Motions of Specularities on Undulating Surfaces

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Motions of Specularities on Undulating Surfaces

Abstract

Traditional studies on the image motion of specularities have concentrated on the motion of isolated specular patches on known specular surfaces, where important parameters such as surface curvature are predetermined. Furthermore the considered surfaces are mostly convex and non-undulating. Here, I attempt to expand on some of the previous results, by examining the frequency characteristics of the global motion of specular markings on random undulating surfaces subject to a lateral camera motion with constant velocity. I first expand the notion of a bowtie in the Fourier domain introduced by Langer and Mann[7] to accommodate the case of a smooth undulating surface, and then attempt to show that in most cases the global motion of the specular components for the considered surface and motion display roughly a similar power spectrum signature as the Lambertian components, characterized by a continuum of motions plane intersecting at the origin, which I call a blurred bowtie. The subtle discrepancies between the frequency profile of the Lambertian and specular components are also examined in the scope of this thesis.
Mouvements des Specularités sur une Surface Ondulée

Résumé

Les études traditionnelles sur le mouvement des specularités se convergent généralement sur l'analyse du mouvement des inscriptions spéculaires individuelles sur des surfaces identifiées où d'importants paramètres comme la courbure de la surface sont pré-déterminés. De plus, la plupart des surfaces considérées sont convexes et non-ondulées. Ici, je cherche à étendre les résultats précédemment obtenus en examinant les caractéristiques dans le domaine Fourier, du mouvement **global** des inscriptions spéculaires présentes sur une **surface aléatoire ondulée**, sujet à un mouvement latéral de caméra à une vitesse constante. En premier, je développe la notion d'un **bowtie** introduit par Langer et Mann[7], afin d'accommoder le cas d'une surface ondulée. Ensuite, je tente de démontrer que dans plupart des cas, le mouvement des inscriptions spéculaires pour le type de surface et mouvement considéré décrit le même profile que le composant Lambertian dans le domaine Fourier, caractérisé par un continuum de plan que j'appelle un **blurred bowtie**. Cette thèse examine également les différences entre le composant Lambertian et spéculaire dans le domaine Fourier.
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Chapter 1

Introduction

In recent years, numerous studies have been conducted in the area of image motion, and many computer vision methods have been developed for analyzing natural motion categories comprising such motions as: discontinuous optical flow, smooth optical flow [2][5], motion transparency and more recently optical snow [7]. However, most of these works are predominantly concerned with real-scene features, such as surface markings that remain fixed to the surface they belong to when undergoing motion. The area of research studying virtual scene features, which are the specular reflections\(^1\) of real features and which are often modeled in computer vision as a separate layer, has been rather limited.

Nonetheless, specularities\(^2\) are highly prevalent in nature, and although not attached to a particular surface, the presence of specular reflections in many cases give the scene its true sense of realism. A canonical example is viewing a wavy water

\(^1\)The term specular reflection in this paper is defined as the non-Lambertian reflection of a scene point off a shiny surface.

\(^2\)The term specularity and specular reflection are used interchangeably in this thesis.
surface on a sunny day. It would be quite difficult for instance to analyze the motion of this scene, without considering the motion of the specular highlights\(^3\) and reflections on the water surface. Although these reflections are not attached to the water surface, their contrast and distribution gives the viewer ample information to extract a reasonable inference on the shape and motion of the underlying water surface.

Several papers have studied the motion of specularities on particular shiny objects. Some of the findings from these papers will be described in the next chapter. From my readings, the entire research on the motion of specularities is based on tracking the motion of individual highlights and reflections on particular objects. It is however important to note that in many scenes, the components of a scene and the components within an object in the scene are subject to very similar motions. This fact is especially true if the scene itself is fairly stagnant-rigid, and the perceived motion depends on the motion of an observer or camera. For the above-mentioned reason, it seems essential to extend the analysis of specular reflections a little further. In this thesis, I will attempt to expand the study of specularities within a scene, by trying to characterize and understand the global motion of the specularities. Instead of looking at particular specular markings on particular objects, I will analyze the field of moving specularities present on random rigid undulating smooth surfaces, as seen from an observer moving laterally with constant velocity. I will also try to provide some possible extensions on the choice of motion when considering some possible future works on the topic in chapter 5.

The aim of this thesis is not to demonstrate any new mathematical results on the optics of specular motion. The main motivation and rationale of my research using

\(^3\)Highlights are the specular reflections of a light source on a given surface.
the above-mentioned framework is to extend some of the findings that were made about specularities on standard geometric shapes undergoing a lateral motion, to a more unstructured and random setting.

It has been verified for instance, that in the case of a sphere, viewed along a ray passing through the center of the sphere, undergoing a motion perpendicular to the line of sight (i.e.: translating across the field of view), the motion of the specularities is parallel to the motion of the sphere [3]. I will attempt to show that, similarly to the case of the sphere, looking at the motion of a "field" of specular markings on a random undulating surface provides much information into the structure of the surface and the make-up of the corresponding motion. I will also attempt to demonstrate that under certain conditions in the case of the considered surfaces and lateral motion, the global motion of the specular components are roughly in the same direction as the Lambertian component, or simply along a straight line, as depicted by plane(s) of energy in the Fourier domain. As we will see, the motion of the specularities do not exactly follow a straight line. The deviation of the motion of the specularities from a straight line is called disparity deviation [4].

My analysis of the motion of the specular markings will be conducted predominately in the frequency domain. Space-times statistics, such as normal velocities, will also support some of the results obtained in the frequency domain. The spatio-temporal frequency analysis carried out on the considered specular field can be summarized as follows.

Recently Langer and Mann [7], showed that an image region, undergoing a one-parameter family of velocities produces a bowtie pattern at the origin in the frequency
domain. The details of their work and findings will also be discussed in the next chapter. Their work was conducted using Lambertian objects moving in a cluttered scene. I first extend the notion of a bowtie from a cluttered Lambertian scene as considered by Langer-Mann, to a continuous smooth surface, which I use in this thesis. The illustration of this extension is presented in chapter 3. Consequently in chapter 4, I will show when and why bowtie patterns are obtained in the frequency domain for smooth undulating specular surfaces undergoing a lateral motion.
Chapter 2

Background

2.1 The Motion of Specularities

As previously noted, a scene is composed of two main categories of features. Real scene features correspond to patches or textures on real objects. Virtual features are merely the reflection by a shiny surface of another scene point. More often we study specular highlights, which are simply the reflection of a light source on a specular object. Since virtual features are not attached to any particular object, their image positions shift and their intensities vary if either the position of the observer or the position of the reflected point changes during motion. There are several ambiguities that can arise from the discrepancies between these two types of features. One of such ambiguities occurs in the discrimination of the two types of features in a real scene. The problem of discrimination is crucial for any type of shape from shading or binocular-stereo algorithm, as the specular reflections, if not accounted for, would lead the algorithm into erroneous conclusions about the morphology or depth of the
underlying surface or object. Several algorithms have been proposed to distinguish between the two types of features. Oren and Nayer [10] considered a method for distinguishing the two components using the notion of the catacaustic surface also known as the caustic surface. Although the objective of their work differs from mine, the notion of caustic curve or surface is central in studying specularities. For the purpose of this thesis, I will consider the latter problem to be solved, and will set-up the experiment in such a way as to easily discriminate between the two types of features. I am mainly concerned with the global behaviour-motion of the field of specularities present on an undulated surface under a lateral observer motion.

