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EXPERIMENTAL AND ANALYTICAL INVESTIGATIONS OF GRANULAR-FLUID MIXTURES DOWN INCLINES

BY

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Short thesis title

Investigations of granular-fluid chute flows

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Finally, deepest thanks to my family for their support during my absence, specially my mother Francia Elena who always encouraged me. I dedicate this thesis to my wife Hilda Marina who gave me love, support and a marvellous daughter Lina Sofia in the course of this work.
Granular-fluid mixtures flowing down an incline in the grain-inertia regime were studied experimentally and analytically. The equations of motion are based on the kinetic theory for granular flow. The boundary conditions are formulated following two methods. The first is a simple mechanical approach based on the energy exchange at the wall. The other is based on the kinetic theory for granular flow. Solutions for dry granular flow down a chute are studied in order to compare both methods. The extension of the kinetic theory presented here, includes drag forces resulting from the interstitial fluid that cushions interparticle collisions and particle-wall collisions. Frictional stresses, produced when long term contacts are present, and fluid turbulent fluctuations are introduced in the model.

The results are compared with measurements from an experimental chute in which the inclination, the solids flow rate and fluid flow rate are all varied. The theory is found to give a good qualitative account of the observed behaviour. Finally an application of the model to the description of the mechanical behaviour of the Nevado del Ruij 1985 debris flow is attempted.
RÉSUMÉ

Les écoulements en chute libre de mélanges entre des fluides et de grains non cohésifs, dans le régime granulaire-inertiel, ont été étudiés expérimentalement et de façon théorique. Les équations de mouvement sont construites à partir de la théorie cinétique des écoulements granulaires. Les conditions de frontière sont formulées suivant deux méthodes. La première est une simple approche mécanique basée sur l'échange d'énergie avec le fond du canal. L'autre est basée sur la théorie cinétique des écoulements granulaires. Des solutions pour les écoulements granulaires en chute libre ont été étudiées afin de comparer les deux méthodes. Le prolongement de la théorie cinétique présenté ici tient compte des forces de trainée causées par le fluide interstitiel qui amortit les collisions entre ces dernières et entre les particules et le fond du canal. Les contraintes de frottement produites lors de contacts prolongés et les fluctuations turbulentes du fluide sont introduites dans le modèle.

Les résultats sont comparés avec les mesures effectuées dans un canal expérimental où la pente, le débit solide et le débit liquide ont été tous modifiés. La théorie donne une bonne description qualitative du comportement observé. Finalement une application du modèle est entreprise afin de décrire le comportement mécanique de l'écoulement de boue du Nevado del Ruiz en 1985.
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### NOMENCLATURE

- **a**  
  Bagnold's stress coefficient

- **A**  
  density ratio (fluid/solid)

- **a_R**  
  Reynolds's pressure drop coefficient

- **A_i**  
  coefficient of the slip velocity (i=j and r)

- **b**  
  body force

- **B**  
  factor of correction to the Maxwellian velocity distribution function

- **B_i**  
  coefficient of the slip velocity (i=j and r)

- **Ba**  
  Bagnold number

- **Ba_v**  
  Bagnold number evaluated near the wall

- **Bch**  
  Chen's constant for consistency indexes

- **b_R**  
  Reynolds's pressure drop coefficient

- **c**  
  cohesion

- **CD**  
  drag coefficient

- **C_{h(v)}**  
  apparent viscosity factor

- **C_i**  
  coefficient of the slip velocity (i=j and r)

- **C_h**  
  Chen's constant for relative viscosity

- **C_r**  
  bed friction factor

- **d**  
  particle diameter

- **d_p**  
  effective particle diameter (for non-uniform particles)

- **D**  
  drag force

- **D_c**  
  collisional energy dissipation at the wall

- **D_v**  
  viscous drag energy dissipation at the wall

- **D_0**  
  Iverson's reference deformation rate

- **e_p**  
  particle-particle coefficient of restitution

- **e_w**  
  particle-wall coefficient of restitution

- **E**  
  Young's modulus of elasticity

- **E_1**  
  elasticity parameter

- **F**  
  specific fluid friction coefficient

- **Fr**  
  coefficient of the solids frictional stress

- **g**  
  acceleration of gravity

- **G_{i}**  
  functions of v and e_p (i=1,2,......14)

- **h**  
  flow depth

- **h^***  
  nondimensional flow depth

- **I**  
  identity matrix

- **Kr**  
  wall geometry factor (for Richman's boundary conditions)

- **K_{ph}'**  
  conversion factor from Richman's method to p' method

- **K_r**  
  proportionality constant for Eddy viscosity

- **l**  
  mixing length

- **L**  
  length scale

- **m**  
  particle mass

- **M**  
  rate of momentum tensor

- **M_{xv}**  
  streamwise component of M

- **n**  
  unit normal vector

- **P**  
  rheological power-law index

- **P**  
  Poisson's ratio for material of colliding spheres
\( p \)  
pressure

\( \mathbf{q} \)  
flux vector of fluctuation energy

\( q_f \)  
flux of fluctuation energy \( (q_f) \)

\( \mathbf{q}^* \)  
nondimensional flux of fluctuation energy, \( \mathbf{q}/\rho_s(gdcos\theta)^{3/2} \)

\( q_{w} \)  
flux of fluctuation energy near the wall

\( Q_w \)  
experimental water flow rate

\( Q_s \)  
experimental solids flow rate (mass flow rate)

\( r \)  
Hui's dimensionless constant

\( \text{Re} \)  
flow Reynolds number

\( \text{Re}_l \)  
particle Reynolds number, \( \rho_d/(gdcos\theta)/\mu_r \)

\( \text{Re}_f \)  
modified particle Reynolds number \( f(U_r) \)

\( \text{Re}_p \)  
particle Reynolds number for fluidized beds

\( s \)  
interparticle spacing

\( \mathbf{g} \)  
deviatoric stress tensor

\( S_c \)  
collisional force per unit area at the boundary

\( S_l \)  
Stokes number

\( T \)  
granular temperature

\( T_w \)  
granular temperature at the wall

\( T^* \)  
nondimensional granular temperature, \( T/gdcos\theta \)

\( T_k \)  
Takahashi's parameter

\( T_l \)  
granular temperature parameter for locked grain-fluid flow

\( \mathbf{u} \)  
vector representing the statistical average of the mixture velocity

\( u \)  
statistical average of the mixture velocity in the streamwise direction at depth \( y \)

\( u_s \)  
slip velocity at the wall

\( u^* \)  
nondimensional mixture velocity, \( u/(gdcos\theta) \)

\( u' \)  
fluctuation velocity

\( u_{\text{avg}}' \)  
mean fluctuating velocity

\( u_{l} \)  
velocity in the bulk of the mixture (locked flow)

\( U_{\text{avg}} \)  
cross sectional average velocity

\( U_f \)  
most probable relative speed between the particle and the fluid in the neighbourhood

\( u_f \)  
friction velocity

\( U_s \)  
superficial fluid velocity in fluidized beds

\( w \)  
channel width

\( x_0 \)  
initial distance between two colliding spheres

\( y \)  
distance from bed

\( y_{l} \)  
position at which the flowing grain-fluid locks

\( y^* \)  
nondimensional distance from bed, \( y/d \)

\( \gamma \)  
collisional rate of energy dissipation

\( \alpha \)  
Shen's factor for particle diameter enlargement

\( \beta \)  
Bagnold's bed load transport constant

\( \delta \)  
friction angle between the particles and the wall

\( \zeta \)  
Shen's solids friction

\( \epsilon \)  
Eddy viscosity

\( \theta \)  
chute angle

\( \kappa \)  
Von Karman's constant

\( \lambda \)  
linear concentration, \( d/s \)
\( \lambda_1 \) exponent of grain temperature profile (locked flow)

\( \mu_f \) fluid viscosity

\( \mu_{fa} \) apparent fluid viscosity

\( \mu_0 \) equivalent Newtonian viscosity

\( \mu_1 \) Chen's consistency index

\( \mu_2 \) Chen's cross-consistency index

\( \nu \) solids volume concentration

\( \nu_{avg} \) average cross sectional concentration

\( \nu_{max} \) closest packed concentration

\( \nu_{min} \) concentration below which frictional stresses vanishes

\( \nu_0 \) kinematic viscosity of water

\( \rho_f \) fluid density

\( \rho_s \) solids density

\( \rho_m \) mixture density, \( \nu \rho_s + (1-\nu) \rho_f \)

\( \sigma_i \) stress tensor (where i is c,f,v,t to represent collisional, frictional, viscous and turbulent stresses)

\( \sigma_{1} \) normal stress (where i is c,f,t to represent collisional, frictional, and turbulent)

\( \sigma_{xy} \) total normal stress in the y direction

\( \sigma \) diameter of wall particles

\( \tau_{xy} \) total shear stress in the streamwise direction

\( \tau_1 \) shear stress (where i is c,f,v or t to represent collisional, frictional, viscous and turbulent stresses)

\( \Delta p \) pressure loss

\( \phi \) solids internal friction angle

\( \phi_d \) dynamic angle of internal friction

\( \phi' \) specularity coefficient

\( \psi \) angle between wall particles and flow particles (Richman)

\( \omega \) Hui's coefficient for the slip velocity

\( \partial u/\partial y \) shear rate

\( (\partial u/\partial y)_w \) shear rate evaluated near the wall
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CHAPTER 1

INTRODUCTION, REVIEW OF PREVIOUS WORK

1.1 INTRODUCTION

Fluid-particle two-phase flow is a branch of fluid mechanics that has seen considerable development in the last two decades. Many parameters are important in this kind of flow. Several flow regimes can occur, depending on the kind of interactions between the particles and the fluid and between the particles themselves, the solids concentration and properties, the rate of deformation and the viscosity of the interstitial fluid.

There is a substantial literature that tries to describe the shear flow of a fluid-particle suspension taking into account separately the fluid and particles velocities, with their local average and random components. Following this approach, a large variety of interactions that depend on the two velocity fields must be included. For rigid non-cohesive particles immersed in a viscous interstitial fluid, the main interactions are the following:

(1) Interaction between particles and fluid that results from the difference in their mean velocity fields, and gives rise to the drag force that propels the non-random part of the particle motion.
(2)- Interaction of the particles with the fluctuating component of the fluid velocity. This may cause a flux of kinetic energy in either direction between the fluctuating components of velocity of the two phases.

(3)- Interaction of the fluctuating part of the particle motion with the mean particle motion through interaction forces between particles. These generate stresses in the particle assembly and give rise to its apparent viscosity.

(4)- Interactions between the turbulent fluctuations of fluid velocity and the mean motion of the fluid, which generate the well known Reynolds stresses.

A treatment including all the above effects is unlikely to be practicable without many arbitrary assumptions, especially in view of the incomplete understanding of the structure of turbulence. However, if we restrict the model to some range of particle sizes, concentrations and flow conditions characterized by the particle Reynolds number \( (Re_p) \) and the Bagnold number \( (Ba) \) that are measures of the fluid and particle inertia forces respectively over fluid viscous forces, the treatment becomes more practicable.

For the case of low particle concentrations or relatively large concentrations but small particle size and relatively high fluid viscous forces compared with the inertia forces (low particle Reynolds number and Bagnold number < 40), the fluid phase governs the dynamics of the system and the first, second and fourth kind of interactions are important. The liquid acts as a buffer which prevents direct contacts between the solid surfaces. This dilute suspension of solid particles has a Newtonian behaviour with a viscosity slightly larger
than that of the interstitial fluid. An interesting study on this kind of flow regime is that of Batchelor (1972).

With further increases in the concentration or in the particle size resulting in an increase of the particle Reynolds number and the Bagnold number but under moderately low shear rates (40 < Ba < 450; these limits will be discussed later on), the third kind of interaction also becomes important. This means that the particle interactions have to be taken into account. This is the most controversial flow regime. The rheological behaviour of a suspension in this regime is far from being completely defined. Some authors (Shibata and Mei 1986) conclude that the mixture still has a Newtonian behaviour and that the stress tensor is linear in the shear rate. Others consider a yield strength term (Pierson 1987). The material will not flow until a minimum shear stress has been exceeded. The difficulty comes from the description of the interactions between particles. The recent work of Coussot (1992) makes a step ahead in this direction by proposing a rheological model that includes three different kinds of interactions between particles. The direct contact, the lubricated contact and interaction without contact. When they are close enough, the direct contact is of the frictional type because under low shear rates particles experience long term contacts. The lubricated contact is present when the particles are separated by a fluid layer lower than the particle diameter. The non-contact interaction between particles occurs when the fluid layer between them is larger than the particle diameter.

When the Bagnold number and the particle Reynolds number are relatively high, the third kind of interactions play a dominant role in the dynamics of the flow. This flow regime can be found when highly concentrated mixtures of water and particles of large size flow under high shear rates. In this flow regime most of the direct contacts are of short duration
(collisions). The larger and heavier particles have a greater momentum able to carry them easily through the intervening fluid. When the viscosity of the fluid is non negligible but the Reynolds number is in an intermediate range (less than 10,000, we will discuss this limit later), the mean velocity in the streamwise direction of the particle phase becomes similar to that of the fluid and the first and second kind of interactions become less important. This research will develop a model restricted to this kind of flow regime, constituted by solid particles of enough size and concentration, under high shear rates so that the Bagnold number becomes larger than 450. This limit was defined by Bagnold (1954) and will be explained in the following paragraph.

1.2 BAGNOLD'S WORK

The first researcher to show experimentally the importance of the collisional interactions in a fluid-particle mixture was Bagnold (1954). He performed experiments with dispersions of solid spherical grains (1.3 mm uniform diameter) that were sheared in Newtonian fluids of varying viscosity in the annular space between two concentric drums. He proved experimentally the existence of a radial dispersive pressure exerted between the grains. This was measured as an increase of static pressure in the inner stationary drum which had a deformable periphery. The torque in the inner drum was also measured to determine the shear stress that was found to be proportional to the dispersive pressure. He also proved that in this dispersion of spherical grains, the shear stress varies with the square of the shear rate, for values of a modified Reynolds number, that subsequent researchers called Bagnold number (Ba), larger than 450. Bagnold called this flow regime the grain-inertia flow regime. He found that for Ba<40 the flow is in the macroviscous regime where viscous stresses are dominant and the shear stress is proportional to the shear
rate. Between these two limits there is a transition regime.

The Bagnold number is defined in the following way

\[ \text{Ba} = \frac{\rho_s \lambda d^2 \left( \frac{\partial u}{\partial y} \right)}{\mu_f} \] ................................. (1.1)

where \( \rho_s \) is the density of the particles, \( \mu_f \) is the interstitial fluid viscosity, \( d \) is the particle diameter, \( \partial u/\partial y \) the shear rate and \( \lambda \) is the linear concentration, defined as the particle diameter over the mean free dispersion distance. For spherical particles of the same diameter it is

\[ \lambda = \frac{1}{\left( \frac{v_{\text{max}}}{v} \right)^{1/3} - 1} \] .................................................. (1.2)

where \( v \) is the solids concentration by volume and \( v_{\text{max}} \) is the closest packed concentration.

For the fully inertial range, from his experiments as well as from theoretical considerations, Bagnold found the following semi-empirical relation for the shear stress caused by dispersive pressures that we will call collisional shear stress \( \tau_c \)

\[ \tau_c = a \sin \phi s \rho_s \lambda^2 d^2 \left( \frac{\partial u}{\partial y} \right)^2 \] ................................................. (1.3)
where $\phi_d$ is the dynamic angle of internal friction and $a$ is an empirical constant that Bagnold evaluated as $a=0.042$ for his experimental conditions. The normal stress is $T_0 \tan \phi_d$. He used water and mixtures of glycerine-water-alcohol as the interstitial fluids and neutrally buoyant particles made of paraffin wax and lead stearate.

1.3 RESEARCH AFTER THE WORK OF BAGNOLD

1.3.1 Gravity-induced mass movements

One of the fields of research where Bagnold’s concepts could be applied is that of gravity-induced mass movements. Even if there is controversy about the classification of gravity-induced sediment-water flows, it is generally accepted that the dispersive stresses are dominant in the so-called debris flows. The origin of the controversy is the definition of the limit between a mixture of water and sediment that possesses a measurable yield strength but that still appears to flow like a liquid (some researchers call this kind of flow hyperconcentrated streamflow (Pierson et al. 1987)) and the mixture where an abrupt increase in resistance to flow is observed. The latter flow is called debris flow. A debris flow is a mixture of water, fine solids, rocks and boulders flowing under high shear rates. In this flow the increase in resistance to flow is attributed to the onset of internal friction mainly caused by the interparticle interactions. Constitutive equations have been developed separately for hyperconcentrated flows and debris flows. The former have the form of viscoplastic models, such as that of Bingham which considers a yield strength and a linear dependence between the stress and shear rate. The latter are essentially dilatant type models that exhibit an increase in viscosity with the
shear rate. Some researchers argue that debris flows are not always in the grain-inertia flow regime and the dilatant type models are not accurate to model debris flows (Chen 1987, Coussot 1992).

Attempts has been made to formulate constitutive equations valid for the entire range of debris-flow regimes. Iverson (1985) proposed a constitutive equation for mass movement behaviour. His rheological model is based in a simple mechanical system with one-dimensional stress-deformation-rate behaviour analogous to that of soil. The friction slider is a rigid block that maintains the applied deviator stress if the yield stress is not exceeded. A nonlinear dashpot provides viscous resistance if the yield stress is exceeded and motion occurs. Iverson's rheological model include a plastic yield component that accounts for a linear dependence of the shear strength on the confining pressure. Also his nonlinear flow component simulated by a power-law equation (to include different flow regimes) introduces a rate-dependent viscosity. The two-dimensional form of the equation is

$$\tau_{xy} = \left( \frac{1}{2\sigma_0} \right)^{n-1} \mu_0 \left( \frac{\partial u}{\partial y} \right)^{\frac{1}{n}} + \sigma_{yy} \tan \phi \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1.4)$$

where $\tau_{xy}$ is the shear stress, $\sigma_{yy}$ is the normal stress, $c$ is the cohesion, $\phi$ is the solids internal angle of friction, $n$ is the power-law index, $\sigma_0$ is a reference deformation rate, and $\mu_0$ is the equivalent Newtonian viscosity observed during material deformation at the deformation rate of $\sigma_0$. The viscosity of the mixture is $\mu = \mu_0 \left( (\partial u/\partial y)/2\sigma_0 \right)^{(1-n)/n}$. The model is very flexible and by varying $n$, we can obtain many rheological behaviours. However the values of $n$, $\mu_0$ and $\sigma_0$ must be obtained experimentally. This limits its application as a
predictive model.

Chen (1988) formulated a generalized viscoplastic model for debris flow. He used two equations, one for the shear stress and the other for the normal stress to fully describe the normal stress effect. His equations are

\[ \tau_{xy} = c_1 \cos \phi + p \sin \phi + \mu_1 \left( \frac{\partial u}{\partial y} \right)^n \]  

\[ \sigma_{yy} = -p + \mu_2 \left( \frac{\partial u}{\partial y} \right)^n \]  

where \( \mu_1 \) and \( \mu_2 \) are the consistency and cross-consistency indexes, \( p \) is the pressure and the other parameters are similar to those of Iverson. Chen argues that \( \mu_1 \) and \( \mu_2 \) are functions of concentration in a form similar to that proposed by Einstein for the relative viscosity of rigid uniform spherical particles, proportional to \((1 - c_0 v)^{-2B_{eh}/c_0}\), where \( c_0 \) and \( B_{eh} \) are undefined constants. When the factors \( B_{eh}/c_0 = 0 \) and \( n = 2 \), the model is similar to Bagnold's. He obtained velocity profiles for uniform, steady flows and found that for values of \( B_{eh}/c_0 = 1.75 \) and \( n = 2 \) the model improves Bagnold's solutions when applied to predict Takahashi's experimental data. The usefulness of the model for predictive purposes is limited by the difficulty in choosing appropriate values of the parameters \( B_{eh}/c_0 \) and \( n \).

Coussot's (1992) debris flow model is inspired by the work of Batchelor (1972) for the definition of the stress tensor assuming laminar flow. To define the kinematics of the flow he used the results of Adler (1985) but included the local
velocity gradient tensor and local relative rotations. The introduction of the lubricated contacts with the proper definition of the forces where the compression effect is dominant in the energy dissipation and when the sliding effect is important are the main contributions of this work. The model is applicable for debris flows outside the grain-inertia flow regime.

