Exploring stochastic optimization in open pit mine design

By

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Contribution of Authors

The author of this thesis is the primary author in all papers included in this thesis. Professor Roussos G. Dimitrakopoulos was the supervisor of the author’s Masters of Engineering program and is included as co-author in the Chapter 2 of this thesis, which has been previously published in the peer reviewed journal Mining Technology: IMM Transactions section A, Volume 118, Number 2, June 2009, pp. 79-90.
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Abstract

Over recent years, new methods have been developed to integrate uncertainty into the optimization of life-of-mine (LOM) production planning. This thesis makes use of two stochastic optimization methods: simulated annealing (SA) and stochastic integer programming (SIP); which are implemented in the context of the requirements of mining applications through the use of stochastic simulation to model uncertainty. For the case of SA, the second chapter of the thesis documents the case of a copper deposit where ten simulated realizations are sufficient to provide stable LOM optimization results. In addition, the study shows that the selected true optimal pit limits are larger than the ones derived through conventional optimization. Stochastically optimized pit limits are found to be about 17% larger, in terms of total tonnage, than the conventional (deterministic) optimal pit limits. The difference adds one year of mining and approximately 10% of additional net present value (NPV) when compared to the NPV of conventional optimal pit limits and a production schedule generated stochastically with the same SA algorithm. In the third chapter of the thesis, the SIP based optimizer is used with the purpose of integrating uncertainty into the process of pushback design. Results show the sensitivity of the NPV to the design of starting and intermediate pushbacks, as well as the pushback design at the bottom of the pit. The new approach yielded an increment of ~30% in the NPV when compared to
the conventional approach. The differences reported are due to the different
scheduling patterns, the waste mining rate and an extension of the pit limits
which yielded an extra ~5.5 thousand tonnes of metal.
Resume

Depuis quelques années, de nouvelles méthodes ont été développées pour intégrer l'incertitude dans l'optimisation de la planification de la production de la vie-de-mine i.e. life-of-mine (LOM). Cette thèse se sert de deux méthodes d'optimisation stochastique : recuit simulé (RS) et programmation en nombres entiers stochastique (SIP); les deux méthodes sont programmées dans le cadre des besoins des applications d'exploitation de la simulation stochastique et modélisation d'incertitude. Pour le cas de RS, le deuxième chapitre de la thèse décrit le cas d'un dépôt de cuivre où dix réalisations simulées sont suffisantes pour fournir des résultats stables d'optimisation de LOM. En outre, l'étude prouve que les véritables limites optimales choisies de mine sont plus grandes que celles dérivées par l'optimisation conventionnelle. Des limites stochastiquement optimisées de mine s'avèrent environ 17% plus grandes, en termes de tonnage total, que les limites optimales (déterministes) conventionnelles de mine. La différence ajoute un an d'exploitation et approximativement 10% de valeur nette additionnel (NPV) une fois comparée au NPV des limites optimales conventionnelles de mine et une cédule de production produit stochastiquement avec le même algorithme de RS. Dans le troisième chapitre de la thèse, l'optimiseur basé sur SIP est utilisé en vue d'intégration de l'incertitude dans le processus de la conception de fosses emboîtées. Les résultats montrent la
sensibilité du NPV à la conception de fosses emboîtées de commencement et intermédiaires aussi bien que la conception de la fosse emboîtée du fond de la mine. La nouvelle approche a produit une augmentation de ~30% dans le NPV une fois comparée à l’approche conventionnelle. Les différences rapportées sont dues aux différents cédules de production, du taux de décapage du stérile et d’une extension des limites de la mine qui ont produit ~5.5 mille tonnes supplémentaires de métal.
1.1 Introduction

A mining operation relates to the extraction of mineral resources from the ground subject to it being a profitable activity. Geological uncertainty, market price fluctuations, technical requirements and others, lead mining companies to face the problem of constantly developing technological advances in order to increase profit, improve the utilization of reserves and meet economic expectations of investors. A considerable part of such developments is related to the optimal design of mining operations; including the definition of optimal ultimate pit limits, pushback design and other sub problems.

The definition of the optimal ultimate pit limit of an open pit mine is a subject of general interest because it decides when to stop mining. Lerchs and Grossman (1965) introduced an efficient algorithm to find the optimal pit limits based on a graph theory approach. Then, within this framework scheduling is carried out by breaking the pit space into pushbacks. Pushbacks are represented by a set of connected blocks that facilitate the mining operation in terms of safety slope requirements, minimum working width required by mining equipments and
maximization of the NPV of the project through the adequate management of stripping ratios. Conventional methods to approach the problem discussed above operate under the assumption of perfect information being available. This means that no uncertainty is taken into consideration. For example, it is assumed that the economic value assigned to a block is perfectly known in advance, but in reality the true value of a block is unknown until it is finally extracted. Consequently, the results are suboptimal; leading the mining operation to face critical problems, e.g. of not meeting production targets nor NPV forecasts.

The literature review consists of a discussion related to the (i) limits of the conventional approaches; (ii) current technologies to improve such approaches; (iii) transition from conventional optimization methods towards stochastic approaches and their applications to the mining industry; (iv) details of the stochastic optimization approaches used in this thesis.

### 1.2 Conventional approaches and their limitations

In open pit mining, optimization techniques are used to define optimal ultimate pit limits and a sequence of extraction. Optimization is a scientific approach to decision making through the application of mathematical methods and the use of modern computing technology. It concerns the maximization or minimization of an objective function, e.g. maximization of profit or minimization of cost, subject
to a set of constraints being imposed by the nature of the problem under study. Depending of the situation under study optimization problems allow their decision variables to assume i) continuous values; ii) integer or binary values; iii) both integer or continuous values; these branches of mathematical programming are known as linear programming (LP) integer programming (IP) and mixed integer programming (MIP) respectively. It is a common practice to generate LOM production schedules through MIP formulations that attempt to maximize the discounted economic value of a mining project. Then, scheduling is carried out as follows:

1. Maximize the undiscounted value of the project through the use of graph theory or maximal flow methods (Lerchs and Grossman, 1965; Picard, 1976; Hochbaum, 2001). This leads to the definition of ultimate pit limits.

2. The net present value, NPV, of the project is then maximized through the optimization of the space within the ultimate pit limits. The idea is to generate a LOM production; an optimal sequence of extraction of ore and waste throughout each production period, so as to maximize the NPV of the project.

A matter of concern when IP and MIP are used to effect mine production scheduling is the time required to obtain optimum solutions. This issue is related to the number of binary variables, the constraints and the formulation of the
objective function; e.g. the maximization of discounted cash flows. An efficient MIP formulation is proposed in Ramazan and Dimitrakopoulos (2004); they discuss the possibility of decreasing the number of binary variables, the linearization of waste variables, the use of different slope constraint formulations and its impact on the solution time. Ramazan (2001) introduces the fundamental tree algorithm to decrease the number of integer variables and constraints in the MIP scheduling formulations in order to reduce the time required to solve these problems. Boland et al. (2007) strengthens an integer programming formulation by adding a constraint which decreases the number of required integer variables before the problem is actually solved.

Optimizers generate optimal solutions assuming that the data given as input is correct, which is not the case in mining due to the uncertainty around the economic value of a block. Conventional approaches to the optimal design of open pit mines, do not incorporate uncertainty into the process because they make use of a single estimated orebody model generated through kriging (Goovaerts, 1997) as input to the optimization model. Past efforts in the area of conventional approaches are Johnson (1968), Dagadalen and Johnson (1986) and Hochbaum (2001). Dimitrakopoulos (1998) highlights that due to the smoothing effect present in any estimated type orebody model, as in the case of a kriged model, the histogram and variogram show lower variability than the
actual data which leads to not meeting production targets and NPV forecasts. Dimitrakopoulos et al. (2002) discuss the effect of estimated orebody models on non–linear transfer functions used to schedule production throughout the whole LOM and the risk that arises from not accounting for geological uncertainty; Figure 1: illustrates the risk of assigning a single grade value to a block and stresses the importance of modeling uncertainty.

Figure 1: (left) shows that the true value is unknown; (right) modeling uncertainty incorporates the unknown, true value, into the mining decision-making process.

1.3 Modelling geological uncertainty: Assessing geological risk in mine designs

Geostatistics provide a methodology for modeling data and process uncertainty, then propagating that uncertainty all the way into a probabilistic reporting.
(Journel, 1996). The grade at a given location is modeled as a random variable with a specific probability distribution. Through stochastic simulation, an alternative equally probable set of simulated grade values, which emulate the unknown true spatial distribution of grades (Journel, 1989), is obtained. The set of alternative realizations provides a visual and quantitative measure of the uncertainty (Goovaerts, 1997) of the grade value at a given location. Each realization of the random variable is equiprobable and is also known as a simulation. Ravenscroft (1992) used simulated orebody models to analyze the risk of not meeting production targets as a function of the use of a given mine design and a LOM production schedule. Dimitrakopoulos et al. (2002) use simulations to analyze the NPV of the project; a range of possible financial outcomes which describes the distribution of NPV is generated. Figure 2 shows that in their case study, the NPV forecast has a 90% probability of being lower than the one predicted by the conventional approach. The technical literature has shown that the risk of not meeting production targets and NPV forecasts is increased due to the effect of smooth estimated orebody models on a non linear transfer function, as in the case of optimization models; therefore, the integration of uncertainty into mine planning is a major requirement.
Figure 2: Uncertainty analysis for the NPV of a set of nested pits based on an estimated orebody model of a gold deposit. Taken from Dimitrakopoulos et al. (2002)

1.4 Moving Forward: Integrating uncertainty into conventional approaches

As discussed above, conventional approaches are misleading due to their deterministic nature. Note an optimal solution is only optimal for the data input to the model; which is rarely the case in real processes e.g. mining. To overcome such shortcomings, first the uncertainty of the spatial distribution of grades must be modeled and second such model must be integrated into the mine design
related problems. Several methods are available to integrate uncertainty into mine planning, e.g. Godoy and Dimitrakopoulos (2004) implemented simulated annealing to handle the uncertainty of assigning a period of extraction to a block in order to minimize the risk of deviating from production targets. Golamnejad et al. (2007) propose a deterministic formulation that is “equivalent” to a stochastic formulation to schedule production throughout the life-of-mine; the idea is to replace stochastic constraints by non-linear deterministic constraints. Unrealistic assumptions are made i.e. the grade distribution of a block is assumed to be normal, this is never the case in mining. Menabde et al. (2007) introduce a stochastic programming formulation to simultaneously optimize the sequence of extraction and the cut off grade policy. In their research it is highlighted that the size of mining problems is prohibitively large, thus imposing computational difficulties on the optimization process. To overcome such issues, blocks are aggregated into bigger units thereby decreasing the number of binary decision variables considered in the model. Such approaches may lead to the loss of information related to the metal quantity in the block, thus leading to suboptimal mining sequences. Dimitrakopoulos and Ramazan (2004) introduce probability constraints to the MIP “conventional” deterministic formulation in order to handle the risk of a block being above or below a cutoff; nevertheless, a probability is a single number which does not describe fully the grade uncertainty. To effectively
handle grade uncertainty, Ramazan and Dimitrakopoulos (2007), introduce a stochastic integer programming (SIP) formulation that attempts to maximize the discounted cash flows and minimize the deviation from production targets throughout the life of the mine. Risk management is accomplished by the introduction of a geological risk discounting rate, GRD, into the calculation of the costs for excess and deficient production. Aside from a grade cut-off, the formulation uses a probability cut-off to classify the blocks as ore or waste. Leite and Dimitrakopoulos (2009) study the variation of the probability cut-off and its impact on the robustness of mine production schedules generated through the use of the formulation introduced by Ramazan and Dimitrakopoulos (2004). The application of these methods generate production schedules that minimize the possibility of deviating from production targets, and result in schedules with a substantial improvement in net present value, shown to be in the order of 25% when compared to conventional scheduling; due to the fact that the value of the stochastic solutions is always greater or equal to the value of deterministic solutions. As shown above, through the effort of several research groups the mining community has gained a better understanding of the problems at hand. The availability of such technologies, allows the integration of geological uncertainty into the process of defining ultimate pit limits and designing
pushbacks. The following section provides a detailed description of the theory and implementations behind the approaches used in this thesis.