For a flat surface, the virtual image of the scene point lies in a single point behind the surface. The projection of that virtual point on to the actual surface represents the specularity as seen by the observer.

However, as shown in several papers [3][4][10], for any curved surface the position of a given specular feature is viewpoint dependant. Starting from some of the earlier works on the topic, Koenderink and Van Doorn [6] gave a crude characterization of the motion of the highlights and showed that highlights travel freely in hyperbolic and elliptic regions and speed up in parabolic regions. Later on, Blake and Bulthoff [3] studied the linear relationship between the motion of specularities on a surface and the motion of an observer moving relative to a fixed point on that surface. The authors argue that the motion parallax \( \Delta t \) of a given specular patch, which is its velocity relative to a fixed surface point at time \( t \), is proportional to:

- the inverse squared of the distance \( \lambda \) between the surface-object and the observer.
• the radius of the surface\(^1\). (i.e: the parallax increases with the radius)

Generally the authors show that for a given surface:

\[ \Delta_t \propto -v_t \ast \frac{r}{\lambda^2} \tag{2.1} \]

where \( v_t \) describes the velocity of the observer, and \( r \) is the radius of curvature of the surface.

The authors conclude that although the specularity parallax vector is parallel to the observer's velocity vector in some special cases, such as a sphere translating across the field of view, this condition does not hold in the case of general surfaces.

More recently Oren and Nayer [10] studied the motion of the specular features along the catacaustic surface. The catacaustic surface is obtained by considering an envelope of tangents to each of the reflected rays coming off the surface. The geometry of the catacaustic surface is fully determined by the geometry of the specular surface and the position of the reflected scene point (figure 2.1).

The morphology and characteristic of the catacaustic surface has been widely studied in recent years. As previously stated, for planar specular objects such as ordinary mirrors, the catacaustic is merely a single point behind the object. For curved surfaces, the profile of the catacaustic surface and the geometry-curvature of the attached surface exhibit an intriguing relationship. For instance, Koendrink and van Doorn [6] showed that for shapes such as sharp corners having a very low radius of curvature, the catacaustic is concentrated in such a small area, that it would be quite difficult to distinguish its profile from a singular point in the presence of noise. More

\(^1\)As the radius of curvature become very large (i.e: the surface-object becomes nearly planar), the relation between the radius and and \( \Delta_t \) is no longer valid.
Figure 2.1: The caustic surface for a curved concave reflector. The locus of the virtual points lie in front of a concave reflector, and behind a convex reflector.

recently, empirical evidence has also confirmed this point, and has shown in general that although highly curved surfaces exhibit a higher angular motion along their catacaustic surface, their motion is enclosed in a very small area. More interestingly, due to the containment of their respective catacaustic surface in small areas, surfaces exhibiting high curvature display a very low disparity deviation [4][10]. The following interesting fact crudely relates surface curvature to disparity. Planar surfaces have a singular catacaustic, and specularities move with the viewer rather than the surface. Highly curved surfaces having a catacaustic surface contained in a small area exhibit very small disparities. Surfaces having curvatures between these two extrema exhibit larger disparities [4].

Moreover, the curvature of the surface is not the only factor influencing disparity. As shown in [4], the orientation of the surface with respect to the scene point has
a direct effect on disparity deviation. An increase of the angle of incidence (angle between the scene point and the normal of the specular surface) causes greater motion along the catacaustic, in turn causing a larger disparity. The minimum disparity deviation is obtained when the angle of incidence is perpendicular to the plane of the surface. This type of reflection, also known as normal reflection, exhibits the smallest disparity deviation.

Yet another factor affecting disparity deviation is the distance of the camera-viewer from the scene. The disparity between two scene-points decreases as the viewer moves further away from the scene. This in turn decreases the motion of the feature along the catacaustic surface, and therefore decreases the disparity deviation [3][4]. Although the above-noted general assumptions have been widely studied, understood, and empirically verified for a variety of surfaces, there remain a few factors that must also be considered.

Criminisi et al. [4] conduct a detailed study and identify several other factors that have considerable effect on the observed disparity deviation for a given surface. The authors derive an equation that relates the motion of a given specularity from time $t_0$ to $t_1$, based on the curvature of the surface, the change of curvature of the surface, the orientation of the surfaces, the distance of the camera to the surface $d_c$ and lastly the virtual distance $d_v$ of the virtual point from the surface, calculated by considering a straight line-segment from the virtual point to the camera. As demonstrated by the authors, the disparity deviation for a curved surface are the greatest when $d_c = -d_v$, which means that the camera is at the focal point of the curved reflector. For a curved convex reflector the virtual depth slowly decreases as the curvature increases.
The locus of the virtual points is located behind the surface and approaches the surface as the curvature increases. A concave reflector, for which the locus of virtual image points lie in front of the surface exhibits a complex behaviour. For a concave reflector, the virtual depth approaches negative infinity as the object distance lies within but approaches the focal distance of the curved surface, and then jumps back into a positive virtual depth as the object moves further out from the focal point. Furthermore, Zisserman et al. [13] showed that specularities "move with" the viewer on convex surfaces, whereas on concave surfaces specularities exhibit a "contrary" motion to the viewer. This argument stems from the proof given by the authors, that specularities moving on a convex reflector, exhibit a positive epipolar velocity, defined as the specular velocity component in the direction of motion with respect to a fixed surface point, whereas specularities moving on a concave reflector, exhibits either a zero or negative epipolar velocity with respect to a fixed surface point.

The obvious difference between the effect of a convex and concave reflector on the virtual images requires us to investigate the behaviour of specular motion with respect to variations of curvature along the undulating surface.

2.2 Optical Snow and the Bowtie Effect

In recent years, many studies have been conducted on the nature of motion in the spatiotemporal frequency domain. Watson and Ahumada[12] introduced the notion of the motion plane property. They showed that if an image frame translates over time with a given velocity \((v_x, v_y)\), then every component of the 2-D Fourier transform of the frame travels with the same velocity. Using the Horn and Schunck's [5] constraint
equation on velocities in a scene, they derived the notion that an image marking that travels with a uniform velocity creates a plane of energy in the frequency domain (See appendix for definitions and the proof of the motion plane property). Langer and Mann [7] extended the simple case of the constant velocity to a more composite case in which, there exists a one-parameter set of velocities within an image region. The authors propose optical snow as one type of motion pattern suited for above framework. Optical snow is an example of motion where a viewer or camera moves relative to a densely cluttered 3-D scene. A static viewer observing falling snowflakes or an observer moving through a cluttered scene such as a forest are examples of such motion. The one-parameter set of velocity for the optical snow stems from motion parallax. The velocity of a given object in the 3-D scene is inversely proportional to the depth of the object with respect to the viewer-camera. The extension of the motion plane property proposed by the authors implies that the one-parameter family of velocities within the image region produce a one-parameter family of motion planes in the frequency domain. Furthermore the authors demonstrate that this one-parameter family of motion planes intersect at a line that passes through the origin in the frequency domain \((f_x, f_y, f_t)\), producing a bowtie, with the line of intersection representing the axis of the bowtie. The signature bowtie obtained in the above manner allows the authors to draw conclusions on the nature of the motion, such as the directions and range of velocities within the scene.
2.3 Making the Connection

Although it might seem at first glance that the analysis carried out by the previous authors is solely applicable to "optical snow" type motion and their derivatives, their analysis can be applied to the type of motion considered in this thesis, which is the motion of specularities on a smooth undulating surface as seen by an observer moving laterally parallel with respect to the axis of the surface. The latter point is quite apparent, by considering the following argument. The only assumption needed to produce a bowtie pattern in the frequency domain is an image region comprising a one-parameter family of velocities. The authors impose no other constraints on the nature of the scene, and although they argue that their framework is ideal for analyzing the motion in a densely cluttered 3-D scene, where a one-parameter family of velocity vectors is spawned by motion parallax, their analysis can be applied to any type of scene where the previous assumption is valid.

