience, I will start by discussing canons of the Omne trinum perfectum type, those that use the proportions 1:2:4 in terms of note length.

Omne trinum perfectum is a canon that instructs the performer to produce three sounding parts from one written part. Its presentation is nearly identical in both of the sources that preserve it: one line of music in the alto clef, prefaced by three mensuration signs. Two of the signs, O and Ø, are a relatively straightforward direction to sing one part by treating the tactus as a semibreve beat in one part, and as a breve beat in another. The third sign, ☐, indicates that the minim should be worth one tactus.¹ All of the parts are in triple meter.

Figure 3.1. Omne trinum perfectum, Ludwig Senfl.²

¹ Normally, ☐ indicates three minims per semibreve; this circumstance is highly exceptional in that the dot indicates augmentation.

² Adapted from Heinrich Glarean, Dodecachordon (Basel, 1547) ed. and trans. Clement A. Miller (American Institute of Musicology, 1965), 274.
My analysis of this type of canon is very much like that of the simpler two-voice canons discussed previously. The same initial conditions are established, and then the possibilities for initial motion in the dux are derived. Once again, I begin with the premise that the initial motion in the dux is crucial. I list all the possibilities in a series of figures, each one of which changes the parameter of which voice moves at which speed. Since there are three speeds, this results in six combinations. The fastest voice is called 1, the voice that is twice as slow 2, and the voice four times as

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24 slow 4. The upper voice is listed first, the middle second, and the lowest third, from left to right in the caption. My examples are in duple meter rather than triple because it simplifies the presentation. From the examples, it is easy to see that there are very few options for initial motion in the *dux*. In the following figures, options that are acceptable are placed in a box, all others result in undesirable counterpoint.

**Figure 3.2. Three voices, 1:2:4 proportion, unison.**

It is easy to see here that unison transposition prohibits all melodic motion between RP 2 and 3 except by third. The following are more usual transpositional arrangements of perfect fifth and perfect fourth, and canons at the octave.

**Figure 3.3. Three voices, 1:2:4 proportion, P4+P5**
Figure 3.4. 2:1:4 proportion.

Figure 3.5. 2:4:1 proportion.
Figure 3.6. 4:2:1 proportion.

Figure 3.7. 4:1:2 proportion.
From the preceding figures, one can see that only the following initial dux motions are possible, given these initial conditions: duple meter, three-voices, 1:2:4 proportions, and P5+P4 voice disposition.

Table 3.1. Possible initial motions in 1:2:4 proportion, P5+P4 canons.

<table>
<thead>
<tr>
<th>Speed ratios (left to right=top to bottom voice)</th>
<th>Initial dux melodic interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:2:4</td>
<td>No options available</td>
</tr>
<tr>
<td>2:1:4</td>
<td>[-3]</td>
</tr>
<tr>
<td>2:4:1</td>
<td>[+3]</td>
</tr>
<tr>
<td>4:2:1</td>
<td>No options available</td>
</tr>
<tr>
<td>4:1:2</td>
<td>[-3] [+4] ^4</td>
</tr>
</tbody>
</table>

Another possibility for three-voice canons is to have two voices at the unison and the other voice at the octave. In this case, I have considered that it is most desirable to keep the voices in unison at the upper octave rather than in the bass, since that is the more normal arrangement. There are only three sets of 1:2:4 openings at the octave, as opposed to six at the P5+P4. This is because I see no need

^4 The [+4] option is the one chosen by Senfl for Omne trinum perfectum.
to invert voices that are at the unison; for example I count both 1:2:4 and 2:1:4 as a single option.

Figure 3.9. 1:2:4 proportions with two upper voices at the unison and the lower voice an octave below. Line one is 1:2:4; line two 2:4:1; line three 4:1:2.

It is good to keep in mind, however, that all of the examples above are the potential contrapuntal moves for three-voice canons at the octave using only duple proportions, but none of them establishes all of the initial conditions that exist in any one of the examples that I have found. The closest is the boxed motion in figure 3.7 (see above), which reproduces the initial conditions of Omne trinum perfectum and in which [+4] is the same interval that occurs in that canon’s opening. These figures are meant to illustrate that there are many unused possibilities for canonic openings with all kinds of initial conditions.
Table 3.2. Possible initial motions in 1:2:4 canons with the upper two voices at the octave.

<table>
<thead>
<tr>
<th>Ratios (left to right=top to bottom voice)</th>
<th>Initial dux melodic interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:1:4</td>
<td></td>
</tr>
<tr>
<td>4:2:1</td>
<td></td>
</tr>
<tr>
<td>1:4:2</td>
<td></td>
</tr>
</tbody>
</table>

Triple mensurations are slightly different from duple mensurations in terms of the dissonances allowed. Dissonance treatment is dependent upon note values.\(^6\) In the case of perfect breves, all divisions should be consonant, since semibreves are not eligible for dissonances. When discussing proportion canons, however, though all voices are nominally in the “same” values, those that are in diminished values are in fact moving in smaller values (and any augmentation would indicate larger values) and the corresponding rules about dissonances apply. What is written as a semibreve in integer valor becomes a minim under diminution, or a breve in augmentation. In two voices this distinction is not of great importance, but in three or more voices, a particular note in any given voice might form consonances with the bass in one place, while the same note in another voice and another rhythmic position might form illegal dissonances with the bass.

The canon *Omne trinum perfectum* is entirely in triple mensuration, of course. It is worth keeping in mind that the dissonances above the bass in this canon are entirely the result of *fusa* motion in the *dux*. The few instances of passing dissonances in the bass are in *seminimens* or *fusas*. The result of putting the slowest voice in the superius is that the long values do not overly restrict the other voices: the rules about

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\(^5\) I consider that when the voices are at the unison, it does not matter which one is “above” the other, so I place them together in the same column in the chart.

consonances with the bass are not so restrictive because the bass moves faster than the upper voices and therefore may have passing dissonances. Because of the canonic requirements, putting the longest note values in the bass would create severe restrictions on the other voices. Dissonance treatment would then become more problematic in triple meter than in duple.

One straightforward method for writing such a canon is to write a piece of double counterpoint, as in the two-voice canons, but to include a third voice: Both *Omne trium perfectum* and the *Agnus dei* II resemble nothing so much as a kind of braid when considered in this way.⁷

**Figure 3.10.** Invertible counterpoint in a three voice proportion canon using 2:1 proportion.

This diagram of rhythmic positions can be modified to show canonic elements, or groups of rhythmic positions, as in figure 3.11.

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⁷ The following discussion has propelled this thesis forward from the beginning. The diagram and explanations of *Omne trium perfectum* are adapted from materials created by Mr. Olivier Trachier for use with his advanced counterpoint classes. I am deeply indebted to Mr. Trachier for sharing with me his own method for composing on Senfl’s model. In the fall of 2005 he graciously invited me to visit him at his home. He was generous in explaining the technique that he had devised and presented to his counterpoint classes in previous years and in providing me with copies of his class materials.
If elements B and CD are written using invertible counterpoint at the twelfth, then they can be inverted, transposed, and augmented, and a third part written against the resulting duo. Correct counterpoint is assured each time the process repeats. Invertible counterpoint at the twelfth is required since one voice must use pitch classes a perfect fifth away from the other two. The only necessity is to take care at the points in the fastest-moving part that will participate in cadences, one-half and one-quarter of the way through the melody. The entire second half of the canonic melody is freely composed over the “automatically composed” duo that results from this process. Canons of the Omne Trium Perfectum type are thus not necessarily strenuous to write, despite their apparent complexity.

Josquin’s *Agnus dei II, Missa L’homme armé super voces musicales.*

More confusing is the *Agnus dei II* by Josquin. In this canon, triple and duple meter are combined at the level of the tactus. What I mean by this is that the beat is divided into two and three simultaneously, which raises questions about where dissonances can fall within the beat. Essentially, the first minim of each tactus group must always be consonant. In Josquin’s canon, there are two different kinds of minimis: triple and duple. Each of these has its own rules, illustrated below.
Figure 3.12. *Agnus dei II*, Josquin. Rhythmic positioning of consonance and dissonance in simultaneous triple and duple minims.⁸

3:2 (c₃) minim 1 2 3
               c    d    c

1:1 (c) semibreve 1

2:1 (c) minim 1 2
               c    d

Example 3.2. *Agnus dei II*, *Missa L’homme armé super voces musicales*, Josquin des Prez.⁹

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⁸ This diagram is borrowed from Lee Rothfarb, “Tinctoris vs. Tinctoris: Theory and Practice of Dissonance in Counterpoint,”

⁹ Adapted from Josquin DesPrez, *Opera Omnia*, vol. I, ed. Albert Smijers (Amsterdam: Vereiniging voor Nederlandse Muziekgeschiedenis, 1952), 30. No ligatures have been notated here, since different sources disagree, though all ligatures are of only two notes. For a complete listing of ligatures in different sources, see the appendix, number 12, page 102.
As with the previous example, Josquin’s canon has identifiable initial conditions. Three proportions are present: 1:2:3, the voices are spaced P5+P4, and the voices are in the order 3:1:2 from top to bottom. Given these initial conditions, only a finite number of options is available, depending upon how long the composer is willing to wait before making the first melodic motion in the *dux*. I will consider three possibilities: rhythmic position 2, 3, or 4. Josquin chose to move on rhythmic position 4, so that is the first set of examples that I will use. Below I consider all of the possible permutations of proportions. Usable options are boxed.

Figure 3.13. 1:3:2 proportion, *dux* motion on rhythmic position 4.
Figure 3.14. 3:1:2 proportion, dux motion on rhythmic position 4.

Figure 3.15. 1:2:3 proportion, dux motion on rhythmic position 4.
Figure 3.16. 2:1:3 proportion, *dux* motion on rhythmic position 4.
Figure 3.17. 2:3:1 proportion, *dux* motion on rhythmic position 4.

Figure 3.18. 3:2:1 proportion, *dux* motion on rhythmic position 4.
Table 3.3. 1:2:3 proportions, possible *dux* motions in rhythmic position 4.

<table>
<thead>
<tr>
<th>Proportions (right to left = top to bottom)</th>
<th>Possible <em>dux</em> motions (RP 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:3:2</td>
<td>[+3]⁹⁻¹⁰⁻⁴</td>
</tr>
<tr>
<td>1:2:3</td>
<td>none</td>
</tr>
<tr>
<td>2:1:3</td>
<td>[+2] [-3]</td>
</tr>
<tr>
<td>2:3:1</td>
<td>[+3]</td>
</tr>
<tr>
<td>3:2:1</td>
<td>none</td>
</tr>
</tbody>
</table>

From this table, we can see that the number of options is miniscule, given all of the restrictions of Josquin’s initial conditions. There are certainly other options for the composer: he is not restricted to moving on rhythmic position 4, there are acceptable solutions that move the dux on RP 2 and 3 as well. Other sets of initial conditions also exist: different mensurations or proportions, for example. Johannes Ciconia’s three-voice canon *Le ray au soleyl* uses a 4:3:1 proportion. Ciconia handles this situation by using several clever moves, including exclusively triadic motion in the opening, and imitation only at the octave and unison. In addition, certain notes (those in coloration) are omitted in the bass because they are harmonically problematic. It is for this reason that I do not pursue an extensive analysis of *Le ray au soleyl* here, but I include the score as number five in the appendix.

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⁹ This is Josquin’s initial motion in the *Agnus dei II*. 
Chapter 4.