1.5 Stochastic optimization: A mining application

This thesis makes use of two optimization methods: simulated annealing and stochastic integer programming. Such methods are implemented in the context of the requirements of mining applications.

1.5.1 Simulated Annealing

There are many interesting optimization problems which require computational efforts that increase exponentially as a function of the number of decision variables. To approach such problems, heuristic methods are considered; nevertheless, there is no guarantee that heuristic procedures provide near optimal solutions for more than one type of optimization problem. Simulated annealing (Kirkpatrick et al, 1983; Geman and Geman, 1984) is a heuristic procedure that integrates the “divide-and-conquer” and “iterative improvement” strategies. An annealing implementation requires four ingredients: (i) a concise description of a configuration system; (ii) a random generator of the rearrangements of the elements in the configuration; (iii) an objective function that quantifies the tradeoffs being made; and (iv) termination criteria for the
algorithm. The algorithm proceeds by weighting the initial configuration of the system by using its Boltzmann probability factor; then, raising the temperature of the system until it is “melted” and cooling it down, by rearranging the system’s configuration, in slow stages until a termination criterion is reached. The Boltzmann probability factor is defined as $e^{-\left(\frac{E(r_i)}{k_b T}\right)}$, where $E(r_i)$ is the energy of the system based on the configuration, $r$, of its i elements, $k_b$ is the Boltzmann constant and $T$ is the temperature. The energy of the system is quantified at each stage by the objective function. The temperature forces the procedure to allow large changes in the configuration at high temperatures while the small changes are deferred until low temperatures; yielding an adaptive form of the “divide-and-conquer” strategy. The temperature also controls the “iterative improvement” strategy by accepting uphill changes in the objective function in order to avoid getting stuck in locally optimal system configurations.

Godoy (2003) introduces a mine scheduling approach based on simulated annealing that accounts for uncertainty by considering fixed pit limits for all the simulated orebody models and constant mining rates. The idea is to find a production schedule that minimizes the deviation from ore and waste production targets by handling the uncertainty of mining a block at a given production period. The stages of such approach are the: (i) definition of the ultimate pit based on an estimated orebody model; (ii) development of a set of production schedules
within the predefined ultimate pit limits for each one of the simulated orebody models; and (iii) use of the implementation of simulated annealing by considering the previously generated LOM schedules to obtain a single schedule that meets production targets. The parameters involved in this simulated annealing implementation may be classified in three groups: the (i) number of input mining sequences; (ii) annealing related parameters; and (iii) production targets. For the case at hand, the objective function is defined as a measure of the difference between the desired characteristics and those of a candidate production schedule. Consider the objective of meeting a series of optimal production rates throughout the LOM. If a production schedule achieves such goal when tested against all the orebody models, there is 100% chance that the production targets will be met given the knowledge of the orebody reproduced in the simulations. An objective function that measures the average deviation from the production targets for a given mining sequence over a series of $s$ simulated orebody models is defined as $O = \sum_{n=1}^{N} O_n$ where $O_n, n = 1, \ldots, N$ are the components of the objective function corresponding to a given production period. Then, for all the $n$ components, the objective function measures the average deviation of the actual ore and waste production $\theta^*(s)$ and $\omega^*(s)$ of the perturbed mining sequence from the production targets $\theta(s)$ and $\omega(s)$ over all $s$ simulated grade models with $s = 1, \ldots, S$.
\[ O_n = \frac{1}{S} \sum_{s=1}^{S} |\theta^*(s) - \theta(s)| + \frac{1}{S} \sum_{s=1}^{S} |\omega^*(s) - \omega(s)| \]

The perturbation mechanism consists of swapping a randomly selected block from its actual period to a candidate period. Candidate periods for a block are determined by the actual periods of the neighbouring blocks, the likelihood that the block will belong to the candidate periods and satisfaction of slope constraints. Then, the decision of whether to accept or reject a perturbation is based on the change of the objective function \( \Delta O = \sum_{n=1}^{N} \Delta O_n \) and the acceptance probability distribution defined as

\[
P\{\text{accept}\} = \begin{cases} 
1, & \text{if } O_{\text{new}} \leq O_{\text{old}} \\
\frac{e^{\frac{O_{\text{old}} - O_{\text{new}}}{T}}}{1 + e^{\frac{O_{\text{old}} - O_{\text{new}}}{T}}}, & \text{otherwise}
\end{cases}
\]

All favourable perturbations \( O_{\text{new}} \leq O_{\text{old}} \) are accepted and unfavourable permutations are accepted with an exponential probability distribution.

The parameters involved in this simulated annealing implementation may be classified in three groups: the (i) number of input mining sequences; (ii) annealing related parameters; and (iii) production targets. The annealing related parameters contain parameters which are specific to this implementation as well as basic parameters of the simulated annealing algorithm. The latter parameters define termination criteria and control the iterative annealing algorithm: the annealing temperature which is usually set to a high starting value; the reduction
factor which is used to lower the temperature whenever enough perturbations have been accepted or too many have been evaluated without any change of the objective function; a maximum number of attempted perturbations at a given temperature; a stopping number, which is a function of the number of times that the maximum number of attempted perturbations at any given temperature is reached. The main parameters specific to this implementation allow the user to: 1) freeze the blocks which have been assigned 100% probability to belong to a given mining period, this will constrain the set of candidate blocks for swapping; and 2) define the initial configuration of the set of candidate blocks for swapping by a given input mining sequence, or candidate blocks may be assigned to candidate periods according to their probability rank. A more detailed discussion is available in Godoy (2003). The issues of the approach are that it ignores the impact of different starting sequences on the quality of the solution, it assumes fixed pit limits and it does not specify the minimum number of required simulations to generate LOM production schedules that meet production targets and NPV forecasts.
1.5.2 Stochastic Programming

A deterministic linear program consists of finding a solution to (Bradley et al., 1977)

\[
\begin{align*}
\min z &= c^T x \\
\text{s.t.} A x &= b \\
x &\geq 0
\end{align*}
\]

The value of \( z \) corresponds to the objective function, while \( \{x | A x = b, x \geq 0\} \) defines the set of feasible solutions. Consider \( x^* \) to define the optimal solution to the problem. If at least one parameter is a random variable, it is of interest to know the distribution of optimal solutions \( x^*(\xi) \) and values of the objective function \( z(x^*(\xi), \xi) \); where \( \xi \), in this case, represents the outcomes of the random variable. Such problem is called the distribution problem; finding the solution to such problem becomes difficult and time consuming since a random variable may assume a large amount of outcomes. Consider that the optimal solutions \( x^*(\xi) \) and values of the objective function \( z(x^*(\xi), \xi) \) are known; then, the expected value of the optimal solution may be calculated. This is known in the literature as the wait-and-see solution and is defined as \( WS = E_{\xi} \{ \min_x z(x, \xi) \} = E_{\xi} z(\bar{x}(\xi), \xi) \).

Problems of this nature have given rise to stochastic programming (Dantzig, 1955), a branch of optimization where at least one variable in the optimization
model is considered to be uncertain. Uncertain data means that such data may be represented by random variables. A probabilistic description of the possible outcomes of such random variable is assumed available under the form of probability distributions, densities or any other measure. The vector $\xi(\omega) = \xi$ of particular values of the various random variables is only known after the random experiment takes place. Then, a number of decisions must be taken before the experiment takes place; these are known as first stage decisions and are taken during the first stage. A number of decisions are taken after the random experiment as a corrective action of the first stage decisions based on $\xi$; these are known as second stage decisions and take place during the second stage. Recourse programs are those that allow decisions or recourse actions to be taken after the random experiment takes place. The first and second stage decisions are represented by the vectors $x$ and $y$. Second stage decisions are a function of the outcomes of the random experiment of random variables in $\xi$ and the first stage decisions $x$. The sequence of events and decisions is thus summarized as $x \rightarrow \xi(\omega) \rightarrow y(\omega, x)$.

An approximation to the distribution problem, defined above, is known as the stochastic linear program with fixed recourse or just the recourse problem RP, defined by Dantzig (1955)
\[
\min z = c^T x + E_\xi[\min q(\omega)^T y(\omega)] \\
\text{s.t. } Ax = b \\
T(\omega)x + Wy(\omega) = h(\omega) \\
x \geq 0, y(\omega) \geq 0
\]

The objective function contains a deterministic component \(c^T x\) and the expectation of the second stage objective \(q(\omega)^T y(\omega)\) taken over all realizations of the random event \(\omega\). The main problem is to find \(x\) such that \(E_\xi[\min q(\omega)^T y(\omega)] = \min E[ q(\omega)^T y(\omega)]\) under the assumption that for each \((\xi(\omega), x)\) there exists at least a \(y\) that represents a feasible solution and the optimal one.

It is tempting to solve a much easier problem, where all random variables are replaced by their expected values; this is known as the expected value solution and is defined as \(EV = \min_x z(x, \bar{\xi})\), where \(\bar{\xi} = E_\xi(\omega)\). The expected result of using the EV solution is defined as \(EEV = E_\xi \left( z(\bar{x}(\bar{\xi}), \bar{\xi}) \right)\); where \(\bar{x}(\bar{\xi})\) is the optimal solution to \(\min_x z(x, \bar{\xi})\). Such quantity measures the performance of \(\bar{x}(\bar{\xi})\) allowing the second stage decisions \(y\) to be chosen optimally. Now, consider any realization \(\bar{\xi}\), which leads to the relation \(z(\bar{x}(\bar{\xi}), \bar{\xi}) \leq z(x^*, \bar{\xi})\); where \(x^*\) is the optimal solution to the RP. Taking the expectation at both sides leads to \(WS \leq RP\). \(x^*\) is the optimal solution to RP and \(\bar{x}(\bar{\xi})\) is just one solution yields
Finally, by defining the value of stochastic solutions as $VSS = RP - EEV$; then it is clear that $VSS \geq 0$. For further details on the topic the reader is referred to Birge and Louveaux (1997).

In the thesis presented here in, LOM production schedules are generated by solving the Stochastic Integer Programming, SIP, model introduced by Ramazan and Dimitrakopoulos (2004). The optimization process is based on the economic value of each block $V_l, l = 1 \ldots L$ that belongs to the set of blocks being scheduled. The expected economic value of a block is calculated by using its expected return, which is defined by the revenue gained by selling the amount of metal contained in it. The objective function consists of 4 parts. The 1st part involves the maximization of the expected net present value, $E[(NPV)_l^t]$, which is generated by mining a block at a given production period and by considering a certain simulation; the 2nd part accounts for the blocks sent to the stockpile such that only the mining cost of extraction is incurred; the 3rd part adds the discounted value generated by processing a given amount of stockpiled material, $k_s^t$; and the 4th part minimizes the risk of not meeting ore, grade and metal production targets. The definition of the objective function follows
Max \( \sum_{t=1}^{T} \left[ \sum_{l=1}^{L} E\{NPV\} b_t^l + \sum_{j} E\{\cdot\} \rho^l_j + \sum_{s=1}^{S} \frac{SV^t}{M} k^l_s \right] \)

\( - \sum_{s=1}^{S} \sum_{\tau=\{\text{ore}, \text{grade}, \text{metal}\}} c_{o\tau}^t d_{so}^{t\tau} + c_{u\tau}^t d_{su}^{t\tau} \)

Where \( l \) is the block identifier; \( t \) is the time period; \( t_0, t_g \) and \( t_q \) flag the ore, grade and metal production target type; \( u \) stands for lower bound; \( o \) stands for upper bound; \( s \) stands for the simulation number; \( T \) is the maximum number of scheduling periods; \( L \) is the total number of blocks to be scheduled; \( b_t^l \) is a variable representing the portion of block \( l \) to be mined in period \( t \); if it is defined as a binary variable, it is equal to 1 if the block \( l \) is to be mined out in period \( t \) and equal to 0 if otherwise; \( E\{\cdot\} \) is the expected NPV to be generated by mining block \( l \) is to be mined in period \( t \) considering simulation \( s \); \( c_{o\tau}^t \) and \( c_{u\tau}^t \) are the unit costs for excess and deficient \( \tau \) type of production, \( \tau = \text{ore} (o), \text{grade} (g) \) and \( \text{metal} (q) \), respectively; and \( d_{so}^{t\tau} \) and \( d_{su}^{t\tau} \) are the excess and deficient amount of \( \tau \) type of production in period \( t \) considering simulation \( s \). Risk management is accomplished by the introduction of a geological risk discounting rate, GRD, into the calculation of the costs for excess and deficient production. Consider a GRD rate of value \( d \), then the unit cost of deviation from ore production targets is defined as
\[ c_t^o = \frac{c_o^0}{(1 + d)^t} \quad c_t^u = \frac{c_u^0}{(1 + d)^t} \]