By means of the background work on specular surfaces from the previous chapter, we observe that the motion of the specularities depend on the motion along the catacaustic, which in turn depends on the curvature of the surface and the distance of the viewer from the scene.

Furthermore as the authors in [4] also noted, factors such as curvature-orientation of the surface and the distance of the viewer from the surface, and fast undulations (changes in curvature sign) affect the disparity deviation of the specular patches. Here we need to specify the possible components of a disparity deviation. In view of the lateral motion, the component of the motion of a specularity is called an epipolar component [1][3][4] along the direction of motion and circumpolar component [3] in the
perpendicular direction. The motion of specular markings on an undulating surface often exhibit both an epipolar and circumpolar component.

As we will show experimentally in chapter 4, although a circumpolar component is present in the case of the considered surface and motion, the parallax of specularities with respect to a fixed surface point are such that the global motion of the specular markings is dominated by an epipolar component. Under such conditions, the global motion of specularities on a smooth undulating surface describes roughly a similar spatio-temporal frequency spectra as the Lambertian components. This fact for instance implies that in the case where the undulating surface presents a range of depths, the motion of the specular markings broadly describe a one-parameter family of velocity vectors in the direction of heading. In such cases, I will try to depict the presence of a blurred bowtie pattern in the power spectrum of the specular motion using the same methods as the authors in [7].

However, the motion of specularities on a separate caustic surface also implies:

- Specularities exhibit both a positive and negative epipolar velocity with respect to fixed surface points of the undulating surface [13].

- The parallax of the specularities with respect to the undulating surface decreases with an increase of the curvature of the undulating surface.

- The parallax of the specularities with respect to the undulating surface decreases with an increase in depth of a given surface point.

The effects of above-mentioned characteristics of the motion of specularities will be analyzed in chapter 4 when comparing the projected power spectra for the Lambertian
and specular components of motion.

Before exploring the experimental results, some theoretical findings required to analyze the experimental data, will be discussed in the next chapter.
Chapter 3

Theoretical Results

Before exploring any experimental results, a few theoretical findings are needed to analyze the motion of the specular markings on the undulating surface. In the first section of this chapter, I extend the bowtie framework proposed by Langer and Mann [7] to accommodate the case of a smooth undulating surface. In the second section, I analyze some of the characteristics of the normal velocity distribution for such surfaces.

3.1 Bowtie for a Continuous Surface

As mentioned in the previous chapter, the aim is to conduct a similar analysis on the motion of specularities on undulating surfaces in the frequency domain, as accomplished by Langer and Mann [7] for the motion of Lambertian objects in highly cluttered scenes. The difference between the two types of scenes requires an extension to the work accomplished by the previous authors. The transition from an "optical
snow" type scene to a continuous smooth surface does alter the appearance of the bowtie pattern.

3.1.1 Case 1: Lambertian Surfaces

To make the transition a little easier, I first consider the case of a Lambertian surface. The simplest kind of Lambertian surface is a planar Lambertian surface. As usual, \( z \) represents the depth of a surface point and \((x, y)\) represent its image position in the image plane. I would like to examine the image velocity field obtained, as the center of projection moves laterally in the \( x \)-direction (i.e., no motion in the \( y \) and \( z \) directions) with respect to the planar surface (this is the type of lateral motion that is considered in the framework of this thesis). Following such a framework, the camera coordinate of any point moves over time in the following way: if the camera coordinates of a 3D scene point at time \( t = 0 \) are \((X_0, Y_0, Z_0)\), then at time \( t \) the coordinates are \((X_0 - T_xt, Y_0, Z_0)\).

Assuming, without loss of generality, that the projection plane is at \( z = 1 \), we can calculate the image-plane coordinates of the point as a function of \( t \), namely,

\[
(x(t), y(t)) = \left( \frac{X_0 - T_xt}{Z_0}, \frac{Y_0}{Z_0} \right)
\]

Taking the derivatives with respect to \( t \) at \( t = 0 \) yields an image velocity vector:

\[
(v_x, v_y) = \frac{d}{dt}(x(t), y(t)) \mid_{t=0} = \left( \frac{1}{Z_0^2}(-T_z, Z_0), 0 \right) = \left( \frac{-T_x}{Z_0}, 0 \right).
\]

As one can clearly see, the image velocity vector is a function of the depth of the planar surface, and therefore, when a wide range of depths are present, such a scene exhibits a high degree of parallax.
The nature of the depth map clearly distinguishes this type of scene from the cluttered "optical snow" framework considered by Langer and Mann. As previously noted, the bowtie pattern is formed due to the fact that different planes of energy representing different one-parameter \((Z_0)\) velocities within the scene, intersect at the origin in the frequency domain. Velocities are proportional to inverse depth, and in the optical snow case the depth map is discontinuous near all image points, resulting in a set of distinct planes intersecting at the origin. For a continuous surface however, there are no depth discontinuities, and therefore, a \textit{continuum} of energy planes is obtained resulting in what I characterize as a \textit{blurred bowtie}. The projection of the 3D bowtie in a direction identical to the axis of the bowtie yields the 2D bowtie pattern presented in figure 3.1 - 3.2. The blurred bowties obtained using my experimental data can be seen in figure 4.6 in the next chapter. The slopes of the motion planes \(\left(\frac{f_t}{f_x}\right)\) represent the value of the different one-parameter \((Z_0)\) velocities within the scene. \(f_t\) and \(f_x\) are respectively the temporal and spatial Fourier coefficients (see appendix).

Using the above, we define the \textit{envelope} of a bowtie as the set of two motion planes representing respectively the highest and lowest one-parameter \((Z_0)\) velocities. The envelope of the bowtie is also depicted in figure 3.2. The envelope will be used in the analysis of the experimental data in chapter 4.

Similar arguments are applied and similar results are obtained by replacing the plane by a smooth undulating surface, since in the latter case, the depth map is also continuous.
Chapter 3: Theoretical Results

Figure 3.1: The projected 2D bowtie pattern for a scene with high degree of depth discontinuities (Optical snow).

Figure 3.2: The projected 2D blurred bowtie pattern of a scene with a continuous depth map (smooth undulating surface).
3.1.2 Case 2: Specular Surfaces

The arguments for the Lambertian case are also applicable to specular surfaces. Specularities are present at all depths and move along the caustic surface between parabolic lines [3][6]. Near parabolic lines, specularities are created and annihilated in pairs [6]. However as the authors in [3] note, long range motion of the viewer is often needed for observing the creation and annihilation of a specular patch. Using the above argument, it would seem that the general motion of specularities is dominated by the "continuous" motion of specularities along the caustic. Discontinuities seldom occur when the viewer "crosses" the caustic surface, which turns out to be at or near parabolic lines. This would imply that in the case of a specular motion, although occasional discontinuities may be present, if and when a bowtie pattern is observed, it would likely have the appearance of a continuous-blurred bowtie, rather than a bowtie having distinguishable motion planes, as is the case of the optical snow which exhibits discontinuities near all image points. For the case of the specular surfaces considered in the scope of this thesis, the above inference was shown to be empirically valid.