An approach widely used among researchers of debris flows is to assume that they are in the grain-inertia flow regime. Kanatani (1980) developed a model in which particle collisions play a dominant role. He developed a thermodynamic analogy. His constitutive equation shows a shear stress dependence on the square of the shear rate. Ogawa (1980) introduced the quasi-thermal energy as a new variable and formulated equations to describe the motion of granular materials. His constitutive equation also shows stresses to depend on the square of the shear rate. Tsubaki et al. (1983) investigated experimentally particle-particle interactions in a solid-liquid shear flow. By means of a video camera, they investigated the contact angle during collisions. They were able to separate different modes of grain-grain interaction. Most of the grains in the flow collide at the upstream quadrant on the surface of the relatively lower grains, and then override them until the grains separate at the downstream quadrant. The stresses arising from such interactions are conveniently divided into collisional stresses and contact stresses respectively, due to the forces acting at the grain collisions and during the subsequent relative movements. In this model the shear stress depends on the square of the shear rate, but the normal stress differs from that of Bagnold by the introduction of a contact pressure term. When the model is applied to Takahashi's experiments, better agreement is obtained than with Bagnold's model (Chen 1987). The main limitation of the model is the neglect of the interstitial
10 fluid effect on energy dissipation.

1.3.2 Takahashi's work on debris flows

Takahashi (1990) revised the previous granular-fluid work in the grain-inertia flow regime and used a generalisation of Bagnold's equations to calculate debris flow velocities down a chute. He was able to compare the analytical results with experimental chute flows. According to his experimental observations, he defined four flow conditions: stony flow, immature flow, hybrid flow and turbulent flow. In a stony flow, the main mechanism of stress generation is the collisional interaction between particles that generates the dispersive pressures. The immature flow is a stony flow with most of the particles concentrated in the lower part of the flow. Above this particle mixture layer, a water layer appears which may contain suspended sediment. The hybrid flow is a turbulent dominated flow with a small layer near the wall where the particles are supported by dispersive pressures. The turbulent flow involves a large turbulent stress generated by the large-scale turbulent mixing of the fluid mass incorporated with the solid and the water. Here, we will describe the equations formulated by Takahashi for flows in two regimes only, because the analytical treatment of the others is based on stony flow and turbulent flow.

Takahashi assumed constant concentration and solved the momentum equation of the stony debris flow analytically. The mixture constitutive equation used by him is similar to Bagnold's equation (1.3). The shortcoming of his model is that the Bagnold constant "a" must be determined experimentally. However, we showed that more elaborated theories have also similar shortcomings. The usefulness of Takahashi's work is the simplified form of the equations together with the
experimental validation. For uniform steady flow down a chute inclined at an angle $\theta$ with the horizontal, assuming constant concentration $v_{avg}$ and no slip boundary condition, he found the following relationship for the average cross-sectional velocity in a stony debris flow

$$U_{avg} = \frac{2}{5d} \sqrt{\frac{g \sin \theta}{\rho_s}} \left( v_{avg} + (1-v_{avg}) \frac{v_{max}}{\rho_s} \right) \left[ \left( \frac{v_{max}}{v_{avg}} \right)^{1/3} - 1 \right] h^{3/2} \quad (1.7)$$

For the turbulent flow, he considered the total shear stress as the sum of the collisional $\tau_c$ (equation 1.3) and the turbulent $\tau_t$, defined as

$$\tau_t = \rho_l l^2 \left( \frac{du}{dy} \right)^2 \quad (1.8)$$

where $l=\kappa y$ is the mixing length and $\kappa$ is the Karman constant. After integration of the momentum equation considering the concentration constant, he found the following equation for the average cross-sectional velocity in a turbulent flow

$$U_{avg} = \frac{\sqrt{gh \sin \theta}}{\kappa} \left[ \ln \left( \frac{1+\sqrt{1+T_k}}{Z_0+\sqrt{Z_0^2+T_k}} \right) - \sqrt{1+T_k+\sqrt{T_k}} \right] \quad (1.9)$$

where $Z_0 = v_0/(9.025h/(gh \sin \theta))$, $v_0$ is the kinematic viscosity of water and $T_k$ is given by
1.3.3 Analytical and experimental research on granular-fluid mixtures under high shear rates

A first theoretical attempt to include the effect of the interstitial fluid in a model based on collisional interactions was made by Shen and Ackermann (1982). They established constitutive relationships that describe the shear and normal stresses as a function of the velocity gradient $\partial u/\partial y$, the solids friction $\zeta$, restitution coefficient $e_p$, the fluid drag coefficient $C_D$ for the particle shape and the density of the solid $\rho_s$ and fluid $\rho_f$. At first, they considered the model of a two particle collision, which resulted in too small a coefficient in comparison with the prior experiments. Then they modified the model for the case of multiple particle collisions by introducing a hypothetical larger particle diameter. The constitutive equations obtained are the following:

\[ \tau_{xy} = \frac{v_{\text{max}}}{2} \lambda ( \frac{1}{1+\lambda} )^{3/2} f_1(e_p, C_D, \zeta) \rho_s (\alpha d)^2 (\frac{\partial u}{\partial y})^2 \] \hspace{1cm} (1.11)

\[ \sigma_{yy} = -\frac{v_{\text{max}}}{\pi^2} \lambda ( \frac{1}{1+\lambda} )^{2} f_2(e_p, C_D, \zeta) \rho_s (\alpha d)^2 (\frac{\partial u}{\partial y})^2 \] \hspace{1cm} (1.12)

Where $f_1 = (1+e_p)^{3/2} (0.05+0.08\zeta)^{3/2} / (\text{den})^{0.5}$, $f_2 = (1+e_p)^2 (0.05+0.08\zeta) / \text{den}$, and den is defined as
Shen’s constitutive equations produce stresses similar to the experimentally reported by Bagnold (1954) and Savage and Sayed (1984), with a value of the coefficient $\alpha=3.16$. Shen et al. argue that this factor accounts for the formation of particle groups when more than two spheres are simultaneously in the collision mode. The difficulty in the determination of this parameter $\alpha$ is a shortcoming of the theory.

Daido (1979) manufactured an apparatus similar to Bagnold’s and experimented using various size materials ranging from 2.95 mm to 5.85 mm. According to his results, the relationship between shear rate and shear stress exhibits a yield stress for $\partial u/\partial y = 0$. The proportionality of the shear stress to the square of the shear rate begins at $Ba=1000$. Daido’s results are in general similar to those of Bagnold but the coefficient "a" was smaller than that of Bagnold by about a factor of ten.

Good experimental data were obtained in an annular, parallel-plate shear cell by Hanes and Inman (1985). This shear cell, constructed after the design of Savage and Sayed (1984), allows for measurements of the shear and normal stress at a measurable volume concentration and shear rate. They were able to confirm the quadratic dependence upon the mean shear rate (at a constant volume concentration larger than 0.5) for sufficiently high shear rates. They used spherical glass beads and sand. They compared the results with the kinetic theory for granular flow that will be explained later, for smooth, slightly inelastic particles, and found an underprediction of the shear stress.
1.4 THE KINETIC THEORIES FOR GRANULAR FLOW

Chute flow solutions based on a model valid for all the sediment-water flow regimes are still difficult to obtain because of the uncertainties in the definition of the many parameters involved. Since most of the researchers agree that fluid-solid mixtures at high shear rates and high concentrations are more likely to be in the grain-inertia flow regime, in this research we are going to limit the scope of the model to this flow regime. The way to proceed is by the formulation of a strong, highly reliable collisional model that includes the effect of the interstitial fluid in the energy exchange. The kinetic theory for granular flow appears to be the appropriate framework for the formulation of this model. The kinetic theory for granular flow was first formulated by Savage and Jeffrey (1981). This theory makes use of statistical mechanics methods analogous to those used in the kinetic theory of gases to calculate the stresses generated in a dry granular flow. The basic assumption of this theory is that the collisions between particles provide the principal mechanism for the transport of the properties such as momentum and energy. The major difference with the kinetic theory of dense gases is that collisions between granular particles are inelastic. This implies that energy dissipation due to inelastic collisions plays an important role in the mechanism of fluctuation energy balance. The kinetic theories for granular flow had a fast development during the last decade. The work of Lun et al. (1984) established the constitutive equations for smooth, slightly inelastic particles in an infinite flow field. One of the difficulties in solving the granular flow equations in real shallow flows is that in general there is a velocity slip at the boundary.
This performs shear work on the flow, generating fluctuation energy. Collisions of particles with the wall boundary dissipate energy at the same time. Thus, the boundary conditions can not be obtained independently of the flow field. Researchers such as Johnson and Jackson (1987) developed boundary conditions to apply the kinetic theories to Couette and chute flows. Richman (1988) improved the boundary conditions calculating the rate at which momentum and energy are transferred across bumpy boundaries. They were able to obtain conditions that ensure the balance of momentum and energy at such boundaries. Johnson et al. (1990) included a frictional component to the stress tensor to take into account the rubbing and sliding of particles with its neighbours when the concentrations are high. Their simple approach adds the frictional stress to the collisional one. A better approach to include surface friction was used by Lun and Savage (1987). The theory includes a parameter that takes into account the roughness of the particles in the collisional part of the constitutive equations. This is valid for high concentrations, where the kinetic contribution to the stress tensor is lower than the collisional one. Recently, Lun (1991) extended this kinetic theory to account for the roughness of the particles in the kinetic and collisional components of the stress tensor and was able in this way to include the dynamics effects of particle surface friction and rotary inertia.

1.5 WORK IN PRESENT THESIS

1.5.1 Analysis to include interstitial fluid effect

The purpose of this research is to develop a theoretical model which describes the transfer mechanisms of momentum and energy for the case of a grain-fluid mixture in the grain-inertia regime. The kinetic theory for granular flow will be used as a framework. But, in order to extend this kinetic theory for
a granular-fluid mixture, this research will include the
effect of the interstitial fluid in the cushioning of the
collisions. This has an important effect on the kinetic energy
transfer. This effect will be introduced by means of a proper
definition of the drag force exerted by the fluid when the
particles fluctuate. To achieve this, it will be necessary to
define a proper drag coefficient. Research conducted by Ergun
(1952), on fluid flow through packed beds, has shown that the
drag force depends on the concentration and on the particle
Reynolds number. The dependence on concentration can be
derived from the concept of apparent viscosity first defined
by Einstein (1906). The work of Frankel and Acrivos (1967) was
an attempt to define the apparent fluid viscosity for a
suspension of uniform spheres in the limit as the
concentration approaches its maximum value. Batchelor (1972)
deﬁned an apparent viscosity for dilute suspensions. One of
the most recent works in this ﬁeld is that of Kamal et al.
(1985). However, there is still not universal agreement about
the law governing the variation of viscosity with
concentration. It has been proved experimentally (Utracki
1988) that the viscosity of the suspension tends to inﬁnity
when the concentration tends towards the closest packed
concentration. Lun and Savage (1992) proposed a drag
coefficient that varies with the Reynolds number and the
solids fraction. The ﬂuid turbulent stresses will be included
in the model to represent more accurately the ﬂows at
relatively low concentrations. The model will use the Prandtl
mixing length theory and will take into consideration the
effects of the particles on the turbulence.

1.5.2 Experiments

The theory will be validated by experimental measurements of
sand-water ﬂows down a chute. For the case of the chute ﬂow,
more accurate experimental results are available for dry
wall but it is not able to provide information inside the
flow. Some researchers in the field of slurry pipe flow (Brown
et al., 1982) developed an electromagnetic probe capable of
taking point velocities in every section of the flow.

We will use a rectangular channel of variable slope with a
hopper placed at the upper end to supply the granular
material. The side walls are clear to allow direct visual
observation. An 8 mm video camera was used to determine the
velocity profiles near the wall and point probes were used to
determine the water depth. The kind of probe used by Brown et
al. (1982) was difficult to use because of the very shallow
flows of only a few millimeters in the present experiments.

It should be noted that a kinetic theory for granular-fluid
mixtures would have applications in many fields: the
industrial handling of solids and slurries; the transport in
pipelines of solids in suspension; in some chemical processes
involving fluidized beds; and in geophysical flows such as bed
load in rivers or debris flow which causes great damage to
lives and property. The results of the present research
provide a contribution to the understanding of the kinetic
energy transfer in a debris flow which is a subject of great
controversy.
CHAPTER 2

ANALYTICAL BACKGROUND

2.1 CONSTITUTIVE EQUATIONS ACCORDING TO KINETIC THEORIES FOR GRANULAR FLOW

Based on the basic ideas of Bagnold on the importance of collisional interactions in the grain-inertia regime and the concept of mean velocity and fluctuation velocity of the particles in motion, some researchers exploited the analogy with the kinetic theory of gases. They developed, during the last decade, the kinetic theory for granular flow (Savage and Jeffrey 1981, Lun et al. 1984). The kinetic theory for granular flow is based on the assumption that, under certain flow conditions, collisions between particles provide the principal mechanism for the transport of properties such as momentum and energy. Some of the statistical mechanical methods developed for gases have been used in the analysis of granular flow. The main difference in the case of granular flows is the need to include energy dissipation that occurs during collisions. To follow with the analogy, a term representing the fluctuation velocity of the particles was called the granular temperature $T$ and is defined as

\[ T = \frac{1}{3}(\langle u'^2 \rangle + \langle v'^2 \rangle + \langle w'^2 \rangle), \]

where $u'$, $v'$ and $w'$ are the three velocity fluctuation components. Again the difference with the case of gases is that in a granular flow, shear work is needed
to provide the energy necessary to maintain the velocity fluctuations.

Lun et al. (1984) developed a kinetic theory for the flow of an idealized granular material consisting of smooth, slightly inelastic, spherical particles of uniform diameter d. Their equations are valid for an infinite flow field over the entire range of solids fraction, provided that the contacts are of short duration. The following equation for the total collisional stress tensor $\mathbf{g}_s$ was proposed by them

$$
\mathbf{g}_s = (\rho_s g_1 T - \rho_s d \frac{\sqrt{2}}{3} v^2 g_0 \sqrt{T} \cdot \mathbf{u}) \mathbf{I} - 2 \rho_s d g_2 \sqrt{T} \mathbf{G} \quad \text{.........(2.1)}
$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{g} = 0.5 (u_{i,j} + u_{j,i}) - u_{k,k} \delta_{i,j}$ is the shear rate tensor and $\mathbf{u}$ is the vector representing the statistical average of the particles velocity. The parameter $e_p$ is the coefficient of restitution for collisions between particles, and $g_0$, $g_1$ and $g_2$ are functions of the concentration that will be defined later.

The flux vector of fluctuation energy $\mathbf{g}$ is

$$
\mathbf{g} = -\rho_s d g_3 T^\frac{1}{2} \nabla T \quad \text{.................................................(2.2)}
$$

The collisional rate of dissipation per unit volume is:
\[ \gamma = \frac{2g_0}{d} \tau^3 \] (2.3)

Here \( g_1, g_2, g_3 \) and \( g_5 \) are functions of \( v \). The corrected first order expressions following Jenkins (1987) are given as follows:

\[ g_1 = v + 4v^2 g_0 \] (2.4)

\[ g_2 = \frac{5\sqrt{\pi}}{96} \left[ \frac{1}{g_0} + \frac{16v}{5} + \frac{64\nu^2 g_2}{25} (1 + \frac{12}{\pi}) \right] \] (2.5)

\[ g_3 = \frac{25\sqrt{\pi}}{128} \left[ \frac{1}{g_0} + \frac{24\nu}{5} + \left( \frac{144}{25} + \frac{512}{25\pi} \right) v^2 g_0 \right] \] (2.6)

\[ g_5 = \frac{24}{\sqrt{\pi}} (1 - e_p) v^2 g_0 \] (2.7)

The radial distribution function at contact, \( g_0 \) is chosen as suggested by Lun and Savage (1987), as follows:

\[ g_0 = \left( 1 - \frac{v}{v_{\text{max}}} \right)^{2.5v_{\text{max}}} \] (2.8)

where \( v_{\text{max}} \), the closest packed concentration, is the maximum shearable solid fraction for spherical particles.
2.2 COLLISIONS OF RIGID SPHERES IN A FLUID

Some research has been done recently on the collisions of spheres immersed in a viscous fluid. The first research to analyze the phenomenon of rebound was that of Davis et al. (1986). Their reasoning was based on a balance of energy. They argued that when physical contact occurs, the incoming kinetic energy of the particles is converted into elastic strain energy as they deform in the vicinity of the contact. The kinetic energy (other than that dissipated by the internal friction or remaining as elastic vibrations) is restored as the particles rebound. The sticking probability is zero when the energy dissipated in the solids and the fluid is negligible. By simultaneously accounting for elastic deformation and viscous fluid forces, they determined the range of conditions for which significant deformation and rebound of colliding spheres occurs when viscous forces are important. They were able to relate the ratio of rebound velocity \( u_{reb} \) to the initial velocity \( u \) (that can be thought of as a coefficient of restitution only it measures the energy dissipated in the fluid rather than in the solid material) to two parameters: the Stokes number \( S_t \) and the elasticity parameter \( E_I \). The Stokes number is a measure of the inertia of the particles relative to the viscous forces

\[
S_t = \frac{mu}{\frac{2}{3} \pi \mu_f d^2} \tag{2.9}
\]

where \( m \) is the mass of the particle and \( u \) the velocity of
approach. The elasticity parameter provides a measure of the tendency of the solids to deform. Its value must be small compared to unity in order for the deformation to be small. It is defined as

\[
E_l = \frac{4\mu \mu (\frac{E}{P})^{3/2}}{\pi E} \frac{2(1-P^2)}{x_0^{5/2}} \text{ ............................................... (2.10)}
\]

where \(P\) is Poisson’s ratio of the spheres, \(E\) is Young’s modulus of elasticity and \(x_0\) is the initial distance between spheres.

They found that the maximum rebound velocity increases as both \(E_l\) and \(S_t\) increase. Typical lower limits of rebound were found: \(S_t=1\) and \(E_l=0.01\), \(S_t=8\) and \(E_l=10^{-6}\).

Later work of Barnocky et al. (1989) studied the influence of the increase in the density and the viscosity of the interstitial fluid due to the high magnitude of the pressures that develop during collisions. They found that although there is a small enhancement of rebound due to fluid compression, there is a much greater effect on the collision between elastic spheres as a result of the increase in viscosity with pressure. For a Stokes number in the range of 4–6 and \(E_l\) in the range of \(10^{-4}-10^{-8}\), the ratio \(u_{reb}/u\) increases by 20%. This enhancement is primarily due to the sharp increase in viscosity near the centerline of the two spheres, which serves to strongly resist their relative motion and to cause additional elastic deformation. The marked increase of viscosity with pressure results in a fluid which behaves much like a solid and prevents the nose of the sphere from approaching further. The fluid near the centre is nearly trapped in a dimple formed around the axis of symmetry; the
Further work of Kytomaa and Schmid (1990) related the collisional parameters of two spheres to those used for granular flows. They found a relationship between the Bagnold number and the Stokes number

\[ S_t = \frac{4}{9 f(v)} B_a \]  \hspace{1cm} (2.11)

where \( f(v) \) is given by the following relationship

\[ f(v) = \frac{4}{\sqrt{2}} \sqrt{\frac{v}{v_{\text{max}}} \left(1 - \left(\frac{v}{v_{\text{max}}} \right)^{1/3}\right)} \] \hspace{1cm} (2.12)

They confirmed the results of Barnocky et al. (1989) that for values of the Stokes number larger than 0.5 rebound occurs.

These notions help us to define the limit of validity of the equations developed in this research. It is clear that a situation of no rebound can not be represented by this model. The occurrence of rebound can be fully determined by the Stokes number when the elasticity parameter is very small (this is the case of rigid particles). The Stokes number is a ratio of inertial forces over viscous forces, as are also the Bagnold number and the particle Reynolds number. In Chapter 6, the way in which \( S_t \) can be related to the parameter used in the numerical model called the particle Reynolds number \( \text{Re}_i \) will be discussed. The viscous dissipation term is a function of a modified Reynolds number as will be shown in Chapter 3.
2.3 GOVERNING EQUATIONS.