Note that the definition of the unit cost of deviation is easily extended for the case of grade and metal production target. The objective function is subject to processing capacity, mining capacity, reserve, grade blending, stockpiling and slope constraints. Consider \( O_{sl}, G_{sl}, Q_{sl} \) to be the ore tonnage, grade or metal quantity of a given block \( l \) conditioned to simulation \( s \), dummy variables \( a_{su}^t, a_{su}^g, a_{su}^q \) and \( a_{so}^t, a_{so}^g, a_{so}^q \) to balance the equality, and the maximum and minimum ore production expected per production period of the LOM; then, ore tonnage, grade and metal production must lie within lower and upper bounds, \( O_{min}, G_{min}, Q_{min} \) and \( O_{max}, G_{max}, Q_{max} \) respectively. The variables and parameters described above, lead to define the processing constraint as \( \sum_{l=1}^{t} O_{sl} b_{li} - \sum_{j} O_{sj} S_{jt}^t + k_s^t + a_{su}^t - a_{su}^o = O_{min} \) and \( \sum_{l=1}^{t} O_{sl} b_{li} - \sum_{j} O_{sj} S_{jt}^t + k_s^t - d_{so}^t + a_{so}^t = O_{max} \). The lower and upper bound grade blending constraints are respectively defined as \( \sum_{l=1}^{t} (g_{sl} - G_{min}) O_{sl} b_{li} - \sum_{j} (g_{sj} - G_{min}) O_{sj} S_{jt}^t + (GST - G_{min}) k_s^t + d_{su}^g - a_{su}^g = O_{min} \) and \( \sum_{l=1}^{t} (g_{sl} - G_{max}) O_{sl} b_{li} - \sum_{j} (g_{sj} - G_{max}) O_{sj} S_{jt}^t + (GST - G_{max}) k_s^t - d_{su}^g + a_{su}^g = O_{max} \). For the case of stockpile constraints, the tonnage in the stockpile at the end of the first production period is first calculated as \( \sum_{j} O_{sj} S_{j1}^1 + h^0 - k_s^1 - h_s^1 = 0 \), then \( \sum_{j} O_{sj} S_{jt}^t + h^{t-1} - k_s^t - h_s^t = 0 \); the capacity of the stockpile is limited by \( h_s^t \leq SC \) and the tonnage available for removal from
the stockpile at a given period is limited by the tonnage at the end of the previous period, such that $k_s^t - h_s^{t-1} \leq 0$. The reserve constraints express that a given block $l$ cannot be mined more than once throughout the LOM. Such constraint is defined as $\sum_{t=1}^{T} b_l^t \forall l = 1 \ldots L$. Two different formulations are available for the slope constraints. Ramazan and Dimitrakopoulos (2003) show that the use of any of the available formulations of the slope constraints affects the solution time depending on the case study but it does not affect quality of the solution. Consider $y$ to be the number of overlying blocks; then, the slope constraints are defined as

$$yb_i^t - \sum_{\gamma=1}^{y} \sum_{\varphi=1}^{t} b_{\gamma}^\varphi \leq 0$$

One constraint for each block per period

$$b_i^t - \sum_{\varphi=1}^{t} b_{\gamma}^\varphi \leq 0$$

$m$-constraints for each block per period

Aside from a grade cut-off, the model introduces a probability cut-off used to classify the blocks as ore or waste. To begin, consider the information about the grade of a block, made available through the set of simulated orebody models, to calculate the probability of a block having a grade greater than the grade cut-off. Then, if the computed probability is greater than the probability cut-off, the block is classified as ore; otherwise, the block is classified as waste. Furthermore, this formulation takes into consideration a different way of defining binary variables introduced by Dimitrakopoulos and Ramazan (2007). This consists of reducing
the amount of binary variables by setting the waste blocks to linear variables and
the ore blocks to binary variables. Dimitrakopoulos and Ramazan (2007) have
shown that such “selective” binary definition not only reduces the solution time
but does not affect the quality of the solution by keeping it optimal.

1.6 Current Challenges
Two recently developed methods that integrate uncertainty into the optimization
of LOM production schedules have been discussed in previous sections. The first
of these methods is based on scheduling with a simulated annealing (SA)
algorithm (Godoy, 2003) and equally probable realizations of a given mineral
deposit. The latter realizations are used to generate production schedules that
minimize the possibility of deviating from production targets, and result in
schedules with a substantial improvement in net present value when compared
to conventional scheduling within the conventionally optimal pit limits. Due to the
computational cost of geostatistical algorithms it is of interest to facilitate the
implementation of this method by decreasing the number of simulated
realizations required to provide stable LOM optimization results. Another issue
that captures the interest of current research trends is that conventional
approaches to define optimal pit limits do not consider geological uncertainty.
Therefore, it is of interest to explore the use of a stochastic optimizer such as SA to derive stochastically optimized pit limits, assess its impact on key technical and economic parameters and compare to the results obtained through the conventional approaches.

Pushback design is a topic of ongoing interest in the mining community. The main objective of pushback design is to break up the overall pit reserve into more manageable planning units such that it enhances the maximization of the net present value of the project. It must be noted that: 1) pushbacks must be sufficiently spaced apart to allow the access of mining equipment; this is known as the minimum mining width; 2) the size of the pushbacks may vary; 3) addition of haul roads out of a pushback and sometimes access roads into a next pushback is required. Whittle (1988) breaks up the pit space to obtain a set of nested pit shells. A set of nested pits is the result of an economic discretization of the space within the ultimate pit limits. The generation of such pits is related to the definition of the economic value of a block; in this case, the amount of metal that should be sold to pay for mining a ton is the variable of importance. Therefore, its variation generates different solutions of the Lerchs – Grossman algorithm (Whittle Software, 1998). Ramazan and Dagdalen (1998) introduce a pushback design algorithm that minimizes the stripping ratio of pushbacks in order to maximize the net present value of the project; their algorithm is a
modified version of the open pit parameterization algorithm found in Seymou
(1994). The algorithm is tested in a small 2D example; an increment of 6 and
16% is reported at 15 and 20% interest rate, respectively, when compared to the
conventional pushback design method proposed in Whittle (1988). Nevertheless,
such results are suboptimal because the algorithm does not account for any type
of uncertainty. Golamnejad and Osanloo (2007) design pushbacks under supply
uncertainty based on a different definition of ore and the implementation of the
approach introduced by Whittle (1988). Nevertheless, their approach is
misleading because it is assumed that the grades of a block are normally
distributed. Another way to design pushbacks is by grouping nested pits. It is a
straight forward idea to use the set of nested pits as a guideline to design
pushbacks; the idea is to group nested pits such that maximizes the NPV of the
project, and that satisfies all the requirements discussed above. LOM PS allows
the transition from pushback design into time period production plans, they
enable the definition of production goals in space such that a physical mining
sequence of extraction is acquired and they give a better definition of the
relationship between the pushbacks as they may overlap in the complete mining
operation. Nevertheless, the primary goal of LOM production scheduling lies in
providing a realistic economic evaluation of the asset. This is possible due to the
effective economic discounting that is allowed by these physical mining sequences.

1.7 Goal and Objectives

The goal of this thesis is to explore the possibility of using mine production scheduling approaches that consider geological uncertainty to design pushbacks, define optimal pit limits and to assess the minimum number of simulations required to generate robust LOM production schedules. In order to achieve this goal, the following objectives must be fulfilled:

1. Review the technical literature related to optimal pit limit definition, pushback design and mine production scheduling methods that integrate uncertainty.

2. Carry out a sensitivity analysis on the simulated annealing algorithm. The sensitivity analysis consists of testing annealing related parameters, the minimum number of simulations required to obtain stable solutions and the variation of the pit limits in order to define the stochastically optimal pit limits.

3. Introduce a multistage methodology that makes use of a stochastic integer programming formulation to integrate uncertainty into the process of designing pushback.
4. Conclusions and recommendations for future work.

1.8 Thesis Outline

The thesis is organized in the following way:

Chapter 1. A literature review is provided with respect to the central topics of this thesis. Details of the mine production scheduling process and current challenges are discussed. The shortcomings of conventional approaches are outlined.

Chapter 2. Describes a LOM production scheduling method based on simulated annealing, SA. A sensitivity analysis is carried out in order to determine the best starting sequence, optimal pit limits and the minimum number of simulations required to generate stable solutions.

Chapter 3. Introduces a multistage methodology to designing pushbacks with uncertain supply based on a stochastic integer programming model. Firstly, the proposed methodology is described; second, an application in a porphyry copper deposit is shown; and to finalize, a discussion of the results is included along with conclusions.

Chapter 4. Overall conclusions and recommendations for future research are provided in order to finalize the thesis.
Chapter 2

Stochastic Mine Design Optimization based on Simulated Annealing: Pit limits, production schedules, multiple orebody scenarios and sensitivity analysis

2.1 Introduction

Assuming perfect knowledge of the spatial distribution of metal content in a mineral deposit is an unrealistic assumption both in general as well as specifically for optimization in mine design and production scheduling. As open pit mine design and production sequencing deal with the management of cash flows in the order of millions of dollars, the above unrealistic assumption has a substantial financial impact. Production targets may not be met as planned, leading to suboptimal management of cash flows, ad-hoc changes to short-term plans that deviate from long-term production plans and forecasts; all of which lead to unfulfilled expectations. Perhaps even more important than the above and other known shortcomings is that assuming perfect knowledge not only misleads the mathematical optimization process but, in addition, deprives planners and optimizers from the ability to manage orebody uncertainty and risk in grades, ore and stripping ratios.

Figure 1 shows the limits of conventional optimization in an application at a copper deposit, which is based on the industry standard implementation (Whittle Software) for optimizing the pit design and scheduling life-of-mine (LOM)
production. The input to the optimization study is the conventionally estimated model of the copper deposit using a common geostatistical estimation technique that, similarly to all estimation techniques, provides a smooth representation of the orebody. The economic and technical parameters used to generate the optimized models are presented in Table 1. The optimization process assumes this input model to be the true deposit being mined (i.e. perfect orebody knowledge). Figure 1(a) shows the expected ore production from the conventional LOM study (green line) that meets a set production targets throughout LOM, and Figure 1(b) shows the corresponding discounted cash flows. A risk analysis of this schedule with respect to the orebody is generated by testing the LOM schedule against mining several stochastically simulated equally probable realizations of the orebody, one after the other. The performance of the schedule is then assessed and the results are summarized in Figure 1. The red line represents the average performance of the schedule and the black dotted lines show the minimum and maximum of the performance of the schedule. The risk analysis shown in Figure 1 suggests that: (a) ore production targets will not be met; and (b) the conventional LOM schedule underestimates the forecasted cash flows and total NPV. Figure 2 shows the same example and comparison, except that instead of the conventionally estimated orebody model, the input to the optimizer is the average of all stochastically simulated models of the copper
deposit (the so called e-type estimate). The results in Figure 2 demonstrate the same behaviour as Figure 1 and, as expected, generate similar results, which is due to the use of a single smooth deposit representation that at the same time are assumed true by the optimizer. This is a well known observation documented and further discussed in the technical literature Dimitrakopoulos et al. (2002) and is attributed to the non-linear nature of the optimization process, that is, an average type input does not generate an average LOM schedule and forecast. Similarly, the non-linearity mentioned above also means that the use of individual simulated realizations of an orebody as an input to a conventional optimization process cannot address the issue at hand. Figure 3 shows the cumulative NPV graph as in Figs. 1 and 2, except that the orebody representation is a simulated realization of the orebody. The related risk analysis profile in Figure 3 shows that the most likely resulting schedule is misleading. Alternatively, if the “lack of perfect knowledge” is quantitatively described and replaces common practices, then (a) the possibility of higher grades and metal content in different locations within the orebody (or, in a sense, the upside potential) is captured and can be utilized during pit optimization; similarly, (b) knowing and separating the possibility of encountering less metal than expected in different locations within the orebody (or downside risk) can be utilized and minimized. This leads to mine designs and production schedules that blend over production periods parts of the
orebody with ‘certain’ grades (metal) with others where high grades (metal) are expected. This leads to higher certainty in meeting production targets and at the same time mining sequences with higher total cash flows.

