AN S-BAND PARAMETRIC AMPLIFIER

by

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ABSTRACT

This thesis describes an S-Band parametric amplifier which was designed and subsequently built to study this type of amplifier at microwave frequencies. The signal, idling and pump were at the frequencies 2975, 7000, and 9975 megacycles respectively. The amplifier worked satisfactorily giving stable gains as high as 20 db. The bandwidth was, however, very low. Some theoretical investigations were performed on such topics as the stability and optimum load of the amplifier, and the negative conductance of parametric diodes. Some technical details about the construction, operation and measurements are also included.
I. **Introduction**

The project for this present thesis got underway at a time when interest was steadily growing for a new type of amplifier at microwave frequencies. This amplifier is now generally known as a parametric amplifier because its action depends on the time variation of one of its circuit parameters. Because of its high-gain-low-noise possibilities, relative simplicity and performance approaching that of the Maser, this type of amplifier is appealing to an ever increasing number of researchers. Its progress seems to be controlled mostly by the rapidity by which new and better parametric diodes can be produced. These are solid state variable capacitance semiconductor diodes and form the basic working element of the amplifier.

At the start of the project, most of the investigations were carried out at signal frequencies up to 1000 Mc. It was therefore decided to build an amplifier to operate around 3000 Mc (signal frequency). As will be seen later, two other frequencies are needed for the amplification process to take place, these are the idling and pump frequencies which were chosen to be around 7000 and 10,000 Mc respectively. The choice of all these frequencies was governed partly by theoretical requirements and partly by the availability of the equipment in our laboratory.

The first part of this thesis presents enough theory for a good understanding of the fundamentals underlying the parametric amplifier; the second part gives details of construction, operation and performance of the amplifier.
II. THEORY

2.1 Historical Introduction

Although a large amount of theoretical and practical work has been done recently towards the development of the parametric amplifier, the basic ideas underlying it are not new. It is generally believed that Lord Rayleigh was one of the first ones to have treated the subject mathematically in the Philosophical Magazine where he published a paper in 1883 under the title of "On Maintained Vibration". In this paper he makes an analysis of a phenomenon demonstrated at the time by Melde's experiment: When a string is given a periodic variation in tension at a given frequency, it will, under some condition, break into strong transverse oscillations at one half this frequency.

In 1916, R.V.L. Hartley made some theoretical calculations to find the effect that currents at different frequencies would have on one another if they were passed in a reactance modulator. He assumed the modulator to be a biased iron-cored inductance with a cubic characteristic for its flux, namely \( \phi = L_0 i(t) + \int i^3(t) \) where \( L_0 \) is the linear inductance.

The idea lay then half dormant until, with the advance of microwave technology, a growing need for high-gain-low-noise amplifier to cover these frequencies made it necessary to investigate new approaches to amplification techniques. The development of the variable capacitance semiconductor diode uncovered the potentialities of the microwave parametric amplifier which was now feasible. This sudden rise in
interest for a relatively unknown principle culminated with the publishing of a paper by Manley and Rowe (8) in 1956 giving a mathematical formulation for the general principle involved in parametric amplification. This paper was quickly followed by many others.

2.2 Simplified Mechanical Analogy

Before getting involved into the many formulae describing the parametric amplifier, it might prove interesting to acquire some sort of physical intuition of the basic principles underlying the negative resistance parametric amplifier. Consider, therefore, the following electro-mechanical system represented by Fig. 2.2.1.

![Fig. 2.2.1. A simple variable capacitance amplifier.](image)

In this diagram a parallel plate capacitor $C_c$ is provided with some mechanical means for varying the distance between its plates. Since the capacitance is inversely proportional to the distance between plates, it can therefore be varied at will. The capacitor is connected to a parallel resonant (or tank) circuit. Assume this combination to resonate at frequency $f_1$. Now, suppose a small sinusoidal signal of frequency $f_1$, placed across the tank, excites the circuit into oscillation; this will in turn cause the voltage which appears across $C_c$ to fluctuate...
at this frequency thus producing an equivalent charging and discharging of its plates. If we now wish to amplify this weak signal by using the mechanical capacitor, this can be done by recalling that the plates of the capacitor, when charged, will contain opposite charges which will try to pull the plates together. If the distance is then varied between plates in a squarewave fashion, i.e. by pulling them apart when the capacitor is fully charged (and hence when the attraction is greatest), and by bringing them back to their minimum separation when the plates are empty (when no opposition is offered to their motion), it is easily seen that maximum mechanical work will be accomplished for a given plate displacement. This work can only appear as electrical energy which will thus build up the original oscillation. It should be remembered that the force between plates is always one of attraction, no matter what the polarity is; and so, we can "pump" twice during each cycle. The result is illustrated in Fig. 2.2.2.