Taking into consideration the ideas just discussed, an analytical model for the flow of a mixture of grains and fluid down a chute will be proposed in this chapter. The model is based on the assumption that the flow resisting forces are of four types: collisional, as a result of short term interactions between particles of adjacent layers; frictional interactions between particles that become important at high solids concentration as a result of long term contacts of the particles; viscous, due to the interaction between adjacent layers of fluid; and turbulent, due to the interactions between the turbulent fluctuations of fluid velocity and the mean fluid motion. The range of applicability of the model is limited to flows where inertial forces are high compared to viscous forces ($Re_t > 100$) and where particle size and concentration are high so that collisional stresses are significant ($Ba > 450$). This range was defined by Bagnold (1954) as the grain-inertia regime. The collisional stresses will be adopted from the kinetic theory for granular materials, especially from the work of Lun et al. (1984).

The model will consider the mean velocity of the fluid and the statistical average velocity of the particles in the streamwise direction as identical. This assumption is only valid for the characteristics of the flow and the solids that we are considering. The equations will be formulated as a function of the velocity of the mixture $u$. The continuity equation, the momentum equation and the translational fluctuation energy equation for the granular-fluid mixture are written in continuum form as follows.
In these equations, \( \rho_m = \rho_s v + (1-v)\rho_f \) is the density of the mixture; \( \rho_f \) and \( \rho_s \) are the densities of the fluid and of the solids respectively. The solids concentration is \( v \). The body force is represented by \( \mathbf{b} \). The stress tensors are represented by \( \mathbf{g}_c \) collisional, \( \mathbf{g}_f \) frictional, \( \mathbf{g}_v \) viscous and \( \mathbf{g}_t \) turbulent. The variable \( T \) is the granular temperature, a concept previously described in the kinetic theories of granular material. The variable \( g \) is the collisional flux of fluctuation energy. Finally \( \gamma \) and \( D_v \) are the dissipation terms, the former due to collisions and the latter due to the interstitial fluid drag that cushions the collisions.

In the translational fluctuation energy equation the term \(-\mathbf{g}_c : \nabla \mathbf{u}\) represents the work done to the system by the collisional stresses, and the so called conduction term, \(-\nabla \cdot \mathbf{g}\), represents the fluctuation energy added to the system through the conduction of granular temperature.
2.4 APPLICATION OF KINETIC THEORY FOR GRANULAR FLOW TO TWO-DIMENSIONAL STEADY FLOW

The purpose of this research is to analyze the mechanical behaviour of the flowing mixture and to formulate appropriate boundary conditions that are able to reproduce experimental results for flow down a chute. The most appropriate experimental approach will be to consider a steady uniform flow down an incline due to the lack of previous experimental results even in this simple flow regime. When we conduct the analysis of a chute flow under uniform and steady flow conditions, the variables of velocity, concentration, granular temperature and flux of energy have variations only in the direction perpendicular to the channel bed. Thus keeping only the derivatives in the y direction the constitutive equations for the collisional component simplifies in the kinetic theory for granular flow as follows: from the collisional stress tensor equation (2.1), the collisional normal stress becomes

$$\sigma_c = \rho g T$$ ......................................................... (2.16)$$

and the collisional shear stress becomes

$$\tau_c = -\rho g T^{1/2} \frac{\partial T}{\partial y}$$ ......................................................... (2.17)

From the flux of fluctuation energy equation (2.2), the flux of energy in the y-direction simplifies to

$$q_y = -\rho g T^{1/2} \frac{\partial T}{\partial y}$$ ......................................................... (2.18)

The collisional dissipation rate is given by equation (2.3).
CHAPTER 3

ANALYSIS OF FULLY DEVELOPED CHUTE FLOW
OF GRANULAR-FLUID MIXTURES

3.1 INTERSTITIAL FLUID EFFECT ON ENERGY DISSIPATION

The numerical model that will be developed in this research is applicable only in the grain-inertia fluid regime. In this regime, the main mechanism of stress generation is due to collisions between particles. In the case of a solid-fluid mixture, the dynamics of collisions is modified by the presence of an interstitial fluid.

When elastic particles collide in a vacuum, or under conditions of negligible fluid resistance, the incoming kinetic energy of the particles is converted into elastic strain energy as they deform in the vicinity of contact. The kinetic energy (other than that dissipated by the internal friction of the solids or remaining elastic vibrations) is restored as the particles rebound. The theory describing the motion and deformation of elastic particles during such a collision is known as Hertz contact theory and can be found in many textbooks of elasticity (Goldsmith 1960). In the case under consideration, when rigid spheres undergo relative motion in a viscous fluid, a portion of the kinetic energy of the spheres is dissipated by non-conservative viscous forces as they approach one another. The rest will be transformed into elastic-strain energy of deformation. Depending on the
fraction of the kinetic energy that becomes stored as elastic-strain energy, the deformation of the spheres may be significant and rebound may occur. These features will be included in the numerical model as a viscous dissipation term in the collisional kinetic energy flux equation.

3.2 Viscous Drag Coefficient Definition

The interstitial fluid effect will be introduced in the kinetic energy equation taking into account the drag force exerted by the fluid when the particle fluctuates. The next step will be to define an appropriate drag coefficient.

The first methodical research on viscous drag was attempted by chemical engineers on the 1940s. They studied fluid flow through packed columns for application on fluidized beds. They based the research on the classical Osborne Reynolds formula for the resistance offered by friction to the motion of the fluid

\[ \frac{\Delta P}{L} = a_R U + b_R \rho_f U^2 \]  \hspace{1cm} (3.1)

where \( \Delta P \) is the pressure loss along the length \( L \), \( \rho_f \) is the density of the fluid, \( U \) is the velocity of the fluid and \( a_R, b_R \) are factors which are functions of the system. The first term represents the flow at low flow rates when viscous effects are important and the second term refers to the kinetic effects at high flow rates.

In 1952, Ergun made a critical revision of the previous chemical researchers and proposed the following formula for
the pressure loss

\[ \frac{\Delta P}{L} = \frac{150}{g} \frac{v^2}{(1-v)^3} \frac{\mu_f U_m}{d_p^2} + \frac{1.75}{g} \frac{v}{(1-v)^3} \frac{\rho_f U_m^2}{d_p} \] .................(3.2)

In this formula the viscous part includes the interstitial fluid viscosity \( \mu_f \) and the influence of the fractional solids volume \( v \), which also affects the kinetic contribution. Here \( U_m \) is the superficial fluid velocity and \( d_p \) is the effective diameter of the particles.

Soo (1967) used this formula to derive the drag coefficient \( C_D \) of a suspension of spheres of diameter \( d \)

\[ C_D = 200 \frac{v}{(1-v)^2} \frac{1}{Re_p} + 7 \frac{1}{3 (1-v)} \] .................(3.3)

where \( Re_p \) is the particle Reynolds number based in the fluid velocity, \( Re_p = \rho_f U_m d / \mu_f \).

Another approach considers that the drag coefficient varies independently with the particle Reynolds number and the solids fraction

\[ C_D = f(Re_p), C_n(v) \] .................(3.4)

The relationship between the drag coefficient and \( Re_p \) has been tested more successfully than that with the solids fraction.
For $f(Re_p)$ the relationship proposed by Kaskas (1970) is

$$f(Re_p) = \frac{24}{Re_p} + \frac{4}{\sqrt{Re_p}} + 0.4 \ldots \ldots \ldots \ldots \ldots (3.5)$$

To define the relationship between the drag coefficient and the solids fraction, this approach uses the concept of apparent fluid viscosity, $\mu_{fa}$. This concept states that the additional resistance to the motion of a particle, caused by the finite size and the relative velocities of its neighbours, is experienced by the particle as an increase in viscosity

$$\mu_{fa}(v) = \mu_f C_n(v) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.6)$$

For extremely dilute dispersions, the Einstein (1906) equation gives the following value for $C_n(v)$

$$C_n(v) = (1 + 2.5v) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.7)$$

Frankel and Acrivos (1967) derived the apparent fluid viscosity for a suspension of uniform spheres in the limit as the concentration approaches its maximum value, $v_{max}$ and found the following apparent viscosity factor
For a dilute suspension, Batchelor (1972) proposed
\[ C_n(v) = (1 + 2.5v + 5.2v^2)^{-2.5} \] \hspace{1cm} (3.9)

Lun and Savage (1992) proposed the following formula for \( C_n(v) \)
\[ C_n(v) = (1 - v - 0.333v^2)^{-2.5} \] \hspace{1cm} (3.10)

A comparison of the drag coefficients proposed by Ergun, Frankel-Acrivos, Batchelor and Lun-Savage shows the following. There is a good agreement for an intermediate range of solids fraction 0.4-0.5 between the Ergun and the Lun-Savage formulas, but large differences for the extreme values. In the intermediate range the Frankel-Acrivos results are 70\% larger than those obtained by Ergun and Batchelor. For small values (\( v=0.2 \)) the Ergun formula gives differences larger than 100\% with respect to the other formulas. The Batchelor formula always gives the smallest value of the drag coefficient. The Lun-Savage formula gives large differences, larger than 100\% compared with that of Frankel and Acrivos, for concentrations near the closest packed concentration.

In this research we are dealing with intermediate and high concentrations, and we will generally use the Batchelor (1972) formula. When the concentrations approach the closest packed
value, it will be necessary to modify this formula in order to obtain a larger drag coefficient as will be shown in Chapter 6.

### 3.3 Governing Equations for Uniform, Steady Flow Down an Incline.

The model will be developed for the flow down an incline making an angle \( \theta \) with the horizontal. A uniform, steady flow regime will be analyzed. In this regime all the derivatives along the streamwise direction will cancel out, and we will only consider variations in the normal to the flow direction.

From the momentum equation (2.14) we obtain the following equation in the direction normal to the flow (where \( \sigma_{yy} \) is the total normal stress)

\[
\frac{\partial \sigma_{yy}}{\partial y} = -v (\rho_s - \rho_f) g \cos \theta \]
\[(3.11)\]

Also from the momentum equation we find the following equation in the streamwise direction (where \( \tau_{yx} \) is the total shear stress)

\[
\frac{\partial \tau_{yx}}{\partial y} = [(1-v) \rho_f + v \rho_g] g \sin \theta \]
\[(3.12)\]

In these equations, \( \sigma_{yy} = \sigma_f + \sigma_c + \sigma_l \), where \( \sigma_f \) is the normal frictional component of the stresses, \( \sigma_c \) is the normal collisional component and \( \sigma_l \) is the normal component of the
fluid turbulence caused by the wall. Also, \( \tau_{xy} = \tau_f + \tau_c + \tau_v + \tau_t \), where \( \tau_f \) is the shear frictional stress, \( \tau_c \) is the shear collisional stress, \( \tau_v \) is the shear viscous stress and \( \tau_t \) is the shear component of fluid turbulence.

From the collisional fluctuation energy equation (2.15)

\[
-\tau_c \frac{\partial u}{\partial y} - \frac{\partial F}{\partial y} + 3 \nu \rho \sigma FT = 0 \quad \cdots \cdots \cdots \cdots \cdots (3.13)
\]

where \( F \) is the specific fluid friction coefficient. It is related to the drag force \( D \) by

\[
D = \left( \frac{4n d^4}{3} \right) \rho \sigma F U_r \quad \cdots \cdots \cdots \cdots \cdots (3.14)
\]

\[
F = \frac{3}{4} C_D \rho \frac{1}{2} U_r \quad \cdots \cdots \cdots \cdots \cdots (3.15)
\]

where \( U_r \) is the most probable relative speed between the fluid in the neighbourhood and the particle and \( C_D \) is the drag coefficient.

From Lun and Savage (1992),

\[
U_r = \left( \frac{RT}{\pi} \right)^{1/2} \quad \cdots \cdots \cdots \cdots \cdots (3.16)
\]
From equations (3.4) and (3.5) we obtain the drag coefficient:

$$C_D = \left( \frac{24}{\text{Re}_f} + \frac{4}{\text{Re}_f^{1/2}} + 0.4 \right) C_n(v) \quad \text{(3.17)}$$

where we introduce a modified particle Reynolds number,

$$\text{Re}_f = \frac{\rho_f d u_r}{\mu_f}.$$ 

The frictional contribution to the stresses represents the rapid increase of the frictional normal stress $\sigma_f$ as the concentration approaches the closest packed concentration $v_{\text{max}}$ of the mixture. It also establishes that $\sigma_f$ vanishes for concentrations smaller that some value $v_{\text{min}}$, at which particles no longer retain long-term contacts. This frictional contribution also depends on the distance from the bed, $y$. In this work we assume a linear variation. The normal stress frictional contribution, proposed by Johnson et al. (1990) following the approach of Savage (1984) and modified with the inclusion of the dependence with depth, is

$$\sigma_f = Fr \frac{(v-v_{\text{in}})^2}{(v_{\text{max}}-v)^2} (h-y) \left( \rho_a - \rho_f \right) g \quad \text{(3.18)}$$

The parameter $Fr$ will be defined appropriately using the following condition on the maximum normal frictional stress: $\sigma_f < \rho_a g h \cos \theta$.

For the frictional shear stress we use the following relationship
$\tau_f = -\sigma f \tan \phi$  .................................................. (3.19)

where $\phi$ is the solids internal angle of friction.

The viscous shear stress will be defined using the concept of apparent fluid viscosity. Thus, using equation (3.6) and the Newton's law for viscous shear stress, we obtain the following

$$\tau_v = -\mu f C_n(v) \frac{du}{dy}  .................................................. (3.20)$$

The components of the turbulent stresses caused by wall roughness are included in the model in a simple way, using Prandtl's mixing length concept. There is no agreement between researchers concerning the effects of particles in the turbulent stresses. Some researchers, such as Iverson (1987) argue that high concentrations dampen the small eddies and the fluid behaves in a more laminar way. Other researchers argue that free turbulence is enhanced by the presence of large particles at high particle Reynolds number due to wake shedding (Hetsroni, 1989). However, the experiments conducted by Hetsroni are in the range of low concentrations less than 0.05. We argue here that turbulent stresses have an increasing importance until some concentration value, which varies according to the flow conditions determined by the particle Reynolds number and the Bagnold number. Therefore, we increase the value of wall turbulent stresses by $C_n(v)$ to take into account the presence of solid particles. It will be shown later that turbulence plays an important role for flows in the grain-inertia regime at an intermediate concentration range and decreases at high concentrations because of the decrease in shear rate. The turbulent shear stress according to Hinze
(1975), with the modification of $C_n$, is

$$\tau_t = -(1-v) \rho_f C_n(v) \epsilon \frac{du}{dy} \quad \text{(3.21)}$$

where $\epsilon$ is the eddy viscosity. The turbulent diffusivity of momentum (eddy viscosity), for a steady flow at normal depth, is related to the local length and the velocity scales (Rastogi et al. 1978), as follows

$$\epsilon = K_e \nu_f \quad \text{(3.22)}$$

where $K_e$ is a proportionality constant, $h$ is the flow depth and $u_f$ is the friction velocity. According to Chu et al. (1988) $u_f = \sqrt{C_f/2} u$, where $C_f$ is the bed friction factor. The normal component of the turbulent stress is, according to Hinze (1975), with the modification proposed here, as follows

$$\sigma_{\epsilon} = \frac{1}{3} (1-v) \rho_f C_n(v) u'_{avg}^2 \quad \text{(3.23)}$$

where $u'_{avg}$ is the fluid mean fluctuating velocity. We use Prandtl's mixing length $l$ to calculate $u'_{avg}$. According to that concept $u'_{avg} = l(du/dy) = \epsilon/l$, where the mixing length is defined as a fraction of the depth of flow $h$. We took $l = 0.1h$. For the value of $K_e$ we use the experimental value of Laufer (1951), $K_e = 0.0765$. The value of $C_f$ depends on the roughness of the bed. Rastogi et al. (1978) used Manning's formula $C_f = n^2 g/h^{1/3}$ for rough beds. Using this formula and assuming $n = 0.014$ and $h = 5\text{mm}$,
we find \( C_f = 0.01 \). This value is reasonably close to the experimental values obtained by Chu et al. (1988).

Now we are ready to establish the governing equations. We will also use the following relationship valid for a uniform steady flow down an incline:

\[
\frac{v_{xy}}{\sigma_{yy}} = -\tan \theta. \tag{3.24}
\]

The parameter \( R_{ef} \) will be considered in the numerical model in the following form: \( R_{ef} = R_{ef1}/(\delta T*/\pi) \), where \( R_{ef1} \) is the particle Reynolds number. In the formulation of the flow equations we will use the following dimensionless variables

\[
T^* = \frac{T}{g d \cos \theta}, \quad Y^* = \frac{Y}{d}, \quad U^* = \frac{u}{\sqrt{g d \cos \theta}}, \quad A = \frac{R}{\rho g}, \quad h^* = \frac{h}{d}
\]

\[
R_{ef1} = \frac{p d \sqrt{g d \cos \theta}}{\mu_f}, \quad q^* = \frac{q}{\rho_d (g d \cos \theta)^{3/2}}, \quad g_9 = \frac{F_r (v-v_{min})^2}{(v_{max}-v)^3}
\]

\[
G_{10} = \frac{F_r (v-v_{min})}{\cos \theta (v_{max}-v)^3} \left[ 2 + \frac{5 (v-v_{min})}{v_{max}-v} \right], \quad G_{12} = \frac{A C_n(v)}{R_{ef}}
\]

\[
g_{13} = \frac{1}{3} \frac{K_e^2 C_f}{0.12} C_n(v) = 0.001 C_n(v), \quad g_{14} = K_e \sqrt{\frac{C_f}{2}} C_n(v) h^* = 0.005 C_n(v) h^*
\]
From Equation (2.18) we have

\[
\frac{\partial T^*}{\partial y^*} = - \frac{q^*}{g_3 \sqrt{T^*}} \tag{3.30}
\]

From Equation (3.18) replacing Equations (2.16), (2.17), (3.18), (3.19), (3.20), (3.21) and (3.23) we obtain

\[
\frac{\partial u^*}{\partial y^*} = \frac{(g_1 T^* + g_{13} (1-v)Au^{*2}) \tan \theta + g_3 (h^*-y^*) (1-A) (\tan \theta - \tan \phi)}{g_{12} + g_2 \sqrt{T^*} + g_{14} (1-v)Au^*} \tag{3.31}
\]

From Equation (3.11) replacing Equations (2.16), (2.18), (3.18) and (3.23) we obtain

\[
\frac{\partial v}{\partial y^*} = \frac{\frac{g_1}{\sqrt{T^*}} g_y^* - 2g_{13} g_3 Au^* (1-v) \frac{\partial u^*}{\partial y^*} + (1-A) g_3 (g_9 - v)}{g_1' g_3 T^* + g_{10} g_3 (1-A) (h^*-y^*) - g_{13} g_3 Au^{*2}} \tag{3.32}
\]

From Equation (3.13) replacing Equations (2.3), (2.17), (3.15) and (3.16) we obtain

\[
q^* = - \int_0^Y \left(-g_2 \sqrt{T^*} \left(\frac{\partial u^*}{\partial y^*}\right)^2 + g_3 T^*^{3/2} + 3.5905C_v T^*^{3/2}\right) dy^* + q_w \tag{3.33}
\]

The boundary conditions will be discussed next in Chapter 4 and the numerical solutions to equations (3.30) to (3.33) will be discussed in Chapters 5 and 6.
CHAPTER 4

BOUNDARY CONDITIONS FOR CHUTE FLOWS
OF GRANULAR-FLUID MIXTURES

4.1 APPROACHES USED FOR DRY GRANULAR FLOW

Real dry granular flows typically have ratios of h/d which are not very large, where h is the flow depth and d is the particle diameter. In these kinds of flows, the particle-wall collisions are important in the dynamics of the flow. Velocity slip and granular temperature jumps can occur at such boundary surfaces. Also, particle fluctuation kinetic energy can be supplied or drained away at the boundary surfaces. The application of boundary conditions such as the no-slip condition and specified values for the granular temperature or its gradient are often crude first approximations to what actually occurs, and in some cases they can be in error.