*Pierre de La Rue’s Agnus dei II in Four Voices*

Pierre de la Rue’s first *Missa L’homme armé* is an impressive example of canonic composition. The *Kyrie* movements use two-voice mensuration canons with two free voices, and the second *Agnus* is a four-voice mensuration canon. It seems highly likely that it was inspired by Josquin’s three-voice *Agnus*. The canon consists of a single notated voice with four mensuration signs before it, $\epsilon$, $o$, $\epsilon_3$, and $c$. As in the Josquin example, these signs serve to show how to interpret the notation in four different ways simultaneously. It is easy to agree with Glarean’s assessment that Pierre de la Rue’s four-voice proportion canon is “far more astonishing” than that of Ludwig Senfl, which precedes it in the *Dodecachordon*.¹ With the added difficulty involved in composing four rather than three voices to be derived from the given voice, it is a more difficult canonic model to follow. To my knowledge, only one author has attempted to write about this piece in any detail.²

In a book entitled *Specchi Invisibili dei Suono: La Costruzione dei Canoni, Risposta a un Enigma*, Gian Carlo Bizzi described numerous types of canons, and proposed analytical models for them, including the La Rue *Agnus dei II*. In his analysis, Bizzi mainly describes the rhythmic interactions that take place between the four different mensurations, using integers to label equivalent rhythmic positions in the four voices.³ He uses the following diagram, and unaccountably includes only three voices.

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¹ Glarean, *Dodecachordon*, 274-5.


³ I have found this approach to be extremely useful. It is also used by Jaap van Benthem and Edward Stam in their articles on Josquin’s *Agnus dei II*, and I appropriated it from them in my discussion of the same piece, as I do here.
He also includes a diagram of the entire piece in schematic form, showing the relative duration of different note values, a version of which I reproduce in Figure 4.2. The diagram does not in fact reproduce the rhythmic relationships in the canon correctly because it does not account for the rhythms that arise as a result of imperfection and alteration under triple mensuration.

Bizzi provides formulas for calculating points of congruence between different voices. The rhythmic positions on the diagram above are numbered (1, 2, 3 …x) in each voice. The formulas are as follows:

1. To find the point of congruence of the tenor with the bassus, take the number of the bassus note ($B_i$), and add the next lower integer ($B_{i-1}$). The result is the number in the tenor series where the bassus note $B_i$ will fall, $k$ being the integer (rhythmic position) attached to the desired tenor note.

$$B_i + B_{i-1} = T_k$$

2. To find the point of congruence of the tenor with the altus, take the number of the altus note ($A_i$), and add the next lower integer to the $n$th member of the series of odd numbers 1, 3, 5, 7, 9, …etc.

$$A_{i-1} + \text{odd-integer member of the A sequence (1,3,5,7, etc.)} = T_k$$

3. The cantus coincides with the tenor throughout.

Figure 4.2 is the illustration of these formulas. The formulas do indeed describe Figure 4.2, but since I do not think that the diagram fits the piece, for reasons that I explain below, I do not find these formulas particularly useful. They do not permit the analyst or composer to perceive more than one relationship between the voices at a time; one is limited to the relationships between the tenor and each

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4 Adapted from Bizzi, *Miroirs*, 120.

5 Bizzi’s presentation is entirely in prose. I translated it, simplified it, and added the formulas.
voice individually. The result is a fragmented understanding of harmonic simultaneities.

Figure 4.2. Chart of rhythmic values in La Rue, *Agnus dei II*. From Bizzi, *Miroirs*, page 119.
Example 4.1. *Agnus dei II, Missa L'homme armé*, Pierre de la Rue.\(^6\)

Note values under \(\text{e}\) and \(\text{e}3\) are halved with respect to O and C. The given bars each represent one tactus unit, one beat of a semibreve under C and O. Note the near-parallel octaves between the cantus and the tenor in bar two.
Example 4.2. Continuation of the previous example.

The problem with this formulation is that the rhythmic positions (though Bizzi does not use the term, he depicts a nearly analogous concept in the diagram) do not behave in the canon in the same way that they behave in his diagram. Figure 4.2 (page 40) shows the layout for a proportion canon in four voices. If we number the rhythmic positions in the opening (Example 4.3) beginning with 1, it is easy to see
that subsequent rhythmic positions do not have corresponding pitch identities, but fall at different points within the “same” melody in the different voices. Since in Bizzi’s diagram (Figure 4.2, previous page), the diagonal lines between the voices that connect notes imply that those notes share at least a transpositional identity, we would expect to find that identity in the score but we do not. While Figure 4.2 indicates only metric positions, even those are incorrect when applied to the metric organization of La Rue’s canon. Pitches at the rhythmic positions indicated in Figure 4.2 are not the same, as Example 4.3 shows. Because of the different mensurations, the opening breve is both perfect and imperfect. The following two semibreves are interpreted in two different ways, which means that RPs 3 and 4 fall in different places within the melody – this is not indicated in Figure 4.2. It is the imperfection and alteration of rhythmic values that causes the piece to work.

Example 4.3. Numbered rhythmic positions in La Rue, Agnus dei II.

Bizzi’s analysis of La Rue’s canon leaves much to be desired as an explanation for the specific pitch and rhythmic content of the piece, with the exception of a diagram that shows pitches as they are transposed and shifted in time by the augmentation process (Figure 4.3), and his basic explanation of the rhythmic values

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7 Bizzi’s note values are halved with respect to my note values.
in the piece. While this is useful as a visual aid to help the reader grasp the temporal and transpositional relationships between the different parts, it amounts to a new graphical representation of the score, and more importantly it still it does not address any questions about compositional principles. How many options does La Rue have for his dux, especially at the beginning?

Figure 4.3. *Agnus dei II*, Pierre de La Rue. Diagram of note names and rhythms as represented by Bizzi.\(^8\)

For example, given a set of initial constraints, there must be limitations on the first motion in the fastest voice, since some motions will cause unwanted parallels or dissonances. Just as I pointed out with respect to Josquin’s *Agnus dei II*, there must be a limited number of such motions due to the extremely strict restraints imposed by the combination of a need for vertical consonances and the initial disposition of the voices.

As Bizzi very correctly points out, there is confusion between cause and effect in pieces of this kind.\(^9\) The fastest-moving voice *causes* the motions in the other

\(^8\) Bizzi, *Miroirs*, 121.

\(^9\) Bizzi, *Miroirs*, 121. “D’u point de vue logique, le canon mensurable est de grand intérêt. Le Ténor (*causa*) détermine en effet toutes les autres voix (*effet*). Mais, du fait que chaque autre voix (*effet*) est
voices, but they in turn have an effect upon the choice of subsequent notes in the duc.
To some extent, this circularity is at work in all kinds of canons. In a piece such as la Rue’s Agnus II, pitches that precipitate a reaction (duc pitches) participate in forming the polyphonic whole along with the pitches that result from the process of its transformation (comes pitches). This is the case in the opening of such a piece, when the same note is overlapping in more than one voice.

La Rue’s Agnus is a much more difficult piece to grasp than Josquin’s Agnus. The principle reason for this is not the extra voice, though that is a significant complication. As van Benthem illustrated, Josquin’s Agnus can also exist in a version for four voices, simply by the addition of another proportion sign, and the result is not significantly less satisfying or “correct” than the more standard three-voice resolution.¹⁰

La Rue’s conception is quite different, because his is not a proportion canon, but a mensuration canon that contains a proportion canon. Two of the four voices have proportional relationships, but the tenor and altus are not just proportionally but mensurally different from the other voices. One major difference between Josquin’s piece and la Rue’s is that Josquin takes care that the rhythms should always be the same within all of the voices, although their individual relationships to one another are constantly changing. This means that the voices can be labeled with integers (RP numbers) as in my discussion of the Josquin Agnus dei II, and the same integer will always fall in the same place within the melody, both in terms of rhythm and pitch. Josquin achieves this by taking care that all breves should be imperfected

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in the triple part, and that no semibreves should be altered. The principles of perfection, imperfection, and alteration are what complicate La Rue’s canon.

To illustrate: the tenor voice of la Rue’s *Agnus* has a mensural relationship rather than a proportional relationship to the *integer valor*, the voice notated in c. The breve is perfect in the voice notated ε3, thus it can be imperfected, and semibreves can be altered. The opening of the piece shows this quite clearly. (B = breve, sb = semibreve, m = minim, sm = semiminim.)

Figure 4.4. Opening phrase of the *Agnus dei II*, Pierre de la Rue. Original and score.

Under C, the first breve is imperfect, the following two semibreves are worth one breve, and the following breve is worth two semibreves. Imperfect tempus holds no surprises here. Under O, the first breve is perfect, worth three semibreves, and is then followed by two semibreves, the second of which is altered, or worth twice the normal value. The two semibreves are thus worth one perfect breve. The next breve is imperfected by the following semibreve, thus creating another group worth three semibreves. Under ε3 the same is true but the note values are effectively halved.

The result is that there is no way to number rhythmic positions so that each number represents the same pitch and rhythmic identity within each voice. This is clear from the very first phrase, in which breves are both perfect and imperfect, and semibreves altered and un-altered, as in RPs 1-4 in Figure 4.4. This fact alone makes

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11 It is possible that Josquin thought of the sign ε3 as a duple mensuration rather than a triple one. In this case, the mensural organization would remain duple but there would be three semibreves in the time of two. This would then be a moot point.

12 See Example 4.1 for a transcription of the piece in score.
it impossible to keep the same Arabic numeral for a given rhythmic position in the
same place within each voice. The superius has, perhaps, rhythmic positions 1 and 2,
while the tenor has 1, 2, and 3. So far, so good. But when we place rhythmic position
3 in the superius, suddenly we see that it does not share the identity of rhythmic
position 3 in the tenor, and the farther we go, the less corresponding Arabic numbers
share an identity of pitch and rhythm in these two voices.

It is difficult to overstate the degree to which the specific rhythmic
arrangements in the opening make the piece “work.”¹³ The particular pattern of
perfected and altered values within the first phrase permits all of the voices to move
independently. If the first four note values had been B sb B sb, then the altus and
bassus parts would suddenly be in parallel octaves in the third and fourth bars (G to
C), as illustrated in Figure 4.5.

Figure 4.5. La Rue Agnus dei II. Parallel octaves between bassus and altus as a result of
imperfecting ternary values.

The simplest way to use perfection, alteration, and imperfection of notes in
triple mensuration is to be sure that all rhythmic values are arranged so that there are
no perfect values in the parts in triple mensuration, as Josquin does in the Agnus dei
II. This simple solution is not available to La Rue because of the initial conditions of
his canon. Given the signs C, O, é, and É3, if all ternary breves are imperfected, then
the voices in C and O will have exactly the same rhythm, and thus move in parallel
octaves, as in the above example. This is actually quite an extreme restriction on the
rhythmic profile of the canonic opening under these initial conditions, and therefore
a very useful thing for the composer to know. The first note must not be

¹³ Also, repeated notes can make this kind of composition much easier to execute.
imperfected or the result will be parallel octaves between the voice in O and the
voice in C.

Remarkably, there are no other options for the canonic opening’s rhythmic
profile. The sequence □ ◊ ◊ ◊ (B sb sb B) requires that the second semibreve be
altered, or doubled in length. This rhythm will create parallel octaves in C and O, and
near-parallels in ě and ě3. This leaves us with only one option for a canonic opening
using La Rue’s initial conditions. Using smaller values results in parallels between C
and O, because all values smaller than the breve are of equal value under both C and
O.

A quick look shows that the initial options available to a composer given the
same constraints as La Rue are quite limited. For the moment, I will expand the
initial constraints to include the rhythmic profile of all voices. Using the rhythms that
he used (to avoid parallel octaves), one is limited to either the leap of a fourth
upward or to remaining on the same note. In the downward direction, however, a
leap of either a third or a fifth is acceptable.

Figure 4.6. Possible first melodic motions in the dux given La Rue’s initial conditions and
rhythmic profile.
The illustration above covers only the option of using a perfect breve; it is
only intended to show the specific results of different melodic motions given La
Rue’s initial conditions and rhythmic profile, including the point at which he makes
the first melodic motion in the dux. There is a very important point to be made here:
it is impossible to make the first melodic motion earlier than this, since the perfect
and imperfect breves of \( \varepsilon \) and \( \varepsilon_3 \) (written as semibreves in score) are aligned. Only by
writing semibreves (minims in the score) can La Rue avoid parallel motion in these
two voices. The following figure illustrates this principle.