![Fig. 2.2.2. Voltage build up across variable capacitor $C_0$.](image)

By pumping at twice the natural frequency of the system, mechanical work can thus be converted into electrical energy while amplifying an electrical signal.
If we wish to pump at a frequency different from twice the signal frequency, this can still be done by introducing a second parallel resonant tank which will make the circuit resonate at a frequency which is equal to the difference between the pump and signal frequency. This third frequency is usually called the idling frequency.

To show the mechanism involved, let us take the simple case where both the signal and idling voltage, \(V(\omega_1)\) and \(V(\omega_2)\) respectively, have equal amplitude. The total voltage appearing across the variable capacitor will therefore be:

\[
V_c = V(\cos \omega_1 t + \cos \omega_2 t) = 2V \cos \left(\frac{\omega_1 + \omega_2}{2}\right)t \cos \left(\frac{\omega_1 - \omega_2}{2}\right)t
\]

The voltage now oscillates at frequency \(\frac{\omega_1 + \omega_2}{2}\), its amplitude being modulated from \(2V\) to 0 at a frequency \(|f_1 - f_2|\). It can be seen that by pumping at the frequency \(2 \times \frac{\omega_1 + \omega_2}{2}\) or \(f_1 + f_2\), energy can still be delivered into the system though at a different rate for each consecutive cycle. The phase of the pumping capacitor is of course important as it was in the first case. This is shown in Fig. 2.2.3.

The result is that power is now being delivered into the two tanks, and some pump power is therefore being wasted at the idling frequency; but this is the cost that one has to be prepared to pay to have amplification at the signal frequency. This is not a serious drawback, in general, for sufficient pump power is usually available.

At microwave frequencies it would be rather difficult if not impossible to vary the capacitance mechanically, and so we resort to a microwave voltage to do the job of varying the capacitance of a solid state device.
2.3 The Solid State Variable Capacitor

Briefly, a p-n junction diode is made up of two semiconductors which are connected together at a "junction". One of the semiconductors (the n-type) contains free electrons, the other (the p-type) contains free holes (or a deficiency of electrons). Practically no mobile charges are present in the small region containing the junction. This region is usually called the depletion layer. It is bounded on either side by regions that do contain mobile charges and so are conductive.

If now a positive bias voltage is applied, i.e. the positive electrode is connected to the p-type semiconductor, and the negative electrode to the n-type, then both electrons and holes will be pushed toward the depletion layer thus reducing its width. Similarly, reversing the field or applying a negative potential will attract the mobile carriers away from the junction, thus widening the width of the depletion layer. The
depletion layer can therefore be looked upon as a dielectric region separating the two plates of an equivalent parallel plate condenser having a capacitance which varies in accordance with the applied voltage. The applied voltage is limited on the positive side to a large forward current, and on the negative side to an avalanche current at a critical "breakdown" voltage (usually -9 to -10 volts).

The range in which the high frequency voltage can be swept without driving into either breakdown or large forward conduction is usually called the dynamic range of the diode.

Approximate formulae for the depletion layer capacitance are:

(1) for a linearly graded junction:

\[ C_{v1} = \frac{C_0}{\sqrt{1 - \frac{V}{\phi_1}}} \]

(2) for an abrupt junction:

\[ C_{v2} = \frac{C_0}{\sqrt{1 - \frac{V}{\phi_2}}} \]

where \( C_0 \) = capacitance under no bias voltage

\( V \) = applied voltage positive or negative - in volts

\( C_v \) = resulting capacitance

\( \phi_1 \) and \( \phi_2 \) are constants which can be determined experimentally.

