Unit Commitment with
Primary Frequency Regulation Constraints
in Electric Power Systems

by

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A Hernán, Piedad y Sergio

"Pensar, analizar, inventar no son actos anómalos, son la normal respiración de la inteligencia... Glorificar el ocasional cumplimiento de esa función, atesorar antiguos y ajenos pensamientos, recordar con incrédulo estupor lo que el "doctor universalis" pensó, es confesar nuestra languidez o nuestra barbarie. Todo hombre debe ser capaz de todas las ideas y entiendo que en el porvenir lo será".

Ficciones, Jorge Luis Borges.
Abstract

The unit commitment problem with secondary and tertiary reserve requirements with time constants ranging from two to thirty minutes has been broadly studied in electric power systems. In contrast, the scheduling of units offering primary frequency regulation reserve deployable within seconds of a contingency has received relatively little attention.

In this dissertation we formulate and solve a unit commitment problem explicitly accounting for the characteristic that primary frequency regulation has a single common degree of freedom, namely, the system frequency deviation.

The simultaneous scheduling of energy, primary and tertiary reserves is then studied through a number of cases of up to 34 generating units.
Résumé

Le problème de la planification de l’opération des unités de production d’électricité comprenant des spécifications des niveaux de régulation secondaire et tertiaire (celles-ci agissant sur un horizon de temps compris entre deux et trente minutes) a été amplement étudié. Le problème de planification de la production comprenant les aspects relatifs à la régulation primaire, agissant en fonction des variations de fréquence du réseau dans un horizon de temps de quelques secondes, n’a cependant reçu que peu d’attention jusqu’à maintenant.

Dans ce mémoire, nous développons et solutionnons le problème de la planification de la production d’électricité en incluant la notion de la régulation primaire. On démontre que celle-ci est gouvernée explicitement et uniquement par la variation de la fréquence du réseau.

Ensuite, la planification simultanée de la production d’énergie et de la fourniture des services de régulation primaire et tertiaire est étudiée via des études de cas sur des systèmes types possédant jusqu’à 34 unités de production.
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Nomenclature

The main symbols appearing in this dissertation are defined below. Other symbols are defined in the text as they appear.

Variables:

- $g_i^0$: Generation set point of unit $i$ in the pre-contingency state (MW);
- $g_i^k$: Generation level of unit under primary regulation after contingency $k$ (MW);
- $r_i$: Primary regulation reserve of unit $i$ (MW);
- $\bar{g}_i$: Maximum generation of unit $i$ under primary regulation (MW);
- $\Delta f^k_i$: Frequency deviation under primary regulation after contingency $k$ (Hz);
- $\Delta f_{bi}$: Frequency deviation when unit operates at $\bar{g}_i$, (Hz);

Primary regulation reserve of unit $i$ (MW).

0 / 1 Binary Variables:
\( w_i^k \) Equal to 1 if and only if unit \( i \) operates at \( \bar{g}_i \) under contingency \( k \);

\( u_i \) Equal to 1 if and only if unit \( i \) is on;

\( v_i \) Equal to 1 when \( \bar{g}_i \) is defined by the ramp-up limit and to 0 when \( \bar{g}_i \) is defined by the unit generation capacity.

**Parameters:**

\( d^n \) System demand in pre-contingency state (MW);

\( d^k \) System demand in after the occurrence of contingency \( k \) (MW);

\( a_i \) Unit \( i \) linear generation cost parameter ($/MWh);

\( b_i \) Unit \( i \) quadratic generation cost parameter ($/MW^2h$);

\( C_{0i} \) Unit \( i \) fixed generation cost ($/h$);

\( q_i \) Unit \( i \) reserve rate ($$/MWh$);

\( \Delta f^{\text{max}} \) Maximum allowed system frequency deviation (Hz);

\( r_i^{\text{max}} \) Unit \( i \) ramp-up limit (MW);

\( D_i \) Droop of unit \( i \) (MW/Hz);

\( g_i^{\text{max}} \) Maximum generation capacity of unit \( i \) (MW);

\( g_i^{\text{min}} \) Minimum generation capacity of unit \( i \) (MW);
Chapter I.

Introduction

The unit commitment problem in electric power systems schedules the on/off status of generating units such that they satisfy the energy demand at minimum operating cost over a daily or weekly time horizon usually divided in discrete periods of one hour. This scheduling problem also considers generation and, possibly, transmission limits, as well as inter-temporal technical restrictions relating the generation status of units at different time periods. Examples of these inter-temporal restrictions are ramp limits, minimum up and down time, must-run requirements, and emissions and environmental constraints. The unit commitment has a major economic impact in any utility or electricity market since a small improvement in operating cost can translate into savings of millions of dollars per year.

In addition to the above mentioned constraints, in a security constrained unit commitment, enough flexibility must be provided for the generation to be able to follow normal time variations and randomness of the load as well as more severe disturbances which may occur with a finite probability. This flexibility is
referred to as reserve.

The type of reserve that a unit can provide depends on its dynamic characteristics, with some generators being able to respond more quickly than others. The fastest reserve type, also called primary reserve or primary frequency regulation, is necessary during the first few seconds following a severe disturbance such as the sudden loss of a generator. This reserve is needed to balance total generation and demand, and to prevent the system frequency from dropping excessively [1], [2]. Primary reserve is provided by the generating units through their local droop characteristic in response to the system frequency deviation from the reference.

The system frequency must be kept within a narrow range for three reasons: i) Customers rely on a constant system frequency since the performance of some industrial processes is highly sensitive to frequency deviations from the nominal level; ii) Generating units are designed to operate most efficiently at a nominal speed. Moreover, units could be damaged if operated beyond certain frequency bounds; iii) Finally, automatic protection systems will disconnect loads if the system frequency drops below a certain value.

In addition to primary reserve, generation must also provide secondary and tertiary reserve [2]. The regulation of secondary generation reserve, also known as automatic generation control (AGC), is a centralized strategy with a response time of the order of minutes whose goal is to regulate the area-control error under load-following conditions[3]. Finally, tertiary reserve provides generation or demand flexibility to ensure that all operational constraints such as line flow and voltage magnitude limits are satisfied following a major disturbance. Tertiary regulation is also centrally implemented and has a response time of the order of tens of minutes[4].

The unit commitment is a complex optimization problem and can be solved
using a wide array of techniques [5] among which mixed integer linear programming (MILP) is one of the most powerful. Among its advantages, is the ready availability of commercial implementations [6], [7], and the fact that MILP algorithms can find an optimal solution or come arbitrarily close to it. Traditionally, in the unit commitment formulation, integer binary variables have been used to model the on/off status of units and other discontinuous constraints such as minimum up and down times. In this dissertation, binary variables will also be used to model the non-linear function that characterizes primary frequency regulation.