Although this thesis focuses on the study of the motion of specularities in the frequency domain, some spatial measurements of motion would be useful to support some of the findings in the frequency domain. The properties of the normal velocity vector, which I use to this end, are described in the next section.
3.2 Normal Velocities

I would like to estimate some local measure of velocity of the specular motion. However calculations of such a local velocity at a given pixel cannot be computed locally, independently of neighboring pixels, since the velocity field at each point in an image is comprised of two components, whereas the change in intensity at a given pixel from one frame to the next only provides us with only one constraint. A useful way to overcome this limitation is to measure the normal velocity at each pixel. The normal velocity is defined as the component of the real image velocity component in the direction of the image intensity gradient, and is described by the equation:

\[ \vec{v}_n = -\frac{\partial I}{\partial t} \frac{\partial I}{\partial x} + \left( \frac{\partial I}{\partial y} \right)^2 \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) \]

(3.1)

where \( I(x, y, t) \) is the image intensity of the point \((x, y)\) at time \( t \), and \( \left( \frac{\partial I}{\partial x} \right), \left( \frac{\partial I}{\partial y} \right) \) are respectively the derivatives taken in the spatial and temporal directions. For the derivation of the above equation please refer to Appendix 1.

3.2.1 Case 1: Lambertian Surfaces

We first consider the case of the continuous Lambertian surface, with no specular component.

Next we argue that for such surfaces undergoing a lateral camera motion, the distribution of these normal velocities plotted as a 2-D histogram, lies:

- on a circle, tangent to the line passing through the origin, and perpendicular to the direction of motion, having as a diameter the projected velocity of the scene, if no motion parallax is present.
Chapter 3: Theoretical Results

- on a *continuum* of concentric circles all tangent to the line passing through the origin, and perpendicular to the direction of motion, having as diameter(s) the various projected velocities dependant of depth, if motion parallax is present.

Consider the camera moving upwards within the scene (positive $y$ direction), with speed $s$ such that $(v_x, v_y) = (0, s)$. The normal velocity is merely the projection of this vector in all possible spatial gradient directions. Since I am considering smooth undulating surfaces for the purposes of this experiment, the gradients are uniformly distributed over all possible directions with only possible foreshortening in a particular direction in case of a slanted surface (There is no bias toward any particular direction often caused in real scenes by the presence of edges, corners, etc.). We would therefore expect the distribution of the normal velocity vectors to lie on a circle where a unique velocity is present (no motion parallax), and on a continuum of concentric circles where a one-parameter family of motion is present (motion parallax). The difference in the distribution of normal velocities between the two types of scene can be seen in Figure 3.3.

Here are a few special results that can be inferred from the previous discussion.

- If the spatial gradient is in the direction of the true velocity vector (the velocity $(v_x, v_y)$ at a given image point dependant on depth), then the normal velocity vector equals the true velocity vector.

- If the spatial gradient has a positive dot product with the true velocity vector but is not in the direction of this vector, then the normal velocity vector lies on the normal velocity circle and is shorter than the true velocity vector.
Figure 3.3: The Lambertian scene on the left exhibits no parallax (unique circle stemming from the unique velocity vector \((0, s)\)). The Lambertian scene on the right, exhibits considerable parallax (continuum of concentric circles stemming from a one-parameter family of motion \((0, s_1)\) to \((0, s_n)\)) dependent on depth.

- If the spatial gradient is orthogonal to the true velocity vector, then the normal velocity is equal to zero.

One subtlety that one must consider, is that case where the spatial gradient has a negative dot product with the true velocity vector. This would imply that the component of the true velocity referred to in the definition must be a negative number. Scaling the intensity gradient vector with a negative number yields a vector in the opposite direction than the intensity gradient. For the above-mentioned reason, the normal velocity and true velocity vector always have non-negative products.

### 3.2.2 Case 2: Specular Surface

When carrying the analysis of normal velocity to surfaces having a specular component, the assumption of a true velocity vector is no longer valid. As previously
Chapter 3: Theoretical Results

noted, specularities move on a separate catacaustic surface, and exhibit both epipolar and circumpolar disparity with respect to a fixed surface point.

This point clearly implies that the normal velocity of specularities may be distributed over a set of directions and magnitudes even if the real velocity of a scene is constant and in one direction. The above argument is shown to be valid in the next chapter. However, as also shown in the next chapter, with the exception of nearly planar surfaces, the dominant components of the specular normal velocity are also roughly the direction of the true velocity component for the general undulating surfaces, and the 2D histogram of the normal velocities of the specular markings traveling on the undulating surface closely resembles the signature of the Lambertian normal velocity histogram.
Chapter 4

Experimental Results

4.1 Generation of an Undulating Surface

As previously mentioned, the purpose of this experiment is to analyze the motion of specularities on an undulating surface undergoing a lateral camera motion.

In order to conduct the experimental investigation, I must first generate an undulating surface. As one might imagine, the use of artificial environments is so prevalent in such areas as animation and game creation, that there have been numerous methods developed for generating such a surface, each trying to improve on the quality and efficiency of the generated environment. There have been several imaged-based techniques based on the notion of fractals, first formalized by Mandelbrot [19]. Several other algorithms using a frequency-based approach have generated various terrain-generating algorithms [18]. In general, most image-based methods rely on the idea of height fields, which is simply a mapping $Z(x, y)$ of all points on an $x - y$ grid. Most algorithms adopt a random perturbation of the height map to generate the artificial
terrain. However, for all the above work, a high degree of fine-tuning is necessary in order to ensure the smoothness of the particular surface. For the purpose of my experiments, I generate a simple height-field using a combination of 2D sinusoids. By controlling the ratio of the maximum amplitude of the sinusoids with respect to their frequency, I am assured to obtain a smooth undulated surface. The randomness of the surface is assured by the direction and number of the generated sinusoids. Examples of the generated surface can be seen in figures 4.3 (oblique view) and 4.9 (top view).

The aim of this thesis is to draw conclusions on the motion of the specularities, by comparing it to the motion of the Lambertian components. The detail of how this will be accomplished is described in the next section.

4.2 Lighting, Motion and Perspective

The issues of lighting, motion and perspective must be considered. The surface was generated and lighting calculations were accomplished using OpenGl. For the purpose of further analysis in later sections, insight into the OpenGl calculations of lighting intensities is necessary. OpenGl uses a standard Lambertian formula for the Lambertian term by using the inner product of the light source vector and the normal at a given pixel. The specular term is calculated using a Phong model. To be exact OpenGL relies on the following equations for the respective terms [16][17].

\[
I_{lamb} = \max(L \cdot n, 0) \cdot \text{Lambertian}_{\text{light}} \cdot \text{Lambertian}_{\text{material}}
\]

\[
I_{\text{spec}} = \max(s \cdot n, 0)^{\text{shininess}} \cdot \text{specular}_{\text{light}} \cdot \text{specular}_{\text{material}}
\]
where \( n \) is the unit vector \((n_x, n_y, n_z)\) at each vertex-pixel, \( L \) is the unit vector \((L_x, L_y, L_z)\) that points from each vertex to the light source and \( s \) is the normalized sum of \( L \) and the unit vector pointing from vertex to the viewpoint. In order to create a realistic environment, such as a scene on a sunny day, a directional light source having an apparent infinite location is used to illuminate the scene. The effect of the infinite location is that the light rays can be considered parallel by the time they reach the object. This is important in the motion of specularities, since it implies the variation of the vector \( s \) is solely caused by the change of the vector between a given vertex and the viewpoint. The term shininess refers to the \( \alpha \) exponent of the Phong model\(^1\). OpenGI calculates the lighting component at each vertex as a sum of the RGB Lambertian and specular components. The RGB components of each of these terms is calculated using the Equations 4.1 and 4.2. As it can be seen in these equations the RGB values for material and lighting are multiplied to obtain the RGB values for the respective terms. In this experiment, I use a single white-light source and therefore \( \text{specular}_{light} \) and \( \text{lambertian}_{light} \), which are respectively the specular and Lambertian component of the light source are regarded as equal. The intensity of the illuminated surface results in an RGB output for each of the pixels of the frame. For the purpose of separating the Lambertian and specular intensity components, separate colour channels are attributed to each component (Figure 4.1). As the camera moves, the intensities at each pixel in the new frames are calculated using equation 4.1 and 4.2.