A typical value for \( \phi_1 \) is 0.32 volts, and for \( \phi_2 \) is 0.40 volts. Fig. (2.3.1) shows a typical graph for \( C_{v2} \) as a function of the applied voltage.

It is important to realize that the variation of the capacitance is a result of an extremely small motion of electrons and holes toward
Fig. 2.3.1. Typical curve showing capacitance as a function of reverse bias voltage.

and away from the depletion layer without appreciable intermingling if the frequency of the applied voltage is kept high enough (above 10 Mc say). Any time lag between the applied voltage and the corresponding change in capacitance is therefore negligible and has not yet been observed for frequencies up to 100 KMc.

At microwave frequencies the characteristics of the variable capacitance diode as a circuit element are usually represented by the following lumped circuit:

![Circuit Diagram](image)

Fig.2.3.2. A variable capacitance diode as a circuit element.
Here $C_0$ is the capacitance under no bias voltage,

$C(v)$ is the change in capacitance due to an applied voltage $V$.

$R_s$ is the series or spreading resistance of the diode. It is due to the impedance of the bulk of the semiconductor material to the flow of majority carriers.

A justification for the above circuit is made in Appendix (1).

So far, one of the most important limitations on the use of the variable capacitance diode as a parametric amplifier element is the spreading resistance $R_s$ which affects both the gain and noise figure of the amplifier to a considerable extent as will be seen later.

2.4 The Manley and Rowe Relations

One of the most important papers which has come up lately in connection with the parametric amplifier is a paper published by Manley and Rowe in 1956(8). Its importance stems from the fact that some general relations are derived in it which form the basis for all parametric amplifier analyses. Evidently one cannot hope that these relations will be general enough to cover all parametric amplifiers and yet be so specific as to allow a complete description of each individual circuit. Nevertheless, they predict the mode of operation, stability and maximum gain that could be expected of each amplifier. Because of their importance, a brief description of the relations and their uses will be given:
Consider the circuit shown in Fig. (2.4.1). Here a non-linear capacitor $C_v$ couples together an infinite number of parallel branches, each consisting of a load, and a filter of zero impedance at the frequency shown in the rectangular box, but of infinite impedance at all other frequencies. The first two circuits also contain an oscillator at frequency $f_p$ and $f_s$ respectively. For definiteness the subscripts denote pump and signal oscillators but their frequencies are quite arbitrary. All other frequencies in the passive circuits are sums or differences of multiples of these two frequencies.

If the relation between the voltage across and the charge in the non-linear capacitor be represented by $v = f(q)$ where $f$ is any arbitrary function with the only restriction that it be single valued, and if the capacitor is non-dissipative (i.e. has no heat loss) then, without any further assumptions, Manley and Rowe show in their paper that these two power relations always hold:

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{m w_{m,n}}{mf_p + nf_s} = 0 \quad 2.4.1$$

$$\sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{n w_{m,n}}{mf_p + nf_s} = 0 \quad 2.4.2$$
where $\dot{W}_{m,n}$ represents power associated with the diode at frequency $mf_p + nf_s$ — positive values of $\dot{W}_{m,n}$ denoting power absorbed by the diode and thus introducing a positive resistance into the circuit; negative values of $\dot{W}_{m,n}$ denoting power given up and thus introducing a negative resistance into the circuit. These power relations are most general and are independent of the power level involved. Note that the first relation contains $\dot{W}_p (= \dot{W}_{1,0})$ but not $\dot{W}_s (= \dot{W}_{0,1})$ while the second relation contains $\dot{W}_s$ but not $\dot{W}_p$. As a result:

1. A flow of a sum frequency (m and n both positive) gives a positive contribution from both generators; in other words, the diode introduces a positive resistance into both generator circuits, and the power is transferred into the passive circuits. The network is therefore always stable.