The purpose of this dissertation is not to develop new unit commitment algorithms but rather to formulate the unit commitment problem with primary frequency regulation as a mixed-integer linear problem. This non-trivial step, detailed in chapters II and III, then allows us to solve this complex problem using standard MILP solvers.

Chapter IV presents some illustrative numerical examples of the unit commitment problem with primary frequency regulation, including two cases from a real power system. Finally, chapter V presents a formulation and solution of the unit commitment problem with simultaneous scheduling of energy plus primary and tertiary reserve, together with a case study.
Chapter II.

Unit Commitment with Frequency Regulation Reserve

Constraints

2.1 Background

The algorithms currently available for scheduling reserve services have been developed primarily for tertiary and secondary reserves [4], [8], [9]. In contrast, the scheduling of primary frequency regulation reserve within the unit commitment problem has received relatively little attention.

One previous important contribution to this subject is the work reported in [10] consisting of an iterative economic dispatch in which frequency excursions outside the allowable range triggered modifications of the scheduled generation and reserve levels. This approach was limited by the iteration heuristics and especially by the fact that the on/off schedule of the generating units was assumed known. A more recent contribution to the dispatch of primary reserve services considers stability and network constraints, and is based on a decision tree solution algorithm [11]. This approach however also assumes that the unit commitment generation
schedule is known a priori. In addition, it does not treat the system frequency deviation as an explicit common optimization variable linking all deployable unit reserves.

What makes the primary regulation problem particularly challenging is that it does not possess the broad freedom present in tertiary or secondary regulation, the latter being limited only by generation capacity and ramp limits. In comparison, subject to saturation, all the primary regulation components are functions of the system frequency deviation, thus sharing a single common degree of freedom. The inclusion of this characteristic in the unit commitment formulation and its solution are non-trivial tasks and form the main contribution of this dissertation.

In this chapter we first formulate a unit commitment problem that schedules power and primary reserves where the latter are all piece-wise linear functions of the common system frequency deviation. This unit commitment is subject to a security criterion defined by a set of pre-specified generation outage contingencies and by the explicit requirement that the system frequency (an optimization variable) must not fall below a specified lower bound under any of the contingencies. We assume a single-period unit commitment problem where some of the units may have already been scheduled to account for multi-period time-coupling constraints as well as for secondary and tertiary reserve requirements. The optimum schedule may be formulated in the context of an electricity market where generating units can submit offers to sell primary reserve.

2.2 Primary Frequency Regulation

Primary frequency regulation is triggered by frequency deviations which arise as a consequence of imbalances between generation and demand. One source of these mismatches is the inherent demand randomness,
which being generally small and relatively slow, can be corrected by secondary regulation or AGC. Under such conditions, primary regulation is also active, helping keep frequency within bounds, but not in a very conspicuous fashion.

In contrast, after a large imbalance between demand and generation, such as the one caused by the loss of a generating unit, primary frequency regulation is vital in limiting system frequency excursions. More specifically, after a contingency \( k \), when the system frequency deviates by \( \Delta f^k \), the participating units respond as illustrated in Figure 1 by incrementing their output by \( -D_i \Delta f^k \) subject to capacity and ramp limits. The parameter \( D_i \) is the frequency regulation constant or governor droop of unit \( i \) in MW/Hz, a value inherent to each unit and usually regulated [12],[13].

\[
\begin{align*}
\Delta f^k & \quad \xrightarrow{+} \quad f^\text{ref} \\
& \quad \xrightarrow{-} \quad D_i \\
& \quad \xrightarrow{\text{Saturation}} \quad g_i^k
\end{align*}
\]

Figure 1: Primary frequency regulation of unit \( i \) after contingency \( k \).

Moreover, the significant time gap between a major contingency and the start of secondary and tertiary corrective actions (of the order of minutes) means that primary frequency regulation is essentially responsible for initially balancing power and stabilizing the system frequency.

Finally, as shown in Figure 2, when referring to system frequency deviation, in this dissertation we mean the steady-state value after the transient dies down.
We also point out that loads may participate in primary frequency regulation as a result of their intrinsic response to changes in frequency. However, in contrast to generators, the frequency regulation provided by the loads is a physical characteristic which is non-controllable.

2.3 General Formulation

The goal of the unit commitment problem with primary frequency regulation is to minimize the overall offered cost of generation and primary frequency regulation reserve,

$$\min \left\{ \sum_i \left[ C_i(g_i^0, u_i) \right] + \sum_i C_{ni}(r_i) \right\}$$

(2.1)

In (2.1), the generation cost offer of unit $i$ is a function of its pre-contingency generator output, $g_i^0$, and of its on/off status, $u_i$. For example,
\[ C_i(g^0_i, u_i) = u_i C_{0i} + a_i g^0_i + \frac{1}{2} b_i (g^0_i)^2; \]  

where the fixed and variable cost parameters, respectively, \( C_{0i} \) and \( a_i \) and \( b_i \), are specified by each unit as part of the offer \([1]\). In addition, each generator may also submit a cost offer, \( C_{r_i}(r_i) \), to supply primary frequency regulation reserve, \( r_i \). For example,

\[ C_{r_i}(r_i) = q_i r_i; \]

where the parameter \( q_i \) is also specified by each unit as part of the overall offer.

The above minimization is subject to the pre-contingency power balance between the generation outputs, \( g_i^0; i = 1, \ldots, n_g \) and the system demand, \( d^0 \). In a lossless network, this means that,

\[ \sum_i g_i^0 - d^0 = 0. \]  

We also assume that in pre-contingency state, the system frequency is at its nominal level or equivalently that the frequency deviation is zero.

The pre-contingency power levels are subject to the generating unit capacity limits,

\[ u_i g_i^{\text{min}} \leq g_i^0 \leq u_i g_i^{\text{max}}, \forall i. \]  

The units belonging to a subset of the generators, \( S^{\text{ps}} \) may also be subject to a pre-specified on/off status,

\[ u_i = u_i^{\text{ps}}; i \in S^{\text{ps}}. \]  

This condition reflects time-coupling and other types of reserve constraints imposed by a previously solved multi-period unit commitment without primary frequency regulation. The reasonable assumption here is that, for computational and practical reasons, primary regulation is not scheduled
jointly with secondary and tertiary reserves, but rather sequentially. In addition, since secondary and tertiary reserves will have been already assigned to the generators, then the new pre-contingency generation levels set by the primary regulation unit commitment step, $g_i^0$, must allow enough room for the deployment of these pre-scheduled reserves, $r_i^{ps}$. Mathematically, this can be expressed by the condition,

$$g_i^{\min} \leq g_i^0 + r_i^{ps} \leq g_i^{\max}; \forall i.$$  

(2.7)

Here, we only consider contingencies defined by the loss of pre-specified combinations of generating units; specifically, contingency $k$ is defined by the loss of all units belonging to a given set $S^k$. This implies that following each contingency, only negative frequency deviations will occur. Since generation and demand must balance under each of the $nc$ post-contingency states, we impose that,

$$\sum_{i \in S^k} g_i^k - d^k = 0; k = 1, ..., nc,$$

(2.8)

where $g_i^k$ and $d^k$ are respectively the generation output of unit $i$ and the system demand, both under contingency $k$.