The scene is viewed in perspective. Details on the perspective of the scene and

\(^1\)\( \alpha \) component ranges from 0 for a non-shiny surface to \( \infty \) for a highly shiny surface such as a mirror
the motion of the viewer relative to the scene will be discussed for each individual example in the following sections.

4.3 Calculation of the Signature Bowtie

The calculations of the signature bowtie pattern were accomplished using a similar approach as the authors in [7]. Here I briefly summarize.

First the mean grey level value from the entire image sequence \( I(x, y, t) \) is subtracted from each pixel. Second, the image sequence is windowed both in space and time by an cosine function,

\[
1 - \frac{\cos(2 \pi)}{N} \ast i
\]

where \( N \) represents the width-height of each frame, or the duration in case of time.

Finally, the power spectrum \( |\hat{I}(f_x, f_y, f_t)|^2 \), of the image sequence \( I(x, y, t) \) is re-
peatedly projected orthographically onto a set of vertical planes,

\[ \cos \theta f_x + \sin \theta f_y = 0 \]  \hspace{1cm} (4.4)

where \( \theta \in [0, \pi] \). The projection is computed by summing the power along lines parallel to \((\cos \theta, \sin \theta)\).

If a bowtie exists, it should become apparent by projecting the power spectrum perpendicularly to the direction of motion (the authors in [7] showed that the axis of the bowtie is perpendicular to the direction of motion). In the experiments conducted in this chapter, the camera describes a lateral motion in the x-direction. The power spectra were therefore projected along the y-axis.

In the last section of this chapter, the power spectrum is projected parallel to the direction of motion, to show that no bowtie exists in this direction.

### 4.4 Results

The experiment is divided into two broad categories, which I believe provides a better framework in studying the motion of the specularities.

Firstly, I examine a case that is highly prevalent in nature. The reflection of a scene point, as seen by a viewer having an oblique angle of view and being on the opposite side of the reflected scene point with respect to the general axis of the surface normals. A canonical example is that of the reflection of the sun-moon on a wavy water surface (Figure 4.2-left). I refer to this as case 1.

Secondly, I analyze a case where the normal axis of the specular surface is viewed along the line of sight, as the observer moves laterally with constant velocity relative
Figure 4.2: (Left) Oblique angle of view: The scene exhibits a wide range of depths. (Right) Perpendicular top view: The scene exhibits a narrow range of depths.

to the undulating specular surface. The scene is constructed in such a way that the distance between the observer and the surface far exceeds the amplitude of the undulations on the surface. An example of this motion is found when an observer tracks the reflections coming off a shiny, bumpy vertical wall (Figure 4.2-right). I refer to this as case 2.

The fundamental difference between case 1 and 2, and the reason I draw a distinction between them, is that in the latter case, the relative distance of the viewer from surface is far greater than the amplitude of the surface, and therefore the difference in distance $\lambda$ between the viewer and different points on the surface is considered negligible. The idea is very easily grasped by imagining a viewer standing in front of a wall like bumpy shiny surface. If the viewer is standing far away from the wall, then the difference in distance from the viewer to the closest and furthest points of the undulations is insignificant.
The same argument does not hold for case 1. The type of scene described in case 1 exhibits a wide range of depths. For instance, there is a great discrepancy in depth between the points close to the viewer, and the ones at far edge of the surface.

This distinction between these two types of cases will allow me to broadly investigate the independent effects of depth and curvature on the motion of specularities on undulating surfaces.

The details of each case are described in their respective sections. I first study Case 1. Such scenes, due to the nature of their depth map, display a high degree of Lambertian parallax. For the lateral motion considered in the scope of this thesis, the direction of the velocity vector of the Lambertian components are parallel to the direction of heading (see section 3.1.1). The actual velocity of the Lambertian components depends on the depth of the considered surface point. I would like to examine whether or not, the same assumption holds for the motion of the specular components. A close comparison of the power spectrum of the two components will allow me to outline the similarities and discrepancies between the respective motions.

4.4.1 Case 1- Slanted surfaces-(High degree of Lambertian parallax)

The scene is constructed in the following manner. The general axis of the undulating surface is rotated by a 45-degree angle about the x-axis and translated to a depth $z$ of 200 units. The surface therefore describes a 45-degree angle with the viewer and an identical angle with the light source (but on the opposite side - Figure 4.2-left). The line of sight is in the positive $z$ axis. The camera is located at $z = 0$. 
and moves in the x-direction with a velocity \( V_x = -1.5 \) unit/frame. The field of view is 60 degrees. The sequence of data contains 64 frames, each of which is a 128 \( \times \) 128 image. Four sequences generated using the above base method are presented in figure 4.3 and are also shown in videos \textit{slanted1.mpeg} to \textit{slanted4.mpeg}. Sequence 1, is a nearly-planar surface, and although I concentrate on undulating surfaces for the purpose of the experiments, this sequence will be used as a basis of comparison with the other sequences and also to demonstrate that a certain minimal amount of undulation is necessary to validate the conclusions of this thesis.

The only difference between the four video sequences is the maximal amplitude of the sinusoids. The maximal amplitude ranges from 0.3 units in the quasi-planar surface in sequence 1 to about 4 units in the undulating surface in sequence 4 (figure 4.3-caption).

Since the distance from the viewer to the scene far exceeds the amplitude of the sinusoids, the Lambertian bowties for all four sequences have roughly the same profile. Figure 4.4 shows the power spectrum for the Lambertian component of sequence 4 projected perpendicularly to the direction of motion, which will serve as the typical Lambertian bowtie of the scene and will be compared to the specular bowties. Figure 4.5 shows the power spectrum for the specular components of all four sequences projected perpendicularly to the direction of motion. In all the projected power figures, the axes range from \([-T/2...T/2 - 1]\), where \( T \) is the number of frames and \([-N/2...N/2 - 1]\) where \( N \) is the image size. Log-energy is plotted, in order to render the energy plots visible.

The first qualitative observation that can be made by looking at the four video
Figure 4.3: One frame of the slanted undulating surface maximal amplitude (top-left) 0.3 units (top-right) 1.25 units (bottom-left) 2.75 units (bottom-right) 4 units
Figure 4.4: The power spectrum for the Lambertian component of sequence 4 projected perpendicularly to the direction of motion. This is considered the typical Lambertian bowtie for all four sequences.