Figure 4.7. Limitation on rhythmic placement of melodic motion in \( \varepsilon \) and \( \varepsilon_3 \).

When composing, one is not obliged to retain the disposition of the
mensurations, their initial starting pitches, or their rhythmic profiles. Whatever
changes are made to these parameters, and many changes are possible, the necessity
for creating consonances when moving the dux naturally remains in effect. If we
retain these mensurations for the present, we see that no matter what rhythmic
values are used in the canonic opening, two of the voices will be moving much faster
than the other two. Also, unless the canon is entirely at the unison, at least one of the
voices will be either a fifth above or below the others. What is most crucial is to
maintain is the proper relationship of consonances and dissonances between the dux
and the lowest sounding voice.

This stipulation, that at least one of the voices imitate at a different pitch
level, means that two pitch classes must be accommodated by the first melodic
motion in the dux. Since the dux is unlikely to be the lowest voice, it will be either an
octave or a fifth above the lowest sounding voice. If it is an octave above, then it can
descend a fourth, or ascend a third or fifth; if a fifth above, then it can ascend a fourth or descend a third or fifth.

It is clear from manipulating pitch variables that given his initial constraints, La Rue had very few choices. The initial dux motion is not the end of the story, of course, but composing out the canon after that point becomes progressively easier, since the voices in integer valor provide a framework for counterpoint in the faster-moving voices.

The compositional challenges of writing a piece like La Rue’s Agnus dei II are daunting but not insurmountable. The canonic parameters are not identical to those in Josquin’s Agnus but are very similar. The main difference is the irregularity that is introduced through the use of mensuration rather than strict proportion canon. Given the initial conditions used by La Rue, strict proportion canon would be impossible since it would result in parallel octaves between the voices in O and C. In contrast to pieces on Josquin’s model, the composer should be sure to write at least one perfectible value or alterable semibreve in order to maintain the rhythmic independence of the voices. Repeated notes are extremely useful as well since they will tend to prolong consonances and help to control dissonance.

In terms of first melodic motion of the dux also, there are a few restrictions posed by the other voices. The dux will be either a fifth or an octave above the bass and its first motion will only be able to produce an octave, a fifth, a sixth, or a third above the bass, resulting in a limited range of first melodic motions.

The question that remains to be resolved is just how restricted the range of melodic motions is, and how much this changes when changes are made to the initial conditions. For example, the position of the mensuration signs in different voices could be changed, giving a total of ten options for mensural combinations. Each of these then may begin with the rhythmic profile: □ ◊ ◊ (B, sb, sb, B). Ten mensural combinations times six melodic intervals (descending and ascending) gives a total of sixty options to investigate for reasonable canonic openings. This is, of course, considering only the mensural options used by La Rue.

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14 The long could also be the first notated value in an opening.
I feel that Bizzi's analysis fails to address the specific compositional constraints imposed by the initial conditions of La Rue's canon. Rather than a simple series of calculations of points of correspondence between the tenor and the other voices individually, it is necessary to develop an analysis that reflects the interactions of all voices. The rhythmic profiles of the voices are not and indeed cannot be the same because of the combination of mensural and proportional processes at work in the realization of the notation into multiple voices. By focusing on the canonic opening, a composer can know exactly what is possible using a given set of initial conditions. The composing-out of the canon is then a controlled process of decision-making based on existing conditions rather than a series of blind guesses about how to start and continue the canon.

To put this chapter and this canon into perspective, we should remember that La Rue's *Agnus dei II* is completely unique. There is no way to compare it to other pieces of its kind, because there are no other pieces of its kind. Therefore, analyzing the canon is in part a process of imagining what the possibilities might be, and seeing how they might compare to the object itself. There are indeed other options, as we have seen. In writing such a canon, the composer must either stumble upon a combination of mensurations and first melodic motions that satisfies contrapuntal restrictions or work through the options to find one that suits his purpose.
Chapter 5.

Transmitting Josquin’s Agnus dei II: A Short Reception History.

From the Petrucci print of 1503.¹

Along with the second Agnus Dei from the Missa L’homme armé super voces musicales and Faulte d’argent, Tu pauperum refugium became one of the representative works of Josquin’s style for several generations of American music students. The influence of HAM as an arbiter of repertory was pervasive. Although Collected Editions were widely available, HAM often provided the point of entry into an œuvre, genre, or compositional style, particularly in the case of early music.²

Although Cristle Collins Judd’s subject in this case is the reception history of the motet Magnus es tu domine/Tu pauperum refugium, the case of the second Agnus from the Missa L’homme armé super voces musicales is similar in many ways. This piece is now probably better known than any other mensuration or proportion canon, and it might even be said to define that type of composition. This view is not really justified by the evidence that I have collected elsewhere in this thesis; Josquin’s canon is not really representative of any group of compositions. As far as I can tell, it is completely unique.

In this chapter I will present a reception history for the Agnus dei II that is similar to Judd’s for Tu pauperum refugium. The record for this canon is not so


extensive as for the motet, probably not least because in contrast to motets, mensuration and proportion canons form a miniscule part of the total repertoire of Renaissance music, and no composer apart from Ockeghem seems to have shown a sustained interest in them.

Josquin’s three voice Agnus dei II from the Missa l’homme armé super voces musicales survives in numerous sources, and was reprinted several times in the sixteenth century, both in collections of masses and on its own. It probably dates from around 1490, since the earliest sources for it are from the early sixteenth century. It is considered to belong to Josquin’s mature style; though it is an early work, it has been dated after several of his other masses. In 1502, Petrucci printed the volume Missa Josquin, including the Missa L’homme armé super voces musicales, which contains three shorter proportion canons in its Benedictus.

Table 5.1. Mensuration signs in sources for the excerpted Agnus dei II.4

<table>
<thead>
<tr>
<th>Mensuration Signs</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>ĕ3, c, ĕ</td>
<td>Petrucci 1502 (J666), s.d., 1514 (J667), 1516 (J668), Giunta 1526 (J669), Petreius (RISM 1539), Heyden 1537, Glareanus (RISM 1547), Ms. Regensburg 878-882, Ms. Toledo 9, Paris Ms. Bourdeney.</td>
</tr>
<tr>
<td>c3, c, ĕ</td>
<td>Grapheus (RISM 1539), Faber 1550, Ms. Berlin 1175.</td>
</tr>
</tbody>
</table>

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3 Sherr, Josquin Companion, 65. According to Blackburn, the earliest source is Rome, CS 197, from c.1492-5. The question of chronology in Josquin scholarship is rather fraught, but the Missa L’homme armé is likely to be an early work. Blackburn suspects that “these two masses [L’homme armé s.v.m. and L’homme armé sexti ton] pre-date Josquin’s entry into the papal chapel, in 1489.” Jesse Rodin, Josquin, 89, 336-337, supports the view that Missa L’homme armé dates from Josquin’s early years in the Papal Chapel.

4 This table is adapted from van Benthem, “Kompositorisches Verfahren.” Other parts of this mass were also excerpted for use as treatise examples. For more information, see Smijers, Josquin, and Sherr, Josquin Companion, 52. These lists do not include the painting that preserves the Agnus dei II discovered by van Benthem in the church of San Sisto in Piacenza, discussed in van Benthem, “Kompositorisches Verfahren.”
Petrucci reprinted the same volume twice more under the title *Liber primus Missarum Josquin*, once in 1514, and twice in 1516. In 1526 it was reprinted with the same title, and the complete mass appears in two prints from Nuremberg, both from 1539.

In addition to the basic timeline for the copying and printing of the entire *Missa L'homme armé super voces musicales*, a large part of the canon’s reception history might well be termed a *resolution* history, since it is not necessarily obvious how to derive three voices from one, and not all resolutions are identical. In resolving the canon, theorists of the sixteenth century made two different types of decisions, especially about how the cantus part should relate rhythmically to the two lower voices. I will examine two of these alternate solutions, as they are reported in Albert Smijers’ 1952 edition of the complete works of Josquin.

It appears that Sebald Heyden was the first to include the *Agnus dei II* in a treatise. His *De arte canendi* of 1540 features all three duos from the Benedictus as well as the *Agnus dei II*.\(^5\) It is most likely that he knew these examples from the 1502 Petrucci edition.\(^6\)

Heyden produced his treatises as working textbooks for his students, boys at the St. Sebald School in Nuremberg during the 1520s and 1530s. His three music publications of the 1530s were essentially elaborations of the same instructional material, each time with more examples, longer explanations, and a new preface. Judd explains Heyden’s relationship to his musical examples, showing that the examples were intended for recreational singing or pedagogical purposes, not for use in the liturgy. Even though they were virtually all taken from masses or motets, Heyden printed no text with his examples. Even more interesting is his inclusion of many *fugae*, which probably appealed to Heyden as pedagogical tools.\(^7\) Since music

\(^5\) Judd, *Reading*, Appendix A 4.1. The 1537 edition also contained all of these examples, with attribution.

\(^6\) Judd, *Reading*, 99. Almost all of the examples in Heyden’s 1537 edition have concordances in Petrucci’s music prints. It is also interesting to note that in 1537 he relied on the private collection of one Ulrich Starck, but in 1540, he had access to Petreius prints of masses by Josquin and others (RISM 1539\(^1\) and RISM 1539\(^2\)).

\(^7\) *Bicinia* and *tricinia*, two- and three-voice untexted pieces, were common in the late sixteenth century.
examples were typically copied out by the teacher and memorized by the students, fugae made the process of both writing and memorizing polyphony much simpler: one part for everyone. Enigmatic notation also saved space, presumably making Heyden’s book cheaper for him to produce and perhaps for his students to buy.\(^8\)

The Benedictus examples appear in Heyden’s chapter on mensuration signs, and serve to illustrate the way that the mensuration signs c, e\(^2\), and reverse-c could be used to indicate proportions. Likewise, Heyden resolved the Agnus and printed it in separate parts, mainly to show the use of mensuration signs as proportions. The written-out resolution is not in score, but shows each of the three parts, using the same note values; the only difference between them is length. This presentation conveys the same information as the original enigmatic notation, save that the parts end where a signum congruentia would have showed a singer where to stop.

Ten years later, in book two of the Dodecachordon, Glarean printed the Agnus dei II, along with two similar examples, discussed elsewhere in this thesis.\(^9\) The canon appears there in the same form that it appears in Petrucci’s 1502 print, as a single melody in mezzo-soprano clef, prefaced by three mensuration signs, and underlaid with its Latin text.\(^10\) Like Heyden, Glarean resolved the canon in separate parts, but his resolution is rhythmically different from the given music. Repeated notes in one part are realized as longer note values in the other parts, and Glarean includes some

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\(^8\) Judd, Reading, 92-96. “There was not a tradition of students in Latin schools buying music books, which from all evidence were very expensive” (95). It is not likely that all of Heyden’s students purchased copies, though some may have done so.

\(^9\)Glarean, Dodecachordon, 271. These three examples, and several other esoteric canonic pieces such as Ockeghem’s Prenez sur moi, are in chapter twenty-six, Concerning the skill of symphonetae (composers of complex polyphony). This chapter, Glarean says, is an afterthought to the rest of the book, to point out some pieces of special interest by composers who either wished to show off, to “please the many,” to instruct the youth, to “support the majesty of ecclesiastical song,” or to “relieve their own compulsion.”

\(^10\) Glarean had a copy of Petrucci’s 1502 Misse Josquin in his library, which we may assume was the source for the Agnus dei II as it appears in the Dodecachordon. See Cristle Collins Judd, “Musical Commonplace Books, Writing Theory, and ‘Silent Listening’: The Polyphonic Examples of the Dodecachordon,” The Musical Quarterly 82 (1998), 482-516, for a list of items known to have been in Glarean’s collection.
dotted rhythms where there are none, and omits others that are in the original notation. He resolved all the parts in such a way as to avoid the direct two-against-three implied by the proportion 3/2, in which three semibreves in the cantus part fit into the time of two in the bassus. Glarean offers no explanation of his resolution, in which no one part retains the rhythms as they are written in the given voice. It is easy to see that his resolution does not even maintain the same rhythms in the opening of all three parts; all are slightly different from one another.