Thus, the primary frequency regulation output of unit $i \not\in S^k$ under contingency $k$ is given by,

$$g_i^k = \begin{cases} g_i^0 - D_i \Delta f^k; & \text{if } g_i^0 - D_i \Delta f^k \leq \bar{g}_i \\ \bar{g}_i; & \text{if } g_i^0 - D_i \Delta f^k > \bar{g}_i, \end{cases}$$

(2.9)

where $\Delta f^k$ is the value to which the system frequency deviation settles (see Figure 2) due to primary regulation after contingency $k$. Furthermore, the power output for all units belonging to the set $S^k$ must be zero, that is,

$$g_i^k = 0; \forall i \in S^k; \forall k.$$  

(2.10)

The upper generation bound, $\bar{g}_i$, used in (2.9) is the maximum output of
unit $i$ under primary frequency regulation, defined by either the unit frequency-regulation ramp limit, $r_i^{\text{max}}$, or by the unit generating capacity limit, $g_i^{\text{max}}$, whichever is smaller, that is,

$$
\overline{g}_i = \min \{ u_i g_i^{\text{max}}, g_i^0 + u_i r_i^{\text{max}} \}. \tag{2.11}
$$

This is a fast ramp limit consistent with the short time constants of primary frequency regulation. It is typically narrower than the ramp limits associated with the slower secondary and tertiary regulations.

![Figure 3. Frequency regulation response of unit $i$](image)

Note that the break-frequency deviation, $\Delta f_{bi}$, at which unit $i$ saturates at $\overline{g}_i$ is another decision variable computed as,

$$
\Delta f_{bi} = \left( g_i^0 - \overline{g}_i \right) / D_i. \tag{2.12}
$$

Now, in order to avoid load shedding, the maximum frequency deviations must be limited,

$$
-\Delta f^k \leq \Delta f_{\text{max}}^k; \forall k. \tag{2.13}
$$
where $\Delta f^{\text{max}}$ is the maximum frequency deviation set by the system operator.

Finally, the scheduled primary regulation reserve for unit $i$, $r_i$, must be greater than or equal to the maximum generation deviation relative to the pre-contingency level over all contingencies.

$$r_i \geq g_i^k - g_i^0; \forall k$$  \hspace{1cm} (2.14)

### 2.4 Summary of General Formulation

In summary, the general formulation of the unit commitment problem with primary frequency regulation is as follows:

Minimize the objective function:

$$\min \left\{ \sum_i \left[ C_i \left( g_i^0, u_i \right) \right] + \sum_i C_{ir_i} \right\}$$  \hspace{1cm} (2.15)

Subject to the power balance relations in the pre and post-contingency states,

$$\sum_i g_i^0 - d^0 = 0$$  \hspace{1cm} (2.16)

$$\sum_{i \in S} g_i^k - d^k = 0; \forall k$$  \hspace{1cm} (2.17)

To the generation capacity and frequency limits,

$$u_i g_i^{\text{min}} \leq g_i^0 \leq u_i g_i^{\text{max}}; \forall i$$  \hspace{1cm} (2.18)

$$-\Delta f^{k} \leq \Delta f^{\text{max}}; \forall k$$  \hspace{1cm} (2.19)

To the pre-committed units and pre-specified reserves,

$$g_i^{\text{min}} \leq g_i^0 + r_i^{ps} \leq g_i^{\text{max}}; \forall i$$  \hspace{1cm} (2.20)
\[ u_i = u_i^m; i \in S^m \] (2.21)

To the post-contingency primary frequency regulation with the single degree of freedom, \( \Delta f^k \),

\[
g_i^k = \begin{cases} 
  g_i^0 - D_i \Delta f^k; & \text{if } g_i^0 - D_i \Delta f^k \leq \bar{g}_i; \forall i \\
  \bar{g}_i & \text{if } g_i^0 - D_i \Delta f^k > \bar{g}_i
\end{cases}
\] (2.22)

where the maximum available generation capacity is given by,

\[
\bar{g}_i = \min \left\{ u_i g_i^{max}, g_i^0 + u_i r_i^{max} \right\}; \forall i
\] (2.23)

vi. To the unavailability of those units making up contingency \( k \),

\[
g_i^k = 0; \forall i \in S^k; \forall k
\] (2.24)

vii. To the inequalities defining primary regulation reserve,

\[
r_i \geq g_i^k - g_i^0; \forall i, k
\] (2.25)
Chapter III.

Unit Commitment with Primary Frequency Regulation:

Formulation as a Mixed-Integer Linear Program

In solving the above unit commitment problem, three difficulties arise: (i) The objective function is non-linear; (ii) The piece-wise nature of the frequency regulation response curve imposed by equation (2.9), particularly the fact that the saturation level, $\bar{g}_i$, and the corresponding break-frequency deviation, $\Delta f_{bi}$, are themselves variables; (iii) The fact that $\bar{g}_i$ is defined as the minimum of two variables by equation (2.11). In this implicit form, the unit commitment problem cannot be solved using standard MILP optimization algorithms. The goal of this chapter is therefore to show how to convert the problem into a form that can be directly solved via commercially available mixed-integer linear programming packages [6], [7].
3.1 Linearization of the Objective Function

The generation cost of a unit can be approximated by a quadratic function of its generation level as in equation (2.2). Since this function is generally convex, it can be linearly approximated by tangents as illustrated in Figure 4, [14].

\[
C_i(g_i, u_i) = L_1 + L_2 + L_3
\]

Figure 4. Quadratic cost function linearization

Minimizing the original quadratic objective function can be approximated by minimizing \( X \), that is,

\[
\min \left[ \sum_i u_i C_{a_i} + a_i g_i^0 + \frac{1}{2} b_i \left( g_i^0 \right)^2 \right] \approx \min \left[ \sum_i X_i \right]
\]

(3.1)

where,

\[
X_i \geq L_j(g_i^0, u_i); \forall j = 1, ..., nl
\]

(3.2)

and where \( L_j(g_i^0, u_i) \) stands for the jth tangent line of the nl tangent lines employed to approximate the cost curve if the ith generator.
As the number of tangents increases, \( \min \left[ \sum_i X_i \right] \) converges to
\[
\min \left[ \sum_i u_i C_{oi} + a_i g_i^0 + \frac{1}{2} \beta_i (g_i^0)^2 \right].
\]

### 3.2 Explicit Formulation of the Non-Linear Condition (2.11)

Next, we recast condition (2.11) into an equivalent linear form explicitly relating the problem variables. In order to do this, we first introduce a new set of binary variables, \( v_i \), one per generating unit, defined so that,

\[
v_i = \begin{cases} 
1; & \text{if } u_i g_i^{\max} > g_i^0 + u_i r_i^{\max} \\
0; & \text{if } u_i g_i^{\max} \leq g_i^0 + u_i r_i^{\max}.
\end{cases}
\] (3.3)

Thus, \( v_i = 1 \) if the capacity limit is greater than the ramp limit, which means that the ramp limit defines the maximum generation output. Alternatively, if \( v_i = 0 \), the capacity limit is less than or equal to the ramp limit, meaning that the capacity limit defines the maximum generation output.