Figure 4.5: The power spectra for the specular components projected perpendicularly to the direction of motion from sequence 1 (far left) to sequence 4 (far right).
sequences from scene-type 1 is that the motion of specularities in sequence 1 exhibit very little correspondence with the motion of the surface and display a free range of motion near the center of the image. For sequences 2-4, there seems to exist a much stronger correlation between the motion of the specularities and that of the surface. Even though specularities in these sequences still move freely on the surface, there seems to be considerable bias in the global motion of specularities in the direction of heading. Furthermore, the actual velocities of the specular components seem to exhibit the same effect with respect to depth as the Lambertian components (i.e. closer specularities move faster than the ones further away).

If the above observation is shown to be empirically valid, it could lead to some interesting results. For sequences 2-4, one could indeed expect the global motion of the specular markings to broadly (although not exactly) describe the same direction and velocities as the Lambertian components, and exhibit a strong bowtie pattern in the frequency domain. For sequence 1, for reasons mentioned in the previous paragraph the above argument may indeed not hold.

The projections of the specular power spectra in figure 4.5 seem to confirm the above inference. For sequences 2-4, a well-defined bowtie pattern exists, closely resembling the bowtie pattern of the Lambertian component of motion (figure 4.4). This is quite an interesting result, since the velocity and direction of the motion of specularities on curved surfaces, is highly related to the curvature and undulation of the surface. Yet, for a complex randomly undulating surface considered in this thesis, the global profile of the motion of specularities seems to display a similar signature power spectrum as the Lambertian components in the direction of heading. The lack
Figure 4.6: Scan line ($f_x = 0$) of the projected power spectra of figure 4.5. (The mean power value has been subtracted) (Left): Sequence 1  (Right) : Sequence 4

of noise in the power projections for sequences 2-4 also validates the fact that there is little sustained motion in directions other than the direction of heading.

For sequence 1, although the projected power spectrum does exhibit energy in the direction of heading, there seems to be a notable amount of energy in other directions, depicted by the cloud of noise present near the origin.

To clarify the latter point for the reader, and to compare the specular bowties, I plot the horizontal scan-line passing through the origin for the projected power spectra for two extreme cases of sequences 1 and 4 (figure 4.6). The mean power value has been subtracted from both spectra.

The discrepancy between the power spectra of the two sequences is apparent on the scan line plot. The plot of sequence 4 presents a narrow band of energy near the origin, representing the point of intersection of the one-parameter motion planes. The plot of sequence 1 also presents this high-energy band. However, a much wider low-energy band is also present in the scan line plot of sequence 1, depicting the cloud of noise caused by the motion of specularities in directions other than the direction
of heading.

The discrepancy between the motion of the specularities in the two sequences can also be shown in the spatial domain using a mesh plot of the normal velocity histogram (figure 4.7). The axis of the mesh plot represents the horizontal velocity $V_x$ and the vertical velocity $V_y$. The profile of the histogram of sequence 4 (figure 4.7-bottom right) closely resembles the profile of the histogram of the normal velocity of the Lambertian component of motion (figure 4.7-top) with the image velocity vector, projected in all possible directions, yielding what can be described as a continuum of concentric circles ranging from the origin to maximal velocity of the scene. In the case of sequence 1 (figure 4.7-bottom left), although noticeable bias is also present in the direction of heading, there are noticeable deviations in other directions, as seen by the presence of velocities in a wide range of directions all around the origin.

I have argued in the previous paragraphs that for undulating surfaces that are not nearly planar, the global motion of the specular components may closely resemble the motion of their Lambertian counterparts. However, the simple fact that according to ray-optics of specular motion, the locus of virtual points lies either in front (concave), or in the back (convex) of curved surfaces, implies the existence of discrepancies between the motion of the virtual and fixed components. As mentioned in chapter 2, Zisserman et al.[13] described the relationship between the epipolar component of the specular parallax with respect to a fixed surface point, and the sign of the curvature of the surface. Notably for the type of motion considered in this thesis, specularities should exhibit a higher epipolar velocity on concave surface patches, and lower epipolar velocity on convex surface patches, in the direction opposite to the
Figure 4.7: Mesh plot of the normal velocity histogram (Top): Lambertian normal velocity. (Bottom-left): Specular normal velocity (sequence 1). (Bottom-right): Specular normal velocity (sequence 4)
camera motion, when compared to a fixed surface point. Using the above argument, one concludes that image motion of the specular components should exhibit a wider range of velocities in the direction of heading than the image motion of the Lambertian components.

In order to illustrate this inference, and to compare the Lambertian and specular bowties, one can use the idea of the envelope of a bowtie, which I defined in chapter 3 as the two planes of energy delimiting the bowtie and representing that highest and lowest one-parameter velocities (figure 3.2). Extending the previous observation, if specularities on undulating surfaces move faster or slower than their Lambertian counterparts in the direction of heading, one would expect the specular bowtie to present a higher range of velocities in direction of motion, than the Lambertian bowtie. This in turn implies that two planes of the envelope of the specular bowtie, should respectively represent a higher and lower velocity than the two planes of the Lambertian bowtie envelope.

In order to illustrate this point using the experimental data, I compare the typical Lambertian bowtie (figure 4.4) for the undulating surface, and the specular bowtie of sequence 4 (figure 4.5- Far right). Similar results were obtained by comparing sequence 2-3 and the typical Lambertian bowtie for a lower frequency. By closely analyzing the respective bowties in figures 4.4 and 4.5, one realizes that the plane of the envelope representing the lowest one-parameter velocity of the Lambertian bowtie coincides almost exactly with the same plane in the specular bowtie envelope. The opposite plane of the envelope however, seems to present a higher velocity in the case of sequence 4, than the plane of the typical Lambertian bowtie. Once again looking
Figure 4.8: Scan line of the projected power spectrum for \( f_x = 40 \). (Dotted): Lambertian (figure 4.4) (Solid): specular (figure 4.5)

at the scan line plot of the projected power clarifies this point. The scan line plot of the energy for \( f_x = 40 \) are given on the same plot in figure 4.8.

As one can clearly see, the right boundaries of the scan line plot, representing the lowest velocity image plane seem to entirely coincide, whereas the left bounding plane of sequence 4, seems to be close to roughly 5 units further out than the left plane of the Lambertian envelope.

Based on the previous observations, it seems that for the given type of scene (slanted surfaces exhibiting a high degree of Lambertian parallax), although the image motion of the specular components do present a higher range of velocity than the Lambertian components, the discrepancy seems biased in the direction of the higher velocities (i.e. Specularities may move faster than the Lambertian components in the direction of heading, but not really slower).

Although at first glance this asymmetrical discrepancy of the specular and Lambertian components might seem intriguing, even somewhat bizarre, the idea can be clarified using the geometry of the scene, and the ray-optics of specular motion. The
one-parameter family of motion planes comprising the Lambertian bowtie, stem from the motion parallax of the scene. The motion plane representing the smallest velocity (the right boundary of the envelope in the scan line), therefore represents the image velocity of furthest points from the viewer (largest depth $Z_0$), and conversely the motion plane representing the largest image velocities (left boundary of the envelope in the scan line) represents the image velocity of the closest points from the viewer (smallest depth $Z_0$). The discrepancy between the Lambertian and specular bowties in the direction of heading is dependant on the sign of the surface curvature, which causes specular patches to move faster or slower than the Lambertian image velocity in the direction of heading. So why is there no discrepancy between the motion of specular and Lambertian components for the image plane, representing the furthest points from the viewer? The answer lies in the ray-optics of specular motion, which was provided in the background work in chapter 2. The parallax of a given specular patch with respect to a fixed surface point has an inversed-squared drop off rate with respect to the distance from the viewer (Equation 2.1). This in turn implies that an increase in depth $Z_0$ of a point decreases the specular patches’ epipolar disparity with respect to a fixed image point with an inversed-squared depth factor, explaining why the plane of energy representing the specular markings that are the furthest away from the viewer, exhibit almost no epipolar disparity with respect to a fixed surface point, and the bowtie of the specular case exhibits an asymmetrical discrepancy with respect to the Lambertian bowtie.