Some research has been conducted during the last decade to provide relationships for the velocities, granular temperature and flux of energy at the walls. Among the first published works was that of Hui et al. (1984), that established relationships for the granular temperature at the wall and the slip velocity. They introduced the particle-wall coefficient of restitution to equate the rate of energy loss to the wall per unit area due to collisions and the energy flux conducted to the wall by virtue of grain-grain collisions. They proposed
the following equation for the fluctuation velocity at the wall

\[ \sqrt{T_f} = \frac{2rd}{(1-e^2)} \frac{d\sqrt{T}}{dy} \]  \hspace{2cm} (4.1)

where \( r \) is a dimensionless constant of order unity left undetermined.

For the slip velocity they introduced a specularity coefficient \( \phi' \) as a measure of wall roughness. If \( \phi' \) is small so that most collisions are nearly specular, corresponding to a smooth wall, then the amount of slip may be relatively large. For a rougher surface, however nearly every grain-wall collision will provide a significant transfer of lateral momentum to the wall; thus, in this case \( \phi' \) is near unity, and the amount of slip is minimized (see figure 4.1). The relation proposed for the slip velocity, defined as the difference between the velocity of the interior particles at the wall and the velocity of the wall itself, i.e. \( u_s = u - u_{\text{wall}} \), is

\[ u_s = \frac{\omega d}{\phi'} \left( \frac{du}{dy} \right)_{w} \]  \hspace{2cm} (4.2)

where \( \omega \) is a dimensionless constant of order unity and \( (du/dy)_w \) is the normal derivative of the flow velocity evaluated near the wall.

Following an approach similar to that of Hui et al., Johnson et al. (1990) derived boundary conditions for the case of not
only collisional interactions but also frictional. The magnitude of the tangential frictional component is assumed to be \( \sigma_t \tan \delta \), where \( \sigma_t \) is the normal frictional component of stress and \( \delta \) is the angle of friction between the surface and the particulate material. This is Coulomb's friction law applied to the material sliding over the surface. Their basic idea is to consider a rectangular control volume of unit area parallel to the wall which encompasses both the wall particles and the interior flow particles. Two boundary conditions are obtained by considering the momentum and energy balances over the control volume in the limit as the depth (perpendicular to the wall) of the control volume tends to zero. Equating the sum of the frictional and collisional contributions to the component of the bulk stress vector in the direction of \( u_s \), they obtain the first boundary condition based on momentum balance.

The second boundary condition is obtained from an energy balance over a control volume near the wall. They equate the flux of energy at the wall to the rate of dissipation of pseudo-thermal energy due to inelastic collisions of the particles with the boundary plus the frictional heating as particles slide over the boundary.

Another approach used in dry granular flows is that of Richman (1988). He used equations derived from the kinetic theory for granular flow to calculate the rate at which momentum and energy are transferred across bumpy boundaries and obtain conditions that ensure the balance of momentum and energy at such boundaries. He defined accurately the roughness of the wall by specifying the geometry of the hemispherical particles that made the wall.

In this work we will use both the Johnson et al., and the
Richman approaches modified to include the frictional stresses, the fluid turbulent effects and the viscous drag dissipation for collisions of particles in the presence of an interstitial fluid.

4.2 BOUNDARY CONDITIONS INCLUDING AN INTERSTITIAL FLUID

4.2.1 Method including the specularity coefficient

The momentum balance in the direction of the slip velocity proposed by Johnson et al. (1990), and modified with the inclusion of the turbulent fluid effect, becomes

\[
\frac{u_s \cdot (\tau_c + \tau_t + \tau_f + \tau_v) \cdot n}{|u_s|} + \frac{\phi' \sqrt{3} \pi \rho_s \sqrt{T} |u_s|}{6 v_{\text{max}} [1 - (\frac{v}{v_{\text{max}}})^{1/2}]} + \sigma_f \tan \delta = 0 \quad \text{......... (4.3)}
\]

where \( n \) is the unit normal from the boundary into the particle assembly, \( \tau_c \) and \( \tau_v \) are the components of the turbulent and viscous stresses in the \( u_s \) direction, and \( \phi' \) is the specularity coefficient as defined by Hui et al. The first term in (4.3) corresponds to the total stress (collisional, frictional, turbulent and viscous) in the interior, the second term is the force per unit area on the boundary due to grain-boundary collisions \( S_c \), and the last term is the magnitude of the tangential frictional stress component at the wall.

Replacing equations (2.17), (3.19), (3.20) and (3.21) into equation (4.3) we obtain after nondimensionalisation

\[
\left( \frac{\partial u^*}{\partial y^*} \right) = \frac{g_0 (h^*-y^*) (1-A) (\tan \phi - \tan \delta) - \phi' \sqrt{T} u_s^* g_6}{-g_2 \sqrt{T^*} - g_{14} A (1-v) u_s^* - g_{12}} \quad \text{......... (4.4)}
\]
where \((du^*/dy^*)_w\) means the velocity gradient at the wall,
\[ g_a = \pi v g_0 / (2/3 v_{\text{max}} g_2). \]

Using equation (3.31) in (4.4) we obtain the following relationship for the slip velocity

\[ u_s^* = -B_j + \sqrt{(B_j^2 - 4A_j C_j)} \]
\[ 2A_j \]

(4.5)

Where \(A_j\), \(B_j\) and \(C_j\) are defined as:

\[ A_j = g_{13} (1-v) \tan \theta \]
\[ (4.6) \]

\[ B_j = -g' g_0 \sqrt{T^*} \]
\[ (4.7) \]

\[ C_j = g_s h^* (1-A) (\tan \theta - \tan \delta) + g_1 T^* \tan \theta \]
\[ (4.8) \]

The other boundary condition equates the flux of fluctuation energy at the wall \(q_w\) to the rate of collisional energy dissipation \(\gamma\) due to inelastic collisions, plus the rate of energy dissipation due to viscous drag \(D_v\) and the frictional heating \(u_s S_c\), where \(S_c\) is the collisional force per unit area on the boundary. The equation is
where the collisional dissipation is (Johnson et al. 1990)

\[
\gamma = \frac{\sqrt{3} \pi}{4} \rho_s T^{3/2} (1 - e_s^2) \frac{v}{v_{\text{max}}} \theta_0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4.10)
\]

The viscous drag is obtained using the relative velocity of the particles \( U_z \) to calculate the drag force per particle. The viscous energy dissipation \( D_v \) is this drag force, times the relative velocity, times the number of particles per unit wall area. The equation is the following

\[
D_v = \left( \frac{1}{2} \rho \frac{C_D}{d^2} \frac{\pi d^2}{4} U_z^2 \right) U_z \left[ \frac{1}{d^2 \left( \frac{v_{\text{max}}}{v} \right)^{2/3}} \right] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4.11)
\]

where \( C_D \), the drag coefficient, is given by equation (3.17) and \( U_z \) by equation (3.16). Replacing \( S_c \) (second left term of equation (4.3)), equations (4.10) and (4.11) into equation (4.9) we obtain

\[
q_w = u_s S_c - \gamma - D_v \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4.9)
\]

where \( q_w \) = \( \pi v g_0 / (2/3 v_{\text{max}}) \). Thus, using the same nondimensional variables of Chapter 3, the boundary condition for the flux of energy at the wall \( q_w \) reduces to
where \( g_6 \) is

\[
g_6 = \frac{\sqrt{3} \pi}{4} (1 - e_w^2) \frac{v}{v_{\text{max}}} g_0 + 1.595 A C_D \left( \frac{v}{v_{\text{max}}} \right)^{2/3}
\]  \hspace{1cm} (4.14)

In this way we can solve the equations of flow using a method similar to the one proposed by Ahn (1989) in his Appendix B. He used a shooting method for the two-point boundary value problem. He was not able to define appropriate boundary conditions and he assumed arbitrarily values for \( u_s, q_w, \) the wall concentration and granular temperature at the wall. In this research by means of equations (4.5) and (4.13) we calculate the slip velocity and the flux of energy at the wall. The equations are solved for the entire flow depth, assuming a value of wall concentration and iterating over the value of \( T \) until the boundary conditions at the flow surface are fulfilled. The boundary conditions at the flow surface are zero flux of fluctuation energy and zero gradient of velocity. A third boundary condition depends on the kind of flow. For a loose flow, the concentration should go to zero and for a dense flow, the granular temperature should go to zero at the surface. Any set of these three boundary conditions satisfies the requirement of vanishing stresses at the flow surface.

4.2.2. Method based on the kinetic theory for granular flow.

Richman (1988) employed methods of statistical averaging to derive conditions for flows of nearly elastic particles that ensure that both the flux of momentum and energy are balanced.
at a bumpy boundary. The roughness of the boundary was determined by the parameters describing its geometry. He assumed rough impenetrable walls made up of hemispherical particles that were assumed to be randomly arranged with constant mean spacing along the wall.

He made an improvement to the Maxwellian velocity distribution function in order to be able to expand it about a location accessible to the center of any flow particle. He obtained an expression for the change in momentum experienced by a flow particle when it collides with the boundary. Then \( \mathbb{M} \), the rate of momentum per unit area supplied to the flow by the boundary through collisions, is obtained by multiplying the previously calculated change in momentum by the collision frequency. To do this he employed the improved velocity distribution function and integrated over all possible collisions. The resulting streamwise component of \( \mathbb{M} \) is

\[
M_{xy} = p \rho g T \mu_0 \left[ \frac{1}{\pi} \left( \frac{\sigma_d}{\sqrt{T}} \right)^{\frac{1}{2}} \left( \frac{1 + \frac{\sigma_d}{\sigma} \sin \psi^2 + K_x}{2} \right) \right] \tag{4.15}
\]

The parameter \( K_x \) depends on the wall geometry. It depends on the angle between the wall and the interior particles \( \psi \), shown in figure 4.2.

\[
K_x = \frac{2}{3} \left[ 2 \csc^2 \psi (1 - \cos \psi)^2 - \cos \psi \right] \tag{4.16}
\]

The variable \( \sigma \) is the diameter of the wall particles and \( B = \pi (1 + 5/(8v_0)) / 12 \) is derived from the correction to the Maxwellian velocity distribution function.
Richman focused attention on a parallelepiped within the flow that has two opposite sides of unit area, one of which remains coincident with a unit area of the boundary while the other four sides shrink to zero. In this limit he established the balance of momentum within the parallelepiped, \( \mathbf{M} = \sigma \cdot \mathbf{N} \), where \( \sigma \) is the stress tensor and \( \mathbf{N} \) is the unit inward normal of the wall. In our case of interest, uniform, steady flow of a granular-fluid mixture the balance of momentum reduces to

\[ M_{xy} = \tau_x + \tau_y + \tau_z - \sigma_z \tan \delta \]  

(4.17)

Using equations (2.17), (3.19), (3.20) and (3.21) for the stresses and (4.15) for \( M_{xy} \), we find the following equation where we use the same dimensionless variables as in chapter 3 and have assumed \( \sigma = d \)

\[ \frac{\partial u^*}{\partial y^*} = -K_\tau g_{11} \frac{2\pi}{\pi} u^* + g_9 (h^* - y^*) (1 - A) (\tan \phi - \tan \delta) \]

\[ \times \frac{g_{14}}{g_{11}} \sqrt{\frac{2\pi}{\pi}} \left[ (1 + B) \sin \psi^2 + K_\tau \right] - g_{14} (1 - \nu) A u^* - g_2 \sqrt{\frac{2\pi}{\pi} - g_{12}} \]

(4.18)

Equating (4.18) with the flow equation (3.31), we find the following equation for the slip velocity at the wall

\[ A_1 u^*_2 + B_2 u^*_2 + C_2 = 0 \]

(4.19)

where

\[ A_2 = (1 - \nu) A (g_{14} - g_{13} \tan \theta) K_\tau g_{11} \frac{2\pi}{\pi} \]

(4.20)
The second boundary condition is the balance of energy at the wall. Introducing the energy dissipation by viscous drag $D_v$, the equation becomes

$$q_v = M_{xy} u_s - \gamma - D_v.$$  

(4.25)

The expression for the collisional energy dissipation is obtained from the kinetic theory for granular flow following the approach of Richman, by multiplying the loss of energy per collision by the frequency of collisions, and integrating over all collisions. Richman gives the following expression that is accurate to first order

$$\gamma = \sqrt{\frac{2}{x}} 2 \rho_s g_1 (1 - e_\psi) T^{3/2} (1 - \cos \psi) \csc \psi^2.$$  

(4.26)

The expression of the viscous drag dissipation during
collisions is obtained by multiplying the drag force by the relative velocity of the particle and the neighbouring fluid, $U_r$. The expression obtained is

$$D_v = \frac{5}{8} \left( \frac{v}{v_{max}} \right)^{2/3} \rho_f C_D (U_r)^3$$

(4.27)

where $C_D$ is the previously defined drag coefficient and $U_r = \sqrt{8T/\pi}$.

After nondimensionalization the expression for the flux of energy at the wall $q_w$ becomes

$$q_w^* = g_1 \sqrt{\frac{2T}{\pi}} \left[ K_r u_g u^2 + u_g \frac{\partial u^*}{\partial y^*} ((1+B) \sin^2 + K_r) - 0.855 g_1 (1 - e_w) T^{*3/2} \right] - 1.595 \left( \frac{v}{v_{max}} \right)^{2/3} AC_D T^{*3/2}$$

(4.28)
CHAPTER 5

SOLUTIONS FOR DRY GRANULAR CHUTE FLOW

5.1 EARLIER SOLUTIONS PRESENTED IN THE LITERATURE

In recent years, many equations for the motion of rapidly deforming granular materials have been proposed, but little work has been done to solve the resulting equations and examine the solutions, over a wide range of conditions, in relation to experimental evidence. Most of the difficulty in doing this work is due to the heuristic approach to the definition of the boundary conditions. In fact the approach to be used in solving granular materials flow must be completely different from the one used in conventional fluid mechanics. The latter is simpler because of the use of wall conditions in which both the velocity and the thermodynamic temperature of the fluid next to a solid boundary take on the values corresponding to the boundary. The wall properties can be specified independently of whatever is happening in the rest of the flow field. In granular materials, the flow behaviour at a solid or free surface is an integral part of the solution for the entire flow field. The slip velocity is determined by the interaction of the flow with the boundary. Also, because there is a slip velocity, the boundary performs shear work and
generates granular temperature. But, at the same time, collisions with the wall dissipate energy. Thus the wall may act as a source or sink of granular temperature.

The most recent review of the solutions for chute flows was conducted by Anderson and Jackson (1992). They used the constitutive equations for the collisional stress tensor proposed by Lun et al. (1984) using kinetic theories for granular flow and a relationship for the frictional contribution to the stresses similar to the one used here. For the boundary conditions, they used the relationships derived by Johnson and Jackson (1987) using the specularity coefficient introduced by Hui et al. (1984). They found that the inclusion of the frictional term eliminates the unrealistic prediction that there is a maximum mass flow rate for which fully developed flow is possible as predicted by models using only the collisional stresses. They also found that for smooth beds (low specularity coefficients), when the mass flow rate increases, the velocity profiles change from one in which shear occurs throughout the depth, to a profile in which most of the layer consists of a high-density block of material that slides, without shearing, over a thin shear layer immediately adjacent to the surface of the plane. They also found that, despite the fact that the layer is not shearing through most of its depth, the particle temperature does not vanish, but increases exponentially on moving down from the free surface. For rough beds, they found very different velocity, concentration and granular temperature profiles. Bulk densities are lower, velocities are much higher and shear now extends much more uniformly through the layer, with comparatively little slip at the surface of the plane. Particle temperatures are also higher and the temperature now increases on moving up through the layer. The flow is more energetic with the plane acting as a sink, rather than a source of pseudo-thermal energy (as in the smooth bed).
5.2 RESULTS OF THIS RESEARCH

Eliminating the viscous dissipation terms and the fluid viscous and turbulent stresses, we go back to the flow equations for dry granular flow with interstitial fluid of negligible viscosity. To solve these equations we used the two kinds of boundary conditions presented in Chapter 4; one using the specularity coefficient, and that of Richman (1988), based on the kinetic theory, where the wall is made of hemispherical particles. The results from both approaches will be compared in order to better understand the physical meaning of the specularity coefficient and give an approximate method of computation of it, because it is very difficult to measure experimentally.

5.2.1 Results using a method based on kinetic theory

The boundary conditions proposed by Richman (1988) were modified by including frictional terms that account for interparticle and particle-wall rubbing and sliding. The method to obtain relationships for the slip velocity and the flux of fluctuation energy at the wall is similar to the one presented in section 4.2.2. Now we assume negligible the viscous and turbulent stresses and we do not include the viscous drag. The final form of the slip velocity is obtained equating (4.18) with the flow equation (3.31) assuming \( g_{12}=g_{13}=g_{14}=0 \)

\[
u_* = \frac{g_T (\tan \theta + (\tan \delta - \tan \phi)) g h^* [g_T^2 - g_T g_T^r ((1+ \delta) \sin \phi^2 + K^s) - g_T g_T^r (\tan \delta - \tan \phi)]}{\frac{1}{4} K_T g_T^r} \]

\[ \cdots (5.1) \]
The final form of the flux of energy at the wall is obtained from equation (4.28) without the viscous dissipation term

\[ g_w = g_1 \sqrt{\frac{2\tau}{\pi}} \left[ u^{*2} K_x + \left( \frac{\partial u^*}{\partial y^*} \right) u^* \left((1+B) \sin^2 \psi + K_x\right) \right] - 0.855 g_4 (1-e^*) T^{*3/2} \ldots \ldots (5.2) \]

With these boundary conditions the flow equations (3.30) to (3.33) can be solved for the velocity, concentration, granular temperature and flux of fluctuation energy.

A complete exploration of all the parameters of the model is difficult, so we used two sets of parameters representing two roughness conditions of the channel bed. The values adopted were similar to those used by Anderson and Jackson (1992) which correspond to experimental measurements made with glass beads. Additionally, this allows us to compare their results with ours. These parameters are the following:

### TABLE 5.1
Parameters used in the calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Moderately Rough</th>
<th>Very Rough</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>28.5°</td>
<td>28.5°</td>
</tr>
<tr>
<td>( \delta )</td>
<td>20.0°</td>
<td>20.0°</td>
</tr>
<tr>
<td>( e_p )</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>( e_w )</td>
<td>0.8</td>
<td>0.94</td>
</tr>
<tr>
<td>( \psi )</td>
<td>30.0°</td>
<td>45.0°</td>
</tr>
<tr>
<td>( v_{\text{max}} )</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>( Fr )</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
</tbody>
</table>
The results for the moderately rough bed conditions, that represent a bed made of hemispherical particles with zero interparticle spacing are presented in figures 5.1-5.4. They show relatively low densities (figure 5.2) and high velocities (figure 5.1). The granular temperature is higher near the surface than at the bed which acts as a sink of pseudo-thermal energy.

In the case of very rough beds ($\psi=45^\circ$), corresponding to a wall interparticle spacing of 0.414d, we were able to get solutions only for very dilute flows. These are presented in figures 5.5-5.8. The overall pattern corresponds to what was expected, high velocities (figure 5.5) and high temperatures (figure 5.7) typical of low bulk density flows. Flow depths are larger than those obtained for the smoother bed. For example, a moderately rough bed with a nondimensional surface velocity of 3.5 has a depth of 12 particle diameters (figure 5.1) and a very rough bed, with a nondimensional surface velocity of 3.0, has a depth of 15 particle diameters (figure 5.5) for the same chute angle and similar parameters. The possible explanation of this behaviour comes from the mass flow rate. In order to reach a high mass flow rate, a very dilute flow needs not only a high speed but also a high flow depth. The results of Richman and Marciniec (1990) corroborate this behaviour. Their figure 8 shows concentration, velocity and fluctuation velocity profiles for chute flows with the same mass flow rate. If we compare the curves for 21.8° (mean concentration 0.5, mean velocity of 10) with that of 20.7° (mean concentration 0.1, mean velocity 28), we find that the flow depth for the dilute flow is 1.8 times that of the more dense flow. Unfortunately Richman did not compare his results with experiments but nevertheless he concluded that the dilute flow results are in qualitative agreement with the experiments conducted by Johnson et al. (1990). For the dense flows, he
found shallower flows, because Johnson et al. reported experimental dense flows of larger depths than the dilute ones. The results of Anderson and Jackson (1992) using a similar model showed that the difference in dense flows was due to the roughness of the wall and the inclusion of the frictional stresses. In fact for smooth walls, a locked dense flow with high flow depth appears. The Richman boundary conditions are not able to model this kind of surface roughness.