Example 5.2. Glarean’s resolution of the Josquin Agnus dei II.

The following example is what I call a standard resolution of the Agnus dei II, in which the proportions indicated by the mensuration signs are followed strictly, and all of the rhythmic positions in each voice have the same relationship to one another as in the other two voices.
There is one other source that transmits a reading similar to Glarean’s: the Berlin theory manuscript.\textsuperscript{11} This resolution of the canon is one of three options printed in Smijers’ edition of Josquin’s complete works. It is entirely binary in character, and like Glarean’s resolution the parts do not agree between themselves on note values.

Example 5.3. A standard resolution of the Josquin \textit{Agnus dei II}. 

Given this feature of the resolution, I can only conclude that the Berlin theory manuscript also features written-out parts, with or without an enigmatic form of the notation in a single voice.\textsuperscript{12}

What are we to make of these rather different solutions? Some of these decisions seem obviously wrong to us today. The resolution from the Berlin theory manuscript changes the rhythm of the cantus part in such a way as to suggest either

\textsuperscript{11} Berlin, Deutsche Staatsbibliothek Mus. Ms. Theor. 1175.

\textsuperscript{12} Unfortunately I have not been able to access any facsimiles of the manuscript.
that the scribe copied from an incorrect exemplar or that he did not understand how to interpret a sign with triple proportion in this context (the sign \(\frac{3}{3}\)).

In any case, these two examples show clearly that in the sixteenth century, opinion varied on how to resolve the canon. To speak of a “correct” resolution is not possible in the strictest sense, for the clearly more than one solution could be considered correct. Although we can be sure that the resolution that respects the apparent intentions of the enigmatic notation, we can never be entirely certain what was intended by the composer, and more than one interpretation is possible. Maybe it is less interesting to establish correctness on this point than to simply note the diversity of solutions.

On the other hand, the existence of such resolutions leads me to the conclusion that the skill of resolving and probably also of writing this kind of composition was not extraordinarily widespread. This conclusion is also supported by the relatively restricted number of composers represented in my list of canons. They tend to be either Franco-Flemish or from German-speaking countries, but there is no particular geographical concentration of mensuration canon activity beyond the locations of individual composers.
Example 5.4. Resolution of the *Agnus dei II* from Berlin Ms. Theor. 1175.  

These two different solutions continue to show up in subsequent discussions of the piece. Although interest in this compositional technique waned during the sixteenth century and after, this particular piece continued to show up in prints and

13 My source for this resolution is *Josquin des Prez: Opera Omnia*, ed. A. Smijers (Amsterdam: Vereinigung voor Nederlanse Musiekgeschiedenis, 1952), 41.
manuscripts. After Heyden (1537), Glarean (1547), Faber (1550), and Finck (1556) printed the *Agnus dei II* out of its context, Jacob Paix did so again in his collection of canons of 1594,

\[14\] and Cerone included it in the extensive collection of canons at the end of *El melopeo y maestro* (1613), which might be considered the ultimate compendium of sixteenth century musical practice.

These anthologists, a predominantly Germanic group, include the *Agnus dei II* in their treatises for different reasons. Heyden’s aim was clearly basic pedagogy and illustration of his lessons. Glarean and Paix seem more focused on musical oddities. Glarean, though he took a somewhat dismissive tone, was probably nearly as fascinated by his finds as was Paix. Paix especially would hardly have bothered to collect so many canons and go to the trouble of having them printed, as a rule in enigmatic form, if he had not thought them to be valuable, fascinating pieces of music. It is at least clear that Paix took the *Agnus dei II* from Glarean, as he did many of the canons in the collection.

\[16\] Hermann Finck and Pedro Cerone presented the canon as part of an extended collection of similar pieces in a separate section of their treatises. Finck provided no explanation at all, and no resolution. He must have taken the canon from Glarean, since it is followed in both sources by a rare canon by Senfl entitled *Omne trinum perfectum*. I know of no other source for this second piece beyond Glarean and Finck, which leads me to conclude that Finck used Glarean as his source for Josquin’s *Agnus dei II*.

\[17\] Cerone provides more explanation, and more helpful commentary, than any other source. In addition to printing the enigmatic notation, preceded by three mensuration signs, he explains the signs verbally, and resolves all three parts.

\[14\] Jacob Paix, *Selectiae Artificiosae et Elegantes Fugae* (1594).

\[15\] Cerone, *El melopeo*. Peter Schubert says that “[*El melopeo*] is the obvious choice if you had to take only one treatise to a desert island.” See Schubert, “Counterpoint Pedagogy,” 505.


\[17\] It is even possible that Glarean had exclusive access to *Omne trinum perfectum*, since it is known that he commissioned at least two examples for *Dodecachordon* from Senfl. Judd, “Commonplace Books” refers to an unpublished study by Harold Powers on the subject.
In this canon all three parts begin together, but with different values in the different voices: because the principal part sings with the c tempus, where there is one semibreve per tactus; the other part sings with this $\textbf{e}3$ where there are three semibreves in one tactus; and the third sings with this sign $\textbf{e}$ where there are two semibreves per tactus. In this way the c part can be sung with the $\textbf{e}3$ in triple and in duple with the $\textbf{e}$; or the $\textbf{e}3$ can be sung in sesquialtera [3:2] with $\textbf{e}$; and the placement of the tempus signs in the spaces and lines shows us where we have to place the voice; a fourth higher in the alto, and a fifth lower than the principal voice. But to put the three parts with the principal tempo, the resolutions are like this.\textsuperscript{18} [resolutions for all three voices are printed] (my translation)

After Cerone, the sources for the piece are less numerous. At least one seventeenth-century manuscript contains a copy of the \textit{Agnus dei II},\textsuperscript{19} but after that the first person to mention it is Charles Burney in the first volume of \textit{A General History of Music} (1789). Burney knew it through Glarean, as he says quite plainly. After printing the three proportion canons of the \textit{Benedictus}, he writes:

After this [the \textit{Agnus dei I}] there is a second movement, to the same words, where three parts, in different measures, are drawn out of one: \textit{tria in unum}. At the beginning of this canon, three characters for time are placed over each other, thus:

$\textbf{e}3$
$c$
$\textbf{e}$

but as it is inserted by Glareanus with its solution, I shall only refer the curious reader to p. 442 of the \textit{Dodecachordon}.$^{20}$

\textsuperscript{18} Cerone, \textit{Melopeo y Maestro}, 1075.

\textsuperscript{19} Paris, Ms. Bourdenay. The manuscript page is reproduced in Van Benthen, “Kompositorisches Verfahren.” This is the earliest presentation of this mass in score.

\textsuperscript{20} Burney, \textit{General History} I, 742.
Example 5.5. From Pedro Cerone, *El melopeo y maestro* (1613).21

**Enigma con tres Tiempos. Num. I.**

Viaron hizo vn Tercio en el *Agnus Dei* segundo, de la Missa _Lomme arme super voces musicales_ a 4. vozes, formado con tres diferentes Tiempos, pero muy lento en el tercer, pues no dice mas de afla.

**TRES INV NUM**

![Musical notation](image)

En este Canon todas tres partes comienzan juntamente, pero con diferentes valores, y en diestas vozes: por cuanto la parte principal canta con este  C, tiempo, en el qual pasa vna. Semibreue al Compas: otra parte canta con este  C, adon de pasa tres Semibreves en un Compas: y la tercera canta con el otro  C, adonde pasa dos Semibreves al Compas. De modo que la parte  C canta  C, en Sexaginta, y con acer afeitado los Tiempos en los espacios y regla que vemos, nos aduerte adonde hauemos de tomar la voz, que es una Quinta mas en alto, y una Quinta mas en baxo de lo principal. Pero para poner todas tres partes debaxo de vn principal Tiempo, hago las Resoluciones en ella manera:

![Musical notation](image)

Example 5.6. Overleaf of the original.

Following Burney, I know of two nineteenth century sources for the *Agnus dei II*. The first is a notebook kept by the composer Luigi Cherubini, in which the enigma is followed by a solution in score.\(^{22}\) The second is the collection of works by Josquin des Prez, edited by Robert Eitner, and printed in 1877.\(^{23}\) Eitner’s collection, which includes the entire *Missa L’homme armé super voces musicales*, retains the original note values, and as well as a full score, shows the canon as a single voice (though not in its entirety) prefaced by the three signs c, ¢, and 3. Interestingly, Eitner’s understanding of the signs c and ¢ is exactly the opposite of the one that is more generally accepted: he places the sign ¢ before the slowest voice, and the sign c before its diminution.

It is difficult to be certain that other copies of this piece do not exist elsewhere. In all likelihood there are at least a few more lurking in unsuspected corners. It is certainly remarkable, however, that so many examples of Josquin’s *Agnus Dei II* are to be found, and from all eras, and that it has regularly attracted the

\(^{22}\) I am indebted to Luciane Beduschi for drawing my attention to this notebook, which she found in the archives of the Bibliothèque Nationale in Paris.

attention of curious musicians almost since its composition. Equally remarkable, from my point of view, is that such a unique piece is still, to all appearances, unique after so much scrutiny. To date, I have found only three others that seem even to be similar in conception: the four-voice *Agnus dei II* by Pierre de la Rue, *Le ray au soleyl*, attributed to Johannes Ciconia, and Ludwig Senfl’s *Omne trigum perfectum*.

*The Agnus dei II in the Twentieth Century.*

In the twentieth century, Josquin’s *Agnus dei II* has circulated repeatedly, in my view for three basic reasons: first, because it was written by Josquin, second, because it uses enigmatic notation, and third because of its symbolic significance. It was first printed in the *Historical Anthology of Music* in 1949, and again in the very first volume of the Josquin *Opera Omnia*, edited by Albert Smijers. In addition to the standard resolution, Smijers included the solutions from Glarean and the Berlin theory manuscript, probably because the three readings are so strikingly different from one another.

As Cristle Collins Judd implies, this piece is one of three by Josquin to have a very wide distribution and a strong influence on the image of Renaissance music created in the minds of North American music students by the *Historical Anthology of Music*. In *HAM*, the piece is reproduced in piano score, as are all of the other examples, but also as an enigmatic single line of music. This notation is not in a facsimile of one of the sixteenth-century prints, but a clean, “modern” period-style typeface, complete with ligatures. Printing in this way highlights the unusual nature of the piece, its clever use of a small amount of notation to present what turns out to be a complex piece of polyphony.

Shortly after the publication of *HAM* in 1949, Willi Apel, its co-editor, included several examples of mensuration and proportion canons in *The Notation of Polyphonic Music, 900-1600*. These include several of the two-voice examples by

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24 The la Rue *Agnus dei* was first linked to Josquin by Glarean, and although there is no way to be sure, the utter uniqueness of these two canons speaks strongly of Josquin’s influence on la Rue. *Le ray au soleyl* is a three-voice proportion canon, but it uses quite different constraints from the *Agnus dei II*. The same is true of Ludwig Senfl’s *Omne trigum perfectum*, discussed in chapter three.
Josquin, a complex three-voice accompanied canon from the Old Hall manuscript (Appendix 16), and the two by now usual suspects: Josquin’s *Agnus dei II* and la Rue’s *Agnus dei II*, in facsimile from Glarean’s *Dodecachordon*.

J.A. Bank’s 1972 work *Tactus, Tempus, and Notation in Mensural Music from the 13th to the 17th Century* prints the *Agnus dei II* using the characteristic shapes of sixteenth-century notation, diamond and square-shaped note heads. Thus, although the music is in score, in an elegant, modern-looking presentation, it retains an “early music” feel. The author also included the “resolutio binaria,” from Berlin 1175, the anonymous sixteenth-century theory manuscript.