Even though the basis of this thesis is to analyze frequency characteristics of the general motion of specularities by comparing them to the motion of fixed surface
points, there are some interesting facts to be noted for the motion of specularities with respect to the increase of amplitudes of undulations. Although not covered in the scope of this thesis, here I argue that in general, the behaviour of specularities as the curvature of the surface increases can be characterized by the following 1-D continuum. At one end of continuum lie the planar or quasi-planar surfaces. For planar surfaces, the specularities move entirely with the viewer rather than the surfaces. For quasi-planar surfaces, although specularities do exhibit a certain correspondence with the motion of the surface, a considerable amount of motion is noted in directions other than the direction of heading (sequence 1). At the other extreme end of the continuum, lie highly curved undulating surfaces. For such surfaces, the locus of the virtual points lie so close at the front or back of the surfaces, that specularities exhibit very little parallax with respect to a fixed surface point(specularities move seemingly entirely with the surface). The surfaces considered in this thesis (Sequence 2-4), which all coarsely describe a medium range of curvature lie between these two extremes. These surfaces exhibit a signature divergence from the direction of heading very similar to the Lambertian components, but specularities still move freely as exhibited by the fact that specular surfaces exhibit a larger range of velocities in the direction of heading than fixed surface points. I conjecture that there exists an optimal zone along this continuum. The surfaces in this zone are characterized as having the same signature motion as the Lambertian components in the direction of heading, yet also display the greatest epipolar discrepancy with respect to these components. I believe that the detection of this zone would require the statistical study of a greater range of amplitudes that is covered in the scope of this thesis. Furthermore, as the
Curvature of the surface is also related to the frequency of the undulating surface, variation of frequency of the undulations should prove useful in the discovery of this optimal zone.

Before considering the second case of the experiment, here is a brief summary of the findings for the type of scenes exhibiting a high degree of Lambertian parallax.

Even though the velocity and direction of the motion of specularities on curved surfaces is highly related to the geometry of the surface, the motion of the *global* field of specular markings on a randomly undulating surface were shown to display roughly the same signature motion in the direction of heading as the Lambertian components. Nonetheless, an asymmetrical discrepancy between the Lambertian and specular bowties was observed in the direction of heading, and was explained using the geometry of the scene and the ray-optics of specular motion.

In the next subsection case 2 is considered. These types of scene exhibit almost no Lambertian parallax. Following the previous argument, one would expect that in such cases, the specular and Lambertian bowtie to exhibit a more symmetrical discrepancy.

### 4.4.2 Case 2- Vertical wall (Little or no Lambertian parallax)

The second type of scene studied displays little or no Lambertian parallax. The scene is constructed in the following manner. The plane of the undulating surface is perpendicular to the line of sight of the camera. The camera is located at $z = 0$ and moves laterally with a velocity $V_x = -1$ units/frame. The surface is located at $z = -70$ units. The field of view is 60 degrees. The aim of this section is to confirm the results
obtained in the previous section, and to understand the discrepancy between the Lambertian and specular components in this type of scene. The nearly planar surface is no longer considered in the analysis of this case. Once again, we generate three sequences of undulating surfaces by solely increasing the amplitudes of the sinusoids from one sequence to the next. The generated surfaces can be seen in figure 4.9, and also in the videos wall1.mpeg - wall3.mpeg.

Here, once again the aim is to conduct an analysis of the projected power spectra, generated by the above-mentioned sequences.

Sequence 3 presents the highest degree of Lambertian parallax, and therefore the power spectrum of sequence 3 will be used as the typical Lambertian power spectrum. For this sequence, the maximum amplitude of the sinusoids is approximately 5 units. Using the fact that the axis of the surface is 70 units from the viewer, the maximum amount of Lambertian parallax represented by this type of surface is \( \frac{70+5}{70-5} \) or approximately 15 percent, which is quite insignificant when considered as the slopes of a bowtie. The projected power spectrum for the Lambertian component of sequence 3 can be seen in figure 4.10. The projected power spectra for the specular components of sequence 1-3 can be seen in figure 4.11.

The observations that were made in the previous case of the slanted surface are confirmed by looking at the projected power spectra for the specular components of motion for sequence 1-3. Once again, the global motion of the specular components seems to display a similar signature as the Lambertian components, as illustrated by the presence of strong motion plane(s) in the power spectra, projected perpendicularly to the direction of heading. The lack of the noise in the projected spectra once more
Figure 4.9: One frame of the wall-like undulating surface maximal amplitude (top-left) 2 units (top-right) 3.5 units (bottom) 5 units
Chapter 4: Experimental Results

Figure 4.10: The power spectrum for the Lambertian component of sequence 3 projected perpendicularly to the direction of motion. This is considered as the Lambertian bowtie exhibiting the highest degree of parallax.

Figure 4.11: The power spectra for the specular components projected perpendicularly to the direction of motion from sequence 1 (far left) to sequence 3 (far right)
seems to display the lack of sustained motion in directions other that the direction of heading.

As explained in the previous subsection, due to the simple fact that the locus of the virtual point lies either in front or in the back of actual undulating surface, specularities exhibit higher and lower epipolar velocities with respect to a fixed surface points in the direction of heading and the specular power spectra in turn exhibit a wider range of velocity than the Lambertian power spectrum when projected perpendicularly to this direction. This observation becomes quite evident when comparing the projected Lambertian power spectrum in figure 4.10, to the specular power spectra in figure 4.11. However, the range of depth in the scene is fairly small and therefore one would expect the discrepancy between the specular and Lambertian component to have a more symmetrical profile. Here is the reasoning behind the previous statement. Since the distance of the viewer to the scene far exceeds the amplitude of the sinusoids, there are no considerable depth differences between the closest and furthest points on the surface with respect to the viewer. This was obviously not the case for type of scenes considered in the previous subsection, where considerable depth differences stemmed from the slanted nature of the surface. One would therefore expect that, contrary to the type of scenes considered in the previous subsection, specular marking that are the furthest away from the viewer should exhibit roughly a similar epipolar parallax with respect to a neighboring fixed surface point, as the points that are the closest to the viewer. The latter point is quite evident by visually comparing the projected Lambertian and specular spectra for sequence 3 (figure 4.10 vs. 4.11 far-right). Using the same scan line method as in the previous case, I compare
Figure 4.12: Scan line of the projected power spectrum for \((f_x = 35)\). (Dotted): Lambertian (figure 4.10) (Solid): Specular: sequence 3 (figure 4.11)

the Lambertian and specular projected power spectra of sequence 3. The scan line plot for \((f_x = 35)\) in figure 4.12 clearly depicts the more symmetrical discrepancy between the two bowties. Once again, please note that the average power value has been removed for both power spectra. Furthermore, note that the bowtie appears at \((f_t = -22)\). The other peaks present near the origin and at \((f_t = 20)\) are the aliasing effects present on the actual projected power spectra. Please refer to [7] for a discussion on aliasing.