The above results show the great importance of the boundary conditions on the flow patterns. The Richman boundary conditions, more accurate and rigorously deducted from the kinetic theory for granular flow, are able to model only rough beds made of hemispherical particles. It is not possible to reproduce some flow patterns found experimentally using smooth beds, even with the inclusion of frictional terms as was done here. The more ad hoc method of the specularity coefficient seems to be appropriate for a wider range of surface roughness. The trouble with using this method is the difficulty in defining the appropriate value to be used for $\phi'$, because no experimental method has been proposed until now. In the next section we will present some numerical results obtained using this method and then make some suggestions to define a value of $\phi'$ for the case of moderately rough beds based on a comparison with the results using Richman's boundary conditions.

5.2.2 Results using the specularity coefficient

The specularity coefficient $\phi'$ was introduced by Hui et al. (1984) as a measure of the bed roughness. If the wall is smooth, the collisions are specular, corresponding to a specularity coefficient near zero. If the bed is rough, nearly every grain-wall collision will provide a significant transfer
of lateral momentum to the wall. The collisions are diffuse which corresponds to a specularity coefficient near unity. The slip velocity is inversely proportional to the specularity coefficient. Thus for a smooth wall, $u_s$ will be large and for a rough wall $u_s$ will be small. The equations are found using a similar method to that of section 4.2.1. For the slip velocity we equate equation (4.4) with the flow equation (3.31) and assuming $g_{12}=g_{13}=g_{14}=0$, we find

$$u_s = \frac{g_1 T \tan \theta + g_2 h^* (\tan \theta - \tan \delta)}{\phi' \sqrt{T^* g_6}}$$

(5.3)

The equation for the flux of energy at the wall is obtained from equation (4.13) making the viscous dissipation term equal to zero.

$$Q_{wall}^* = \phi' g_7 \sqrt{T^*} u_s^* 2 - \frac{\sqrt{3}}{\pi} (1-e_y^2) \frac{v}{v_{max}} g_0 T^{3/2}$$

(5.4)

The parameters used were similar to those used by Anderson and Jackson (1992), in order compare the results and also because the properties of the particles were measured during the experiments performed by Johnson et al. (1990). They are listed in the following table:
TABLE 5.2

Parameters used in the calculations

<table>
<thead>
<tr>
<th></th>
<th>smooth plane</th>
<th>rough plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>28.5°</td>
<td>28.5°</td>
</tr>
<tr>
<td>$\delta$</td>
<td>12.3°</td>
<td>12.3°</td>
</tr>
<tr>
<td>$e_p$</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>$e_w$</td>
<td>0.91</td>
<td>0.8</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>0.25</td>
<td>0.85</td>
</tr>
<tr>
<td>$v_{\text{max}}$</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>$Fr$</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

For rough beds (figures 5.9-5.12), the results are very similar to those obtained by Anderson and Jackson (1992). Bulk densities are relatively low (figure 5.10), decreasing rapidly from a maximum value at the wall to zero at the surface. The velocities are high and increasing with the flow depth (figure 5.9). The granular temperature is higher near the surface (figure 5.11) showing that the wall acts as a sink of pseudo-thermal energy.

For smooth beds (figures 5.13-5.16), the results show a very different behaviour. Bulk densities are large and nearly constant throughout the depth of the flow (figure 5.14). Velocities are lower as are flow depths (figure 5.13). The granular temperature is larger near the bed (figure 5.15), showing that the bed acts as a source of pseudo-thermal energy, but the magnitude of the granular temperature is very much lower than for a rough bed. The positive flux of fluctuation energy near the bed (figure 5.16), shows that the work done by the shear stresses is larger than the collisional...
energy dissipation. Our model was not able to predict the behaviour for very large concentrations as demonstrated by Anderson and Jackson. We believe this is because the numerical method used here experiences difficulties when the granular temperatures are very small ($10^{-4}$).

### 5.2.3 Comparison of boundary conditions.

It is clear for the above results that Richman's boundary conditions are only applicable for rough beds and that the specularity coefficient method could be useful for some kind of surface roughness not covered by that of Richman. The question remains how to define the appropriate value of $\phi'$. If we compare equations 5.1 and 5.2 with 5.3 and 5.4 we find some similarities. Comparing, for example, the slip velocity equations, we can define the specularity coefficient in the following way

$$
\phi' = \sqrt{\frac{2}{\pi}} \frac{K_\tau g_1}{g_6}.
$$

(5.5)

The equation for the slip velocity using the boundary conditions following the approach of Richman can be expressed as a function of the slip velocity obtained with the specularity coefficient method. The following relationship is found

$$
(u_s)_{Richman} = (u_s)_{\phi'} (1 - K_{\mu\phi}').
$$

(5.6)

where
From these equations we can establish that the specularity coefficient not only depends on the wall characteristics but also on the flow characteristics. Particularly the concentration near the bed has a large effect on the value of the specularity coefficient. Table 5.3 shows the values of $\phi'$ and $K_{\phi'}$ for different values of the spacing between particles at the wall, $\psi$. We used a value of granular temperature of 0.4 typical of the numerical results shown in section 5.1.

**TABLE 5.3**

<table>
<thead>
<tr>
<th>$\nu$ at wall</th>
<th>$\psi$ deg</th>
<th>$\phi'$</th>
<th>$K_{\phi'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>30.0</td>
<td>0.21</td>
<td>0.30</td>
</tr>
<tr>
<td>0.35</td>
<td>45.0</td>
<td>0.48</td>
<td>0.63</td>
</tr>
<tr>
<td>0.35</td>
<td>60.0</td>
<td>0.85</td>
<td>1.00</td>
</tr>
<tr>
<td>0.40</td>
<td>30.0</td>
<td>0.14</td>
<td>0.80</td>
</tr>
<tr>
<td>0.40</td>
<td>45.0</td>
<td>0.32</td>
<td>1.66</td>
</tr>
<tr>
<td>0.40</td>
<td>60.0</td>
<td>0.57</td>
<td>2.65</td>
</tr>
</tbody>
</table>

In table 5.3 we used two values of concentration at the wall to show the decrease in the specularity coefficient with the
concentration at the wall, even if the geometry of the wall remains unchanged. When the interparticle wall spacing is increased (increase in $\psi$), the specularity coefficient increases as expected because there is an increase in rugosity, but also the term $K_{xx}$ increases sharply. This allows us to conclude the following. First, the definition of the specularity coefficient (after 5.5) is only valid for low interparticle spacing (moderately rough beds), a situation in which the slip condition following Richman's definition and that based on the specularity coefficient are similar. The other conclusion is that for large interparticle spacing the two boundary conditions differ greatly. For some values of concentrations, the specularity coefficient should be larger than one to give a value of slip similar to that of the other method. This does not agree with the definition of specularity coefficient and we conclude that one of the methods fails to describe this situation accurately. If we go back to the numerical results we find that using the specularity coefficient method with $\phi' = 0.85$ (close to one), the results are similar to those obtained with Richman's method for interparticle spacing of zero. In light of the impossibility of defining $\phi'$ larger than one, we can conclude that the specularity coefficient method is unable to describe the situation for very rough walls. For those situations it is necessary to use the method of Richman.

However, the previous analysis has shown that for the case of smooth walls and moderately rough walls, the specularity coefficient method is able to give appropriate results and the specularity coefficient can be calculated by means of equation 5.5.
6.1 NUMERICAL SIMULATIONS.

The set of equations 3.30-3.33 was solved for the mixture velocity $u^*$, the concentration $v$, the granular temperature $T^*$ and the flux of fluctuation energy $q^*$. The boundary conditions on $u^*$ and $q^*$ at the wall (4.19) and (4.28) were used. For this granular-fluid chute flow we decided to use only the boundary conditions derived from the kinetic theory (modified Richman) because of the uncertainties in the definition of the specularity coefficient. Two additional boundary conditions at the surface are needed. One condition is that the flux of energy there is zero and the other is that the concentration is zero. The latter condition can sometimes be replaced by the condition of zero granular temperature at the surface when the flow is very dense. Both sets of boundary conditions ensure that the stresses are zero at the surface.

The differential equations were solved by the Euler method and the trapezoidal rule was used for the numerical integration. The boundary value problem was solved by a shooting method. The energy flux and velocity at the wall were
calculated by the derived boundary conditions; values of concentration and temperature at the wall were chosen by trial and error to satisfy the boundary conditions at the free surface.

Exploration of the complete set of parameters, \( e_p \), \( e_w \), \( \theta \), \( \phi \), \( \delta \), A, Fr and Re, is very difficult and time consuming. We decided to focus our attention on the three parameters that govern the energy dissipation. These are: \( e_p \), which controls the interparticle collisional dissipation; \( e_w \), which controls the collisional wall-particle energy dissipation; and \( Re \), which controls the viscous drag energy dissipation. The other parameters were varied but only in a very limited range that was fixed according to the experimental set-up and the work of Anderson and Jackson (1992) for comparison purposes. The following table shows these ranges

<table>
<thead>
<tr>
<th>PARAMETERS RANGE USED IN CALCULATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \delta )</td>
</tr>
<tr>
<td>( \theta )</td>
</tr>
<tr>
<td>( A )</td>
</tr>
<tr>
<td>( Fr )</td>
</tr>
<tr>
<td>28.5°</td>
</tr>
<tr>
<td>12.3° - 20.0°</td>
</tr>
<tr>
<td>21.0° - 23.0°</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.0002</td>
</tr>
</tbody>
</table>

Preliminary runs showed that particle-particle coefficients of restitution less than 0.9 produced very low flow velocities. Also, particle-wall coefficients of restitution larger than 0.4 produced the same effect. The explanation of this behaviour is related to the collisional energy dissipation. High \( e_p \) diminishes the loss of flow energy in the grain-inertia regime explaining the increase in velocity. Low \( e_w \) is usually associated with a high slip velocity at the wall. This
high slip velocity together with the wall shear stress perform work, which explains the increase in velocity. However, the appropriate values of \( e_w \) seem very low and later on we will provide an explanation to this observation.

The variation with \( e_p \) is shown in figures 6.1-6.4. To perform this simulation we used values of \( e_w=0.05 \) according to the observations of the last paragraph. The value of \( Re_1=200 \) was chosen to reproduce the experiments, it results from a mean particle diameter of 1.2 mm and the viscosity of water. The variation of the drag coefficient with the concentration \( (C_o) \) was chosen according to the work of Batchelor (1972). The relationships proposed by Lun and Savage (1989) and Ergun (1952) were used during preliminary runs for an intermediate range of concentrations (0.3-0.5) and we found an extremely large viscous dissipation decreasing the velocity and granular temperature to unreasonable values. However, for very dense flows, the relationship proposed by Lun and Savage provides better results as will be shown in section 6.2.

The velocity profiles obtained (figure 6.1) show an increase in velocity and flow depth with increasing \( e_p \). The slip velocity also increases with \( e_p \). The model shows a high sensitivity to the particle-particle coefficient of restitution. For large flow depths (large \( e_p \)), a dense flow appears near the surface (figure 6.2). This behaviour is accompanied by a high increase of granular temperature near the bed (figure 6.3). The bed acts as a source of pseudo-thermal energy despite the low wall-particle coefficient of restitution. This is caused by the slip velocity that has increased and together with the shear stresses near the wall, perform slip work. The relationship between the slip velocity and \( e_w \) is not obvious from the boundary conditions, but
computations in which all the parameters were fixed except $e_w$ will confirm this behaviour. This is the main difference between the behaviour of a dry granular flow and a granular-fluid flow. As we showed in chapter 5 for dry granular flow, higher $e_w$ values are associated with lower slip velocities. By contrast, when we keep the other parameters constant and we change only the wall-particle coefficient of restitution, we find the results shown in figures 6.5-6.8. They show that for diminishing $e_w$, the flow velocity, the slip velocity and the depth of flow increase. Without considering the effect of the interstitial fluid, this behaviour appears contradictory. The interstitial fluid allows a more dense flow near the bed by cushioning the collisions. This generates high frictional particle-bed stresses. These frictional stresses depend on the flow depth and as a consequence are larger for high $h$. In denser flows, the slip velocity is also larger, and with higher stresses at the bed more work is done. This is reflected in the granular temperature profile (figure 6.7). The flux of fluctuation energy near the bed, as we can see in figure 6.8, is the largest for the smallest $e_w$. This means that the total energy generated at the wall is the largest for the smallest $e_w$.

The influence of the particle Reynolds number ($Re_1$) is also shown in figures 6.9-6.12. We obtain the largest velocity and flow depth for the largest $Re_1$. The mean concentration decreases with the increase in particle Reynolds number (figure 6.10). Therefore, the trend is that the velocity increases with a decrease in mean concentration. The question now is to define how representative $Re_1$ is of the flow behaviour. To do this, we examine the definition of $Re_1=\rho_d d/(g\cos\theta)/\mu_r$. This is not the standard definition of the particle Reynolds number. However, from figure 6.7 we know that when $Re_1$ increases so does the granular temperature.
Thus, a particle Reynolds number $Re_f$ based on the relative velocity $U_r$ (that is a function of $\sqrt{T}$) follows the same trend with the Reynolds number as defined here. Based on our definition of $Re_f$, the magnitude of the inertia force depends on the particle diameter and that of the viscous force depends on the viscosity of the interstitial fluid. For a larger particle diameter and a lower $\mu_f$ value, we find the largest $Re_f$. If we keep the particle diameter constant, an increase in $Re_f$ means a decrease in the interstitial fluid viscosity. If we consider that the fluid is the same, this decrease in viscosity can be associated with the effect of concentration (and therefore granular temperature) on the fluid viscosity. In fact, an increase in $Re_f$ is found to coincide with a decrease in mean concentration (figure 6.10).

An interesting result is found when $Re_f$ is lower than 200. This is shown in figures 6.13-6.16. The trend of increasing concentration with decreasing $Re_f$ is maintained (see figure 6.14). However, the trend of an increase in flow depth for increasing $Re_f$ is inverted (see figure 6.13). A similar behaviour is found for dry granular dense flows in the case of very smooth walls (Anderson and Jackson, 1992). It is surprising that this behaviour could be found for this kind of bumpy wall made of hemispherical particles. The explanation is the effect of the interstitial fluid in the energy dissipation at the wall. As will be shown in Chapter 7 this behaviour is related to the Bagnold number and will be corroborated with the experiments.

There are two limits of validity for the trends that we show in the figures. One relates to particle diameter. When the particle diameter becomes very small and $\mu_f$ remains large, contacts between the particles when they approach each other no longer occur ($Re_f$ very small). The other limit is when the
viscosity of the interstitial fluid is very small ($Re_l$ very large). To discuss the former limit it is interesting to make reference to the definition of the Stokes number (section 2.4). The Stokes number was found to be linearly proportional to the Bagnold number. The Bagnold number decreases with a decrease in the Reynolds number because of the decrease in the shear rate (see figure 6.9). The lowest Stokes number is found for the lowest $Re_l$. For the runs shown when the Bagnold number approaches 450, applying equation 2.11 for a mean concentration of 0.35, we find a Stokes number of 55. The work of Davis (1985), showed that rebound occurs for $St > 0.5$. The results shown are in the range of rebound. But, if the Bagnold number is decreased below 450 (obtained with a decrease in particle diameter), despite an increase in the mean concentration the resulting value of $St$ approaches the limit of no rebound. But this situation is outside the grain-inertia regime where the kinetic theory is applicable.

The discussion of the limit of validity of the results concerning high particle Reynolds number can be made using a comparison with the dry granular flow. In those flows, the interstitial fluid viscosity has a very small effect. The velocities are lower than those obtained for a granular-fluid flow because of the absence of turbulent fluid stresses, but the framework used in constructing the model does not allow us to think in terms of higher viscosities. The kinematic viscosity of air at 1 atmosphere pressure and 27° C is 44 times lower than that of water at the same temperature. If we consider particles of 1.6 mm, an upper limit of particle Reynolds number is $Re_l = 10,000$. This is so because this value of $Re_l$ is obtained with a kinematic viscosity 44 times lower than that of water. If we use a diameter of 3 mm in order to have the same ratio of kinematic viscosities we need $Re_l = 27,000$. Depending on the particle diameter under
consideration, the range of validity of the model can be extended to higher particle Reynolds numbers of the magnitude already shown. But in reality this upper limit is further reduced because of the effect of the decrease in concentration on the reduction of the Bagnold number. The results of figure 6.9 for Re_l=1000 show that for d=1.6 mm, with a mean concentration 0.32, a Bagnold number Ba=458, which is close to the limit of the grain-inertia flow regime, is obtained.

6.2 EXPLICIT SOLUTION FOR LOCKED GRAIN-FLUID FLOW

6.2.1 Governing equations.

It is very difficult to extract information about the flowing system without a full solution of the momentum and energy equations using the numerical method outlined in paragraph 6.1. However there is a special situation when the flow concentration approaches the closest packed concentration. For this kind of flow, the concentration is approximately constant throughout the layer, while the velocity gradient is non-zero only in a thin sublayer adjacent to the plane surface. This situation is outside the grain-inertia regime, Ba<450, because of the low values of shear rate. The mechanical behaviour of the mixture is dominated by frictional stresses. The numerical model of this research is not appropriate to reproduce this situation. However, we will conduct an analysis of this flow regime because, during the experiments, flows were observed with characteristics that can only be explained by such a locked grain-fluid flow. The equations describing the system may be simplified to yield an explicit solution that gives us information about the flow without the use of numerical computations.
We use a treatment similar to the one used by Nott and Jackson (1990). The details of the calculations are given in Appendix A. The equation finally obtained for the nondimensional velocity $u_1$ in the bulk of the material is

$$u_1^* - u_0^* = \frac{\tan \theta}{1} \left( \tan \theta - 1 \right) \left( T_w^{*1/2} - T_l^{*1/2} \right) \left( v(1-A) \left( h^* - y_1^* \right) + \frac{1}{6} \left( \frac{y}{v} \right)^{2/3} + v(1-A) h^* \right) + \frac{\tan \theta}{1} \left( 1 - A \right) y_1^* T_2^{*1/2} \tan \theta \tan \theta \left( T_w^{*1/2} - T_l^{*1/2} \right) \right) \left( 6.2.1 \right)$$

where $y_1^*$ is the position at which the flowing material locks and the velocity gradient becomes zero. The granular temperature in the bulk of the locked flow $T_1^*$ is

$$T_1^* = T_w^* \exp \left( \frac{-2h y_1^*}{3} \right) \left( 6.2.2 \right)$$

where $\lambda_1$ is given by equation A9 (Appendix A). To solve the above equations it is first necessary to calculate the value of the concentration by means of equation A14 (Appendix A).

### 6.2.2 Calculations for locked grain-fluid flow.

Some calculations were performed for this flow regime using almost the same parameters as in section 6.1. It was necessary to diminish the particle-particle coefficient of restitution to $e_p = 0.6$ in order to obtain reasonable values of velocity. It was also found that the apparent fluid viscosity term $C_n$ should be increased. This is understandable since we are
dealing with extremely high concentrations larger than 0.55 ($v_{max}=0.64$). The Batchelor term is not appropriate to describe the high increase in viscosity with concentration. We used the expression proposed by Lun and Savage (1992) with modification of the constant affecting the concentration squared. We used 1 instead of 0.333 to further increase the apparent viscosity. The apparent viscosity factor becomes

$$C_n=(1-n-n^2)^{-2.5} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6.2.3)$$

It was found that for $Re_1$ below 100 the granular temperature at the bed decreases so much that $Re_f$ based on the fluctuation velocity decreases sharply. This causes a high increase in the drag coefficient and the flow velocity decreases to near zero. At the other extreme when $Re_1$ increases above 140 the granular temperature at the bed increases very much and the depth $y_1$ increases to a value where the assumption of zero shear rate in the bulk of the flow is no longer valid. This flow regime is therefore confined to a very narrow range of particle Reynolds number: 100<$Re_1<$140. The following table shows the results for the limiting cases:

**TABLE 6.2**

RESULTS OF LOCKED GRAIN-FLUID FLOW

$\theta=22^\circ$, $\phi=28.5^\circ$, $\delta=12.3^\circ$, $e_f=e_p=0.6$

<table>
<thead>
<tr>
<th>$Re_1$</th>
<th>$T_w^*$</th>
<th>$Re_f$</th>
<th>$C_0$</th>
<th>$v$</th>
<th>$h^*$</th>
<th>$y_1^*$</th>
<th>$U_1^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.06</td>
<td>39</td>
<td>247</td>
<td>0.556</td>
<td>4.4</td>
<td>0.49</td>
<td>1.42</td>
</tr>
<tr>
<td>120</td>
<td>0.23</td>
<td>91</td>
<td>162</td>
<td>0.555</td>
<td>4.3</td>
<td>1.13</td>
<td>2.20</td>
</tr>
</tbody>
</table>

Even if the sensitivity of the model is extremely high
relatively to the concentration, the main trends of the high concentration situation obtained with the numerical model are reproduced. There is an increase in velocity with a decrease in concentration and the flow depth decreases when the velocity increases.