The implicit definition of the binary variables, \( v_i \), in (3.3) can now be expressed as an equivalent explicit linear relation by [15],

\[
1 - \frac{(g_i^0 + u_i r_i^{\max})}{g_i^{\max}} \leq v_i \leq 2 - \frac{(g_i^0 + u_i r_i^{\max})}{g_i^{\max}};
\] (3.4)

where we have reasonably assumed that \( r_i^{\max} \leq g_i^{\max} \).

The equivalence between the implicit relation, (3.3), and its explicit form (3.4) can be easily verified. For example, if \( g_i^0 + u_i r_i^{\max} > g_i^{\max} \), then from the right-hand-side of (3.4), we see that \( v_i < 1 \), while the left-hand-side of (3.4) imposes the condition that \( v_i \) must be greater than a number less than zero. Since \( v_i \) is a binary variable, the only choice is \( v_i = 0 \). A similar reasoning applies for the case when \( g_i^0 + u_i r_i^{\max} < g_i^{\max} \).

With this explicit characterization of the binary variable \( v_i \), we can now
express the generator saturation level as an explicit function of the problem variables,

$$
\bar{g}_i = v_i g_i^0 + (v_i + u_i - 1)r_i^{\text{max}} + (1 - v_i)g_i^{\text{max}}; \forall i.
$$

(3.5)

Note that if unit $i$ is off then $u_i = 0$ then $g_i^0 = 0$. Furthermore, from condition (3.4), it is imposed that $1 \leq v_i \leq 2$, implying that since $v_i$ is a binary variable the only choice is $v_i = 1$. Thus from (3.5), the maximum power available for primary frequency regulation when unit $i$ is off is $\bar{g}_i = 0$.

At this stage, we note that (3.5) contains the nonlinearity $v_i g_i^0$. However, it is always possible to convert the product of a binary and a continuous variable into an equivalent set of explicit linear relations through,

$$
0 \leq x_i \leq v_i g_i^{\text{max}}
$$

$$
x_i = g_i^0 - s_i
$$

$$
0 \leq s_i \leq (1 - v_i)g_i^{\text{max}}.
$$

(3.6)

Here, the product $v_i g_i^0$ is represented by the continuous variable $x_i$, and $s_i$ is a new continuous slack variable [15].

### 3.3 Explicit Formulation of the Non-Linear Condition (2.9)

Now, in order to recast condition (2.9) in a linear explicit form, we first define new binary variables $w_i^k$ as follows,

$$
w_i^k = \begin{cases} 
1; & \text{if } \Delta f^k > \Delta f_{bi}^k; \forall i, k \\
0; & \text{if } \Delta f^k \leq \Delta f_{bi}^k.
\end{cases}
$$

(3.7)

Thus, if the system frequency deviation after the occurrence of contingency $k$, $\Delta f^k$, falls below the break-frequency of unit $i$, $\Delta f_{bi}^k$, then $w_i^k = 1$. From (2.9), this condition also means that such unit will be at its maximum, $g_i^k = \bar{g}_i$. Alternatively, if $w_i^k = 0$, the system frequency deviation after the occurrence of contingency $k$ does not fall below the break-
frequency of unit $i$ and consequently its output follows the linear primary frequency regulation behavior, $g_i^k = g_i^0 - D_i \Delta f^k$.

The implicit definition of the binary variables, $w_i^k$, in (3.7) can now be expressed as an equivalent explicit linear relation,

$$\frac{(\Delta f_{bi} - \Delta f^k)}{L} \leq w_i^k \leq \frac{(\Delta f_{bi} - \Delta f^k)}{L} + 1;$$

(3.8)

where $L$ is an arbitrary large number ($L = r_i^{\max} / D_i$ is one such number that is sufficiently large). The equivalence between (3.7) and (3.8) can be verified by first considering the case when $\Delta f_{bi} < \Delta f^k$, making $w_i^k$ greater than a small positive number and less than a number greater than one. Since $w_i^k$ is a binary variable, the only choice is $w_i^k = 1$. A similar reasoning applies for the case when $\Delta f_{bi} > \Delta f^k$ and $w_i^k = 0$.

With the explicit characterization of $w_i^k$ in (3.8), it is now possible to express the output of unit $i$ given by the implicit form (2.9) through the equivalent linear explicit inequalities,

$$-w_i^k g_i^{\max} \leq g_i^k - g_i^0 + D_i \Delta f^k \leq 0$$

(3.9)

and,

$$(w_i^k - 1) g_i^{\max} + \overline{g}_i \leq g_i^k.$$  

(3.10)

We must also impose the condition that the output of unit $i$ under any contingency, $g_i^k$, must be less than or equal to the maximum permitted by the ramp or capacity limit, $\overline{g}_i$,

$$g_i^k \leq \overline{g}_i; \forall k.$$  

(3.11)

The equivalence between (2.9) and both (3.9), (3.10), and (3.11) can be seen as follows:

First, consider the case with $w_i^k = 0$ under which unit $i$ after contingency $k$ should respond to frequency deviations within its linear zone. Condition
(3.9) does indeed impose this characteristic since \(0 \leq g_i^k - g_i^0 + D_i \Delta f^k \leq 0\), which implies that \(g_i^k = g_i^0 - D_i \Delta f^k\). In addition, with \(w_i^k = 0\), constraint (3.9) says that \(-g_i^{\text{max}} + \bar{g}_i \leq g_i^k\), which is a non-binding condition.

In the case with \(w_i^k = 1\), unit \(i\) should be saturated after contingency \(k\). This is confirmed by noting that condition (3.9) states that \(-g_i^{\text{max}} \leq g_i^k - g_i^0 + D_i \Delta f^k \leq 0\), which is not binding. Condition (3.10) however states that \(\bar{g}_i \leq g_i^k\), which, when combined with necessary condition (3.11) that \(\bar{g}_i \geq g_i^k\), imposes the expected saturation condition, \(g_i^k = \bar{g}_i\).

### 3.4 Summary of Mixed-Integer Linear Formulation

The set of linear relations describing the unit commitment problem with primary frequency regulation can be summarized by:

All the constraints expressed in section 2.4 with the exception of equations (2.22) and (2.23).