4.4.3 One final note

In this thesis, my aim was to investigate the behavior of the global field of specularities in the direction of heading. The power spectra were therefore repeatedly projected perpendicularly to this direction. It was also argued, that due to the lack of noticeable noise in the projection in the direction of motion, no significant motion is present in other directions. However, in some natural scenes having biased spatial derivatives in a given direction, significant amounts of energy may appear in that
direction with no correlation to any type of motion. Even though it is clear that this is not case for the undulating surfaces considered in this thesis, the power spectra were also projected in the direction parallel to the direction of motion for each of 7 sequences (4 slanted surfaces, 3 wall-like surfaces) studied in this thesis. As expected no energy plane was found in this direction. A typical noisy power projection is provided in figure 4.13.
Chapter 5

Conclusions - Future Work

The results in this thesis broaden the study of the motion of specularities. Previous work concentrated on the local motion of specular markings on simple surface geometries such as 3D ellipsoids. Here, I introduced a method for studying the global motion of specular markings on randomly undulating surfaces, as seen by a camera moving laterally with constant velocity relative to the surface. The analysis was based on the frequency domain techniques introduced by Langer and Mann in [7] for the motion of Lambertian objects in cluttered scenes.

The main original research contributions of this thesis are as follows:

- In chapter 3, the notion of a bowtie in the frequency domain, introduced by Langer and Mann [7] for optical snow type scenes was extended to smooth undulating Lambertian and specular surfaces using the idea of the depth map. Arguments were provided that for such surfaces, the signature bowtie should it exist, would have the appearance of a continuum of planes that I called a blurred bowtie.
• In chapter 4, evidence was provided as to the similarities and discrepancies between the motion of the specular and Lambertian components. It was empirically verified that in the case of the lateral motion and considered surface, the global field of specular components exhibit broadly the same spatio-temporal frequency signature (same direction and velocities) as the Lambertian components. However, it was also demonstrated, due to the simple fact that locus of the virtual points lie either in front or back of the undulating surface, the specular components exhibit a greater range of velocities that the Lambertian components in the direction of heading. It was also shown that the profile of this discrepancy depends on the geometry of the scene.

I believe that the structure of specular motion, exhibited frequently in such natural scenes as the reflection of landscapes on wavy water surfaces or the reflection of objects on shiny bumpy surfaces as seen by an observer moving parallel to the surface, lends itself quite well to this of type analysis. Moreover, using the above framework there is no need to separate the Lambertian and specular components of the motion. Although my work was conducted on separate Lambertian and specular layers, analogous results were obtained by conducting the analysis on the combined layering of these components.

I consider that this type of analysis is ripe for important applications such as motion and curvature estimation, which are often challenging in the spatial domain in presence of specularities. As noted by Waldon and Dyer [11] when studying the motion of a specular surfaces in the spatial domain, discontinuities of the specular motion occur near or at parabolic lines, making it impracticable to use such princi-
Chapter 5: Conclusions - Future Work

tives as an "optical flow field" in estimating the motion of the scene. The spectral analysis of specular motion from this thesis, combined with a frequency-domain motion estimation method similar to the one used by the authors in [7], may eventually help overcome some of the limitations of the spatial domain techniques. However, the scope of the work must be further expanded before such applications become feasible. Some of the possible extensions are discussed next.

The first obvious extension that can be envisioned is in the nature of the considered motion. The lateral motion considered in this thesis, imposes restrictions on the motion of specularities. Viewer motion in natural scenes often exhibits both a lateral and non-lateral component. Recently Langer and Mann [9] proposed an extension to the study of the optical snow framework, by considering more complicated non-lateral type motions. Considering a similar extension for the case of specularities, will allow for a much more realistic spectral analysis of the specular motion.

The second possible extension is in the nature of the reflected source. Here, a unique directional light source was placed far away from the surface, such that light rays are considered parallel when hitting the surface. In most natural scenes however, some of the reflected sources may actually lie within the scene (reflection of a landscape in a water surface). In such cases, the motion of the specularities is no longer solely caused by the change of angle between the surface point and the viewpoint (reflected angle), but also on the angle of incidence between the reflected source and the surface point. Replacing the unique light source by a generated environment map should be a useful way to depict a more realistic motion of the specularities within a scene.
Another possible extension lies in the type of considered surface. The undulating surfaces considered in the scope of this thesis, represent rigid bodies as seen by a camera moving laterally in a 3D scene. However, natural surfaces are much more fluid in nature. More realistic physical surface models corresponding to simple natural undulating surfaces such a simple water waves should prove to be a nice complement to the work presented in this thesis.
Bibliography


Appendix A

Appendix

A.1 Motion Plane Property

For any 3D function $I(x, y, t)$ with $x$ and $y$ in $0, 1, \ldots, N-1$ and $t \in 0, 1, \ldots, T-1$, the spatio-temporal Fourier transform is defined:

$$
\hat{I}(f_x, f_y, f_t) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \sum_{t=0}^{T-1} I(x, y) e^{-i\frac{2\pi}{N}(fx+fy)-i\frac{2\pi}{T}ft} \quad (A.1)
$$

If the image is translating with velocity $(v_x, v_y) = (dx/dt, dy/dt)$, then one can assume that

$$
I(x, y, t) = I(x + v_x dt, y + v_y dt, t + dt). \quad (A.2)
$$

From the image flow constraint equation [5], the velocity $(v_x, v_y)$ is constrained by:

$$
v_x \frac{\partial I}{\partial x} + v_y \frac{\partial I}{\partial y} + \frac{\partial I}{\partial t} = 0. \quad (A.3)
$$

To derive the motion plane property, one can use the derivative property of Fourier transforms:
\[ \int \frac{\partial I(u)}{\partial u} e^{-2\pi i f_u du} = -2\pi i \int I(u) e^{-2\pi i f_u du}. \]  

(A.4)

Treating \((v_x, v_y)\) as a constant, apply Eq. (A.4) to Eq. (A.3) which yields:

\[ -2\pi i (v_x f_x + v_y f_y + f_t) \hat{I}(f_x, f_y, f_t) = 0 \]  

(A.5)

Eq. (A.5) implies that whenever \(\hat{I}(f_x, f_y, f_t) \neq 0\).

\[ v_x f_x + v_y f_y + f_t = 0 \]  

(A.6)

That is, all motion energy lies on the plane of Eq. (A.6). This is the motion plane property.

### A.2 Calculation for Normal Velocities

The normal velocity is considered as the image velocity component in the direction of the spatial gradient.

First consider the unit vector in the direction of the gradient:

\[ \frac{1}{\sqrt{(\frac{\partial I}{\partial x})^2 + (\frac{\partial I}{\partial y})^2}} \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right). \]  

(A.7)

The normal velocity is the inner product of the unit gradient and the velocity vector and therefore equals:

\[ \|v_n\| = \frac{1}{\sqrt{(\frac{\partial I}{\partial x})^2 + (\frac{\partial I}{\partial y})^2}} \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) \cdot (v_x, v_y) \]  

(A.8)

Using the brightness constant relation, we substitute of the normal length of the velocity vector, and we obtain


\[ ||v_n|| = \frac{-\frac{\partial I}{\partial t}}{\sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}} \]  

(A.9)

Scaling the spatial gradient vector, with \( ||v_n|| \), we obtain the normal velocity vector:

\[ \vec{v}_n = \frac{-\frac{\partial I}{\partial t}}{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2} \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) \]  

(A.10)