2. A flow of a difference frequency (m and n of opposite signs) introduces a positive contribution from one generator (the pump) and a negative contribution into the other generator circuit, i.e. the pump power is absorbed by the diode and is returned partly into the passive circuits, the remainder being delivered to the signal circuit. The effect of the non-linear capacitor as far as the signal generator is concerned is then to see the total resistance of its associated circuit diminished by a certain amount. This effect can be shown to be equivalent to a negative resistance added to the circuit. If this negative resistance is equal to the total positive resistance, then the signal generator can be removed and the current at that same frequency will continue to flow. Under these circumstances the circuit is said to be unstable.
Applying the M. and R. relations to two simple but actual cases will suffice to illustrate the two important classes of parametric amplifiers which are now used. One is stable but offers only limited gain, while the other is capable of much higher gains but can become unstable.

**Class 1:** Here, the only 3 frequencies which are allowed to flow are \( f_p, f_s \) and \( f_p + f_s = f_+ \) corresponding to pump, signal and output frequencies respectively.

Substituting into the M. and R. relations we get:

\[
\frac{W_p}{f_p} + \frac{W_+}{f_+} = 0 \quad 2.4.3
\]

and

\[
\frac{W_s}{f_s} + \frac{W_+}{f_+} = 0 \quad 2.4.4
\]

It can be seen from these that \( W_+ \) has opposite sign to both \( W_p \) and \( W_s \). This means that if \( W_p \) and \( W_s \) are positive, then power is absorbed from both the pump and signal oscillators to be given up to the output circuit at frequency \( f_s + f_p \). If gain is defined as \( \frac{W_+}{W_s} \), then gain = \( \frac{W_+}{W_s} \) = \( \frac{f_+}{f_s} \). If \( f_+ \gg f_s \) then considerable gain can be achieved. Since both generators see positive resistances, this amplifier will be stable.

**Class 2:** The 3 frequencies permitted are now \( f_p, f_s \) and \( f_p - f_s = f_- \) corresponding to pump, signal and idling frequencies respectively. The M. and R. relations in this case give:

\[
\frac{W_p}{f_p} + \frac{W_-}{f_-} = 0 \quad 2.4.5
\]

\[
\frac{W_s}{f_s} - \frac{W_-}{f_-} = 0 \quad 2.4.6
\]
Equation (2.4.6) shows that $W_s$ and $W_p$ have the same sign while $W_p$ has a sign opposite to both from Equation (2.4.5). This means that if $W_p$ is positive, then power will be absorbed from the pump generator to be given up to both signal and idling circuits. Under these conditions the signal generator circuit experiences a negative resistance effect which can produce instability as was discussed before. The output is usually taken at the same frequency $f_s$ as that of the signal. It can be shown in actual circuits that the gain can be varied between zero and infinity (at which time the amplifier becomes unstable).

Because of the very large gain possible, the class 2 amplifier is usually preferred. Due to its potential instability however, it may prove sometimes difficult to use in practice. Unfortunately, the high gain has to be paid for in terms of very small bandwidth, in fact the product of gain times bandwidth is approximately constant. Note, as was mentioned previously, that power is being absorbed at the idling frequency whether it can be used or not.

2.5 **A Class 2 Parametric Amplifier**

**Part I. Simplified Theory**

A quantitative expression for gain will now be derived for a simplified class 2 amplifier. The result will be discussed and then applied to a more realistic model.

Suppose two tank circuits are resonating at frequencies $V_1$ and $V_2$ respectively and are connected across a variable capacitor

$$C_c = C_3 \sin(\omega_3 t + \theta_3).$$
If 2 filters are now introduced (Fig. 2.5.1), each offering zero impedance to a small band of frequencies containing the resonant frequency of their associated tank circuit, but infinite impedance to all others, then the variable capacitor will get currents only at frequencies close to or equal to \( \omega_1 \) and \( \omega_2 \).

Let the voltage across tank 1 be:

\[ V(\omega_1) = V_1 \sin (\omega_1 t + \phi_1) \]

and that across tank 2 be:

\[ V(\omega_2) = V_2 \sin (\omega_2 t + \phi_2) \]

where \( \omega_1 \) and \( \omega_2 \) are very close to \( \nu_1 \) and \( \nu_2 \) respectively and \( \omega_3 = \omega_1 + \omega_2 \).