The linear constraints that replace non-linear condition (2.22), consisting of:

**Determining whether a unit is saturated or not,**

\[
\frac{(\Delta f_{bi} - \Delta f^k)}{L} \leq w_i^k \leq \frac{(\Delta f_{bi} - \Delta f^k)}{L} + 1.
\]  

(3.12)

**Defining \(g_i^k\) when unit \(i\) is in its linear frequency response region,**

\[
-w_i^k g_i^{\text{max}} \leq g_i^k - g_i^0 + D_i \Delta f^k \leq 0
\]

(3.13)

**Defining \(g_i^k\) when unit \(i\) is in its saturated region,**

\[
(w_i^k - 1)g_i^{\text{max}} + \bar{g}_i \leq g_i^k
\]

(3.14)

\[
g_i^k \leq \bar{g}_i; \forall k.
\]

(3.15)
Linear constraints that replace non-linear condition (2.23) consisting of:

Determining whether the ramp-up or the capacity limit is binding for a given unit,

\[ 1 - \frac{(g_i^0 + u_i r_i^\text{max})}{g_i^\text{max}} \leq v_i \leq 2 - \frac{(g_i^0 + u_i r_i^\text{max})}{g_i^\text{max}} \quad ; \]  

(3.16)

Defining \( \bar{g}_i \),

\[ \bar{g}_i = v_i g_i^0 + (v_i + u_i - 1) r_i^\text{max} + (1 - v_i) g_i^\text{max} ; \forall i . \]  

(3.17)

Defining the linear equivalent of the product \( v_i g_i^0 \),

\[ 0 \leq x_i \leq v_i g_i^\text{max} \]

\[ x_i = g_i^0 - s_i \]

\[ 0 \leq s_i \leq (1 - v_i) g_i^\text{max} \quad . \]  

(3.18)

3.5 Case Studies

Through the following three case studies, we illustrate the characteristics of the proposed unit commitment with primary frequency regulation. First, we show that the primary frequency regulation generation reserves, \( r_i \), and the pre-contingency production levels, \( g_i^0 \), are strongly coupled. This coupling arises primarily because of the dependence of the reserves on the post-contingency system frequency deviation.

It is therefore difficult, if not impossible, for the system operator to schedule these quantities while respecting all security constraints at minimum cost without a systematic tool such as the one proposed here. One of the trickier steps in this type of scheduling is to identify in a systematic way whether, in the post-contingency states, a unit is free to regulate or whether its output is bound by its capacity or ramp limit.

A 4-generating-unit study is used to illustrate in detail the various
characteristics of the proposed unit commitment scheme. Two larger cases are also studied, one with 17 and another with 34 generating units, the data of which are based on an existing power system [10]. All cases are subject to the N-1 system security criterion characterized by the loss of any single generating unit which defines the set of \( nc \) contingencies. For simplicity, it is also assumed that \( u_i \) and \( g_i^0 \) are free variables not subject to any pre-commitment or specification as given by conditions (2.6) and (2.7).

3.5.1 4-Unit Case-Study

The following data is provided for the 4-unit system,

<table>
<thead>
<tr>
<th>Unit</th>
<th>( a ) ($/MW\text{h})</th>
<th>( C_0 ) ($/h)</th>
<th>( g^\text{max} ) (MW)</th>
<th>( g^\text{min} ) (MW)</th>
<th>( r^\text{max} ) (MW)</th>
<th>( q ) ($/MW\text{h})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9.8</td>
<td>10</td>
<td>155</td>
<td>10</td>
<td>45</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>10.7</td>
<td>10</td>
<td>200</td>
<td>40</td>
<td>52</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>15.6</td>
<td>10</td>
<td>250</td>
<td>10</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>40</td>
<td>10</td>
<td>100</td>
<td>0</td>
<td>40</td>
<td>1</td>
</tr>
</tbody>
</table>

The cost functions of energy and reserve are as defined in equations (2.2) and (2.3), with the quadratic energy cost parameter, \( b_i \), assumed to be zero. All units have a regulation droop of 4% with a system nominal frequency of 60 Hz and a maximum frequency deviation of 600 mHz. The set of contingencies consists of the loss of any single generating unit. The demand of 170 MW is insensitive to frequency.

First, we solve the unit commitment without frequency regulation,
imposing generation ramp and capacity limits as well as the power balances in the pre and post-contingency states. In this benchmark case, reserve is not restricted to follow the regulation curve shown in Figure 2. This case therefore has considerably more freedom than the one that restricts frequency regulation and reserve to the regulation curve and to the prescribed system frequency limits.

Table 2 summarizes the results of the unit commitment without frequency regulation showing that all the prescribed requirements are met with only units A, B and D turned on. For example, the outage of unit A leads to the loss of 92 MW which is compensated by the deployment of 52 MW plus 40 MW of reserve from units B and D respectively.

Table 2. Unit Commitment: 4-Unit Case without Primary Frequency Regulation

<table>
<thead>
<tr>
<th>Unit</th>
<th>u</th>
<th>g₀</th>
<th>r (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>92</td>
<td>38</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>78</td>
<td>52</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>0</td>
<td>40</td>
</tr>
</tbody>
</table>

However, there exist at least two situations that make this solution unacceptable if we also consider frequency regulation and the frequency deviation limit of – 600 mHz: (i) Unit D can provide its full scheduled reserve of 40 MW only if the system frequency deviation is – 960 mHz or lower, thus violating the maximum frequency deviation allowed. (ii) After the loss of unit B, both units A and D would need to deploy their full scheduled reserves. Under frequency regulation, this is however infeasible,
since the system frequencies needed for these units to deploy their full reserves are different, unit A producing 38 MW of primary reserve at –588 mHz, while unit D produces 40 MW at –960 mHz.

Next, we solve the proposed unit commitment with primary frequency regulation, obtaining the results summarized in Table 3.

Table 3. Unit Commitment: 4-Unit Case with Primary Frequency Regulation

<table>
<thead>
<tr>
<th>Unit</th>
<th>u</th>
<th>$g^0$</th>
<th>r</th>
<th>$\bar{g}$</th>
<th>$\Delta f_{bi}$</th>
<th>$\Delta f_{bk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>120</td>
<td>12</td>
<td>155</td>
<td>–541</td>
<td>–560</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>40</td>
<td>47</td>
<td>92</td>
<td>–624</td>
<td>–190</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>10</td>
<td>50</td>
<td>60</td>
<td>–480</td>
<td>–53</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>0</td>
<td>23</td>
<td>40</td>
<td>–960</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that since the break-frequencies of units B and D are lower than the maximum system frequency deviation, these units never saturate and therefore do not deploy their full reserve capability. In order to provide sufficient primary reserve, it then becomes necessary to turn on unit C.