The above is documented in the technical literature and mine studies (Dimitrakopoulos et al, 2007; Leite and Dimitrakopoulos, 2007). The approach used in the above examples and past work employ similar concepts to that shown by Ravenscroft (1992), who suggests the use of simulated orebodies to probabilistically assess the performance of mine production schedules. Given the inability of conventional pit optimizers to deal with uncertain inputs, Godoy and Dimitrakopoulos (2004) show a stochastic optimizer based on simulated annealing (SA) that can accommodate the joint use of multiple simulated orebody representations and shows a 28% improvement in cash flows generated from the stochastic LOM schedule versus the conventional one in a large gold mine; the value of the schedule is also accompanied by substantially higher changes to meet production targets. Leite and Dimitrakopoulos (2007) use a variant of the same method to optimize the production schedule of a copper mine, and report similar order of improvements. The comparisons made in both these studies, as well as others, are based on the use of the same pit limits for both the conventional and stochastic optimization studies, thus it is of interest to further explore whether or not the stochastic approach based on SA has an effect or can
be used in selecting stochastically optimal pit limits. Stochastic approaches for pit optimization have also been implemented using stochastic integer programming (SIP) in several cases. Dimitrakopoulos and Ramazan (2008) proposed a SIP formulation for LOM production scheduling; Dimitrakopoulos and Jewbali (2008) integrated future data to an SIP scheduling formulation, and Leite and Dimitrakopoulos (2009) use an SIP formulation to generate LOM production schedules of a copper deposit. The related applications document similar results as the above SA approach, and are the subject of ongoing research.

Stochastic optimization approaches have their own intricacies and computational needs. For instance, the SA approach further explored in this study is relatively labour intensive when compared to the conventional optimization approaches. It is of interest to examine the sensitivity of the results to the number of simulated orebody used and the subsequent design and scheduling process. This paper first assesses the sensitivity of the stochastic mine design and production scheduling optimization process based on SA, aiming to document the critical components required to generate practical pit designs and schedules, thus minimizing the potential labour intensiveness. Then, the sensitivity of the fixed pit limits used in the past implementations is visited to explore the possibility that stochastically derived pit limits are larger than those generated from conventional deterministic optimizers. In the following sections, Godoy’s (2003) stochastic
optimizer based on SA is briefly outlined. Then, the sensitivity analysis of the optimization process and related key aspects LOM schedule are explored. Finally, the optimization of pit limits based on the stochastic approach discussed herein is presented and compared to the deterministic pit limits, followed by conclusions.

Table 1: Economic and technical parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper price (US$/lb)</td>
<td>1.9</td>
</tr>
<tr>
<td>Selling cost (US$/lb)</td>
<td>0.4</td>
</tr>
<tr>
<td>Mining cost ($/tonne)</td>
<td>1.0</td>
</tr>
<tr>
<td>Processing cost ($/tonne)</td>
<td>9.0</td>
</tr>
<tr>
<td>Slope angle</td>
<td>45°</td>
</tr>
<tr>
<td>Processing recovery</td>
<td>0.9</td>
</tr>
<tr>
<td>Block Dimensions (m)</td>
<td>20x20x10</td>
</tr>
<tr>
<td>Ore Production Rate (M x tonnes per year)</td>
<td>7.5</td>
</tr>
<tr>
<td>Waste Production Rate (M x tonnes per year)</td>
<td>20.5</td>
</tr>
</tbody>
</table>
Figure 3: Risk analysis of the ore (left) and (right) cumulative NPV of a LOM production schedule based on an estimated orebody model

Figure 4: Risk analysis of the ore (left) and (right) cumulative NPV of a LOM production schedule based on an E-type orebody model
2.2 Stochastic Mine Design Optimization based on Simulated Annealing: A Brief Recall

The simulated annealing algorithm is a heuristic optimization method that integrates the iterative improvement philosophy of the Metropolis algorithm with an adaptive “divide and conquer” 9, 10. When several mine production schedules are under study, there will be a set of blocks that will always be assigned to the same production period throughout all production schedules; these are referred to as the certain or 100% probability blocks. The set of uncertain blocks is sufficiently large to require special attention. To handle the uncertainty in these blocks, simulated annealing swaps blocks between candidate production periods.
in order to minimize the average deviation from the production targets for N mining periods and for a series of S simulated orebody models:

\[
MinO = \sum_{n=1}^{N} \left( \sum_{s=1}^{S} |\theta_n^*(s) - \theta_n(s)| + \sum_{s=1}^{S} |\omega_n^*(s) - \omega_n(s)| \right)
\]

where, \(\theta_n^*(s)\) and \(\omega_n^*(s)\) are the ore and waste production targets, respectively, and \(\theta_n(s)\) and \(\omega_n(s)\) represent the actual ore and waste production of the perturbed mining sequence. Each swap of a block is referred to as a perturbation. The probability of acceptance or rejection of a perturbation is given by:

\[
prob\{\text{accept}\} = \begin{cases} 1, & \text{if } O_{new} \leq O_{old} \\ e^{\frac{O_{old} - O_{new}}{T}}, & \text{otherwise} \end{cases}
\]

This implies all favourable perturbations \((O_{new} \leq O_{old})\) are accepted with probability 1 and unfavourable perturbations are accepted based on an exponential probability distribution, where \(T\) represents the annealing temperature.

**2.2.1 Key Aspects of the Simulated Annealing Implementation**

As outlined in Figure 6, the SA implementation requires (i) a set of S simulated orebody models; (ii) the definition of pit limits and mining rates for the S
simulations; and (iii) a mine production schedule corresponding to each one of the $S$ simulations. Furthermore, simulated annealing requires the definition of specific parameters, e.g. parameters to define termination criteria and to control the iterative annealing algorithm. This simulated annealing implementation allows the user to (i) freeze the blocks which have been assigned 100% probability to belong to a given mining period, as this will constrain the set of candidate blocks for swapping; and (ii) define the initial configuration of the set of candidate blocks for swapping by 1) a given input mining sequence, or 2) blocks may be assigned to candidate periods according to their probability rank.

![Diagram](image)

Figure 6: Stages of the simulated annealing mine production scheduling approach (Leite and Dimitrakopoulos, 2007)

### 2.2.2 Sensitivity Analysis

The implementation of the simulated annealing algorithm can be labour intensive compared to standard conventional pit optimization, thus it is of interest to
improve the application of the method. To explore this possibility, the robustness of SA production schedules is tested by analyzing the sensitivity to (i) the ‘freezing’ of mining blocks that have a 100% probability to belong to a given mining period; (ii) the initial input mining sequence, or starting sequence, which affects the blocks being swapped according to their probability rank; (iii) the number of mining sequences required, where the less needed the more practical the annealing approach is; and (iv) the extension of the pit limits, which allows the optimizer to stop mining when negative cash flows are obtained.

### 2.2.3 Locking Blocks in a Period or Not?

The decision to not freeze blocks will change the initial configuration of the annealing process such that the set of candidate blocks for swapping will consist of all blocks within the ultimate pit limits. Consider 20 mining sequences given as an input to SA and the decision of whether or not to freeze blocks having 100% probability of belonging to a given period. The 7 to 15% variation of this parameter yields no differences in the resulting mine production schedules; obviously a 100% probability is more efficient in terms of computing time, as Godoy (2003) also documents at a substantially larger deposit.
2.2.4 Selection of a Starting Sequence

An initial mining sequence requires careful selection because such sequence provides an initial configuration to the annealing process. The annealing process requires a sequence which involves the least possible risk and yields the maximum NPV out of the set of available mining sequences given as an input. For this case study, the Whittle Software (1998) yields mining sequences with seven or eight production periods. All eight period mining sequences perform poorly in terms of meeting production targets and NPV forecasts when tested against a set of twenty simulated orebody models. On the other hand, all seven period mining sequences yield the same total average deviation from ore production targets. The average cumulative NPV of each mining sequence is considered as the selection criterion because it provides a set of values from which the sequence that yields the highest NPV is evident.

Figure 7 illustrates the performance of the best-case and worst-case starting sequences when tested against a set of 20 simulations. The red line represents the worst-case starting sequence, while the green line represents the best-case starting sequence. Both sequences have been used as a starting sequence to run simulated annealing.
Figure 7: Comparison the average performance of the best, and worst case sequence in order to choose a starting sequence.

Figure 8 shows the performance of the production schedule when the worst-case sequence is used as the starting sequence. It does not meet the ore production targets and yields a low cumulative NPV value when compared to the production schedule which considers the best-case sequence as the starting sequence. The best-case production schedule is discussed in the next section of the paper (Figure 17). Both the best and worst sequences considered the same number of input mining sequences and annealing parameters.
Figure 8: Performance of annealing sequence when the worst-case sequence is given as the starting sequence

2.2.5 The Number of Mining Sequences Used

The set of candidate blocks for swapping, which constitutes the initial configuration of the annealing process, is a function of the number of mining sequences that are given as an input and will be more constrained as the
number of mining sequences decreases. Figure 9 shows the overall decreasing proportion of 100\% probability blocks as the number of mining sequences is increased; the dark red colour represents the frozen blocks. Different probability ranks are assigned to each block based on the number of input mining sequences. A proof of this is the variation of the size of the set of blocks considered for freezing, as shown in Figure 9 (a) and the variation of the probability rank of candidate blocks for swapping as shown in Figure 9 (b). A relationship between the number of input mining sequences and the performance of simulated annealing exists when the initial configuration is defined by a “probability rank”, such as the one in Figure 9 (b), instead of a given starting sequence. Figure 9 (a) shows that, for the case study, the set of blocks considered for freezing remains almost constant after 10 mining sequences are considered, implying that there exists a set of blocks which, regardless of the number of simulated orebody models being scheduled, will assume a high probability value of being assigned to the same production period. This explains the link between the performance of the different simulated annealing production schedules and Figure 9 (a).
The simulated annealing algorithm generated LOM production schedules that minimizes the risk of not meeting production targets, and is largely independent of the number of mining sequences given as an input to the algorithm. The algorithm was tested for 2, 3, 4, 5, 10, 15 and 20 mining sequences. Figure 10 shows an annealing LOM schedule that only requires two mining sequences as input and whose initial configuration is based on the probability rank. The waste production rates do not meet the waste production targets; in fact it is, on average, lower than the waste production targets. Figure 11 and Figure 12 show the risk analysis of annealing sequences when 3 and 4 mining sequences, respectively, are given as an input to the simulated annealing algorithm. In both cases, the initial configuration of the system is defined by a given starting
sequence. The optimizer is sensitive to the starting sequence, which, for the case in hand, does not constitute the best-case starting sequence, thus the ore production target was not met throughout the first production period. For the case of 5 input mining sequences (Figure 13), the ore production targets are not met after the 4th production period. The annealing sequence in Figure 14 is produced based on a probability rank. Figure 10 to Figure 17 show risk analyses of annealing sequences with variations in the number of input mining sequence ranging from 2 to 20. Figure 14 and Figure 15 show that when 10 mining sequences are given as input, simulated annealing will produce mining sequences that will meet the production targets regardless of how the initial configuration of the system is obtained; based on a “probability rank” or defined by a given mining sequence. Furthermore, the comparison of the risk analysis in Figure 14 and Figure 15 against Figure 16 and Figure 17 show that the annealing sequences in the case study meet production targets when 10 or more mining sequences are given as an input. Figure 18 shows that LOM schedules present practically no physical differences; however, visual inspection should not be considered to provide a measurement of the risk involved.
Figure 10: Risk analysis of the ore (upper left), waste (upper right), metal (lower left) production and (lower right) cumulative NPV of a LOM production schedule generated through the simulated annealing implementation and considering 2 input mining sequences with the initial configuration based on the probability rank.
Figure 11: Risk analysis of the ore (upper left), waste (upper right), metal (lower left) production and (lower right) cumulative NPV of a LOM production schedule generated through the simulated annealing implementation and considering 3 input mining sequences with the initial configuration defined by a given mining sequence.
Figure 12: Risk analysis of the ore (upper left), waste (upper right), metal (lower left) production and (lower right) cumulative NPV of a LOM production schedule generated through the simulated annealing implementation and considering 4 input mining sequences with the initial configuration defined by a given mining sequence.
Figure 13: Risk analysis of the ore (upper left), waste (upper right), metal (lower left) production and (lower right) cumulative NPV of a LOM production schedule generated through the simulated annealing implementation and considering 5 input mining sequences with the initial configuration based on a probability rank.
Figure 14: Risk analysis of the ore (upper left), waste (upper right), metal (lower left) production and (lower right) cumulative NPV of a LOM production schedule generated through the simulated annealing implementation and considering 10 input mining sequences with the initial configuration based on a probability rank.
Figure 15: Risk analysis of the ore (upper left), waste (upper right), metal (lower left) production and (lower right) cumulative NPV of a LOM production schedule generated through the simulated annealing implementation and considering 10 input mining sequences with the initial configuration defined by a given mining sequence
Figure 16: Risk analysis of the ore (upper left), waste (upper right), metal (lower left) production and (lower right) cumulative NPV of a LOM production schedule generated through the simulated annealing implementation and considering 15 input mining sequences with the initial configuration defined by a given mining sequence.
Figure 17: Risk analysis of the ore (upper left), waste (upper right), metal (lower left) production and (lower right) cumulative NPV of a LOM production schedule generated through the simulated annealing implementation and considering 20 input mining sequences with the initial configuration based on a probability rank.
Figure 18: Sections of annealing mining sequences given an input of 2, 3, 4, 5, 10, 15 and 20 (left to right) mining sequences respectively