Unfortunately, I am not in a position to offer a history of analysis for the *Agnus dei II* like Judd’s history of *Tu pauperum refugium*, for the simple reason that the *Agnus dei* is less analysed. I am aware of only two analyses, written by Edward Stam and Jaap van Benthem, in three articles from the mid nineteen-seventies.

*The Agnus dei II in Musicological Literature.*

There is not a great deal of musicological commentary on the *Agnus dei II*. The earliest twentieth century reference to it that I have found is in Curt Sachs’ *Rhythm and Tempo*, from 1953. His discussion is very brief, and merely describes the familiar rhythmic relationships, and the way that they result from the mensuration signs. In his review of this book, Otto Gombosi went so far as to print a transcription, reproduced below. He found that

…when transcribed in a reasonable manner, it is transparent, easy to read and to grasp and is constructed on what modern architects call a *module*: in multiples of 3. The three parts enter at the octave and the fifth (ratios 1:2:3) in the temporal relationship 1:2:3. Three measures of once, twice, or three times three tones [semibreves] form a period, three periods the whole movement. The piece is cast, in spite of the imperfect signatures, in plain triple time. […] …the organization is on a high level, the ‘engineering’ astonishing…

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25 Apel did not actually reprint the Old Hall Credo, but he described at some length the problems involved in its transcription.

26 Apel refers to both of these examples as mensuration canons, a term that he also applies to canons that use sequential repetition in one voice part, as I explain in chapter one.
Gombosi’s observation that the piece has triple divisions at all levels, and is cast in three periods of equal length, was not a new one, though he was probably not aware of this.  


In 1974, Jaap van Benthem published an article describing a fortuitous find: while visiting a church in Piacenza, he noticed a painted panel in the choir stalls that depicted musical notation. Upon closer examination, this proved to be the *Agnus dei II*, but preceded by four, rather than three mensuration signs: C, ¢, ¢3, and 3. Van Benthem found that a four-voice resolution of the canon was indeed possible, and only somewhat more problematic from a contrapuntal point of view than the three-voice version. The fourth voice is a triple version of the voice in C, indicated simply by the figure “3” and puts three semibreves of integer valor in the time of two, an octave above the voice in C.

As far as I know, van Benthem’s are the most detailed analytical comments to be published on the *Agnus dei II*. They reveal certain unusual aspects of the

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28 From Otto Gombosi, review of *Rhythm and Tempo, A Study in Music History*, by Curt Sachs, *Journal of the American Musicological Society* 7 (Fall 1954): 224. Observe that this transcription is in 1/8 note values!

counterpoint in the piece, including thinly disguised parallel fifths and unprepared sevenths. Neither of these, in my opinion, are particularly obvious to the ear.

Indeed, as van Benthem points out, a four-voice resolution exacerbates these problems, because the note values in the fourth voice are longer. The harmonic seventh in bar 8 is a semiminim long, but when it recurs in bar 15 it is a minim. This kind of departure from the usual rules for counterpoint is sometimes encountered in canonic writing, as are unusual cadences.30

Van Benthem’s 1974 article got a response from Edward Stam in 1976, in which Stam proposed that, unlike Omne trinum perfectum, a three-voice proportion canon by Ludwig Senfl, Josquin’s Agnus dei II could not be explained or composed using invertible counterpoint.31 Instead, he proposed that the piece could be composed by using Wechselkontrapunkt (changing counterpoint), or a voice that works as a counterpoint to multiple voices. Stam’s proposal involves a Herstellungstimme, a fourth voice in 2:1 ratio with the voice in $\times 3$, given the sign $\times 3$. This voice exists only to ensure that good counterpoint results between the voices in $c$ and in $\times$, since they are respectively in an identical configuration to the voices in $c_3$ and $\times_3$. Ultimately, this explanation merely deflects the problem, since the question of note choice at specific moments of the composition remains an issue.

In his second article, van Benthem also mentions that the Agnus dei II melody is exactly the same as the first nine semibreves of Ockeghem’s chanson Ma bouche rit.32 This striking coincidence is in fact extremely surprising: how would Josquin know that this particular melody would fit simultaneously into the proportions that he chose? Did he have some reason for using this particular melody? Though it is galling to admit, the piece may have been an entirely fortuitous discovery on Josquin’s part, and we will very likely never be able to answer these questions.

30 See the Benedictus of the Josquin Missa L’homme armé super voces musicales, and for a set of enigmatic canons from Cerreto, some of which have unusual cadences. See also Peter Schubert, Modal Counterpoint, Renaissance Style, (New York: Oxford University Press, 2008), 210-212.

31 I explain this in chapter three of this thesis.

Example 5.8. A four-voice resolution of the *Agnus dei II* with my annotations.  

Van Benthem in turn replied to Stam’s article, disagreeing with his colleague’s interpretation of the fourth voice. He thought of the fourth voice as a natural result

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33 Van Benthem ends his resolution by finishing the phrase in the cantus, so that the other three voices hold their final pitches for many beats. I prefer the solution given above. Erratum: what I have called unprepared sevenths above are technically unresolved sevenths.
of the process of composition. In addition, he offered an analysis of the canon’s opening up to rhythmic position 13 in ε, or the amount of material borrowed from *Ma bouche rit*. Van Benthem’s explanation of the compositional fitness of this selection for treatment as a mensuration canon seems well-founded, but while he explains how this particular canon works, his explanation does not translate into principles for the composition of other canons of the same type.

The most recent discussion of the *Agnus dei II* is by Jesse Rodin, in his 2007 dissertation *Josquin and the Polyphonic Mass in the Sistine Chapel*. Rodin includes a thorough description of all four of the proportion canons in the Missa *L’homme armé super voces musicales*, the three two-voice examples from the Benedictus as well as the *Agnus dei II*. He touches on all of the points that I mention, including a thorough discussion of the borrowing of the contratenor of Ockeghem’s *Ma bouche rit*. Josquin may well have intended the canon as homage, joke, display, and challenge for singers, all at once.\(^3^4\)

Rodin also delves into compositional process and the possible reasons behind individual note choices. He concludes, as I do, that “the most difficult section of any mensuration canon from a compositional point of view is the opening, where the voices gradually pull apart from one another.”\(^3^5\) His conclusion about the initial conditions and note choice of the opening is that it fit Josquin’s needs perfectly. In addition to having wide currency, “the repeated semibreves at the beginning ensure that the altus will essentially hold on a pedal for the first three measures; the subsequent ascending motion by thirds is a perfect complement to that a when sung in the outer voices beginning on d/d’. Melodic motion by a third also ensures that the outer voices remain consonant with one another. All in all, Josquin could not

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\(^3^4\) Jesse Rodin, *Josquin and the Polyphonic Mass in the Sistine Chapel*, Ph.D. diss., Harvard University, 2007, 338-343. Josquin’s reference to Ockeghem is unmistakable due to the use of mensural canons (as in the *Missa Prolationum*), modal manipulation (as in *Missa Cuiusvis toni*), and in the particularly obvious borrowing of motive from Ockeghem’s motet *In hydraulic*.

\(^3^5\) Rodin, *Josquin*, 318.
have asked for a better motive on which to base the opening of a three-out-of-one canon.’’

There is one article that mentions the *Agnus dei II* in terms of its symbolic significance. H. Colin Slim was apparently the first person since the sixteenth century to identify this canon in Dosso Dossi’s *Allegory of Music*, in an article published in 1990. This painting contains two canons: one anonymous, notated in a circle, the other Josquin’s *Agnus dei II*. He attributed his find to the poor detail in previously available reproductions of the painting, in which the canons are not clearly visible.

Josquin’s canon is depicted in the painting in the form of a triangle, a figure that, in addition to having three sides corresponding to the canon’s three voices and its 3:2 mensural ratio, signifies the three-in-one Holy Trinity, and the perfection that this last imprinted onto Creation. Conveniently, the enigmatic notation of the piece divides into three groups of nine semibreves, each of which fits onto one side of the triangular staff. The triangular notation complements the circular notation used for the other canon, since both are symbols of perfection. The various manifestations of the number three in the canon and the three-sided notation have allegorical functions within the context of the painting, which depicts the Biblical blacksmith Tubalcaïn and two Venus figures. The blacksmith, in a reference to the well-loved Pythagorean myth, forges musical materials with his weighted hammers, and the two women hold the more refined end-products of harmony, melody, and counterpoint, wrought into the shapes of perfection on stone for eternal preservation.

Josquin’s canon has thus been put to a wide variety of uses in its more than five hundred year history. Initially a clever canonic mass movement, it quickly caught the attention of theorists and other anthologists, who reproduced it in print for their own ends. It was particularly popular with writers on music in German-speaking areas, where its compact notation reappeared in print, often without text, for music pedagogical purposes. The *Agnus dei II* has never quite lost currency, probably because of its astonishing cleverness, but strangely enough it did not inspire many, if any, similar compositions.

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36 Rodin, *Josquin*, 320. Indeed, he could not. Rodin’s is the most complete commentary on the *Agnus dei II* to date, and makes a fascinating read alongside this chapter.

37 Slim, “Dosso Dossi.”
While it has hardly inspired a flurry of musicological activity, the *Agnus dei II* has gained some attention in the nineteenth and twentieth centuries. Robert Eitner and Albert Smijers included the *Missa L'homme armé super voces musicales* in their publications of Josquin’s music. In both the *Historical Anthology of Music* and *The Notation of Polyphonic Music 900-1600*, standard books in the second half of the twentieth century, Willi Apel included the *Agnus dei II* as a mensuration canon. In 1953-1954, in Curt Sachs’ book *Rhythm and Tempo*, and the review of it by Otto Gombosi, the *Agnus dei II* made a minor appearance, but one that, like Apel’s publications, reached a relatively wide audience within the academic music world. It was not until the 1974-1976 exchange between Jaap van Benthem and Edward Stam that any in-depth analysis of the piece was published. Van Benthem’s discovery of and speculation on a four-voice resolution for the canon was another step in the long series of interpretations that the canon has inspired so far. Rodin’s recent comments are particularly useful as an overview of the entire piece, its history, and its likely significance for its composer and audience. Chapter three of this thesis, which I conceived of and wrote without knowing Rodin’s work, nevertheless forms a continuation of the speculation on compositional process that surrounds this piece.
Conclusion.

When I began research on this project, I was expecting to find many more examples of proportion and mensuration canons than I did. I imagined that with a little effort I could uncover at least fifty with no trouble. One year later, I had found only thirty-nine, and of those, only three of the type that I thought would be most common: the three voice unaccompanied proportion canon.

Partly, the reason for this discrepancy lies in wishful thinking on my part. I simply wanted there to be more examples, and especially more consistency. What I found was an incredible variety of procedures, which I have attempted to document in the Appendix, meant to be a reference to imitative compositions within the class of procedures that relies upon mensural transformation of melodic material.

Based upon my assessment of these pieces, I conclude that they make up a heterogeneous body of music that cuts across all generic boundaries. Each one is slightly different from all of the others. Only the two-voice canons, which are in the greatest number, show signs of being a unified group of pieces. In my opinion, the most interesting find is documented in chapters two, three, and four, in which I describe contrapuntal structures thoroughly in two and three voices and to some extent in four. Also of interest is chapter five, in which I explain how the second Agnus from Josquin’s Missa L’homme armé super voces musicales became an often used example of a mensuration canon. What is particularly intriguing about this piece’s history is that it became an example of a category of piece that, as far as I can ascertain, never existed as a category. When I read in Apel that it was “an example of a mensuration canon,” I naturally assumed that it was a representative example, but such is not the case. It is the only piece with the combination of mensurations c, ς, and ς3, and one of only three unaccompanied three-voice proportion canons that I collected.

While searching for and collecting these canons, I found an incredible variety of forms, and indeed formal structures, within which mensural changes may occur. From DuFay’s chanson Bien veigne vous to the anonymous Credo of Bologna 2216, these pieces are diverse. The earliest is probably Ciconia’s Le ray au soleyl, dating from about 1400, and the latest are probably the three pseudo-Byrd canons from the
Some of the canons use accompanying voices, some do not. Some of them incorporate temporal delays, others bring in all of the voices simultaneously. My overriding impression is of a compositional culture exploiting the ways in which one symbol can be realized into sound in multiple ways. The same note value may mean two different things in two different contexts. When these contexts occur simultaneously, maintaining proper harmonic relationships between two voices becomes a compositional challenge.