We also note that although unit A offers primary reserve at the relatively high incremental cost of 10 $/MWh (compared to 1 $/MWh for the other units), it is still scheduled to produce 12.27 MW of primary reserve. Even though the remaining units offer cheaper reserve, unit A cannot be excluded because of the coupling imposed by the common system frequency in primary frequency regulation. Another example of this strong coupling can be seen from the fact that the expensive reserve from unit A is required even though the frequency deviation after any contingency (see last column in Table 3) never reaches the 600 mHz maximum level.
The worst contingency in this case is the outage of unit A producing 120 MW, which is compensated by the remaining primary reserves resulting in a frequency deviation of –560 mHz. Finally, except for unit C that saturates at its ramp limit, all other units operate within their linear region at all times.

3.5.2 17-Unit Case Study

The unit commitment scheme with primary frequency regulation was also applied to a larger system described in [10], the characteristics of which are given in Table 4.

The cost functions of energy and reserve are as defined in equations (2.2) and (2.3). All units have a regulation droop of 5% with a system nominal frequency of 50 Hz and a maximum frequency deviation of 500 mHz. The offered cost for primary regulation, $q$, is assumed to be zero. The set of contingencies consists of the loss of any single generating unit, and the system demand of 1500 MW is insensitive to frequency.

Table 5 summarizes the results of applying the proposed unit commitment scheme with frequency regulation. The minimum cost schedule calls for units $H$, $K$, $L$, $O$, and $P$ with relatively high operating and fixed costs to be turned off.
Table 4. 17-Unit Case Input Data

<table>
<thead>
<tr>
<th>Unit</th>
<th>$b$ (£/MW²h)</th>
<th>$a$ (£/MWh)</th>
<th>$C_0$ (£/h)</th>
<th>$g_{i}^{\text{max}}$ (MW)</th>
<th>$r_{i}^{\text{max}}$</th>
<th>$g_{i}^{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.00045</td>
<td>9.8</td>
<td>200</td>
<td>330</td>
<td>100</td>
<td>26</td>
</tr>
<tr>
<td>B</td>
<td>0.00045</td>
<td>10.7</td>
<td>157</td>
<td>298</td>
<td>130</td>
<td>25</td>
</tr>
<tr>
<td>C</td>
<td>0.00045</td>
<td>13.6</td>
<td>800</td>
<td>154</td>
<td>120</td>
<td>20</td>
</tr>
<tr>
<td>D</td>
<td>0.00073</td>
<td>14.8</td>
<td>547</td>
<td>123</td>
<td>95</td>
<td>15</td>
</tr>
<tr>
<td>E</td>
<td>0.00066</td>
<td>15.2</td>
<td>532</td>
<td>234</td>
<td>37</td>
<td>16</td>
</tr>
<tr>
<td>F</td>
<td>0.00252</td>
<td>16.1</td>
<td>532</td>
<td>246</td>
<td>37</td>
<td>15</td>
</tr>
<tr>
<td>G</td>
<td>0.00252</td>
<td>16.1</td>
<td>590</td>
<td>91</td>
<td>17</td>
<td>5</td>
</tr>
<tr>
<td>H</td>
<td>0.0162</td>
<td>16.4</td>
<td>612</td>
<td>95</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>I</td>
<td>0.0162</td>
<td>17.1</td>
<td>580</td>
<td>274</td>
<td>54</td>
<td>21</td>
</tr>
<tr>
<td>J</td>
<td>0.00235</td>
<td>17.1</td>
<td>377</td>
<td>276</td>
<td>58</td>
<td>19</td>
</tr>
<tr>
<td>K</td>
<td>0.00175</td>
<td>17.7</td>
<td>670</td>
<td>82</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>L</td>
<td>0.01059</td>
<td>18.3</td>
<td>910</td>
<td>159</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>M</td>
<td>0.00279</td>
<td>19.5</td>
<td>155</td>
<td>114</td>
<td>55</td>
<td>10</td>
</tr>
<tr>
<td>N</td>
<td>0.00044</td>
<td>20</td>
<td>170</td>
<td>126</td>
<td>64</td>
<td>15</td>
</tr>
<tr>
<td>O</td>
<td>0.00047</td>
<td>22.1</td>
<td>658</td>
<td>100</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>P</td>
<td>0.00122</td>
<td>24.8</td>
<td>297</td>
<td>118</td>
<td>52</td>
<td>10</td>
</tr>
<tr>
<td>Q</td>
<td>0.00038</td>
<td>25.2</td>
<td>103</td>
<td>62</td>
<td>28</td>
<td>5</td>
</tr>
</tbody>
</table>

Note that the magnitudes of the unit break-frequencies, $\Delta f_{bi}$, are smaller than the maximum allowed deviation of 500 mHz. This implies that all units can deploy their maximum frequency regulation reserve without exceeding the 500 mHz limit. Furthermore, all units operate at pre-contingency generation levels, $g_{i}^{0}$, significantly below their maximum capacities, $g_{i}^{\text{max}}$. This is economically inefficient but it is the best that can be done with the given relatively low levels of ramp limits for primary reserve, $r_{i}^{\text{max}}$. 
Table 5. 17-Unit Case Unit Commitment

<table>
<thead>
<tr>
<th>Unit</th>
<th>( g^0 )</th>
<th>( r )</th>
<th>( \bar{g} )</th>
<th>( g^{\text{max}} )</th>
<th>( \Delta f_{hi} )</th>
<th>( \Delta f^k )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(MW)</td>
<td></td>
<td></td>
<td>(mHz)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>166</td>
<td>26</td>
<td>192</td>
<td>330</td>
<td>– 196</td>
<td>– 500</td>
</tr>
<tr>
<td>B</td>
<td>167</td>
<td>25</td>
<td>192</td>
<td>298</td>
<td>– 209</td>
<td>– 500</td>
</tr>
<tr>
<td>C</td>
<td>134</td>
<td>20</td>
<td>154</td>
<td>154</td>
<td>– 324</td>
<td>– 155</td>
</tr>
<tr>
<td>D</td>
<td>108</td>
<td>15</td>
<td>123</td>
<td>123</td>
<td>– 304</td>
<td>– 122</td>
</tr>
<tr>
<td>E</td>
<td>176</td>
<td>16</td>
<td>192</td>
<td>234</td>
<td>– 170</td>
<td>– 500</td>
</tr>
<tr>
<td>F</td>
<td>177</td>
<td>15</td>
<td>192</td>
<td>246</td>
<td>– 152</td>
<td>– 500</td>
</tr>
<tr>
<td>G</td>
<td>86</td>
<td>5</td>
<td>91</td>
<td>91</td>
<td>– 137</td>
<td>– 96</td>
</tr>
<tr>
<td>H</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>95</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>I</td>
<td>70</td>
<td>21</td>
<td>91</td>
<td>274</td>
<td>– 191</td>
<td>– 85</td>
</tr>
<tr>
<td>J</td>
<td>173</td>
<td>19</td>
<td>192</td>
<td>276</td>
<td>– 172</td>
<td>– 500</td>
</tr>
<tr>
<td>K</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>82</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>L</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>159</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>M</td>
<td>104</td>
<td>10</td>
<td>114</td>
<td>114</td>
<td>– 219</td>
<td>– 117</td>
</tr>
<tr>
<td>N</td>
<td>111</td>
<td>15</td>
<td>126</td>
<td>126</td>
<td>– 297</td>
<td>– 126</td>
</tr>
<tr>
<td>O</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>100</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>P</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>118</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Q</td>
<td>28</td>
<td>5</td>
<td>33</td>
<td>62</td>
<td>– 201</td>
<td>– 30</td>
</tr>
</tbody>
</table>