The most interesting result of this model is that, at very high concentrations, the energy dissipation by viscous drag is even larger than for the dilute flow situation. This is caused by the very high apparent viscosity of the mixture.

The difference of the drag coefficient between the case of high concentration and that of locked grain-fluid flow as shown in figure 6.17, demonstrates the need for improvement of the models describing the variation of viscosity with concentration. Most of them are empirical; only the work of Frankel and Acrivos (1967) produced an analytical model but it is only valid for the case of extremely concentrated suspensions. However the factor \( C_n \) of Frankel and Acrivos is lower than the one adopted in this locked grain-fluid flow model (see figure 6.17).
CHAPTER 7

EXPERIMENTAL OBSERVATIONS

7.1 EXPERIMENTAL SET-UP

Experiments of steady, uniform flow down an incline, of sand-water mixtures were conducted in a 4 m long, 10.2 cm wide channel (figure 7.1). The chute angle was varied between 19 and 23 degrees. Sand particles with a mean diameter of 1.2 mm were used. The granulometric curve is presented in table 7.1:

<table>
<thead>
<tr>
<th>Sieve diameter</th>
<th>% retained</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>9.9</td>
</tr>
<tr>
<td>1.5</td>
<td>37.7</td>
</tr>
<tr>
<td>1.0</td>
<td>35.0</td>
</tr>
<tr>
<td>0.5</td>
<td>15.6</td>
</tr>
<tr>
<td>0.25</td>
<td>1.6</td>
</tr>
<tr>
<td>0.125</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The channel bottom was roughened with sand particles of the
same kind glued with epoxy. The granular material was supplied by a hopper located at the upper end of the channel. Three hole diameters were used at the exit of the hopper to control the granular flow rate, 4.5 cm, 3.5 cm and 3.0 cm. The mass flow rates were measured in separate sets of experiments by measuring the time to fill a box of known volume. They are respectively 765, 377 and 255 cm$^3$/s.

The experiments were conducted for different chute angles, keeping the mass flow rate constant and varying the water flow rate. The mass flow rate is $Q_s = U_{avg}v_{avg}hw$, where $U_{avg}$ is the mean cross-sectional velocity, $v_{avg}$ is the mean concentration, $h$ the flow depth and $w$ is the channel width. The water flow rate was measured by a flow meter. The flow depth was measured by a point probe, located 1 m upstream of the channel exit, to ensure that the flow was fully developed. The uniformity of the flow depth was confirmed by marking the flow depth all along the length of the channel side wall which was clear and allowed for direct visual observation.

Approximately 10% of the silica sand was painted in black to be used as tracers during the visual velocity measurements. The velocity profiles were obtained near the wall by means of a Hi 8 mm video camera.

7.2 EXPERIMENTAL RESULTS

The first measurements were limited to the mixture flow depth. With these data we tried to establish a relationship between the average cross-sectional velocity and the mixture flow depth. The average cross sectional velocity can be estimated by the volume fluxes (the water flow rate $Q_w$ and the mass flow rate $Q_s$) and the sectional area ($wh$). An average value of the mean concentration can also be estimated: $v_{avg} = Q_s/(Q_w+Q_s)$. 
The following table shows the results obtained for each mass flow rate $Q_s$ varying the water flow rate $Q_w$. We present the results for three values of mass flow rate $Q_s$, each one associated with a chute angle $\theta$.

**TABLE 7.2**

**RESULTS BASED ON FLOW DEPTH MEASUREMENTS**

<table>
<thead>
<tr>
<th>$Q_w$ (cm$^3$/s)</th>
<th>$h$ (mm)</th>
<th>$v_{avg}$ (cm/s)</th>
<th>$v_{avg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_s = 255$ cm$^3$/s, $\theta = 22.9^\circ$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>183.7</td>
<td>7.9</td>
<td>54.4</td>
<td>0.58</td>
</tr>
<tr>
<td>202.7</td>
<td>7.3</td>
<td>61.5</td>
<td>0.56</td>
</tr>
<tr>
<td>266.0</td>
<td>6.7</td>
<td>76.2</td>
<td>0.49</td>
</tr>
<tr>
<td>380.0</td>
<td>6.4</td>
<td>97.3</td>
<td>0.40</td>
</tr>
<tr>
<td>487.3</td>
<td>6.7</td>
<td>108.6</td>
<td>0.34</td>
</tr>
<tr>
<td>582.2</td>
<td>7.0</td>
<td>117.3</td>
<td>0.30</td>
</tr>
<tr>
<td>797.4</td>
<td>7.5</td>
<td>137.6</td>
<td>0.24</td>
</tr>
</tbody>
</table>

| $Q_s = 377$ cm$^3$/s, $\theta = 21.6^\circ$ |          |                 |          |
| 386.0            | 7.4     | 101.1           | 0.49     |
| 405.0            | 7.0     | 109.5           | 0.48     |
| 436.7            | 6.7     | 119.1           | 0.46     |
| 506.3            | 7.0     | 123.7           | 0.43     |

| $Q_s = 765$ cm$^3$/s, $\theta = 20.1^\circ$ |          |                 |          |
| 760.0            | 12.9    | 115.9           | 0.50     |
| 823.0            | 11.3    | 137.8           | 0.48     |
| 886.0            | 10.7    | 151.3           | 0.46     |
The experimental results are plotted in figures 7.2, 7.4 and 7.6. Since the width \( w = 10.2 \) cm is constant, we present the figures as a function of \( Q_s/w \). The values of \( Q_s/w \) corresponding to \( Q_s = 255 \text{ cm}^3/\text{s}, 377 \text{ cm}^3/\text{s} \) and \( 765 \text{ cm}^3/\text{s} \), are respectively 25 cm\(^2\)/s, 37 cm\(^2\)/s and 75 cm\(^2\)/s.

Table 7.2 shows a trend in the values of mean velocity against flow depth very different to that of pure water. There is a U-shaped curve when we plot \( U_{avg} \) against \( h \), for a constant mass flow rate. The depth decreases with an increase in the flow velocity (at high concentrations) and then increases with further increase in the flow velocity (at intermediate concentrations). This is better observed in figure 7.2 where the results obtained for \( Q_s/w = 25 \text{ cm}^2/\text{s} \) are presented. The behaviour is also present for the other mass flow rates. For \( Q_s/w = 75 \text{ cm}^2/\text{s} \), our experimental set-up did not allow us to obtain larger water flow rates and all the results are in the range of high concentrations. For high concentrations (larger than 0.45) the velocity increases when the flow depth decreases. This behaviour is opposite to that of the water flow typically represented by the Chezy or Manning equation. This behaviour is independent of the mass flow rate or the chute angle. It seems to depend on the Bagnold number as will be discussed later.

The next step of the experimental program was to establish typical velocity profiles. We decided to use two mass flow rates \( Q_s = 255 \text{ cm}^3/\text{s} \) and \( Q_s = 765 \text{ cm}^3/\text{s} \) to define the shape of the velocity profiles in the increasing part of the curve (intermediate concentrations) and in the decreasing part of the curve (high concentrations).

At intermediate concentrations (0.3-0.45), the velocity
profiles show a relatively low slip velocity. The slope of the curve is higher near the bed therefore the shear rate is relatively high near the bed and decreases near the surface (see figure 7.9 for $Q_s=255 \text{ cm}^3/\text{s}$ and $Q_w=316.4 \text{ cm}^3/\text{s}$). For high concentrations (larger than 0.45), the slip velocity becomes higher and the shear rate decreases sharply near the bed in comparison with the intermediate concentration case (see figure 7.8 for $Q_s=255 \text{ cm}^3/\text{s}$ and $Q_w=228 \text{ cm}^3/\text{s}$).

7.3 ANALYSIS OF EXPERIMENTAL RESULTS

7.3.1 Simple model applied to experimental results

In order to reproduce the experimental results it is necessary to keep constant the values of mass flow rate, $Q_s$. Because of the difficulty to obtain this condition during the numerical simulations, we decided to build a simple model based on the kinetic theory but using the averaged values of the variables. This model uses as variables $v_{avg}$, $U_{avg}$ and $h$, which define the mass flow rate, $Q_s$. The simple model is only expected to give the trend and the orders of magnitude of the results. The model equates the driving forces and the resisting forces in an infinitesimal channel segment $dx$, per unit width of the channel. For the steady, uniform flow situation that we are considering, the driving force is the component of the gravitational force in the streamwise direction, $\rho mgh \sin \theta dx$. The resisting forces are the collisional component in the streamwise direction $\tau_c dx$, the frictional component $\tau_f dx$ and the bed frictional force caused by turbulence in the streamwise direction $C_f \rho_w U_{avg}^2 dx/2$, where $C_f$ is the bed friction factor. The balance of forces is

$$\phi_m g h \sin \theta dx = \tau_c dx + \tau_f dx + \frac{1}{2} C_f \rho_w U_{avg}^2 dx ................................................(7.1)$$
where \( \tau_f \) is related to the normal frictional stress by Coulomb's law; \( \tau_f = \sigma_f \tan \phi \). After the assumption of collisional dominated flow, the normal frictional force can be represented by the difference between the total normal force and the collisional normal force

\[
\sigma_f = (\rho_s - \rho_f) g h \cos \theta \nu_{avg} - \sigma_c \quad \ldots \quad (7.2)
\]

The normal collisional stress component is related to the collisional shear stress by \( \sigma_c = \tau_c / \tan \phi_d \), where \( \phi_d \) is the dynamic internal friction angle of the solids. For the frictional shear stress we use a form similar to that proposed by Bagnold

\[
\tau_c = \rho_s d^2 g \left( \frac{\nu_{avg}}{h} \right)^2 \tan \phi_d \quad \ldots \quad (7.3)
\]

where the term in parenthesis is the mean shear rate. The expression for \( g_2 \) is given by equation (2.5) replacing \( v \) by \( \nu_{avg} \).

The balance of forces equation (7.1), using (7.2) and (7.3) gives the following expression for the mean cross-sectional velocity

\[
U_{avg} = \sqrt{\frac{gh \sin \theta [(A-1) \nu_{avg} (1-\frac{\tan \phi}{\tan \theta}) + 1]}{\frac{C_f}{2} \frac{4 \rho_s d^2 g}{h^2} (\tan \phi_d - \tan \phi)}} \quad \ldots \quad (7.4)
\]
With equation (7.3) we can obtain the values of \( U_{\text{avg}} \) corresponding to values of \( h_{\text{avg}} \). To reproduce the experimental results we need a constant value of \( Q_s = U_{\text{avg}} \nu_{\text{avg}}hw \) for each chute angle. A graphical solution can be obtained plotting \( Q_s/w \) against \( h \) for different values of \( \nu_{\text{avg}} \). Then on a plot of \( U_{\text{avg}} \) against \( h \), the value of \( h \) corresponding to the previously defined values of \( Q_s/w \) and \( \nu_{\text{avg}} \) is localised. To reproduce the experimental results the parameters shown in table 7.3 were used in the calculations:

**TABLE 7.3**  
Parameters used in the calculations with the simple model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>35°</td>
</tr>
<tr>
<td>( \phi_d )</td>
<td>36°</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>2500 kg/m³</td>
</tr>
<tr>
<td>( C_f )</td>
<td>0.01</td>
</tr>
<tr>
<td>( d )</td>
<td>1.0 mm</td>
</tr>
</tbody>
</table>

Figures 7.2, 7.4 and 7.6 show the plots of \( U_{\text{avg}} \) against \( h \), corresponding to the values of \( Q_s/w = 25 \text{ cm}^2/\text{s} \), \( Q_s/w = 37 \text{ cm}^2/\text{s} \) and \( Q_s/w = 75 \text{ cm}^2/\text{s} \) respectively. The analytical values of \( Q_s/w \) are shown in figures 7.3, 7.5 and 7.7. In figures 7.2, 7.4 and 7.6 the values of \( U_{\text{avg}} \) against \( h \), obtained experimentally, are also plotted for comparison. The agreement is good. These results confirm the assumption that the collisional stresses are dominant in the grain-inertia regime at high concentrations. The confirmation of the latter assumption is better observed in the plot of Bagnold number versus concentration. The \( B_a \) against mean concentration curve is shown in figure 7.10. Table 7.4 shows the experimental values obtained for \( Q_s = 255 \)
cm³/s and θ=22.9° and those predicted by the simple model for the same conditions. The values of du/dy are obtained from table 7.2 with the relationship du/dy=2U_{avg}/h. The values of Ba predicted by the simple model are obtained with the same relationship, but with the values of U_{avg}, h and v_{avg} corresponding to Q_{s}/w=25 cm²/s, plotted in figure 7.2.

<table>
<thead>
<tr>
<th>v_{avg}</th>
<th>Experimental</th>
<th>Simple model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>du/dy</td>
<td>Ba</td>
</tr>
<tr>
<td>1/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.24</td>
<td>368</td>
<td>1399</td>
</tr>
<tr>
<td>0.30</td>
<td>333</td>
<td>1451</td>
</tr>
<tr>
<td>0.34</td>
<td>322</td>
<td>1531</td>
</tr>
<tr>
<td>0.40</td>
<td>303</td>
<td>1645</td>
</tr>
<tr>
<td>0.49</td>
<td>226</td>
<td>1542</td>
</tr>
<tr>
<td>0.55</td>
<td>167</td>
<td>1421</td>
</tr>
<tr>
<td>0.58</td>
<td>136</td>
<td>1280</td>
</tr>
</tbody>
</table>

It is observed in table 7.4 that with the simple model a peak is obtained for a concentration similar to the one observed experimentally. This peak is found to coincide with the onset of large turbulent stresses. It is however surprising to find such large turbulent stresses generated by the channel boundary roughness at intermediate concentrations (v_{avg}=0.4), even considering a small friction factor. This is perhaps caused by the approximate definition that we gave here of the bed frictional stresses. It has been shown experimentally that the bed frictional resistance in highly concentrated suspensions differs from the conventional resistance of
Newtonian fluids. Nevertheless, the results confirm the important role of the turbulent stresses at moderately high concentrations.

7.3.2 Analysis considering variation of shear rate with distance from bed

The difference in velocity profiles between the intermediate concentration range (figure 7.9) and the high concentration range (figure 7.8) generates different effects in the calculation of the flow Bagnold number. In this paragraph we consider the Bagnold number near the bed, because the shear rate near the bed \((du/dy)_w\) is the largest. For high concentrations, the high reduction in the shear rate near the bed decreases the Bagnold number despite the high linear concentration. For intermediate concentrations, the small increase in the shear rate near the bed is not large enough to eliminate the decrease in \(Ba_w\) because of the low linear concentration. As a consequence, the Bagnold number has a maximum value for some concentration and shear rate and decreases for either larger or lower concentrations. This is shown in table 7.5 where the Bagnold number near the bed, \(Ba_w\), is shown for the experimental case of \(Q_w=255\ \text{cm}^3/\text{s}\) and \(\theta=22^\circ\). The corresponding trend in the shear rate is also shown in table 7.5:

<table>
<thead>
<tr>
<th>(Q_w) (\text{cm}^3/\text{s})</th>
<th>(h) (mm)</th>
<th>(v_{avg})</th>
<th>(\lambda)</th>
<th>((du/dy)_w) (1/s)</th>
<th>(Ba_w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>227.8</td>
<td>5.4</td>
<td>0.52</td>
<td>9.45</td>
<td>230</td>
<td>2545</td>
</tr>
<tr>
<td>253.1</td>
<td>4.9</td>
<td>0.50</td>
<td>8.02</td>
<td>357</td>
<td>3640</td>
</tr>
<tr>
<td>316.4</td>
<td>7.1</td>
<td>0.44</td>
<td>5.90</td>
<td>392</td>
<td>3430</td>
</tr>
</tbody>
</table>
The results show a maximum value of $B_{aw}$. The significance is that the ratio of collisional stresses over total stresses has a maximum value for some concentration value. This point coincides with that of minimum value of flow depth. The minimum flow depth is also found, for the maximum $Ba$ (table 7.2), in the data corresponding to $Q_s=255 \text{ cm}^3/\text{s}$. We can state that the minimum depth is obtained for the maximum value of the Bagnold number. The difficulty is to define the concentration for which we will obtain the maximum $Ba$.

For example, when we increase $Q_s$, even for a similar chute angle, a lower value of $(du/dy)_w$ is obtained for a fully developed flow of the same concentration. This can be shown comparing the velocity profile of the flow for $Q_s=255 \text{ cm}^3/\text{s}$ and $Q_w=253.1 \text{ cm}^3/\text{s}$ with that for $Q_s=765 \text{ cm}^3/\text{s}$ and $Q_w=760 \text{ cm}^3/\text{s}$ in figure 7.8. The consequence is that the maximum value of $Ba_w$ will be obtained for a much lower value of concentration. Therefore with the concentration only, we cannot define the location of the maximum Bagnold number. The other important variable, the shear rate, depends on the concentration, flow rates and chute angle. Therefore it is very difficult to define analytically the occurrence of the maximum $Ba$. We can only establish the shape of the $v$ against $Ba$ curve and offer some hypothesis about the behaviour of the mixture when it approaches the closest pack concentration or the dilute flow situation.

If we extrapolate the trend of the curve $v$ against $(du/dy)_w$ for concentrations near the closest pack, we find that the shear rate near the bed decreases to a minimum value and we get the "locked grain-fluid flow". This flow is dominated by frictional stresses and it is likely that $Ba_w$ for this flow is lower than 450. Our experimental set-up did not allow us to
observe a fully developed locked grain-fluid flow, possibly because of the energetic way in which the granular material is supplied. In fact Johnson et al. (1990) observed that for dry granular flows, the locked granular flow is obtained only with "dense entry conditions" obtained with a pre-chute chamber previously filled with the grains and then controlled by a gate valve. If the grains leaving the hopper fell freely, they obtained a low density, highly energetic state at the chute entrance. The resulting flow is called by them "loose entry condition flow". For this kind of entry condition they were not able to obtain the locked flow. Our entry condition is similar to the latter.

At the other extreme, when the concentration is very low the shear rate increases. The Bagnold number decreases because of the decrease in linear concentration and the flow is no longer in the grain-inertia regime but in the macroviscous flow regime.

7.3 COMPARISON WITH TAKAHASHI'S EQUATIONS

We used Takahashi's equations (1.7 and 1.9) to predict the mean cross-sectional velocity for the experiments with the 22.9° chute angle and $Q_s=255 \text{ cm}^3/\text{s}$. To use the equation valid for the stony flow regime (1.7), since we know the flow depth and average concentration, we only need to know the value of the parameter "a". We made several calculations and we found that the best agreement is obtained with a value $a=0.002$. This value is very much lower than that proposed by Bagnold of 0.042, but we must keep in mind that this value was an empirical fit of his own experiments with paraffin wax and lead stearate. To apply the equation valid for turbulent flow regime (1.10) we used a value of the Von Karman's constant $\kappa=0.4$ and a value $a=0.042$. The following table shows the
results obtained:

TABLE 7.6

EXPERIMENTAL RESULTS USING TAKAHASHI'S EQUATIONS

\( Q_s = 255 \ \text{cm}^3/\text{s}, \ \theta = 22.9^\circ \)

<table>
<thead>
<tr>
<th>( Q_s ) (cm(^3)/s)</th>
<th>( h ) (mm)</th>
<th>( v_{sw} )</th>
<th>VELOCITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>STONY</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>m/s</td>
</tr>
<tr>
<td>183.7</td>
<td>7.9</td>
<td>0.579</td>
<td>0.54</td>
</tr>
<tr>
<td>202.7</td>
<td>7.3</td>
<td>0.555</td>
<td>0.65</td>
</tr>
<tr>
<td>266.0</td>
<td>6.7</td>
<td>0.487</td>
<td>1.04</td>
</tr>
<tr>
<td>487.3</td>
<td>6.7</td>
<td>0.342</td>
<td>2.28</td>
</tr>
<tr>
<td>582.2</td>
<td>7.0</td>
<td>0.302</td>
<td>2.89</td>
</tr>
<tr>
<td>797.4</td>
<td>7.5</td>
<td>0.240</td>
<td>4.15</td>
</tr>
</tbody>
</table>

Comparing the velocities of table 7.2 with these velocities, we observe that only the stony model is able to reproduce the trend of the increase in average cross-sectional velocity with a decrease in the average concentration for the entire range of concentrations. However for low concentrations, the predictions are far from the estimated velocities. The model is only valid for high concentrations. The difficulty to apply the model, even for high concentrations, is the definition of the constant "a", that as was shown varies according to the experimental conditions. In these experiments a is 20 times lower than the value proposed by Bagnold. The turbulent model is not able to reproduce the shape of the curve of mean velocity against flow depth.