2.2.6 Stochastic Pit Limits: The Impact of Managing Uncertainty

The conventional algorithm used for pit optimization (Lerchs and Grossman, 1965) defines the optimal ultimate pit limit as the equivalent to finding a set of orebody blocks such that the sum of the economic value of the set’s blocks is maximal. The assumption of ‘perfect knowledge’ is that the algorithm assumes that the metal content of any given block is the true value of the same block in the ground. This means that in the presence of uncertainty, a conventional
(deterministic) optimization cannot provide an optimal solution, i.e. truly optimal pit limits. The limiting contour of that set of blocks defines the ultimate pit limit, or when to stop mining. The size of the ultimate pit is highly dependent on economic factors such as the commodity price. The variations of such factors lead to an economic discretization of the space known as the set of nested pits 11. Figure 19 shows the discounted cumulative economic value of each nested pit for the copper deposit under consideration. Standard practice is to select a pit as the ultimate pit limit based on its economic value. As shown in Figure 19, regardless of the relatively narrow range of prices of copper considered (1.6 to 2.0$/pound of Cu), selecting a given pit as the ultimate pit limit it is not always an obvious task.

An alternative approach to the conventional selection of pit limits is to use a stochastic optimizer, such as the simulated annealing algorithm discussed in this paper. To do this, different production schedules are generated for different pits (or potential pit limits). In this case study, the pit limits examined are pits 16 and 18; the former being considered as the conventionally optimal pit limit. The size difference between the pit limits is shown in Figure 20. The production schedule, generated using simulated annealing, is tested by allowing the optimizer to continue mining beyond the previously defined pit limits at pit 16. Figure 22 indicates that mining beyond the conventionally selected pit limits results in a 9%
increase in NPV over the NPV attained by scheduling the blocks within the fixed pit limits. Furthermore, LOM production has been increased by a year. Figure 21 shows the differences of scheduling patterns due to the different pit limits.

![Graph showing the differences of scheduling patterns due to the different pit limits.](image)

**Figure 19:** Cumulative discounted economic value of the open pit as a function of the price of the commodity in the market

![Image showing physical difference between 16 pits (left) and 18 pits (right).](image)

**Figure 20:** Physical difference between 16 pits (left) and 18 pits (right)
Figure 21: Sections of annealing mining sequences that show different scheduling patterns due to the variation of the pit limits.

Figure 22: Risk analysis of the ore (upper left), waste (upper right), metal (lower left) production and (lower right) cumulative NPV of a LOM production schedule generated through the simulated annealing implementation and extending the pit limits.
2.3 Conclusions

The present study addresses (i) the impact of critical parameters of the simulated annealing stochastic mine schedule optimizer on mine production schedules, thus potentially reducing the labour intensiveness of the SA approach required in the past; and (ii) the assessment of pit limits optimized with conventional deterministic optimizers, compared to larger stochastically optimal pit limits.

Production schedules based on SA were first assessed by studying the impact of freezing (i.e. locking in a given period) blocks with 100% probability to be mined in the same given period. The results indicate that whether or not the blocks are frozen, there is no impact on the production schedules other than the expected additional running time. A similar approach was used to investigate the effects of using a probability rank on a starting mining sequence. Furthermore, a guide to select an adequate starting sequence was given.

The implementation of simulated annealing is shown to not be particularly sensitive to increasing the number of input mining sequences (or simulated orebodies) above 10. The modelling of uncertainty of any mineral deposit requires a number of simulated realizations to describe the block’s grade distribution; such description is satisfactory when the probability distribution remains stable, regardless of the number of realizations considered. On the other hand, when mining sequences are considered instead of simulated realizations,
the total proportion of 100% probability blocks may serve as a parameter that provides information regarding the evolution of the set of mining sequences as more mining sequences are added to the set. The level of uncertainty in the spatial distribution of grades is different from the uncertainty in assigning a block to a production period; this case study shows that some mining blocks will be assigned to the same production period long before it is noticeable that the probability distribution of grades remains unchanged by adding more realizations. Finally, it is concluded that the minimum number of realizations required to obtain stable solutions is assessed through the analysis of the uncertainty that lies in assigning blocks to a given production period.

To address the delineation of the ultimate pit limits through deterministic or stochastic approaches, the study compares stochastically generated optimal pit limits to conventional ones, along with the related production schedules both generated stochastically. It is shown that deterministic optimization and the implicit assumption of ‘perfect knowledge’ of the metal content in an orebody has a substantial cost, which amounts to 10% of NPV and one year of additional LOM in the presented case study.

In the future, other stochastic optimizers, such as a stochastic integer programming optimizer, could be compared to the performance of conventional
pit optimizers. This would assist, similarly to the present study, in quantifying differences in terms of optimal pit limits, and LOM production scheduling.
Chapter 3

Stochastic pushback design: Method, application, comparisons and value added

3.1 Introduction

A pushback is an aggregation of mining blocks. A pushback design is used to guide the sequence of extraction of an orebody, from the point where the mining operation begins and where it stops. A starting pushback has a significant impact over the distribution of cash flows throughout the life of the project. Figure 23 (a) shows a comparison of NPV forecasts of two life of mine, LOM, production schedules based on different pushback designs in an application at a porphyry copper deposit, and, Figure 23 (b) shows the physical differences of the pushback designs (left to right). Designs (a) and (b) differ on the design of starting pushbacks which contain ~10 and ~22 million ore tonnes respectively. The conventionally generated LOM production schedules show a 5% (~$10 Millions) difference in the achieved NPV due to the location, tonnage and grade distribution of subsequent pushbacks exclusively as a consequence of the design of the starting pushback. Conventional production schedules are generated through optimizers that do not account for geological uncertainty. Regarding the pit limits, a mining operation should be stopped before negative cash flows are obtained. Ultimate pit limits are also dependant on the pushback design. Figure 24 shows an example where ultimate pit limits are extended to assess the risk
associated to the uncertainty in pit limits. The extension of pit limits is carried out through the addition of pushbacks at the bottom of the pit. Figure 24 shows the NPV of LOM production schedules based on different pit limits; and, Figure 24 shows the physical difference in scheduling patterns. A 5% (~$10 Millions) NPV difference between the schedules is shown. These production schedules were generated through the Milawa Algorithm of the Whittle Software. Due to the deterministic nature of conventional production schedules, there is a risk of not meeting production targets and therefore NPV forecasts (Dimitrakopoulos et al, 2002). A solution to such issue is available through stochastic optimizers which have the ability to integrate uncertainty into the scheduling process. Godoy (2003), Jewbali (2006) and Leite and Dimitrakopoulos (2007) report NPV increments in the order of 10 to 25% due to the use of stochastic optimizers; on such cases, production targets are met. Following, relevant topics to the methodology proposed in this paper are briefly described.
Figure 23: (a) Difference of NPV forecasts due to the difference in the design of starting pushbacks and (b) cross sections of pushback designs.

Figure 24: (a) Difference of NPV forecast due to different pit limits and (b) cross sections of the corresponding scheduling patterns.

The problem of defining optimum pit limits was first addressed in optimization as finding the maximum closure of a graph (Lerchs and Grossman, 1965). Later, the problem was formulated as that of finding a minimum cut (Picard, 1976) which
may be solved by maximum flow algorithms (Gallo et al, 1989). However, the
design of an open pit is not complete because one encounters the problem of
defining the best way to reach the final contour. Considering that there are many
ways to do so, it is of interest to define the sequence of extraction to reach the
final contour that maximizes the value of the project. The design of intermediate
pits is traditionally used to guide the sequence of extraction. To generate such
pits it is a common practice to find maximal closures on a graph with resource or
capacity constraints; each closure representing a different intermediate pit.
Solving this problem requires the dualization of the constraints, which yields a
Lagrangian relaxation problem (Fischer, 2004). Now, the problem changes into
finding the value of the lagrangian multiplier which reduces the gap of optimality.
A number of methods are readily available to solve this problem (Tachefine and
Soumis, 1997). Another approach to designing intermediate pits relates to what
is known as parameterization (Lerchs and Grossman, 1965) which consists of
generating closures on a graph as a function of another parameter related to the
properties of the blocks of the orebody model e.g. finding pits that maximize the
economic value for different desired volumes. The process is carried out through
a search for nested pits at incremental depths, in other words maximal closures
of a graph that lie on the convex hull. The reader is referred to (Seymour, 1994)
for further details on this topic.
Although the design of nested pits may be used to define a sequence of extraction, it is not necessarily optimal for the problem at hand. This is due to: i) not considering grade blending or metal production requirements, ii) large variations in the size of nested pits, iii) ignoring uncertainty of the economic value of a block, iv) nested pits not being sufficiently spaced apart to provide a minimum mining width that allows the access to mining equipment, and v) the lack of economic discounting in the objective function. An objective function that maximizes the discounted value of the project leads to the definition of a production schedule. Ore, waste, metal, grade blending and stockpiling requirements are easily integrated into production scheduling approaches. Nested pits may undergo a conversion into pushbacks; they can be naively grouped into pushbacks by analyzing a pit by pit graph, a graph that plots the cumulative economic value and tonnage versus each pit. Such an approach relies on the subjectivity of the planning engineer and is based on the assumption that each pushback is mined out completely before moving on to the next. In mining operations, different pushbacks may be mined out simultaneously. Designing an open pit with the goal of maximizing the NPV of the project is a conventional practice. Consider the grouping of nested pits into pushbacks; then, finding the design that maximizes NPV and satisfies operational constraints is an
intensive task. To illustrate this, consider N to be a finite set of strictly positive integers. If N represents the number of nested pits, then they can be grouped into as much as N pushbacks. Then, all possible groupings of a pushback design must meet the following requirements:

1. $\sum_{j=1}^{n_i} a_{ij} = N$, where $n_i$ is the number of elements of the $i^{th}$ possible groupings; $a_{ij}$ is an integer that indicates the $j^{th}$ element of the $i^{th}$ possible groupings; $a_{ij} \in [1,N] \forall i$.

2. The number of all possible groupings is $i_{max} = 2^{N-1}$.

For example consider 4 nested pits, all their possible groupings are shown in Table 2. To find the optimum pushback design, 8 production schedules corresponding to each pushback design must be generated. Considering that in practice, the number of nested pits may be very large; instead of generating LOM production schedules for each one of the $2^{N-1}$ possible pit groupings, a search for attractive groupings may be conducted before hand. In other words, if a set of 13 pits is available, 4096 LOM production schedules need to be generated to find the grouping that yields the highest NPV and therefore the optimal pushback design; thus, it is of interest to reduce this number before proceeding with the production scheduling process.
Table 2 Possible groupings of 4 nested pits.