The last column in Table 5, \( \Delta f^k \), is the system frequency deviation after the loss of the unit associated with the corresponding row. We note that the loss of any one of the units A, B, E, F and J results in a \( \Delta f^k \) equal to the maximum allowed 500 mHz. This means all of these contingencies affect the system security and the cost of operation. Thus, defining security on the basis of only the loss of the unit with the largest generation or ramping capacity, as proposed in other approaches, would be inadequate in this case. Furthermore, basing the security criterion on the
loss of the largest generation requires predicting which unit or units meet this condition, something that is not a priori evident.

Finally, note that, unlike the previous 4-unit case, here all units reach their post-contingency limit, \( \bar{g}_i \), for at least one contingency, thus making use of the entire available primary reserve. Furthermore, for all units, the limit, \( \bar{g}_i \), is defined by the ramp rather than by the capacity limit. In this system therefore the ramp limits are overly restrictive.

### 3.5.3 34-Unit Case Study

This case differs from the previous in that the system load has doubled and each unit has been duplicated, defining a new system with 34 units (numbered from A to Q, with two units per letter) and with additional primary reserve capability.

Table 6 shows the schedule of each of the two identical units corresponding to each letter. Here more units are turned off (G, H, I, K, L, O, P, Q) than in Case II since there is considerably more reserve availability. More significant is that the cheaper units operate in the pre-contingency state closer to their maximum capacity. For example, unit A with 330 MW capacity and a low marginal cost of approximately 10 £/MWh produces 296 MW compared to only 166 MW in Case II.

The other notable difference between cases II and III is that here, for some generating units, the post-contingency limit, \( \bar{g}_i \), is defined by both the ramp and the capacity limits. Thus, the optimum unit commitment schedule can make better use of the available generation capacity and still meet the primary regulation security constraints.
Table 6. 34-Unit Case Unit Commitment

<table>
<thead>
<tr>
<th>Unit</th>
<th>$g^0$ (MW)</th>
<th>$r$</th>
<th>$\bar{g}$</th>
<th>$g^\text{max}$</th>
<th>$\Delta f_{hi}$ (mHz)</th>
<th>$\Delta f^k$ (mHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>296</td>
<td>26</td>
<td>322</td>
<td>330</td>
<td>– 196</td>
<td>– 500</td>
</tr>
<tr>
<td>B</td>
<td>273</td>
<td>25</td>
<td>298</td>
<td>298</td>
<td>– 209</td>
<td>– 235</td>
</tr>
<tr>
<td>C</td>
<td>134</td>
<td>20</td>
<td>154</td>
<td>154</td>
<td>– 324</td>
<td>– 91</td>
</tr>
<tr>
<td>D</td>
<td>108</td>
<td>15</td>
<td>123</td>
<td>123</td>
<td>– 304</td>
<td>– 73</td>
</tr>
<tr>
<td>E</td>
<td>218</td>
<td>16</td>
<td>234</td>
<td>234</td>
<td>– 170</td>
<td>– 152</td>
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<tr>
<td>F</td>
<td>231</td>
<td>15</td>
<td>246</td>
<td>246</td>
<td>– 152</td>
<td>– 163</td>
</tr>
<tr>
<td>G</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>91</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>H</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>95</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>I</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>274</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>J</td>
<td>220.5</td>
<td>19</td>
<td>239.5</td>
<td>276</td>
<td>– 172</td>
<td>– 156</td>
</tr>
<tr>
<td>K</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>82</td>
<td>–</td>
<td>–</td>
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<tr>
<td>L</td>
<td>–</td>
<td>–</td>
<td>–</td>
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</tr>
<tr>
<td>M</td>
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<td>10</td>
<td>65</td>
<td>114</td>
<td>– 219</td>
<td>– 37</td>
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<td>15</td>
<td>79</td>
<td>126</td>
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Chapter IV.

Simultaneous Scheduling of Primary Frequency Regulation and Tertiary Reserves

4.1 Overview

In a competitive pool-based electricity market, energy and reserve can be scheduled simultaneously or sequentially. Sequential procedures assume weak coupling of energy and different reserve types, therefore scheduling each service or product independently. However, this independence does not always reflect the real behavior of reserve and energy which, as seen in the previous chapter, could be strongly coupled. Sequential methods have been broadly applied due to their simplicity, transparency, and the independently obtained prices for energy and each reserve service. Nevertheless, they can lead to suboptimal solutions and to cases where lower quality services such as reserve can be priced higher than energy [16].

In sequential scheduling approaches, the use of heuristic rules such as turning on extra units in each subsequent step is employed to avoid
infeasibilities and mimic the coupling behavior between services. The use of different rules can lead to different solutions. For example, if in a given step, a unit is required to be turned on for security reasons, some already turned-on units would need to withhold power. The rule to decide how to re-dispatch generation could be based, among other possibilities, on minimizing either the new resulting generation cost or the generation set-point deviations from the ones found in the previous steps. All of these heuristics add to the overall inefficiency in terms of social welfare.

On the other hand, simultaneous approaches jointly schedule energy and the various reserve types, and, in theory, lead to optimal solutions. However, one drawback of simultaneous approaches is their increased complexity.

Under simultaneous scheduling, it is possible to define a price for energy and a unique marginal cost for security [17], which under marginal pricing will define a unique reserve price. This aspect will be studied more extensively in section 5.3.

### 4.2 Formulation of Unit Commitment with Simultaneous Primary and Tertiary Reserves

In this chapter we formulate and solve the unit commitment problem simultaneously scheduling energy as well as primary and tertiary reserves. As before, we assume a single period unit commitment without network constraints or losses.

First, the overall generation and reserve costs are simultaneously minimized,

$$\min \left\{ \sum_i C_i \left(g_i^0, u_i \right) + \sum_i C_i^p (r_i^p) + \sum_i C_i^t (r_i^t) \right\},$$

(4.1)
where \( r^p \) and \( r^t \) are the scheduled primary and tertiary reserve with respective costs \( C_n^p(r'_i) \) and \( C_n^t(r'_i) \). For example,

\[
C_n^p(r'_i) = q'_i r^p \\
C_n^t(r'_i) = q'_i r^p ;
\]

while the energy cost, \( C_i^t(g_i^0, u_i) \), can be of the same quadratic form as in equation (2.2).