The example discussed shows the need of a theory that is able to describe more accurately the behaviour of the mixture in the high concentration range. The theory presented in this research, even if limited to idealized monosized spheres, is a first step towards the solution of the difficulties just
described. The application of the theory to the experiments will be discussed in section 7.5.

7.4 LIMITS OF VALIDITY OF THE MODEL.

We will discuss the limits of validity of the theory developed in this research. The most obvious happens when the role of the viscous stresses becomes dominant and the collisions are fully dampened. An introduction to this discussion was made in section 2.2. It was shown that the Stokes number determines the occurrence or not of rebound in the case of rigid particles. It was also shown that the Stokes number can be related to the Bagnold number by means of equation (2.11). When the concentration is low, the flow $Ba$ is also low and $f(v)$ approaches unity. The Stokes number is less than 10, for Bagnold numbers lower than 5. This is the macroviscous regime defined by Bagnold. The model developed in this research is valid only for the grain-inertia regime. Another limit of validity occurs when the concentration is near the closest packed where it was also shown that $Ba$ decreases because of the decrease in the shear rate. In this situation $f(v)$ increases rapidly and the Stokes number is greatly reduced. This is the locked grain-fluid flow where the frictional interactions become dominant. In our experimental range (concentrations larger than 0.3 and lower than 0.6), the Stokes number is always larger than 10. For example, the data of table 7.2 give values of Stokes number of 144, 227 and 260 for average concentrations of 0.55, 0.49 and 0.40 respectively. We can conclude that the situation of no rebound for the case of rigid particles in the grain-inertia regime is unlikely.

7.5 APPLICATION OF THE NUMERICAL MODEL TO EXPERIMENTAL RESULTS

The experiments performed do not agree with some basic
assumptions of the numerical model. The particle size is not uniform and the silica sand is not perfectly spherical. We decided to use silica sand because of its availability and because we are interested in explaining some features of geophysical flows such as debris flows where the solids are heterogeneous. The numerical simulations do not try to reproduce quantitatively the experimental results but only provide a qualitative description of the mechanical behaviour observed during the experiments.

The main experimental trend is the increase in mean velocity with a decrease in the mean concentration. The reproduction of this trend is observed in figures 6.9-6.10 and 6.13-6.14. The figures show the results obtained for a 22° chute angle and we observe an increase in velocity with the particle Reynolds number \( \text{Re}_p \) followed by a reduction in the mean concentration with the increase of \( \text{Re}_p \). It is interesting to observe that the granular temperature increases also with particle Reynolds number even if the concentration decreases. This trend in the granular temperature is found for both the intermediate concentration range and the high concentration range.

Another trend in the experimental results is the U-shape of the average velocity against flow depth curve for a constant value of \( Q_s = U_{\text{avg}} v_{\text{avg}} h_w \). It was shown that this shape of the curve can be explained by the variation of the flow Bagnold number. When we consider an average concentration and a uniform shear rate, it is easy to define a flow Bagnold number. However when the concentration and shear rate change with depth there is also a variation of Bagnold number with depth. It is thus difficult to control the variation of the Bagnold number during the numerical simulations. One way to avoid this difficulty is to exploit the similarity between the Bagnold and the flow Reynolds number. The Bagnold number can be approximated to \( \text{Ba} = 2\text{Re}/\lambda d/Ah \).
with $\frac{du}{dy} = 2U_{av}/h$. Therefore the variation of $Ba$ can be explained with that of the flow Reynolds number $Re = \frac{\rho U_{av} h^2}{\mu_f}$ and the ratio of linear concentration to flow depth. Below some concentration value corresponding to the maximum value of $Ba$, the Reynolds number increases but the linear concentration decreases very fast and the flow depth increases (the ratio $\lambda/h^*$ decreases) and therefore the Bagnold number decreases. Above the concentration corresponding to maximum $Ba$, the Reynolds number decreases with an increase in concentration (see figures 6.13-6.14) and the depth of flow also increases eliminating the effect of the increase in linear concentration. The Bagnold number therefore decreases. Exploiting the similarity between $Re$ and $Re_i$ it is thus possible to find the concentration corresponding to maximum $Ba$ by means of $Re_i$. Plotting the velocity profiles for different $Re_i$, it is possible to find the value of Reynolds number where the depth of flow is the smallest.

The application of the numerical model to reproduce this curve presents some difficulties. It is difficult to control simultaneously the flow depth, mean concentration and mean velocity. The only method available is by trial and error. The examples shown in Chapter 6 present similar values of mass flow rate, but not exactly the same. For example for the velocity profiles of the $22^\circ$ chute flow shown in figures 6.9 and 6.13, where the mass flow rate is close to 100 cm$^3$/s (considering a particle diameter of 1.6 mm) the minimum value of flow depth is found for $Re_i=200$. The mean concentration corresponding to this situation is found in figure 6.10 (for $Re_i=200$) to be equal to 0.4. This value is similar to the value found in the experiments with $Q_o=255$ cm$^3$/s. The discrepancies could be due to the fact that the closest packed concentration in the numerical simulations is 0.64 (corresponding to monosized spheres) and the silica sand has
a value of \( v_{\text{max}} = 0.71 \). However the overall trend is well reproduced by the numerical simulations shown in figures 6.9 through 6.16. The dimensional numerical results corresponding to a particle diameter of 1.6 mm and using the value of the shear rate near the bed \((du/\text{dy})_w\) to calculate the Bagnold number, are summarized in table 7.7:

### TABLE 7.7
**DIMENSIONAL NUMERICAL RESULTS**
*(Figures 6.9-6.16)*

<table>
<thead>
<tr>
<th>( \text{Re}_1 )</th>
<th>( v_{\text{avg}} )</th>
<th>((du/\text{dy})_w)</th>
<th>( \text{Ba}_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0.32</td>
<td>56</td>
<td>458</td>
</tr>
<tr>
<td>500</td>
<td>0.34</td>
<td>44</td>
<td>695</td>
</tr>
<tr>
<td>200</td>
<td>0.40</td>
<td>40</td>
<td>722</td>
</tr>
<tr>
<td>180</td>
<td>0.42</td>
<td>36</td>
<td>679</td>
</tr>
<tr>
<td>160</td>
<td>0.43</td>
<td>34</td>
<td>672</td>
</tr>
</tbody>
</table>

The shape of the concentration against \( \text{Ba}_w \) curve is well reproduced. The shear rate near the bed increases monotonically with \( \text{Re}_1 \) as in the experiments. The decrease in the shear rate with increase in the average concentration is also well reproduced. The ratio \( u_{\text{slip}}/u_{\text{surface}} \) is lower in the numerical results than in the experiments but the overall trend in the slip velocity is well reproduced (see figure 6.9).

We can conclude that the main experimental features of the experimental results are well reproduced by the numerical model. The quantitative agreement with respect to surface velocities, slip velocities, and concentrations for maximum \( \text{Ba} \) is difficult to obtain because of the many parameters involved.
in the model corresponding to idealized material and experimental conditions that we do not respect.

7.6 APPLICATION OF THE NUMERICAL MODEL TO THE NEVADO DEL RUIZ 1985 DEBRIS FLOW

7.6.1- Nevado Del Ruiz physical environment.

The Nevado del Ruiz Volcano is located at the northernmost end of the Andean Volcano chain, Colombia, South America. Its elevation is 5200 m over sea level. On November 13, 1985 this eruption caused the second-worst volcano disaster of this century killing 22 000 people (Mount Pelee Martinique 1902, killed 28 000). The disaster was actually caused by a debris flow originating from this eruption. I will describe the main characteristics of the debris flow. Then by means of the numerical model and Bagnold's bed load transport equation, based on the assumption that the sediment transport capacity rate is proportional to the rate of energy dissipation within the shearing bed load layer, explain the powerful transport capacity of the debris flow.

Five rivers originate at the Nevado del Ruiz summit and flow down to their mouth in the Magdalena and Cauca river which flows in valleys located at 400 m and 600 m over sea level respectively. Two of these rivers are the Azufrado and Lagunillas. The Azufrado is a tributary of the Lagunillas 40 km downstream from the Ruiz summit. They flow in steep (channel gradients average as much as 0.3 in the upper reaches to as low as 0.02 in the lower reaches) and narrow valleys for 65 km before reaching the Magdalena valley. At the canyon mouth, in the Magdalena valley, was located the town of Armero with a population of 27 000 in 1985.

7.6.2- Debris flow description.
On November 13, 1985 at 9:08 P.M the eruption began, Pierson (1990) explains that hot energetic pyroclastic flows and surges destabilized parts of the ice cap and helped to trigger mixed avalanches. According to the missing water from the ice cap nearly 18 Mm$^3$ began the debris flow. The debris flow continued to erode and increased in volume for at least 70 km until an estimated final volume of 80 Mm$^3$. Average bulking rates of the debris flow in upstream reaches ranged from 100 to 700 m$^3$ of sediment eroded per lineal m of channel. The scour was more than 6 m in some reaches of the channel.

Deeb and Ordonez (1986) tried to define the hydrograph of the flow to apply the HEC-1 model. They estimated a peak flow discharge of 17,400 m$^3$/s at the canyon mouth. If we use this discharge and the cross-section at the canyon mouth the maximum depth of flow at the canyon mouth was 11 m for the estimated average velocity of 8 m/s.

The flow in the Azufrado valley remained narrowly confined all the way to the debris fan where Armero was located. Pierson reports considerable flow turbulence and possibly large-vertical axis eddies. Evidence of hydraulic jump occurrence was found in the steep upper flanks.

7.6.3 Debris-flow deposits.

Debris flow deposits in the canyon show three flow zones:
- Lower flow zone: The true modal size of this zone was in the cobble to boulder range.
- Intermediate flow zone: Deposits in this zone go from ungraded to normally graded. They tend to be finer at higher positions on the valley walls, specially further down.
- Upper flow zone: Constituted of thin coatings or ponded deposits of very poorly sorted, pebbly muddy sands with
matrix-supported clasts seldom larger than 5 cm.

The Armero deposit shows two fan deposits which were caused by the multiple debris flow pulses: At the bottom a first deposit shows a large textural variation. Field observers report boulders of diameter larger than 1 m. Over this deposit a grayer and better sorted muddy gravelly sand was found. It seems to be deposited by the Lagunillas delayed debris flow.

The granulometric characteristics are in % by weight:

- gravel: 40%
- sand: 53%
- silt: 4.5%
- clay: 1.5%

7.6.4 Model application to sediment transport.

There are many unknowns in the Nevado del Ruiz debris-flow, but we can try to apply the numerical model to explain the large sediment transport capacity of the debris-flow.

The main unknown is the constitution of the mixture. Granulometric analysis of the Armero deposit can give us a rough idea, but there are many limitations of these data. Only the material smaller than 1 cm in diameter was sampled. Boulders as large as 5 m diameter were observed in the debris flow, and field observers estimate that nearly 40% by volume of the deposit is coarser than 10 cm. There is however a good description of the fine matrix which can give us an idea of the interstitial fluid viscosity. The percentage of clay and silt with diameter less than 10 microns is 8% in the upper zone and 16% in the lower zone. We can take an average value of 12%. This value of fine content gives the following value of viscosity after laboratory experiments with water-clay...
mixtures conducted by O'Brien et al (1984); \( \mu_z = 0.5 \) poises.

To apply the numerical model we consider the debris flow material as granular material of mean diameter 1 m dispersed in a basic fluid consisting of a mixture of fine particles in water with a clay concentration of 12%. This is a crude representation of the heterogeneous constitution of the Nevado del Ruiz flow material, but we take into account the mean fines content and the field observations of coarse material. The total solids concentration of the debris flow is the sum of the grains concentration and the fines concentration of 12%, and it can not exceed the estimated total solid concentration of 0.6. With such large particle diameters and with the estimated viscosity of the interstitial fluid of 0.5 poises we obtain a particle Reynolds number \( \text{Re}_p = 20000 \). The simulations with the numerical model were carried out with this value of \( \text{Re}_p \) and a chute angle of 10°, a little higher than the mean slope of the Lagunillas river (5°), in order to take into consideration the variation of slope in the downstream direction. The other parameters were chosen similar to those used in the previous calculations because of the difficulties to measure the parameters of the real flow. The results are shown in figures 7.11 through 7.14. The depth of flow is 8 m and the surface velocity 5.2 m/s. These values are lower than the estimated peak values of the Nevado del Ruiz debris flow (\( h=11 \) m and \( U_{avg}=11 \) m/s) and can be considered representative values. They agree with our purpose. We attempt to establish the mechanical characteristics of the debris flow that allowed the transport capacity of 48 Mm³ of solids in approximately two hours.

The sediment transport capacity according to Bagnold's (1966) bed load transport equation is \( Q_s = \beta \tau_c U_{avg}/\tan \phi \), where he assumes that the main mechanism of energy dissipation is the
collision between particles. If we make a calculation of the flow Bagnold number near the bed using the velocity profile 7.11 we obtain \( B_{aw} = 24,000 \), that clearly shows that we are in the grain-inertia regime. If we plot the variation of the shear stresses with the distance from the bed using the velocity, concentration and granular temperature profiles, we find (see figure 7.14) that the collisional stresses are dominant. This is true along almost all the flow depth, with the exception of a small layer near the surface where the concentration diminishes sharply. Therefore Bagnold's bed-load equation is valid for the case in consideration. We need however to modify Bagnold's equation because we are not here considering the small bed load layer usual in conventional rivers but all the depth of the flow. The parameter \( \beta \) accounts for the part of the whole stress which is transferred to the bed via the saltating solids. In the debris flow case we are concerned with the energy transfer in all the layers. Therefore we replace the mean effective fluid velocity in Bagnold's equation with the velocity at the depth \( y \) and we consider the variation of stresses with the vertical. The equation becomes

\[
Q_s = \frac{1}{\tan \phi} \int_0^h \tau_c \, u \, dy
\]

(7.6.1)

With this equation we find that the granular material transported per meter of channel width is \( Q_s = 115,000 \) kg/(s.m). Assuming a channel width of 90 m and a flow duration of 2 hours, the total material transported is 76,000,000 kg. Assuming a density of 2,500 kg/m\(^3\) the total volume transported is 30,040,000 m\(^3\). This value is of the same order of magnitude to that estimated in the Armero deposit (48 Mm\(^3\)).
We conclude that the model was able to explain the powerful transport capacity of the flow and to show the fundamental role of the dispersive stresses in the mechanics of the Nevado del Ruiz debris flow.
CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 THE THEORETICAL MODEL AND NUMERICAL COMPUTATIONS

A theoretical model for the flow down an incline, in the grain-inertia regime, of a granular-fluid mixture was formulated. The rheological model of the mixture includes the dispersive and frictional stresses caused by the interaction between particles and the turbulent stresses caused by the interaction between the turbulent fluctuations of fluid velocity and the mean motion of the mixture. A balance of kinetic energy equation is needed because of the inelasticity of the particle collisions. To formulate this equation a viscous dissipation term is introduced to modify the equations coming from the kinetic theory for granular flow, to take into account the cushioning effect of the interstitial fluid during the collisions between particles.

The momentum and kinetic energy equations for the case of uniform and steady conditions were solved using appropriate boundary conditions on the velocity, granular temperature, concentration and flux of fluctuation energy at the bed and at the surface. The formulation of the boundary conditions was
made using methods coming from the kinetic theory for granular flow to calculate the rate at which momentum and energy are transferred across bumpy boundaries. After introducing the cushioning effect of the interstitial fluid on the particle-wall collisions by means of a viscous dissipation term the conditions that ensure the balance of momentum and energy at the boundaries were obtained.

Velocity, concentration, granular temperature and flux of fluctuation energy profiles were obtained. The influence of the parameters $e_p$, $e_w$ and $Re_1$ was analyzed. It was found that the particle-particle coefficient of restitution has a large effect on the collisional energy dissipation. Only large values of $e_p$ give reasonable velocities. There are increases in the mean velocity and the slip velocity with increasing $e_p$. Also the granular temperature increases as a result of smaller collisional energy dissipation when $e_p$ is increased.

The effect of $e_w$ is more surprising. For diminishing $e_w$ the mean flow velocity, the slip velocity and the depth of flow increase. The more dense flow, allowed by the cushioning of the particle-wall collisions by the fluid, tends to have a higher slip velocity and therefore performs more work. This is reflected in the increase in the granular temperature for decreasing $e_w$.

The particle Reynolds number $Re_1$ is the parameter that helps to explain the surprising results obtained during the experiments. An increase in velocity with an increase in $Re_1$ was observed for nearly constant mass flow rate. However the flow depth decreases with an increase in $Re_1$ for small values of $Re_1$ ($<200$), but for further increase in $Re_1$ it increases. The concentration decreases with increasing $Re_1$ for all the range of values of particle Reynolds number. When we analyze
the variation of the maximum shear rate (near the bed) with \( \text{Re}_i \), it is found that there is a monotonic increase. The Bagnold number is proportional to the particle Reynolds number, the linear concentration and the shear rate. The combined effect of the monotonic increase in shear rate and linear concentration with the decrease in \( \text{Re}_i \) with concentration, generates a concave upwards \( \text{Ba} \) versus \( v \) curve. The experimental results confirmed the finding that the concentration corresponding to minimum flow depth corresponds also to the maximum Bagnold number.

The locked grain-fluid flow is a limiting case when the concentration of the mixture approaches the closest packed concentration of the solids. It corresponds to a narrow range of \( \text{Re}_i \) and to \( \text{Ba} \) less than 450. It was useful to study this case because the equations could be solved explicitly and it was found that the viscous drag dissipation attains maximum values in this flow regime because of an abrupt increase in the apparent viscosity of the mixture.

8.2 EXPERIMENTAL OBSERVATIONS.

When the chute angle and the mass flow rate \( (Q_s) \) are kept constant and only the fluid flow rate \( (Q_w) \) is varied, two main trends are observed in the experiments down an incline. The first is a decrease in concentration with an increase in flow velocity for all the range of mixture flow rates \( (Q_s + Q_w) \). The other is a concave downwards \( U_{\text{avg}} \) versus flow depth \( (h) \) curve. For high concentrations there is a decrease in flow depth with an increase in the flow velocity. When the concentration decreases the inverse behaviour is observed. The trend observed for high concentrations is not found in pure water flows and is related to the increase in the frictional
interactions between particles. Some researchers in the field of dry granular flows, Savage and Sayed (1984), have observed a similar behaviour. Their explanation is that the shear rate is not strong enough to generate large grain temperatures and the large concentrations together with the gravitational forces tend to collapse the material.

When the experimental Bagnold number near the bed is plotted against concentration, a concave upwards curve (n-shape) is observed. The peak of the curve was shown to correspond to the minimum flow depth. Therefore, the concentration corresponding to minimum flow depth (maximum Ba) represents the situation where the ratio of collisional stresses over frictional and turbulent stresses has a maximum value.

The experimental velocity profiles show a monotonic decrease in shear rate near the bed with increasing concentration. This explains the decrease in Ba with the increase in concentration.