<table>
<thead>
<tr>
<th>Grouping [number of pushbacks]</th>
<th>Details of pushback design</th>
</tr>
</thead>
<tbody>
<tr>
<td>1[4]</td>
<td>1  2  3  4</td>
</tr>
<tr>
<td>2[3]</td>
<td>1  3  4</td>
</tr>
<tr>
<td>3[3]</td>
<td>1  2  4</td>
</tr>
<tr>
<td>4[3]</td>
<td>2  3  4</td>
</tr>
<tr>
<td>5[2]</td>
<td>1  4</td>
</tr>
<tr>
<td>6[2]</td>
<td>2  4</td>
</tr>
<tr>
<td>7[2]</td>
<td>3  4</td>
</tr>
<tr>
<td>8[1]</td>
<td>4</td>
</tr>
</tbody>
</table>

3.2 An Approach to Pushback Design: Grouping Nested Pits

The approach presented here in, is a multistage approach to designing pushbacks based on the grouping of nested pits resulting from the parameterization of the pit space. The basic inputs required by this methodology consist of a set of simulated equally probable orebody models and an estimated ore body model. The stages of the proposed approach are i) generation of a set of nested pits based on the estimated orebody model; ii) group nested pits based on a target number of pushbacks that satisfy operational constraints and maximize the economic value of the project; iii) generation of LOM production
schedules based on each pushback design generated in Stage 2. The stages of the pushback design approach used herein are shown Figure 25.

**Stage I – Discretizing the orebody/pit space**

In the first stage, the space within the pit limits is discretized to generate a set of nested pits. It is a common practice in the mining industry to generate a set of maximum valued nested pits as a function of another parameter defined by the properties of the blocks in the orebody model; this is known as parameterization. Each value of the parameter generates a single pit. A wide range of values of the parameter provides enough information to analyze the sensitivity of the pit size to the parameter of interest.

**Stage II – Grouping Nested Pits**

Consider the example described in Table 2, where \( N = 4 \) nested pits. The following groupings are available: 1 possible combination of \([4]\) pushbacks, 3 possible combinations of \([3]\) pushbacks, 3 possible combinations of \([2]\) pushbacks and 1 possible combination of \([1]\) pushback. Then, \( J \) possible combinations of \([J_{PB}]\) pushbacks are available; where \( J \) is the number of possible pushback designs.
that may be achieved given a required or target number of pushbacks, \([T_{PB}]\).

The number of possible pushback designs for a given \([T_{PB}]\), \(J(T_{PB})\), may be calculated as follows: first consider \([T_{PB} = 1]\), then regardless of \(N \cdot J(1) = 1\); then for \([T_{PB} = 2], J([T_{PB} = 2]) = \frac{J(T_{PB} - 1 = 1)}{[T_{PB} - 1 = 1]} \cdot (N - [T_{PB} - 1 = 1])\). Then \(J([T_{PB}]) = \frac{J(T_{PB} - 1)}{[T_{PB} - 1]} \cdot (N - [T_{PB} - 1])\) and \(\sum_{T_{PB} = 1}^{N} J([T_{PB}]) = 2^{N-1}\). If \(M[T_{PB}] = \{m_j = m_1 \ldots m_J\}\) is a finite set of elements that represents the set of possible pushback designs given a target \([T_{PB}]\); then \(m_j\) is the \(j^{th}\) element of \(M\) and \(m_j^* = \max_{j=1\ldots J} \{f(m_j)\}\) represents the best grouping, where \(f(\cdot)\) is a function that evaluates the value of each design. \(f(\cdot)\) may evaluate the economic value of the design, the stripping ratio or any other criteria of interest for pushback design.

**Stage III – LOM Production Scheduling**

Having defined the best grouping of pits \(m_j^* \forall T_{PB} = 1\ldots N\), the next step relates to which is the most favourable number of pushbacks. In other words, the problem of pushback design based on grouping nested pits is reduced to finding the optimal number of pushbacks. For such purposes, recall that convenient pushback designs should guide the LOM production schedule to meet production targets, maximize the overall discounted cash flows, minimize the stripping ratio and guarantee safety slope requirements. Then, the optimal number of pushbacks is defined by the LOM production schedule that yields the best performance in terms of the requirements stated above. To assess this, for each
one of the generated LOM production schedules, risk analysis on relevant project indicators is carried out through the use of the available simulated orebody models. Such approach is discussed in detail by Ravenscroft (1992); in the approach herein, the idea is to evaluate the impact of uncertainty on the performance of an open pit design and its related economic parameters. The schedules are generated based on the model introduced by (Ramazan and Dimitrakopoulos, 2007). The optimization process is based on the economic value of each block $V_l, l = 1 \ldots L$ that belongs to the set of blocks being scheduled. The expected economic value of a block is calculated by using its expected return, which is defined by the revenue gained from selling the amount of metal contained in it. The objective function involves the maximization of the expected net present value, $E\{(NPV)_t\}$, which is generated by mining a block at a given production period and by considering a certain simulation and minimizes the risk of not meeting ore production targets. The definition of the objective function follows:

$$\text{Max} \sum_{t=1}^{T} \left[ \sum_{l=1}^{L} E\{(NPV)_t\} b_l^t - \sum_{s=1}^{S} c_{t_0}^s a_s^t + c_u^t a_u^t \right]$$

where $l$ is the block identifier; $t$ is the time period; $t_0$ flags the ore production target type; $u$ stands for lower bound; $o$ stands for upper bound; $s$ stands for the simulation number; $T$ is the maximum number of scheduling periods; $L$ is the total
number of blocks to be scheduled; \( b^t_l \) is a variable representing the portion of block \( l \) to be mined in period \( t \); if it is defined as a binary variable, it is equal to 1 if the block \( l \) is to be mined out in period \( t \) and equal to 0 if otherwise; \( E\{(NPV)^t_l\} \) is the expected NPV to be generated by mining block \( l \) is to be mined in period \( t \) considering simulation \( s \); \( c^{t_0}_o \) and \( c^{t_0}_u \) are the unit costs for excess and deficient ore production respectively; and \( d^{t_0}_{so} \) and \( d^{t_0}_{su} \) are the excess and deficient amount of ore production in period \( t \) considering simulation \( s \). The objective function is subject to reserve, mining, processing, slope and grade constraints. For purposes of completeness, the processing constraints are defined below. Consider \( O_{st} \) to be the ore tonnage of a given block \( l \) conditioned to simulation \( s \), dummy variables \( a^{t_o}_{su} \) and \( a^{t_o}_{so} \) to balance the equality, and the maximum and minimum ore production expected per production period of the LOM; then, ore tonnage production must lie within lower and upper bounds, \( O_{min} \) and \( O_{max} \). The variables and parameters described above, lead to define the processing constraint as

\[
\sum_{l=1}^{L} O_{st} b^t_l + d^{t_o}_{su} - a^{t_o}_{su} = O_{min} \quad \text{and} \quad \sum_{l=1}^{L} O_{st} b^t_l + d^{t_o}_{so} - a^{t_o}_{so} = O_{max}.
\]

Risk management is accomplished by the introduction of a geological risk discounting rate, GRD, into the calculation of the costs for excess and deficient production (Dimitrakopoulos and Ramazan, 2004). Aside from a grade cut-off, a probability cut-off is used to classify the blocks as ore or waste. Furthermore, this SIP implementation takes into consideration a different way of defining binary
variables (Ramazan and Dimitrakopoulos, 2004). This consists of reducing the amount of binary variables by setting the waste blocks to linear variables and the ore blocks to binary variables. Ramazan and Dimitrakopoulos (2004) also show a case study where such “selective” binary definition not only reduces the solution time but does not affect the quality of the solution by keeping it optimal. The formulation can be easily extended to generate production schedules that minimize the risk of deviating from metal and grade targets; the reader is referred to an example in Dimitrakopoulos and Ramazan (2008).

3.3 Case Study at a Porphyry Copper Deposit

The approach described in the previous section is tested on a porphyry copper deposit. The technical and economic information related to the case study are shown in Table 3.

Stage I

To discretize the pit space, the industry common practice is to make use of an implementation of the Lerchs-Grossman algorithm available commercially in Whittle software. Their idea is to parameterize the space through the variation of the ratio of metal price to extraction cost (Whittle, 1988). In this case, a wide range of values of the parameter provides enough information to analyze the sensitivity of the pit size to the price of the commodity. The information in Table 3
and a conventionally estimated orebody block model created through ordinary kriging are provided to the Whittle Software in order to generate a set of 17 nested pits.

**Stage II**

The total number of possible pushback designs is \(2^{16}=65536\). Figure 26 shows the possible number of designs as a function of the target number of pushbacks for the case where the number of available nested pits is 17. For the case study, the nested pits are grouped based on the maximization of the economic value of each design. The value of each pushback is discounted based on its life; time required for depletion. Note that since there are 17 nested pits available, at the most there are 17 different pushback designs that may be tested in Stage III. In other words, there exists a unique pushback design, \(m^*_j = \max_{j=1\ldots T_{PB}} \{ f(m_j) \} \), for each \(T_{PB}\) that yields the highest economic value. Then, in this case of study, the number of possible groupings of nested pits reduces to 17.

**Stage III**

20 simulated orebody models are fed to the SIP model described in the previous section to generate a LOM production schedule for each one of the pushback designs in Table 4. The orebody models were generated using the Direct Block Simulation Algorithm (Godoy, 2003). The SIP model described above could be solved for each one of the 17 different pushback designs available from the set of
17 nested pits. Nevertheless, and as it will be explained the approach is tested only for 6 different pushback designs, \( T_{PB} = 3, 5, 6, 7, 9, 10 \). The case of \( T_{PB} = 1 \) is ignored because it equals to not considering pushback design. Though the cases of \( T_{PB} = 2, 4, 8 \) were initially part of the case study; solutions in Stage III were not obtained in a feasible amount of time. Regarding a \( T_{PB} > 10 \), recall that defining a sequence of extraction based on the available set of nested pits is neither necessary optimal nor feasible for the case at hand. As \( T_{PB} \) approaches the number of available nested pits the production schedule presents greater deviations from production targets and leads to smaller ultimate pits; this is illustrated in the cases of \( T_{PB} = 9, 10 \). Therefore, the approach is tested until \( T_{PB} = 10 \).
Table 3 Economic and technical parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper price (US$/lb)</td>
<td>1.9</td>
</tr>
<tr>
<td>Selling cost (US$/lb)</td>
<td>0.4</td>
</tr>
<tr>
<td>Mining cost ($/tonne)</td>
<td>1.0</td>
</tr>
<tr>
<td>Processing cost ($/tonne)</td>
<td>9.0</td>
</tr>
<tr>
<td>Slope angle</td>
<td>45°</td>
</tr>
<tr>
<td>Processing recovery</td>
<td>0.9</td>
</tr>
<tr>
<td>Block dimensions (m)</td>
<td>20x20x10</td>
</tr>
<tr>
<td>Ore production target (tonne/year)</td>
<td>7.5</td>
</tr>
<tr>
<td>Waste production target (tonne/year)</td>
<td>20.5</td>
</tr>
</tbody>
</table>
Figure 26: Number of pushback designs as a function of the target number of pushbacks and the number of available nested pits.

Table 4 Grouping of pits based on a given target number of pushbacks. The grouping criterion is the maximum economic value.

<table>
<thead>
<tr>
<th>Target number of pushbacks [TPB]</th>
<th>Resulting grouping of pits</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>[3 6 17]</td>
</tr>
<tr>
<td>5</td>
<td>[2 5 7 13 17]</td>
</tr>
<tr>
<td>6</td>
<td>[2 4 6 7 12 17]</td>
</tr>
<tr>
<td>7</td>
<td>[1 3 4 6 8 12 17]</td>
</tr>
<tr>
<td>9</td>
<td>[1 3 4 5 6 7 12 16 17]</td>
</tr>
<tr>
<td>10</td>
<td>[1 3 4 5 6 7 12 14 16 17]</td>
</tr>
</tbody>
</table>
3.3.1 Analysis of solutions

For the case of three pushbacks, $T_{PB} = 3$, the grouping that yields the highest economic value is shown in Figure 28. The starting pushback contains the 1st, 2nd and 3rd nested pits. The starting pushback allows the scheduling of the first and part of the second production period. Regarding the performance of the production schedule, during the 3rd and 4th periods there is a $\sim$5% deviation (over production) from the ore production target and a $\sim$2% deviation (under production) during the 5th period, as shown in Figure 28. The risk of not meeting the production target during the 6th period is higher than that observed in the other production periods; note that the risk profile is wider.