It is this challenge that I have explored in chapters two, three, and four, the music-theoretical part of this thesis. Following my initial observation that the opening of the dux in a simultaneous canon determines what follows in the other voices, I decided that the canonic opening is essential in the writing of a mensuration or proportion canon. The canonic opening is the place in which the initial conditions of transposition and mensuration come together and affect note choice in a very precise way. Defining those initial conditions is an important step in understanding the compositional processes that may be at work in these canons. Given starting pitches and mensurations, the dux must make a motion that satisfies one melodic and two harmonic constraints: it must be an appropriate melodic interval, it must make an appropriate consonance with the other voices, and it must make appropriate consonances when it is transferred into comes voices. These restrictions are severe ones, and they limit the available first melodic motions in the dux, thereby shaping the entire canon. The comes voices of the opening provide a cantus firmus for the other voices. The farther the dux progresses, the more cantus firmus is established, until the point at which the final cadence is established, after which the limitations on the dux become progressively less severe as the comes voices reach the final cadence.

My goal in collecting these pieces, and of analysing a few of them, is ultimately to provide some focus to issues of counterpoint and style composition. After all, one of my initial questions was “how does one write a piece like Pierre de La Rue’s Agnus dei II?” I have provided a manual for composers to follow, and I

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1 There do exist some examples later than this, but they fall outside of the time reference of the present study. The chapter headings to Martini’s Storia della musica (1757-1781) contain several canons that use mensural oddities. J.S. Bach wrote several canons, notably in the Musical Offering, that use augmentation or diminution, including at least one in contrary motion.
have tried to be mindful of style composition as a goal in my analyses and speculations. In several cases, I have written canons modeled on those that I found; some of these have made their way into the thesis, some have not. I hope that interested composers will be able to do so as well.

I feel that it has been worthwhile to undertake this project, if only to create a list of simultaneous mensuration and proportion canons, which may serve later researchers as a starting-point even if, or especially if, they should disagree with my conclusions.

I can see a number of possibilities for continuation of this research. The first and most obvious is a clearer picture of truly mensural procedure as opposed to strictly proportional canons. A thorough analysis of the Old Hall Credo and Missa Prolationum would be the beginning point for such an investigation. A second issue to pursue is the mathematical nature of proportion canon composition. I have likened it to the game of chess, a somewhat misplaced comparison, but like chess, I believe that proportion canons especially could be modeled mathematically to very good effect. These compositions in particular are like little musical logic machines: given a set of initial conditions and contrapuntal rules, there should be a very large, but definable, set of possible compositions, or at least canonic openings. A final possibility related to the preceding one is a connection to the recent work of Tymoczco et al., in which the authors propose a modeling of voice-leading patterns using complex mathematical spaces.\textsuperscript{2} What kind of spaces might proportion canons create when subjected to the same kind of analysis? There may be many more questions to ask of these canons. I hope that in due course some of them may be asked and answered.

Appendix.

This appendix is a series of short descriptions of each of the canons that I collected during my research. The canons are numbered, but because they can be classified in so many ways I elected to present them in the order in which I found them. The appendix is provided with an index that is searchable by composer name, number of canonic voices, and presence of non-canonic or accompanimental voices.

Each description begins with the title or incipit, the composer’s name, and an indication of the number of canonic voices and accompanying voices. Then the source or sources are given, and the editions of the piece or movement. I have not included every edition of every piece, since some older editions have been superseded by more recent ones. When canonic rubrics appear, they are given in the original form and English translation. Following this, I give a brief description of the piece, or of each canonic movement within a larger piece, including information about mensuration, proportion, intervals of imitation, notation, resolutions, and other pertinent details.

The number of voices given is the number of voices related by canonic imitation, not the overall number of parts. In instances of double canon, the canon is counted by the overall number of canonic voices, not the number of voices derived from a given part. Four-voice examples from the Missa Prolationum, for example, are considered to be for four voices because four voices are sounding in canon, even though there are two written canonic voices.
Index to the appendix.

Number of Voices

Two-voice Unaccompanied:
8, Agnus dei .................. Heinrich Fink
9, Benedictus ................. Josquin des Prez
10, Qui venit ................. Josquin des Prez
11, In nomine................. Josquin des Prez
21, Pleni sunt coeli .......... Johannes Ockeghem
25, Agnus dei II ........... Johannes Ockeghem
33, Eslongsie sy ............ Anonymous
35, Christe .................... Jacob Obrecht
36, Puzzle canon VI ........ Anonymous
38, Benedictus............... Josquin des Prez
39, Benedictus............... Pierre de La Rue

Two-voice Accompanied:
1, Credo ........................ Anonymous
3, Per naturam. Synophe .... Anonymous
4, Per naturam. Synophe ...... Anonymous
6, Indita stella maris .... Johannes Ockeghem
7, Bien veignes vous .......... Guillaume Dufay
28, Kyrie I ...................... Pierre de La Rue
29, Christe ...................... Pierre de La Rue
30, Kyrie II ..................... Pierre de La Rue

Three-voice Unaccompanied:
5, Le ray au soleyl .......... Johannes Ciconia
12, Agnus dei ................. Josquin des Prez
37, Omne trinum perfectum .... Ludwig Senfl

Three-voice Accompanied:
2, Miserere........................ Anonymous
32, Benedicta sit sancta trinitas.. Anonymous
34, Credo ........................ Anonymous

Four-voice examples continued:
18, Patrem ................. Johannes Ockeghem
19, Et resurrexit .... Johannes Ockeghem
20, Sanctus ................. Johannes Ockeghem
22, Osanna ................. Johannes Ockeghem
23, Benedictus .......... Johannes Ockeghem
24, Agnus dei ...... Johannes Ockeghem
26, Agnus dei III ...... Johannes Ockeghem
31, Agnus dei II .......... Pierre de La Rue

Six-voice examples:
27, Magnificat I toni (verse 10) Pierre de La Rue
Composer

Anonymous:
1, Credo
2, Miserere
3, Per naturam. Synophe
4, Per natumram. Synophe
32, Benedicta sit sancta trinitas
33, Eslongies suy
34, Credo
36, Puzzlet-canon VI

Ciconia, Johannes:
5, Le ray au solyel

Dufay, Guillaume:
6, Inclita stella maris
7, Bien veignes vous

Finck, Heinrich:
8, Agnus dei

des Prez, Josquin:
9, Benedictus
10, Qui venit
11, In nomine
12, Agnus dei
38, Benedictus

La Rue, Pierre de:
27, Magnificat I toni (verse 10)
28, Kyrie I
29, Christe
30, Kyrie II
31, Agnus dei II
39, Benedictus

Obrecht, Jacob:
35, Christe

Ockeghem, Johannes:
13, Kyrie I
14, Christe
Type of Opening

Simultaneous:
1, Credo.........................Anon.
2, Miserere....................Anonymous
3, Per naturam. Synophe.........Anon.
4, Per naturam. Synophe.........Anon.
5, Le ray au soleil...........Johannes Ciconia
6, Inclita stella maris...Guillaume Dufay
7, Bien veignes vous.....Guillaume Dufay
8, Agnus dei...............Heinrich Finck
9, Benedictus..........Josquin des Prez
10, Qui venit........Josquin des Prez
11, In nomine..........Josquin des Prez
12, Agnus dei...........Josquin des Prez
13, Kyrie............Johannes Ockeghem
14, Kyrie II...........Johannes Ockeghem
15, Et in terra*........Johannes Ockeghem
16, Sanctus........Johannes Ockeghem
17, Qui tollis........Johannes Ockeghem
18, Patrem........Johannes Ockeghem
19, Et resurrexit*...Johannes Ockeghem
20, Osanna........Johannes Ockeghem
21, Pleni sunt coeli..Johannes Ockeghem
22, Benedictus........Johannes Ockeghem
23, Agnus II........Johannes Ockeghem
24, Magnificat I toni.....Pierre de La Rue
25, Kyrie I..............Pierre de La Rue
26, Kyrie II.............Pierre de La Rue
27, Christe.............Pierre de La Rue
28, Agnus III........Pierre de La Rue
29, Benedicta sit sancta trinitas.....Anon.
30, Benedictus II.......Pierre de La Rue
31, Eslongies suy...........Anon.
32, Christe.............Jacob Obrecht
33, Puzzle canon VI.........Anon.
34, Omne trinum perfectum...Ludwig Senfl
35, Benedictus..........Josquin des Prez
36, Agnus II...........Josquin des Prez

Imitative:
1, Credo.........................Anon.
14, Christe ........Johannes Ockeghem
16, Et in terra*...Johannes Ockeghem
17, Qui tollis......Johannes Ockeghem
19, Et resurrexit*...Johannes Ockeghem
20, Sanctus...........Johannes Ockeghem
21, Pleni sunt coeli..Johannes Ockeghem
23, Benedictus........Johannes Ockeghem
24, Agnus II........Johannes Ockeghem
26, Agnus III....Johannes Ockeghem
34, Credo.........................Anon.

*Numbers 17 and 19 have a simultaneous entry in one duo and an imitative entry in another duo.
Mensural Technique

Mensuration Canons:
6, *Inclita stella maris*........Guillaume Dufay
13, Kyrie..................Johannes Ockeghem
14, *Christe*.............Johannes Ockeghem
15, Kyrie II............Johannes Ockeghem
16, *Et in terra*...........Johannes Ockeghem
17, *Qui tollis*........Johannes Ockeghem
18, *Patrem*.............Johannes Ockeghem
19, *Et resurrexit*......Johannes Ockeghem
20, *Sanctus*............Johannes Ockeghem
21, *Pleni sunt coeli*.....Johannes Ockeghem
22, *Osanna*.............Johannes Ockeghem
23, *Benedictus*.........Johannes Ockeghem
24, *Agnus dei*.........Johannes Ockeghem
26, *Agnus dei III*.....Johannes Ockeghem
32, *Benedicta sit sancta trinitas*........Anon.
34, *Credo*................Anon.

Proportion Canons:
1, *Credo*..................Anon.
2, *Miserere*................Anon.
5, *Le ray au soleyl*.........Johannes Ciconia
7, *Bien veignes vous*........Guillaume Dufay
8, *Agnus dei*............Heinrich Finck
9, *Benedictus*...........Josquin des Prez
10, *Qui venit*............Josquin des Prez
11, *In nomine*...........Josquin des Prez
12, *Agnus dei*...........Josquin des Prez
25, *Agnus II*............Johannes Ockeghem
27, *Magnificat I toni*.......Pierre de La Rue
28, *Kyrie I*..............Pierre de La Rue
29, *Christe*...............Pierre de La Rue
30, *Kyrie II*.............Pierre de La Rue
31, *Agnus dei II*........Pierre de La Rue
33, *Elongies suy*.........Anon.
35, *Christe*.............Jacob Obrecht
36, *Puzzle canon VI*........Anon.
37, *Omne trinum perfectum*...Ludwig Senfl
38, *Benedictus*.........Josquin des Prez
39, *Benedictus*.........Pierre de La Rue
1.

*Credo*

Anonymous

Two-voice, accompanied

Source:

Bologna, Biblioteca Universitaria 2216.

Edition:


The canonic rubric is unfortunately not clear enough for me to make it out in the facsimile, and no translation of it is available.

This proportion canon is of the “accompanied tenor fuga” type in Newes’s classification. The *tenorista*, or middle voice, and the triplum enter at the same time, and the tenor imitates the *tenorista* at a time interval of eight breves. The tenor voice is in 1:2 augmentation at the lower fifth.
2.