8.3 CONCLUSIONS.

The analytical model developed is able to reproduce qualitatively the overall behaviour of a flowing granular-fluid mixture down a chute. The velocity profiles show a relatively large slip velocity at the wall and a decrease in the shear rate as the concentration increases. The concentration profiles show higher densities near the bed than the corresponding dry granular flows. This effect is caused by the interstitial fluid and was shown to be of great importance in the dynamics of the flow. The concentration profiles also show large densities in the bulk of the flow and a sharp decrease near the surface. From the granular temperature profiles it was found that the bed acts as a source of pseudo-thermal energy even when the wall-particle collisional
dissipation of energy is large. The particle-particle coefficient of restitution $e_p$, the particle-wall coefficient of restitution $e_\nu$, and the particle Reynolds number $Re_1$ are found to have a fundamental role in the dynamics of the mixture because they control the collisional and viscous drag dissipation terms. The effect of the rigid boundary on the dynamics of the mixture was found to be very important. The large slip velocity present at the boundary performs work and is able to generate granular temperature there.

The quantitative reproduction of the experimental results was not attempted because of the experimental conditions that do not agree with the assumptions of the model (spherical monosized particles) and the measurements devices that were not able to inspect inside the flow. However qualitative results, like the shape of the velocity profiles, the decrease in shear rate with increase in concentration, and the shape of the $Ba$ against $v$ curve are encouraging.

8.4 RECOMMENDATIONS FOR FUTURE RESEARCH.

The analytical model needs improvement; in particular, the way in which the frictional interactions are included in this research is rather crude. A way to extend the kinetic theory to account for the roughness of the particles into the kinetic and collisional components of the stress tensor is necessary.

Also improvement in the formulation of the boundary conditions is needed. It was found that the values of the particle-wall coefficient of restitution that give reasonable values of velocity are very low. These low values should be interpreted as parametric curve fits rather than physically meaningful values. It is possible that we have neglected some energy dissipation mechanism at the wall caused by the presence of
the interstitial fluid. The difficulty can come from the fact that we are using statistical averages of the mixture velocity. At the wall it is known that there is a fluid no-slip condition. It is possible that there our assumption of similar average velocities of the fluid and the solids fails. To consider separate boundary conditions for the fluid velocity and the solids velocity at the wall would overcome the problem, but this changes the entire structure of the model. It is a topic of future research to introduce the fluid no-slip boundary condition in a model formulated with a statistical average mixture velocity.

The accurate definition of the fluid turbulent stresses is necessary. The effect of the particles in the turbulence is an open field of research. It was shown in this research that for moderately high concentrations, the increase in turbulent stresses can be taken into account with the use of the apparent viscosity factor $C_n$, however for low concentrations it is not appropriate. A better definition of the proportionality constant for the Eddy viscosity is also worthwhile.

For the experimental program it is recommended that more sophisticated probes be developed to inspect the interior of the flow. The use of optical techniques limits the kind of materials that can be used. We attempted to build a conductivity probe, but were unable to build a probe of less than 3 mm diameter. This diameter was too large to permit its use in the very shallow flows observed in the present experiments. It is possible that better materials or the increase in the experimental scale could overcome the trouble.
8.5 STATEMENT OF ORIGINALITY.

This is the first time that the dominant role of the dispersive stresses, in a flowing granular-fluid mixture down an incline, is analyzed theoretically by means of a model based on the kinetic theory for granular flow, and confirmed experimentally.

The flux of fluctuation energy was modified by the introduction of a viscous drag dissipation term $D_v$, that was expressed in terms of an original definition of the drag coefficient. To do this we used the apparent viscosity factor $C_n$ and a factor depending on the particle fluctuation Reynolds number, $Re_f$ that includes the relative velocity between the particles and the fluid in the neighbourhood.

The boundary conditions take into account the frictional and turbulent contributions to the stress tensor. They are also affected by the viscous drag dissipation term.

The effect of particles on fluid turbulence was taken into account using the apparent viscosity factor $C_n$.

A link between the specularity coefficient method to calculate the boundary conditions and the method coming from the kinetic theory for a bumpy wall made of hemispherical particles was established. In this way it is possible to have a physical description of the wall roughness when the specularity coefficient is used.

An apparent viscosity factor is proposed for the locked grain-fluid flow.
REFERENCES


Tsubaki, T. and Hashimoto, H. 1983 Interparticle stresses and characteristics of debris flow. Journal of Hydroscience and Hydraulic Engineering, Japanese Society of Civil Engineers, the Committee on Hydraulics 1, #2, 67-82.

APPENDIX A

EQUATIONS FOR GRAIN-FLUID LOCKED FLOW

In this high density case, the concentration has a very steep gradient immediately adjacent to the surface in order for it to go to zero. The surface layer of particles is considered separately from the bulk of the material and we equate the normal stress at the surface to the weight of this layer, giving

\[ n.\sigma = \frac{\pi}{6} (\rho_s - \rho_f) d \left( \frac{v}{v_{max}} \right)^{2/3} g \]  

(A1)

Assuming that the turbulent stresses are small in this dense flow, considering only the viscous dissipation during collisions and using A1, the integrated form of the momentum equations (3.11) and (3.12) is

\[ g \frac{T^*}{\sqrt{\tan \theta}} \frac{du^*}{dy^*} + \frac{\tan \phi}{\tan \theta} g_s (1-A) = (1-A) \left( \int_y^{h^*} v dy^* + \frac{\pi}{6} \left( \frac{v}{v_{max}} \right)^{2/3} \right) \]  

(A2)

\[ \frac{g_s}{\sqrt{\tan \theta}} \frac{du^*}{dy^*} + \tan \phi \frac{g_s (1-A)}{\tan \theta} = (1-A) \left( \int_y^{h^*} v dy^* + \frac{\pi}{6} \left( \frac{v}{v_{max}} \right)^{2/3} \right) \]  

(A3)

From the energy equation (3.13) we obtain

\[ \frac{d}{dy^*} \left[ g_s \frac{T'}{\sqrt{\tan \theta}} \frac{du^*}{dy^*} \right] + g_s \sqrt{T^*} \left( \frac{du^*}{dy^*} \right)^2 - g_s T'^{3/2} - 3.5905 C_p v T'^{3/2} = 0 \]  

(A4)

Replacing \( g_s (1-A) \) from A2 into A3 the following equation is obtained

\[ \frac{g_s}{\sqrt{\tan \theta}} \frac{du^*}{dy^*} = \left[ v (1-A) (h^* - y^*) + \frac{\pi}{6} \left( \frac{v}{v_{max}} \right)^{2/3} \right] \left( 1 - \frac{\tan \phi}{\tan \theta} \right) + \frac{\tan \phi}{\tan \theta} g_s T'^* \]  

(A5)
Where we have used the fact that for constant concentration:
\[ \int_y^y v dy = y (h' - y') \] .......................... (A7)

Using the assumption that the velocity gradient is zero in the bulk of the flow, we obtain from equation A4
\[ g_3 \frac{d}{dy} \left[ \sqrt{T^* \frac{dx^*}{dy}} \right] - \left( g_5 + 3.5905 v C_d \right) T^{*3/2} = 0 \] .......................... (A6)

The solution of equation A6 is
\[ T^* = T_w^* \exp \left( - \frac{2 \lambda_1 y^*}{3} \right) \] .......................... (A8)

where using the definitions of \( g_5 \) and \( g_3 \) we obtain

\[ \lambda_1 = \sqrt{\frac{40.62 (1 - e_p) v^2 g_0 + 10.772 v C_d}{8.502 v^2 g_0}} \] .................................. (A9)

By using equation A8 in the boundary conditions (4.19) and (4.28), the slip velocity is found as a function of \( T_w^* \). This granular temperature at the wall varies only with \( u_s^* \) since the concentration is a known constant. We used \( \Psi = 30^\circ \) to simplify the equations, thus

\[ T_w^*3/2 = \frac{3}{2 \lambda_1} \left[ \frac{g_1 \sqrt{T_w^*}}{0.7979 (u_s^*2 + 0.1372 + (\frac{\partial u}{\partial y})_w)} (0.3872 + 0.25B) u_s^* \right] \\
- (0.855 g_1 (1 - e_w) + 1.595 \left( \frac{v}{v_{max}} \right)^{2/3} A_C) T_w^*3/2 \] .................................. (A10)

where the shear rate at the wall is given by equation (3.31) by replacing \( T^* \) by \( T_w^* \). The values of granular temperature at
the wall and slip velocity are obtained directly. With equation A5 and A9 we can calculate the position \( y_{l}^{*} \) at which the flowing material locks and the velocity gradient becomes zero. By taking the left hand side of A5 to be zero and replacing \( T' \) in A5 by equation A9 we obtain a nonlinear algebraic equation which may easily be solved by successive substitution

\[
\frac{-2}{3} \lambda_{l} y_{l}^{*} = \ln (1 - \tan \theta) + \ln [n(1 - A) (h^{*} - y_{l}^{*}) + \frac{2}{5} (\frac{-y}{y_{\max}})^{2/3}] - \ln (g_{b} T_{l}^{*}) \ldots \ldots \ldots (A_{11})
\]

Then we can integrate equation A5 from \( y^{*} = 0 \) to \( y^{*} = y_{l}^{*} \) obtaining the velocity \( u_{l}^{*} \) in the bulk of the material. The resulting equation is the one presented in chapter 6 as (6.2.1)

\[
u_{l}^{*} - u_{o}^{*} = \frac{\lambda}{\sigma_{o}^{2}} \left\{ \frac{\tan \theta}{\tan \theta} - 1 \right\} (T_{l}^{*} - 1/2 - T_{l}^{*1/2}) [n(1 - A) (h^{*} - y_{l}^{*}) + \frac{2}{5} (\frac{-y}{y_{\max}})^{2/3}] + v(1 - A) h^{* -1/2} \frac{1}{\lambda_{l}^{2}} \]

\[+ \left( \frac{\tan \theta}{\tan \theta} - 1 \right) v(1 - A) y_{l}^{*} T_{l}^{* -1/2} + \frac{\tan \theta}{\tan \theta} \frac{\tan \theta}{\tan \theta} \frac{\tan \theta}{\tan \theta} \frac{\tan \theta}{\tan \theta} (T_{l}^{*1/2} - T_{l}^{*1/2}) \ldots \ldots \ldots \ldots (A_{12})
\]

The granular temperature in the bulk of the locked flow \( T_{l}^{*} \) is

\[T_{l}^{*} = T_{l}^{*} \exp \left( \frac{-2 \lambda y_{l}^{*}}{3} \right) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\]

To estimate the constant value of the volume fraction we use equation A3, by taking the shear rate to be zero obtaining

\[
v = v_{\max} \left[ \frac{\tan \theta}{\tan \theta} \frac{Fr}{(v - v_{\min})^2} \right]^{1/5} \ldots \ldots \ldots \ldots (A_{14})
\]

With this volume fraction then known, all the other quantities may be calculated.
Figure 4.1 Description of specular and diffuse collisions, concepts used in boundary conditions based on the specularity coefficient method.
Figure 4.2 Bed geometry used in boundary conditions based on the kinetic theory, following the approach of Richman.
Figure 5.1 Velocity profiles for dry granular flow using Richman's boundary conditions, moderately rough bed, ($\varphi=30^\circ$).
Figure 5.2 Concentration profiles for dry granular flow using Richman's boundary conditions, moderately rough bed, ($\psi=30^\circ$).
Figure 5.3 Granular temperature profiles for dry granular flow using Richman's boundary conditions, moderately rough bed, ($\psi=30^\circ$).
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Dry granular chute-flow

![Graph showing dry granular flow profiles](image)

Figure 5.8 Flux of fluctuation energy profiles for dry granular flow using Richman's boundary conditions, very rough bed, ($\psi=45^\circ$).
Figure 5.9 Velocity profiles for dry granular flow using boundary conditions based on the specularity coefficient, moderately rough bed ($\phi'=0.85$).
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Dry granular chute-flow

Figure 5.13 Velocity profiles for dry granular flow using boundary conditions based on the specularity coefficient, smooth bed (\(\phi'=0.25\)).
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Figure 6.1 Velocity profiles for granular-fluid chute flow showing the variation with $e_p$, $e_w=0.05$, $Re_1=200$, $\psi=30^\circ$. 

Granular-fluid chute-flow

$y^*$ - nondimensional distance from bed

$u^*$ - nondimensional velocity

$\theta = 22^\circ$, $\delta=12.3^\circ$, $\phi=28.5^\circ$
$e_w=0.05$, $Re_1=200$, 

$e_p=0.99$
$e_p=0.97$
$e_p=0.95$
Granular-fluid chute flow

Figure 6.2 Concentration profiles for granular-fluid chute flow showing the variation with $e_p$, $e_w=0.05$, $Re_1=200$, $\psi=30^\circ$.
Figure 6.3 Granular temperature profiles for granular-fluid chute flow showing the variation with $\varepsilon_p$, $\varepsilon_w=0.05$, $Re_1=200$, $\psi=30^\circ$. 

Granular-fluid chute flow

$T^*$ - nondimensional granular temperature

$y^*$ - nondimensional distance from bed

$\theta = 22^\circ$, $\delta=12.3^\circ$, $\phi=28.5^\circ$
$\varepsilon_w=0.05$, $Re_1=200$, $\psi=30^\circ$. 
Figure 6.4 Flux of fluctuation energy profiles for granular-fluid chute flow showing the variation with $e_p$, $e_w=0.05$, $Re_1=200$, $\psi=30^\circ$. 

Granular-fluid chute flow

$\theta = 22^\circ$, $\delta=12.3^\circ$, $\phi=28.5^\circ$

$e_w=0.05$, $Re_1=200$, $\psi=30^\circ$.
Figure 6.5 Velocity profiles for granular-fluid chute flow, showing the variation with $e_w, e_p=0.91, Re_1=200, \psi=30^\circ$. 

Granular-fluid chute flow

$y^*$ - nondimensional distance from bed

$u^*$ - nondimensional velocity

$\theta=22^\circ, e_p=0.91, Re_1=200, \delta=12.3^\circ, \phi=28.5^\circ, \psi=30^\circ$.
Granular-fluid chute flow

Figure 6.6 Concentration profiles for granular-fluid chute flow, showing the variation with $e_w$, $e_p=0.91$, $Re_1=200$, $\psi=30^\circ$. 

The concentration profile is shown for various values of $e_w$. The parameters used in the simulation are $\theta=22^\circ$, $e_p=0.91$, $Re_1=200$, $\delta=12.3^\circ$, $\phi=28.5^\circ$, and $\psi=30^\circ$. The y-axis represents the nondimensional distance from the bed, and the x-axis represents the concentration.
Figure 6.7 Granular temperature profiles for granular-fluid chute flow, showing the variation with $e_w$, $e_p=0.91$, $Re_1=200$, $\psi=30^\circ$. 

Granular-fluid chute flow

$y^*$ - nondimensional distance from bed

$T^*$ - nondimensional granular temperature

$\theta=22^\circ$, $e_p=0.91$

$Re_1=200$, $\delta=12.3^\circ$, $\phi=28.5^\circ$

$\psi=30^\circ$

$e_w=0.2$, $e_w=0.18$, $e_w=0.16$
Granular-fluid chute flow

Figure 6.8 Flux of fluctuation energy profiles for granular-fluid chute flow, showing the variation with $e_w$, $e_p=0.91$, $Re_1=200$, $\psi=30^\circ$. 
Figure 6.9 Velocity profiles for granular-fluid chute flow, showing the variation with Re for an intermediate concentration range, $e_p=0.95, e_w=0.2, \psi=30^\circ$. 
Figure 6.10 Concentration profiles for granular-fluid chute flow, showing the variation with $Re_1$ for an intermediate concentration range, $e_p=0.95$, $e_w=0.2$, $\psi=30^\circ$. 
Figure 6.11 Granular temperature profiles for granular-fluid chute flow, showing the variation with $Re_1$ for an intermediate concentration range, $e_p=0.95$, $e_w=0.2$, $\psi=30^\circ$. 
Figure 6.12  Flux of fluctuation energy profiles for granular-fluid chute flow, showing the variation with $Re_1$ for an intermediate concentration range, $e_p=0.95$, $e_w=0.2$, $\psi=30^\circ$. 

$q^*$ - nondimensional flux of fluctuation energy

$y^*$ - nondimensional distance from bed

Re$_1$=200
Re$_1$=500
Re$_1$=1000

$\theta=22^\circ$, $e_p=0.95$
$\delta=12.3^\circ$, $\phi=28.5^\circ$
$\phi=30^\circ$, $e_w=0.2$
Figure 6.13 Velocity profiles for granular-fluid chute flow, showing the variation with $Re_1$ for a high concentration range, $e_p=0.95$, $e_w=0.2$, $\psi=30^\circ$. 

Granular-fluid chute flow

$y^* -$ nondimensional distance from bed

$u^* -$ nondimensional velocity

$\theta=22^\circ$, $e_p=0.95$
$\delta=12.3^\circ$, $\phi=28.5^\circ$
$\psi=30^\circ$, $e_w=0.2$
Granular-fluid chute flow

Figure 6.14 Concentration profiles for granular-fluid chute flow, showing the variation with Re₁ for a high concentration range, e_p=0.95, e_w=0.2, ψ=30°.
Figure 6.15 Granular temperature profiles for granular-fluid chute flow, showing the variation with Re for a high concentration range, $\varepsilon_p=0.95$, $\varepsilon_w=0.2$, $\psi=30^\circ$. 
Figure 6.16  Flux of fluctuation energy profiles for granular-fluid chute flow, showing the variation with $Re_1$ for a high concentration range, $e_p=0.95$, $e_w=0.2$, $\psi=30^\circ$. 

Granular-fluid chute flow

$q^* -$ nondimensional flux of fluctuation energy

$y^* -$ nondimensional distance from bed

$Re_1=160$

$Re_1=180$

$Re_1=200$

$\theta=22^\circ$, $e_p=0.95$

$\delta=12.3^\circ$, $\psi=28.5^\circ$

$\psi=30^\circ$, $e_w=0.2$
Figure 6.17 Apparent viscosity factor $C_n$ versus concentration from different researchers.
Figure 7.1 EXPERIMENTAL SET-UP.
Simple model and experimental results

![Graph showing average cross-sectional velocity versus flow depth after experimental flow depth measurements and after simple model calculations.](Image)

Figure 7.2  Average cross-sectional velocity versus flow depth after the experimental flow depth measurements and after simple model calculations, $Q_s/w=25 \text{ cm}^2/\text{s}$, $\theta=22.9^\circ$. 
Simple model results

Figure 7.3 Values of $Q_s/w$ versus flow depth after simple model calculations, $Q_s/w=25 \text{ cm}^2/\text{s}$, $\theta=22.9^\circ$. 
Figure 7.4 Average cross-sectional velocity versus flow depth after the experimental flow depth measurements and after simple model calculations, \( Q_s/w = 37 \text{ cm}^2/\text{s} \), \( \theta = 21.6^\circ \).
Simple model results

Figure 7.5 Values of $Q_s/w$ versus flow depth after simple model calculations, $Q_s/w$=37 cm$^2$/s, $\theta$=21.6 $^\circ$. 
Figure 7.6 Average cross-sectional velocity versus flow depth after the experimental flow depth measurements and after simple model calculations, $Q_{w}/w=75 \text{ cm}^2/\text{s}$, $\theta=20.1^\circ$. 
Figure 7.7 Values of $Q_s/w$ versus flow depth after simple model calculations, $Q_s/w = 75 \text{ cm}^2/\text{s}$, $\theta = 20.1^\circ$. 
Experimental results

Figure 7.8 Experimental grain-fluid velocity profiles using a video method for the high concentration range
Figure 7.9 Experimental grain-fluid velocity profile using a video method for the intermediate concentration range

Experimental results

\[ Q_a = 255 \text{ cm}^3/\text{s} \]
\[ Q_w = 316 \text{ cm}^3/\text{s} \]
\[ \theta = 22^\circ \]
Figure 7.10 Experimental variation of Bagnold number with concentration and simple model predictions, $Q_s = 255 \text{ cm}^3/\text{s}$, $\theta = 22.9^\circ$. 
Figure 7.11 Velocity profile from the numerical model for the simulation of the Nevado del Ruiz debris flow.
Figure 7.12 Concentration profile from the numerical model for the simulation of the Nevado del Ruiz debris flow.
Figure 7.13 Granular temperature profile from the numerical model for the simulation of the Nevado del Ruiz debris flow.
Figure 7.14 Variation of shear stresses with the vertical in the Nevado del Ruiz debris flow.