Figure 27: (a) 3 pushback design and (b) its corresponding LOM production schedule
Figure 28: Performance of a LOM production schedule based on a 3 pushback design in terms of (a) ore production, (b) waste production, (c) cumulative metal production and (d) cumulative discounted cash flows.

The design of five pushbacks, $T_{PB} = 5$, is shown in Figure 29. The starting pushback contains the 1st and 2nd nested pits; there are clear physical differences, regarding the starting pushback, between this case and the previous design leading to different scheduling patterns as shown in Figure 27. The risk
analysis of the LOM production schedule in Figure 30 shows that there are no major deviations of the ore production target. Nevertheless, the schedule is not necessarily feasible due to the increment of the waste mining rate during the 4th production period; the waste tonnage extracted in the 3rd period increases from 11.3 Mtones to 29.2 Mtones in the 4th period. This result relates to the grouping of pits selected for the five pushback design; recall that the grouping criteria is the maximization of the economic value, a different criteria would have suggested a different design. Furthermore, for the case study under consideration the waste production rate is not constrained, allowing it a free variation. Scheduling the first two pushbacks satisfied the ore production target of the 1st, 2nd and 3rd periods; the schedule defers waste mining. When the third pushback is scheduled, the optimizer is not able to meet the ore production target for the 4th period without mining the huge amounts of waste that were left behind from the previous pushbacks; leading to an infeasible waste production rate.

Figure 29: (a) 5 pushback design and (b) its corresponding LOM production schedule
Figure 30: Performance of the LOM production schedule based on a 5 pushback design in terms of (a) ore production, (b) waste production, (c) cumulative metal production and (d) cumulative discounted cash flows

The design of 6 pushbacks, $T_{PB} = 6$, is shown in Figure 31. The starting pushback is designed as in the previous case leading to the same scheduling patterns during the 1st production period. The intermediate pushbacks contain different nested pits from the previous cases; as shown in Table 4. This leads to different
scheduling patterns in the subsequent production periods. Figure 32 shows a 3% and 4% deviation (over production) from the ore production target during the 4th and 5th periods and a 3% deviation (under production) during the 6th period; this results in a ~15% reduction in the metal production during the 6th period, when compared to previous periods. Table 5 shows that the overall ore-waste tonnage extracted in this case is the same as the one in previous cases; this is reflected in the identical ultimate pit contours as shown in Figure 27.

Figure 31: (a) 6 pushback design and (b) its corresponding LOM production schedule
Figure 32: Performance of the LOM production schedule based on a 6 pushback design in terms of (a) ore production, (b) waste production, (c) cumulative metal production and (d) cumulative discounted cash flows.

The design of seven pushbacks is shown in Figure 33. Figure 34 shows that its corresponding LOM production schedule presents a ~6% deviation (over production) from the production target during the 3rd and 4th periods. The average grade of the ore fed to the mill is constant throughout the LOM resulting
in a constant metal production rate. Table 5 shows that the overall tonnage extracted in this case is higher than the previous cases resulting in a different ultimate pit contour as shown in Figure 33. This is due to the difference in the pushback design which yields a different cash flow pattern and therefore a different physical point in which the cash flows become negative.

Figure 33: (a) 7 pushback design and (b) its corresponding LOM production schedule
Figure 34: Performance of the LOM production schedule based on a 7 pushback design in terms of (a) ore production, (b) waste production, (c) cumulative metal production and (d) cumulative discounted cash flows.

The design of nine pushbacks is shown in Figure 35. The total tonnage extracted with this schedule is 4% lower than the previous cases, which implies that the operation is stopped at different times. A proof of this result is the physical difference in the pit limits between this case and the previous cases, Figure 35.
The waste mining rate of the 3rd production period, Figure 36, affects the performance of the schedule by restricting the availability of ore during the 5th production period. Due to the pushback design which causes the ore shortage during the 5th production period, the optimizer extracted the ore within the 17 nested pits in eight periods. The 8th production period yielded negative cash flows; therefore, mining stops at the 7th period. This highlights the importance of pushback design and its effect on the definition of ultimate pit limits.

Figure 35: (a) 9 pushback design and (b) its corresponding LOM production schedule.
Figure 36: Performance of the LOM production schedule based on a 9 pushback design in terms of (a) ore production, (b) waste production, (c) cumulative metal production and (d) cumulative discounted cash flows.

The design of ten pushbacks is shown in Figure 37. The main difference between this and the previous design lies on the pushbacks that discretize the bottom of the pit. In this case, the bottom of the pit is broken up into three pushbacks, defined by the nested pits [12 14 16 17]. Such discretization provides more
information about when the cash flows become negative. This leads to a smaller ultimate pit than the one obtained in the previous case; Figure 37 shows the ultimate pit contour and Figure 38 shows the differences in the performance of the LOM production schedule.

Figure 37: (a) 10 pushback design and (b) its corresponding LOM production schedule
3.3.2 Selecting the Pushback Design

Table 5 shows the overall tonnage extracted, NPV, the maximum deviation from the ore production target and the maximum stripping ratio presented throughout the LOM for the production schedules discussed above. The positive (+) and
negative (-) signs used in the column of the maximum deviations from ore production targets express over and under production respectively. The overall tonnages may be used as a basis for comparison of the different pit limits presented in the cases above. The cases of 9 and 10 pushback designs extract considerably less tonnage than the previous cases and the NPV differences are of the order of ~3%. To analyze these results one must consider that deviations from ore production targets are in the order of 13%; leading to an erroneous NPV forecast. Similar observations apply for the design with 5 pushbacks; erroneous NPV forecast caused by the operational infeasibility due to the high stripping ratio, 3.89%.
Table 5 Global proportions of ore-waste tonnage extraction, NPV, Maximum deviation from ore production targets and Maximum stripping ratio of LOM production schedule based on different pushback designs.

<table>
<thead>
<tr>
<th>Pushback design (number of pushbacks)</th>
<th>Overall tonnage extracted (Mtones)</th>
<th>NPV (M x $)</th>
<th>Maximum deviation (%)</th>
<th>Maximum stripping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>160</td>
<td>277</td>
<td>5(+)</td>
<td>2.92</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
<td>275</td>
<td>2(+)</td>
<td>3.89</td>
</tr>
<tr>
<td>6</td>
<td>160</td>
<td>277</td>
<td>4(-)</td>
<td>2.95</td>
</tr>
<tr>
<td>7</td>
<td>161.5</td>
<td>268</td>
<td>6(+)</td>
<td>2.79</td>
</tr>
<tr>
<td>9</td>
<td>156</td>
<td>271</td>
<td>13(-)</td>
<td>3.20</td>
</tr>
<tr>
<td>10</td>
<td>143</td>
<td>274</td>
<td>13(-)</td>
<td>3.20</td>
</tr>
</tbody>
</table>

3.3.3 Design of starting, intermediate and bottom-pit pushbacks

The design of starting pushbacks, in the case of study, consists of three possible groupings: i) 1st, 2nd and 3rd nested pits, ii) 1st and 2nd nested pits or iii) just the 1st nested pit. For the case study, the impact of the design of starting pushbacks on LOM production scheduling is better assessed through the analysis of stripping
ratios and cash flows of the 1\textsuperscript{st} and 2\textsuperscript{nd} production periods because the life of the starting pushback for all cases is at the most 2 years. Figure 39 shows the impact of different starting pushback designs on the stripping ratio and cumulative NPV of the project; the risk of not meeting the NPV forecasts is also shown. The risk profiles show the minimum and the maximum NPV that may be achieved by the schedules under consideration. Figure 39 shows a starting pushback design that yields the highest value even though the stripping ratio increases from 0.5 in the 1\textsuperscript{st} period to 2.1 in the 2\textsuperscript{nd} period. For the case study, initiating the mining operation with a low stripping ratio is best.

The difference in scheduling patterns is a direct consequence of the pushback design. The fact that the designs of nine and ten pushbacks differ only at the bottom of the pit and that the NPV is higher in the latter case suggests that, for the set of pits available in this case study, breaking the bottom of the pit into more pushbacks provides a better assessment of the uncertainty in the ultimate pit limits. Regarding the performance of the LOM production schedule based on nine and ten pushbacks, it is important to recall that, for the case study, the pushback design depends on a previous discretization generated through the nested pit implementation of the Whittle Software. Increasing the target number of pushbacks will augment the similarity of the pushback design to the initial nested pit discretization. The objective function of the optimization model in the
Lerchs Grossman algorithm maximizes cash flows, this represents a greedy approach that does not necessarily maximize the NPV of the project and does not control the ore-waste tonnage relationship on each nested pit. This explains the ore shortage during the 5th production period in the cases that consider nine and ten pushbacks.

The discussion above provides guidelines to select the pushback design. Then, the starting pushback is defined by the 1st nested pit and the bottom of the pit should be discretized based on the results of the design of ten pushbacks shown on Figure 37. In the case of five pushbacks, shown in Figure 30, the 1st and 2nd pushbacks provide enough material to schedule the ore production of the 1st, 2nd, 3rd and part of the 4th period but it allows the optimizer to defer waste mining in order to maximize the cash flows obtained up to 3rd production period; leaving behind large amounts of waste that affect the stripping ratio of the 4th production period. Decreasing the size of the 2nd pushback forces the optimizer to extract more waste during 2nd and 3rd production period leading to a stable stripping ratio throughout the LOM. Therefore the 2nd pushback is redefined by the nested pits [2 4]. The following pushbacks are designed to avoid ore shortages as presented in the cases of six, nine and ten pushbacks; shown in Figure 32, Figure 36 and Figure 38 respectively. In the case of six pushbacks, there is an ore shortage during the 6th production period; this is a consequence of
the pushback design. The 5th period is scheduled mainly on the 4th pushback defined by the nested pits [6 7] and the 6th period is scheduled mainly in the 5th pushback defined by the nested pits [7 12]. To avoid ore shortage issues, the 4th and 5th pushbacks are enlarged to envelope the 8th and 14th nested pit respectively; which thus forces the optimizer to mine additional waste and ore tonnage during the 5th production period in order to facilitate the access to ore blocks during the 6th production period. Note that the extension of the 4th pushback provides ore and waste alike, hence avoiding ore shortage issues as in the cases of nine and ten pushbacks. Following the previous discussion, another pushback design is considered through the following grouping of nested pits: [1 4 6 8 14 16 17]. The design is based on seven pushbacks and it is shown in Figure 40. The physical patterns of the LOM production schedule are shown in Figure 40. Figure 41 shows the performance of the LOM production schedule; Figure 41 shows that the ore production rate meets the ore production target except for the 4th and 6th production periods which present a ~4% deviation (over production).
Figure 39: Shows the stripping ratios and cumulative NPV associated to the LOM production schedule based on the cases of (a) three pushbacks, grouping of 1st, 2nd and 3rd nested pits; (b) five pushbacks, grouping of 1st and 2nd nested pits; and (c) seven pushbacks, just the 1st nested pit (left, right and bottom respectively)

The discussion above provides guidelines to select the pushback design. Then, the starting pushback is defined by the 1st nested pit and the bottom of the pit
should be discretized based on the results of the design of ten pushbacks shown on Figure 37. In the case of five pushbacks, shown in Figure 30, the 1st and 2nd pushbacks provide enough material to schedule the ore production of the 1st, 2nd, 3rd and part of the 4th period but it allows the optimizer to defer waste mining in order to maximize the cash flows obtained up to 3rd production period; leaving behind large amounts of waste that affect the stripping ratio of the 4th production period. Decreasing the size of the 2nd pushback forces the optimizer to extract more waste during 2nd and 3rd production period leading to a stable stripping ratio throughout the LOM. Therefore the 2nd pushback is redefined by the nested pits [2 4]. The following pushbacks are designed to avoid ore shortages as presented in the cases of six, nine and ten pushbacks; shown in Figure 32 respectively. In the case of six pushbacks, there is an ore shortage during the 6th production period; this is a consequence of the pushback design. The 5th period is scheduled mainly on the 4th pushback defined by the nested pits [6 7] and the 6th period is scheduled mainly in the 5th pushback defined by the nested pits [7 12]. To avoid ore shortage issues, the 4th and 5th pushbacks are enlarged to envelope the 8th and 14th nested pit respectively; which thus forces the optimizer to mine additional waste and ore tonnage during the 5th production period in order to facilitate the access to ore blocks during the 6th production period. Note that the extension of the 4th pushback provides ore and
waste alike, hence avoiding ore shortage issues as in the cases of nine and ten pushbacks. Following the previous discussion, another pushback design is considered through the following grouping of nested pits: [1 4 6 8 14 16 17]. The design is based on seven pushbacks and it is shown in Figure 40. The physical patterns of the LOM production schedule are shown in Figure 40. Figure 41 shows the performance of the LOM production schedule; Figure 41 shows that the ore production rate meets the ore production target except for the 4th and 6th production periods which present a ~4% deviation (over production).