_Miserere_
Anonymous
Three-voice, accompanied

Source:
Brit. Mus. Add. Mss. 31391

Edition:

This canon on the plainsong _miserere_ is in four parts: a three-in-one canon and the plainsong cantus firmus. The four parts correspond to SATB ranges. The parts begin together, with the cantus firmus in the tenor, and the canonic subject in the superius. The bassus has a canonic realization of the superius in doubled note values two octaves below it, and the altus has quadrupled note values a fourth below.

This canon, as well as the next two (2 and 3), were thought by Edmund Fellowes in 1941 to be by Byrd because of the initials “WB” attached to the manuscript book. By implication, this attribution was called into question by Philp Brett in 1972 and is likely to be incorrect.¹

3.

*Per naturam. Synopbe*

Anonymous

Two-voice, accompanied

Source:


Edition:


This canon is in four parts, and all parts begin simultaneously. Two parts are in canon, the superius and the altus, which is in 1:3 augmentation, or three times slower than the cantus.

The cantus firmus is in the tenor, and there is a freely composed *ad placitum* bass line. The cantus firmus that appears to be synthetic because it is essentially six repetitions of a three-note motive, and each repetition is a step higher than the previous one. This is one of the only augmentation canons that I have found to feature the 1:3 proportion. The proportion is indicated by the Arabic numeral “3” below the sign c.
4.

*Per naturam. Synope*

Anonymous

Two-voice, accompanied

Source:

Brit. Mus. Add. Ms. 31391

Edition:


This canon is on the same cantus firmus as the preceding one, but it is in only three parts. The cantus firmus is in the superius, and the two canonic voices are below it. The fastest-moving voice is in the tenor, and the canon is realized at the fifth below in 1:2 augmentation. The given canonic voice is prefixed by two mensuration signs, c and ã.
5.

*Le ray au soleil*

Johannes Ciconia

Three-voice, unaccompanied

Unique source:

Lucca Codex (Codice Mancini) Lucca, Archivio di Stato, MS 184, Perugia, Biblioteca Comunale “Augusta,” MS 3065.

Editions:


Facsimile:


Rubric:

*Dum tria percurris quatuor val<et> Tertius unum. Subque diapas<on> sed facit alba moras.*
While one voice sings three (minims), there are four in another and one in
the third; the latter part sings at the lower octave but reads the white notes as
rests.²

A number of solutions have been proposed for this canon. To date, the most
successful one, and apparently the intended one, appears in the Bent and Hallmark
edition. The canon is for three voices derived from one notated part. The upper two
voices are in 4:3 proportion so that four minims are sung by the superius in the time
of three in the altus. Thus, there is no note value that is equal between the two
except the longa.

The tenor voice moves much more slowly than the two upper voices (the
combined ratios of the voices are 4:3:1) and has the added complication of omitting
those notes that are written as void notes in the notated voice. Two of these occur in
a ligature, allowing the clever avoidance of dissonances early on in the piece, and the
third is just before the end of the piece in the tenor.

Although Bukofzer had reservations about the attribution, *Le Ray au soleil* is now
thought to be the work of Johannes Ciconia, since it shares a manuscript page with
*Una panthera*, a work attributed to him in the source. That the pieces were copied
together makes it is reasonable to assume that they are related.³

The solutions to the canon vary quite widely. Those by Ghisi and Bukofzer
are for two voices only, though Bukofzer suspected a different solution: “In view of
the rests it seems likely that a third part is missing. The remark ‘tertius unum’ may
possibly hint at a third canonic voice, but if it does I do not know how it should
come in.”⁴

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2 I borrowed this translation directly from Newes, *Fuga*: 332, and she in turn cites Richard
Hoppin’s review of Suzanne Clercx, *Johannes Ciconia: Un Musicien Liégeois et son Temps*, in the *Musical

3 For a full discussion, see Nádas and Bent, *Lucca Codex*, 43-45.


Four of the dux equal three of the corpes I and end of the corpes II.
Inclita stella maris
Guillaume Dufay
Two-voice, accompanied

Source:
Bologna, Civico Museo Bibliografico Musicale, Cod. Q15.

Editions:


Rubric:
Et fuga de se cane(n)do de tempo p(er)ficto et simul incipiando et est c(on)corda(n)s sy plac(et) absq(ue) contratenor(es).

Contra : (con)cordan(n)s cu(m) fuga et cuili(bet) p(er) se.

Secund(us) co(n)tratenor: (con)corda(n)s cu(m) o(mn)i(bu)s no(n) potest cantari nisi pueri dica(n) t fuga(m).

The [other] *fuga* [part] is sung in perfect tempus and begins simultaneously on a concord without the contratenor [*fuga* is at the unison].
The *Contra* is in concord with the *fuga* and is pleasing in itself
The *Second Contra* is in concord with all the others and cannot be sung unless boys say the *fuga*.

The canon is between the two cantus parts in this motet. It is a mensuration canon in which the Cantus I is in ɛ and the Cantus II, which is not notated, is in ø, sung as though the breves were perfect rather than imperfect. The two contratenors are in ø and ɛ respectively, mirroring the arrangement of the two cantus parts.

As Besseler notes, *Inclita stella maris* dwells extensively on d minor sonorities. It begins with the convenient descending triadic outline that is so well suited to conflicting mensurations (see also *Le ray au soleil*).\(^6\)

Besseler gives several possibilities for the performance of this piece in both duets and trios, although none of them relieves the piece of its harmonic monotony, in which, in his words, “the tonality of d-minor ever recurs, without noticeable contrast.” Therein probably lies one way of conceiving and understanding such a piece: the fewer harmonic changes, the easier it is to unfold the same melodic sequences at two different speeds.

Fallows says that the two *Contra* canons are what indicate the performance possibilities given by Besserler: voices I and II, voices I, II and III, voices I and III, voices I, II and IV, voices II and III, or all four voices.\(^7\)

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\(^6\) This is due to the consonances that automatically result from “unfolding” a triad, in this case from top to bottom, or from the fifth to the root.

Bien reigne vous, amoureuse liesse
Guillaume Dufay
Two-voice, accompanied

Source:

Edition:

Rubric:

Hunc discas morem
Si vis cantare tenorem:
Ut iacet attente
Cantetur subdiapente

If you wish to sing the tenor
Sing it at the fifth below [author’s paraphrase]

This chanson is a very short rondeau, having only two verses in the form ABA. The edition inserts all of the text within one statement of the canon melody, which has a signum to show the final cadence after the repeat of A. The canon is between the cantus and the tenor, with the contratenor filling in harmonies. It is a straightforward 1:2 proportion canon, with the comes at the lower fifth.

Fallows finds that “the results are perplexingly unpleasant. […] the contratenor in several places produces dissonances that Dufay seems unlikely to have countenanced. Were it not ascribed to him in [Oxford, Bodleian Lib., Cod. Canonici misc. 213] it would surely be judged spurious.” He thinks that the text is actually
incomplete as it appears in the source, and that line one was the “short refrain” of the rondeau.\(^8\)

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8.

*Agnus dei (Missa a 3 voce)*

Heinrich Finck

Two-voice, unaccompanied

Source:

Regensburg, Bischöfliche Bibliothek Proske Ms. B 216-219. This is a set of three partbooks from the first half of the sixteenth century.

Edition:


This part of the Agnus dei from an untitled three-voice Mass is a 1:2 proportion canon at the octave. The faster voice is in the bass in this case. It is relatively long compared to the other two-voice examples presented in this study. It appears to be Finck’s only use of this technique, judging from the complete works edition of 1962.
9, 10, 11.

*Benedictus (Missa L’homme armé super voces musicales)*

Josquin des Prez

Two-voice, unaccompanied

Source:

Misse Josquin. Ottaviano Petrucci, Florence, 1502.9

Edition:


Sebald Heyden’s *Musicae* (RISM 15371) reprinted all three of these canons as examples of the use of mensuration signs to indicate proportions. Glarean also reprinted part of the Benedictus in *Dodecachordon* (RISM 15471).

This Benedictus movement is in three parts. All three are 1:2 proportion canons at the unison. All three are quite short, fewer than twenty breves long. Only the first features a standard sixth-to-octave final cadence.

9. *Benedictus*. Canon at the unison in the proportion 2:1. Both the medial and final cadence are sixth-to-octave cadences.
10. *Qui venit*. Unison canon in the proportion 2:1. No clear internal cadence; the final cadence is not standard.
11. *In nomine*. Unison canon in the proportion 2:1. No clear internal cadence; the final cadence is not standard.

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9 See appendix 12, page 102.
12.

Agnus dei II, *(Missa L’homme armé super voce musicales)*
Josquin des Prez
Three-voice, unaccompanied

Sources:
- Ottaviano Petrucci 1502, 1514, 1516
- Sebald Heyden 1537
- Heinrich Glareanus (RISM 1547),
- Paris Ms. *Bourdeney*

There are many other extant copies and prints of this work; some of them print the entire mass, others print just one movement. For a complete list, see the edition by Smijers given below and page 55 of this thesis.10

Edition:

This example may be the proportion canon with the widest distribution in the twentieth century. It uses both 1:2 and 1:3 proportions, and the relationship of the note values from highest to lowest voice is 3:1:2. These proportions are derived from the three mensuration signs that precede the first note of the piece: ε3, c, and ε. The sign c indicates *integer valor*, or the basic note values, which are diminished in the lower voice, and diminished 3:1 in the upper voice. Like *Le ray au soleyl* and *Inclita stella maris*, it features a largely triadic opening, although in this case an ascending rather than a descending one. There is an internal cadence, and a non-standard final cadence, which is not too unusual in view of the third *Agnus* movement that follows.

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10 The volume containing the *Missa L’homme armé super voce musicales* in the *New Josquin Edition* was not yet available as of this writing.
This piece was printed by Heyden, Glarean, and Jacob Paix in the sixteenth century. In the twentieth century it was used as an illustration of “mensuration” canon by Willi Apel. There are three different printed solutions, from Glarean, Heyden, and Berlin Ms. 1175. These printed solutions differ from the far simpler solution expressed by the mensuration signs, because they involve a re-interpretation of the triple proportion in the upper voice.
13-26.

*Missa Prolationum*

Johannes Ockeghem

Sources:

- Rome, Biblioteca Vaticana, Chigi Codex. C. VIII. 234.
- Vienna, Nationalbibliothek, Codex 11883.

Editions:


This mass is a completely canonic cycle. All but one of the canons are double canons, notated as four voices in two, and the interval of imitation increases for each movement from the unison through the octave. Jaap van Benthem postulates that the original notation, now lost, used flats (fa-signs) and mensuration signs rather than clefs to indicate the intervals of imitation. Below I discuss each segment separately.

Many of the canons in *Missa Prolationum* do not begin in all voices simultaneously, but are both mensuration canons and time interval canons. In these cases, the most common device is to use the two *dux* voices to form a non-imitative duo that is repeated in the *comes* voices at the designated interval above or below the original pitch. ¹¹ In the case of the difficult intervals such as the second and the

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¹¹ See Schubert, “Hidden Forms.” A non-imitative duo is a contrapuntal combination of two voices that repeats at least once within a polyphonic framework.
seventh, Ockeghem writes only a series of duos in which the imitating parts do not overlap at all.\footnote{This procedure is similar to some of the canonic chansons by Josquin, in which phrases are imitated in one voice while another voice rests. <find an example. AMS paper, 5-voice chanson?>}

13. Kyrie. Double canon, all voices enter simultaneously. The voices in each canonic pair are at the unison, the two notated voices are at the octave.

14. Christe. Canon at the second above. Ockeghem manages this by causing the imitation not to overlap at all between the dux and comes voices in each part.

15. Kyrie II. Canon at the third above. All voices enter simultaneously.

(Gloria)

16. Et in terra. Canon at the fourth above. The upper duo enters simultaneously with the tenor, but the imitating contratenor is delayed by two imperfect breves.

17. Qui tollis. Canon at the fourth below. As in the Christe, the comes voices enter after the dux, so the time interval of imitation is quite long. The dux voices are both in imperfect tempus and the comes voices are in perfect tempus, ensuring that the counterpoint remains the same within each non-imitative duo.