As in previous formulations, energy and demand must balance at all times. First, in the pre-contingency state,

\[
\sum_i g_i - d = 0; \quad (\mu^0) \quad (4.3)
\]

and, then, in the post-contingency states, of which there are two per contingency, one due to the primary frequency regulation response,

\[
\sum_i g_i^{p^k} - d^{p^k} = 0; \quad k = 1, ..., nc \quad (\mu^{p^k}) \quad (4.4)
\]

and, finally, a few minutes after a contingency, by the actions of tertiary regulation,

\[
\sum_i g_i^{t^k} - d^{t^k} = 0; \quad k = 1, ..., nc \quad (\mu^{t^k}) \quad (4.5)
\]

In the above power balance relations, \( g_i^{p^k} \) and \( g_i^{t^k} \) represent the post-contingency generation levels of unit \( i \) after the occurrence of contingency \( k \) due respectively to primary and tertiary regulation.

The inequalities that define tertiary reserve are given by,

\[
r'_i \geq g_i^{t^k} - g_i^0 \\
r'_i \geq 0
\]
Additionally, tertiary reserve must respect the generation ramp-up limit which is larger than the primary reserve ramp-up limit,
\[ r_i^t \leq u_i r_i^{t-\text{max}}. \] (4.7)

The tertiary generation is subject to the same maximum capacity limit as the primary, that is,
\[ u_i g_i^{t \text{min}} \leq g_i^{t k} \leq u_i g_i^{t \text{max}}. \] (4.8)

Finally, as shown in previous chapters, primary reserve must be subject to all the constraints enumerated in section 3.4.

Each \( \mu \) in brackets next to each power balance equation represents the Lagrange multiplier related to that equality. Specifically, the pre-contingency power balance will have one \( \mu^0 \), while each post-contingency power balance will have \( nc \) different Lagrange multipliers equal to the number of contingencies considered. Some of the Lagrange multipliers can be zero if the corresponding contingency has been already covered by more severe “umbrella” contingencies [17], [18].

The marginal cost of energy can then be defined as,
\[ \lambda^E = \sum_k \left[ \mu^{t k} + \mu^k \right] + \mu^0, \] (4.9)
and the marginal cost of security as,
\[ \lambda^S = \sum_k \left[ \mu^{t k} + \mu^k \right], \] (4.10)
which under marginal pricing will result in the respective prices for energy and security [17].
4.3 Case Study

We have applied the formulation for the simultaneous scheduling of energy, primary and tertiary reserve to the 17-unit system described in section 3.4.2 with the same assumptions regarding demand and frequency behavior. Tertiary ramp-up limits have been assumed to be twice those for primary reserve. Also, the offered cost for tertiary regulation by unit \( i \) has been assumed to be one tenth the linear cost, \( a_i \). The simulation results obtained are shown in Table 7.

As additional results from this study, we note that the Lagrange multiplier associated with the pre-contingency power balance is \( \mu^0 = 16.1 \, \text{\pounds/MWh} \). From (4.9) and (4.10) it is also possible to calculate the prices for energy and security, respectively \( \lambda^E = 54.5 \) and \( \lambda^s = 38.4 \, \text{\pounds/MWh} \), noting in particular the relatively high price of security and the considerably higher price of energy.

From Table 7, one can also observe under the various contingencies the dramatically larger Lagrange multipliers associated with the post-contingency primary power balance compared to those multipliers associated with the post-contingency tertiary power balances. This indicates the scarcity of primary reserve compared to tertiary reserve. Thus, most of the marginal cost of security is due to the supply of primary reserve and very little to the deployment of tertiary reserve. This is so since tertiary reserve is more abundant and much less tightly coupled to the supply of energy, in other words, much less constrained by the need to meet the power balance under the pre-contingency state. In contrast, the need to meet the power balance in the pre-contingency state puts a severe restriction in the scheduling of primary reserve.
The unit commitment studied in section 3.4.2. has the same characteristics as the one studied in this section, with the only difference that the former does not take into account tertiary regulation. It is interesting to observe that these two cases present no difference in the units committed and in the energy scheduled. This result further points out that primary frequency regulation imposes a much stricter condition than tertiary regulation. One important conclusion is that where the use of sequential procedures is necessary, primary reserve should be dispatched.
first since this is the most restrictive service, and the scarcer services should be dispatched first.
Chapter V.

Conclusions

We have formulated and solved a unit commitment problem subject to primary frequency regulation reserve. This problem has been first studied simultaneously scheduling energy and primary reserves and secondly with the joint scheduling of primary and tertiary reserves in addition to energy.

The proposed unit commitment scheme with primary frequency regulation takes into account not only the system frequency limits and the generation ramp and capacity constraints but also the single-degree-of-freedom relation between the unit reserves and the post-contingency system frequency deviation. This characteristic is particularly difficult to model and forms one of the main contributions of this dissertation.

The case studies indicate that the scarcity of primary reserve can force the scheduling of expensive generation and the under-use of cheaper units. The scarcity of primary reserve can result in excessive market power by some generators and, although the proposed scheme is general enough to allow generating units to submit reserve bids, in order to restrict market power, we agree with previous studies that frequency regulation reserve should be regulated [19].
In the systems studied in this dissertation, primary regulation poses a strong constraint on the scheduled generation levels in the pre-contingency state. Therefore, in order to make use of cheap efficient generation and meet all security constraints, not only large efficient units have to be available, but also fast units that will guarantee a fast system response under a contingency.

The strong coupling between primary reserve and the pre-contingency generation levels suggests that unit commitment schemes should examine joint, rather than sequential scheduling of primary, secondary and tertiary reserves together with the pre-contingency generation levels. Such joint scheduling is computationally more demanding but in some cases there may be no choice in order to achieve optimality or even feasibility.

If sequential approaches are to be employed, the case studies suggest that the scarcer primary regulation resource should be scheduled first. There may be exceptions to this rule in cases where transmission congestion is present, making tertiary reserve scarcer at some buses.

One alternative to scheduling primary reserve, taken by most utilities, is where primary regulation is not a marketable service but, instead, a mandatory one. Under this scenario, primary frequency regulation is no longer an optimization variable and is not included in any of the market clearing procedures. Specifically, the National Grid Company in the United Kingdom has taken the approach where primary frequency regulation is mandatory [12]. Generating units receive a cost-based payment for providing primary frequency regulation as well as an energy payment for the expected energy deviation due to primary frequency regulation. These payments are based on tables which reflect the characteristic response of generating units to the system frequency deviations.

However, a competitive model for the provision of primary regulation has been proposed by some market participants. Among the main concerns
addressed in the implementation of competitive procedures for primary frequency regulation has been the inelasticity of primary reserve. One suggested solution is the demand-side provision of primary regulation. This possibility has not been studied in this dissertation but its formulation could be included with the aid of additional binary variables.
Bibliography