Figure 40: (a) Design with 7 pushbacks and (b) corresponding LOM production schedule
Figure 41: Performance of the LOM production schedule based on design with 7 pushbacks in terms of (a) ore production, (b) waste production, (c) cumulative metal production and (d) cumulative discounted cash flows

3.4 Conventional Vs. stochastic approach: Performance and comparison

Conventional practices do not integrate geological uncertainty into the process of pushback design. In Figure 42 and Figure 43 the dotted line shows the expected performance of a conventional LOM production schedule, generated by the
Milawa Algorithm in the Whittle Software, based on an estimated orebody model and an Etype model respectively; the risk analysis shows that ore production targets are never met and that grades are underestimated at the bottom of the pit resulting in a misleading NPV forecast as shown in Figure 42 and Figure 43. Furthermore, consider a pushback design obtained through the methodology presented herein, if a LOM production schedule is generated by a scheduler that does not account for uncertainty the performance of its key parameters will be deficient. To illustrate this, Figure 44 shows the performance of a LOM production schedule based on the pushback design shown in Figure 40. The schedule is generated by the Milawa Algorithm in the Whittle Software. The ore production targets are never met. In comparison to the production schedule shown in Figure 40, the conventionally generated schedules shown in Figure 42: Performance of a conventional LOM PS based on an ordinary kriging model in terms of (a) ore production and (b) cumulative discounted cash flows, Figure 43 and Figure 44 present the following issues: i) they require one extra year to mine the deposit, ii) they do not meet ore production targets, and iii) the NPV forecasts are misleading because the extra maintenance and operational costs incurred through idle mineral processing capacity are not considered. When compared to the conventional approach, making use of the approach proposed herein has substantial economic and operational implications. Figure 45 shows that there is
a ~30% difference between the NPV achieved by the conventional and the approach introduced in this paper. This is due to the differences in the physical patterns of production scheduling; which lead to different mining rates, an extension of the pit limits and ~5.5 thousand tones of extra metal produced. For the case study, the overall waste production is lower than that of the conventional approach; the cost structure of the stochastic optimizer controls the waste production rate through maximizing the NPV.

Figure 42: Performance of a conventional LOM PS based on an ordinary kriging model in terms of (a) ore production and (b) cumulative discounted cash flows
Figure 43: Performance of a conventional LOM PS based on an Etype model in terms of (a) ore production and (b) cumulative discounted cash flows.
Figure 44: Performance of a conventional LOM PS based on the design with 7 pushbacks, shown in Figure 17, in terms of (a) ore production, (b) waste production, (c) cumulative metal production and (d) cumulative discounted cash flows
Figure 45: Cumulative NPV of the conventional and stochastic approach to pushback design.

3.5 Conclusions

Pushback design has a significant impact on $i)$ the physical scheduling patterns of an operation; $ii)$ the definition of the ultimate pit limits; in other words, the moment where negative cash flows are obtained; and $iii)$ the performance of key project indicators, e.g. ore, waste and metal production rate and related economic parameters. This paper introduces an approach that makes use of a SIP model to integrate geological uncertainty and economic discounting into the process of pushback design. The approach consists of $i)$ open pit
parameterization in order to generate a set of nested pits,\textit{ii})\ group the resulting nested pits into pushbacks based on a required number of pushbacks and \textit{iii}) the use of a SIP to generate life-of-mine (LOM) production schedules that maximize NPV, while meeting production targets and NPV forecasts, based on the pushback designs obtained in the previous stage.

The methodology was tested on a porphyry copper deposit. Different pushback designs were used to guide the production scheduling process. The optimal number of pushbacks, for the case study under consideration, is seven. The pushback design enables the deferment of waste mining during the initial production periods without generating infeasible stripping ratios or mining rates in the remaining periods, allowing the distribution of the initial investment throughout the LOM. The pit limits were defined by allowing the optimizer to mine until negative cash flows were generated. The new approach yielded an increment of $\sim$30\% in the NPV when compared to the conventional approach. The differences reported are due to the different scheduling patterns, the waste mining rate and an extension of the pit limits which yielded an extra $\sim$5.5 thousand tonnes of metal.

The final pushback design in the case study above is a function of the initial discretization of the pit space. Then, it follows that a different discretization of the pit space will yield different pushback designs and therefore different production
schedules. In the case study, the function used in the second stage discounts the value of the pushback based on its life. This is an approximation of reality because the life of pushbacks varies depending on the number of blocks; then, a year by year discounting is not considered. Other approaches can be considered to group the nested pits in the second stage. Further research should aim to study the sensitivity of the value of the project and the operational feasibility of the production schedules to the initial discretization and the way nested pits are grouped in the second stage.

Conventional optimizers generate production schedules that do not meet production targets and NPV forecasts regardless of the pushback design considered. The use of stochastic optimizers yields pushback designs that guide the sequence of extraction into meeting production targets and NPV forecasts.
Chapter 4

General Conclusions and recommendations for future work

4.1 General Conclusions

This thesis examines the intricacies of two stochastic optimizers that account for metal uncertainty when generating mine design and LOM production schedules. The first optimizer is based on an implementation of simulated annealing. The second optimizer is based on a stochastic integer programming (SIP) model. The main goal of this thesis is to explore the possibility of using mine production scheduling approaches that consider geological uncertainty to design pushbacks, define optimal pit limits and to assess the minimum number of simulations required to generate robust LOM production schedules.

It is a common practice to model geological uncertainty through the use of conditional simulation algorithms available in the field of geostatistics. Each simulated orebody model is an equiprobable representation of the spatial distribution of grades in the area of study, where a grade value is assigned to each block of the orebody model. A set of simulations provides a description of the grade uncertainty in the block and its neighbours, in terms of a probabilistic distribution. The running time of conditional simulation algorithms is affected by the size of the problems; in mining i.e. the number of mining blocks. The set of
simulated orebody models are the main input to a stochastic optimizer; therefore, it is of interest to evaluate the minimum number of simulated orebody models required to generate stable solutions, LOM production schedules that meet production targets and NPV forecasts. Then, it follows that the first objective of this thesis is to carry out a sensitivity analysis on an implementation of the simulated annealing algorithm in order to assess the minimum number of simulations required to generate stable production schedules. To analyze the sensitivity, the numbers of simulated orebody models fed to the optimizer were 2, 3, 4, 5, 10, 15 and 20. The implementation of simulated annealing is shown to not be particularly sensitive to increasing the number of input mining sequences (or simulated orebodies) above 10. The modelling of uncertainty of any mineral deposit requires a number of simulated realizations to describe the block’s grade distribution; such description is satisfactory when the probability distribution remains stable, regardless of the number of realizations considered. On the other hand, when mining sequences are considered instead of simulated realizations, the total proportion of 100% probability blocks may serve as a parameter that provides information regarding the evolution of the set of mining sequences as more mining sequences are added to the set. The level of uncertainty in the spatial distribution of grades is different from the uncertainty in assigning a block to a production period; this case study shows that some mining blocks will be
assigned to the same production period long before it is noticeable that the probability distribution of grades remains unchanged by adding more realizations. Finally, it is concluded that the minimum number of realizations required to obtain stable solutions is assessed through the analysis of the uncertainty that lies in assigning blocks to a given production period.

The selection of a starting sequence has a significant impact on the quality of the solution. This is due to the sensitivity of the simulated annealing approach to the quality of the initial configuration of the system provided to initialize the optimization process. Therefore, a starting mining sequence requires careful selection. The annealing process requires a sequence which involves the least possible risk and yields the maximum NPV out of the set of available mining sequences given as an input. A set of 20 mining sequences was the input available to be fed to the simulated annealing algorithm; the set contains sequences that require seven or eight mining production periods to deplete the deposit. 20 different schedules are generated by varying the starting sequence. All eight period mining sequences perform poorly in terms of meeting production targets and NPV forecasts when tested against a set of twenty simulated orebody models. On the other hand, all seven period mining sequences yield the same total average deviation from ore production targets. The average cumulative NPV of each mining sequence is considered as the selection criterion.
because it provides a set of values from which the sequence that yields the highest NPV is evident.

To define ultimate pit limits, the orebody block model is represented as a graph. The idea is to find maximum closures or minimum cuts on a graph. It is a common practice to evaluate the sensitivity of the size of the pit to the variation of the price of the commodity. This results in an economic discretization of the pit space in the form of maximum closures (or equivalent minimum cuts) on a graph. The set of closures is known as a family of nested pits. A conventional practice is to use one of the available nested pits to define the ultimate contour. To do so, engineers analyze a pit by pit graph, which plots the economic value of each nested pit. Due to the fact that this approach does not account for uncertainty, the second objective of this thesis is to make use of a stochastic optimizer to define optimal pit limits. The basic idea is to test the sensitivity of the simulated annealing implementation to the variation of the size of the ultimate pit. The basic idea is to extend the pit limits to include the biggest available nested pit. Then, implement simulated annealing to generate a schedule that minimizes the risk of not meeting production targets. If negative cash flows are generated before the sequence of extraction reaches the ultimate pit limit, the process is repeated by considering the smaller adjacent nested pit as the ultimate contour of the pit. The process is repeated until negative cash flows are no longer generated; and the
nested pit being used as ultimate contour defines the pit limits. It is important to note that the use of a stochastic optimizer for such purposes yields pit limits defined under geological uncertainty. Stochastically optimized pit limits are about 17% larger, in total tonnage than the conventional optimal pit limits. The difference adds one year of mining and approximately 10% of additional NPV, compared to the NPV of conventional optimal pit limits and a production schedule generated stochastically.

The problem of pushback design under uncertainty is addressed. A multistage method that makes use of a stochastic integer programming based optimizer is proposed in order to find the optimal pushback design. The approach consists of: i) open pit parameterization in order to generate a set of nested pits, ii) group the resulting nested pits into pushbacks based on a required number of pushbacks and iii) the use of a SIP to generate life-of-mine (LOM) production schedules that meet production targets and NPV forecasts based on the pushback designs obtained in the previous stage. The method was tested on a porphyry copper deposit. Different pushback designs were considered to guide the production scheduling process. The optimal number of pushbacks, for the case study under consideration, is seven. The risk analysis of the resulting LOM production schedule based on the pushback design yielded by the multistage methodology described above shows that ore production targets are met in average with ~4%
deviations during the 4th and 6th periods. The pushback design enables the deferment of waste mining during the initial production periods generating no infeasible stripping ratios or unrealistic mining rates in the remaining periods, thus allowing the controlled distribution of the initial investment throughout the LOM. The pit limits were reached by allowing the optimizer to mine until negative cash flows were generated. The new approach yielded an increment of ~30% in the NPV when compared to the conventional approach. The differences reported are due to the different scheduling patterns, the waste mining rate and an extension of the pit limits which yielded an extra ~5.5 thousand tonnes of metal when compared to the results obtained through the conventional approach.

4.2 Recommendations for future work

In the case of simulated annealing, the method should be extended to make the objective function minimize the risk that arises due to the uncertainty surrounding other production aspects such as metal production, to include weighted components in the objective function, optimal mining rates and changes in the cut off grade policies.

For the case of SIP models, it is of relevance to study the sensitivity to the number of simulated orebody models considered as input due to the computing cost of the geostatistical algorithms available to generate the orebody models;
implement algorithms that are specifically designed to solve stochastic programs; introduce cutting planes to the SIP formulations in order to decrease the computational efforts required to solve stochastic programs. Formulate flexible mathematical models that may be easily extended to account for other type of uncertainties.

Further testing of the pushback design methodology presented in the thesis should be carried out; such as the sensitivity of the approach to a different starting discretization and different criteria to grouping nested pits.
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