(Credo)

18. Patrem. Canon at the fifth above, all voices enter simultaneously.

19. Et resurrexit. Canon at the sixth above. The upper voices enter simultaneously, but the lower voices do not stand in the same relationship as the upper two. The lower contratenor follows the upper voices after sixteen long rests, and in turn the upper contratenor follows the lower contratenor after twenty-four semibreve rests.

(Sanctus)

20. Sanctus. Canon at the sixth above. As in the Christe and Qui tollis, the opening is made of two non-imitative duos. The movement is only long enough to allow both pairs of voices to sing through the opening duo one time; in the repetition the imitating voices are accompanied by two free voices.

22. *Osanna*. Canon at the octave below. The two notated voices are a perfect fifth apart, causing a standard octave and fifth voice disposition. All voices enter simultaneously.

23. *Benedictus*. Canon at the fourth below. The *dux* and *comes* non-imitative duos are at the unison with one another, however. There is no overlap between the non-imitative duos.

*(Agnus dei)*

24. *Agnus dei*. Canon at the fourth below; the *dux* and *comes* non-imitative duos are at the unison. The *dux* voices provide counterpoint during the *comes* duo.

25. *Agnus II*. Canon at the fifth below. This is a two-voice canon in the proportion 2:1, the only proportion canon in *Missa Prolationum*.

26. *Agnus III*. Canon at the fifth above. Non-imitative duos between the *dux* and *comes* pairs are at the octave. After the imitative entry all voices continue until the final cadence.
27.

_Magnificat I toni_ (verse 10)

Pierre de la Rue

Six-voice, (six-from-three) unaccompanied

Source:

Jena, Universitäts-Bibliothek, Chorbuch 20, ff.1v-7r. Dated 1512-1525.

Edition:


The last verse of this magnificat is set as a triple proportion canon, or six voices from three. All of the voices are realized using the same proportion (1:2 augmentation of the given voices) and are realized at the unison within each pair. The opening sonority is not an octave divided into a fifth and a fourth, but a tenth with a fifth above the bass.

La Rue makes liberal use of rests, which is surely helpful in view of the number of contrapuntal relationships involved. He also incorporates some imitative entries in between voice pairs. The final cadence is perfectly prepared and entirely standard in all six voices.

The unique source for this piece is a copy made between 1512 and 1525.
Kyrie and Agnus dei (Missa L’homme armé I)

Pierre de la Rue

Sources:

Brussels, Bibliothèque Royale, MS 9126, ff.28v-43r.¹³
London, British Library, Reference division, Ms. Add. 4911. Agnus II only.
Vienna, Österreichische Nationalbibliothek, Ms. 1783.
Jacob Paix, Selectae artificiosae et elegantes fugae. Laungien: I. Reinmichel, 1590 (and 1594). Agnus II only.
Giovanni Battista Rossi, Organo de Cantori. Venice: Gardano, 1618.

Editions:


This mass contains four proportion canons, three in the Kyrie and one in the Agnus dei. The three Kyrie canons are for two canonic voices and a pair of accompanying voices.

¹³ This is the principal source used in Davison, Pierre de la Rue. All other information on sources and editions is from the 1996 Davison edition.
28. *Kyrie* I: the canonic duo is in the bassus and tenor, and the altus and superius are in the same mensurations, respectively, as the lower pair. All voices begin simultaneously.

29. *Christe*: the two inner voices are in 2:1 proportion, (c3 and c) but they begin after six breve rests, during which the bassus and superius (both in) present a duo.

30. *Kyrie* II: the proportion canon moves back to the lower pair of voices (c and 3), and the superius and altus have an accompanying duet. The entire *Kyrie* movement presents the paraphrased *L’homme armé* theme continuously.

31. *Agnus dei* II: in four mensurations simultaneously (c, c, c3, and o). The proportional relationship between the voices is 4:6:3:2. This does not express completely the changes in mensuration that are indicated in the alto and tenor by the signs o and c3 in relation to the bass *integer valor* in c. Those voices must re-interpret the notation, applying the principles of alteration and imperfection that are automatically operative in triple mensurations. This *Agnus dei* II was presumed by Glarean to be a composerly response to Josquin’s *Agnus dei* II from the *Missa L’homme armé super voces musicales*. It is a more complex version of the same thing, incorporating an extra proportional relationship of 3:2 in *integer valor*, or between the mensurations c and o. This canon is resolved in the Petrucci print of 1503.

Davison writes “…the *L’homme armé* melody, perhaps because of its inherent canonic possibilities, was the most widely accepted model for Mass Ordinaries in which composers sought to demonstrate their contrapuntal skills in friendly competition with one another.”  

14 Recent research has indicated that there were widespread liturgical preoccupations with the Armed Man that probably encouraged this kind of competition.  

15 The notion that this melody has more inherent canonic possibilities than others is worth further investigation.

This piece exists in multiple sources from the sixteenth century. Brussels, Bibliothèque Royale 9126 is dated to 1505 and is likely to be the earliest.

14 From Nigel St. John Davison’s introductory remarks to Pierre de la Rue: *Opera Omnia*, LV.

32.

_Benedicta sit Sancta Trinitas_

Anonymous

Three voice, accompanied

Source:

Trent 89, 131v-132r.

Edition:


Rubric:

_Primus novem, secundus sex, tertius quatuor, solum in primus duabis figuris._

Nine in the first, six in the second, four in the third, only in the first two figures.

[the numbers refer to semibreves]

This canon is for three voices at the unison, accompanied by three non-canonic voices. It is a proportion canon, but exceptionally the proportion applies only to two note values, the long and the breve, and even then, as Loyan says, “the rhythmic proportions enumerated for the first three voices are valid only for the first two instances of _longae._” In other words, the first two longs are worth four breves in one voice, six in another, and nine in the third. Thus the piece is like a series of standard imitative canons whose time interval of imitation is not fixed at the beginning, but stabilizes at a distance of six breves between each voice.
33.

Eslongies susy
Anonymous
Two-voice, unaccompanied

Source:
Trent 87, fol. 34r.

Edition:


Rubric:

Qui cupit accipiat cantus cantare tenorem
Istius minimas cantet simul et semibreves
Sub dyapaison, sed in dyatessaron sint tibi breves.

He who wishes to sing the tenor of this piece
May take the minims and semibreves of the
Cantus under the dyapason and sing at the
Same time, but let your breves be in dyatessaron.\textsuperscript{16}

This chanson for two voices is a proportion canon at the octave, with a simultaneous beginning. The upper voice is the faster-moving of the two voices. Loyan explains that the term “dyapason” means the octave in the usual intervallic

\textsuperscript{16} Translation from Loyan, Trent, xvii.
sense, but the term “dyatessaron” refers rather to the ratio between the rhythmic values of the voices.

There is some confusion about this ratio. Loyan believed it to mean 4:3, but he said that the composer intended 1:4. Neither of these interpretations seems to make much sense, since the ratio of speed between the voices is clearly 2:1 in the edition, which gives a very good solution.
34.

_Credo_
Anonymous
Three-voice, accompanied

Source:

Editions:
Andrew Hughes and Margaret Bent. _The Old Hall Manuscript_, CMM 46.
American Institute of Musicology, 1969.

Facsimile:
Available online at the Digital Image Archive of Medieval Music,
www.diamm.ac.uk/

Other literature on the source:
Willi Apel, _The Notation of Polyphonic Music, 900-1600_. Cambridge, MA:
Mediaeval Academy of America, 1949.

The Old Hall Credo 75 probably warrants a complete compositional study in itself. This canon is extremely complicated both in its conception and its notation. It is for five voices, the upper three of which are in canon. The tenor and contratenor provide harmonic support. The three upper voices enter imitatively, each in a different mensuration, and then proceed through a series of overlapping mensural and proportional changes.

The canonic rubric is quite long, since many instructions are needed for the three different parts. Hughes and Bent provide an English explanation in lieu of a direct translation. The realization involves many proportional relationships that
operate within each voice, none of which apply to all voices simultaneously. Thus, the different voices change mensuration at different times. Black, red, black-and-red, void red, and blue coloration is used for the canonic voices, and black and red in the tenor and contratenor. The meanings of these colorations are discussed in detail by Apel and by Hughes and Bent.
35.

*Christe (Missa Petrus Apostolus)*

Jacob Obrecht

Two-voice, unaccompanied

Sources:


Saint Gall, Stiftsbibliothek. Ms. 461. (early 16th century, probably copied in Italy by Netherlandish scribes. Contains music by composers born between c. 1420 and c. 1460)\(^{17}\)

Edition:


This *Christe* is a short, two-voice canon at the fifth in the proportion 2:1. The lower voice is the faster moving one. There is one internal cadence, and the final cadence is standard 6-8. The proportions are indicated by the signs $\text{c}$ and $\text{c}$. 

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\(^{17}\) Hudson, *Obrecht*, xxvii.
36.

*Puzzle-canon VI*

Anonymous

Two-voice, unaccompanied

Source:

British Museum, Add. Ms. 31922.

Edition:


This relatively short two-voice 2:1 proportion canon at the fifth is, at a surface level, slightly more complex than some of the others that I have included. The faster voice is uppermost, and it is more rhythmically complex than many other examples, with many syncopations and triplet figures. The final cadence is standard, sixth-to-octave.

There are several other puzzle canons in the manuscript book from which this collection was compiled. Some of them are lengthy rounds and some, like this one, are more esoteric and unusual.
Omne trinum perfectum
Ludwig Senfl
Three-voice, unaccompanied

Source:
Heinrich Glarean. Dodecachordon, 1547.
Hermann Finck. Practica Musica, 1556.

Edition:

Omne trinum perfectum is a three-voice canon in the proportion 1:4:2. That is, the tactus is on the minim in the upper voice, the breve in the middle voice, and the semibreve in the lowest voice.

The earliest source for this piece is Dodecachordon (RISM 15471) in which Glarean prints the signs O, Ø, and ◊. The use of the circle-dot sign ◊ is very unusual for the purpose of 2:1 augmentation. Glarean is probably the source for Hermann Finck, which is the only other source that I know of.
38.

Benedictus, (Missa sine nomine)
Josquin des Prez
Two voice, unaccompanied

Sources:
Jena, Universitäts-Bibliothec, Cod. Mus. 3.
Modena, Biblioteca Estense, Ms. α N 1.2.
Vienna, National-Bibliothek, Ms. 4809.
Missarum Josquin liber tertius. Petrucci, 1514, repr. 1516. (RISM
Missarum Josquin liber tertium. Giunta, 1526. (RISM
Liber quindecim Missarum. Rome, 1516 (RISM

Edition:
Josquin des Prez. Opera Omnia. Edited by Albert Smijers. Amsterdam:
Vereiniging voor Nederlandse Muziekgeschiedenis, 1952.

This is a short, straightforward two voice 2:1 proportion canon, at the fifth
below. It has three short phrases, each with its own cadence. Notated in ε and c.
39.

*Benedictus (Missa iste confessor domini)*

Pierre de la Rue

Two voice, unaccompanied

Sources:

Coimbra, Biblioteca Geral de Universidad, Mus. Ms. 2.

Munich, Bayerische Staatsbibliothek, Mus. Ms. 7.

Vienna, Österreichische Nationalbibliothek, Sup. Mus. 15497.

Editions:


This canon is a very familiar type, in 2:1 proportion at the fifth below. It is quite short, as are the other canons like it that appear in other *Benedictus* movements, but it is clearly divided into four phrases. Its final cadence is to a unison rather than an octave.

There is some chance that this mass is not by La Rue at all, but by Antoine de Févin or even an anonymous composer.
Bibliography.


———. *Compositional Approaches to Canon from Ockeghem to Brahms.* PhD diss., Harvard University, 2000.


**Music Editions and Facsimiles**


Hughes, Andrew and Margaret Bent, eds. *The Old Hall Manuscript* (CMM 46), part II. American Institute of Musicology, 1969.