The voltage developed across the variable capacitor is then:

\[ V_c = V_1 \sin (\omega_1 t + \phi_1) + V_2 \sin (\omega_2 t + \phi_2) \]

If we define currents flowing in \( C_c \) from B to A (Fig. 2.5.1) as positive and \( V_c \) as positive if the plate on A side is positive,
then
\[ i_c = -\frac{d}{dt} (C_c V_c). \]

By differentiating this expression it can be found that the current contains components at \((\omega_3 \pm \omega_1)\) and \((\omega_3 \pm \omega_2)\). Because of the filters again, only the components of current at \(\omega_3 - \omega_1 = \omega_2\) and \(\omega_3 - \omega_2 = \omega_1\) are allowed to flow. These are:

\[ i_c(\omega_1) = \frac{\omega_1 C_3}{2} V_2 \sin (\omega_1 t + \phi_3 - \phi_2) \]

and
\[ i_c(\omega_2) = \frac{\omega_2 C_3}{2} V_1 \sin (\omega_2 t + \phi_3 - \phi_1). \]

If these are put in complex notation (remembering that the actual current is the real of \(-j\) times the complex number) then:

\[ V(\omega_1) = V_1 e^{j(\omega_1 t + \phi_1)} \]

and
\[ i_c(\omega_1) = \frac{\omega_1 C_3}{2} V_2 e^{j(\omega_1 t + \phi_3 - \phi_2)} \]

\[
\text{Fig. 2.5.2. Admittance as seen by tank 1 at frequency (}\omega_1). \]

Now the equivalent admittance of the circuit at AB, as far as tank 1 is concerned, looking towards the variable capacitance is, for the frequency \(\omega_1\):

\[ Y(\omega_1) = \frac{i_c(\omega_1)}{-V_1(\omega_1)} = \frac{\omega_1 C_3}{2} \frac{V_2 e^{j(\omega_1 t + \phi_3 - \phi_2)}}{-V_1 e^{j(\omega_1 t + \phi_1)}} = -\frac{V_2}{V_1} \frac{\omega_1 C_3}{2} e^{j(\phi_3 - \phi_2 - \phi_1)} \quad (2.5.1) \]
Looking now from tank 2 we have for the frequency $\omega_2$:

\[
\frac{i_c(\omega_2)}{V(\omega_2)} = \frac{V_1}{V_2} \frac{\omega_2 C_2}{2} e^{j(\theta_3 - \theta_2 - \theta_1)}
\]

(2.5.2)

but

\[
\frac{i_c(\omega_2)}{V(\omega_2)} = Y_2 = G_{T2} + j(\omega_2 C_2 - \frac{1}{\omega_2 L_2})
\]

(2.5.3)

where $G_{T2}$ is the total conductance in tank 2 circuit.

Therefore

\[
\frac{V_2}{V_1} = \frac{\omega_2 C_3}{2} \frac{j(\theta_3 - \theta_2 - \theta_1)}{Y_2}
\]

(2.5.4)

Substituting the complex conjugate of (2.5.4) into (2.5.1) we get

\[
Y(\omega_1) = -\frac{\omega_1 \omega_2 C_3^2}{4 Y_2^*}
\]

(2.5.6)

In the same way

\[
Y(\omega_2) = -\frac{\omega_1 \omega_2 C_2^2}{4 Y_1^*}
\]

(2.5.7)

Fig. 2.5.3. Admittance as seen by $C_0$ at frequency $(\omega_1)$.

Note first that both $Y(\omega_1)$ and $Y(\omega_2)$ are negative admittances in their respective circuits; and that the greater $C_3$ or the smaller $Y_2^*$ the greater the negative admittance $Y(\omega_1)$ which is seen by tank 1. If $\sigma_c[Y(\omega_1)]$ is large enough to equal the total conductance of tank 1 then at resonance, oscillations will set in even though no source is connected to tank 1. The same of course is true for tank 2. The circuits are then unstable as is predicted by the Manley-Rowe relations.
It can easily be verified that the magnitudes of the two tank voltages have to satisfy \( \left( \frac{v_2}{v_1} \right)^2 = \frac{\omega_2 y_1}{\omega_1 y_2} \) so that the ratio of the powers given to each circuit is

\[
\frac{P_2}{P_1} = \frac{v_2^2 Y(\omega_2)}{v_1^2 Y(\omega_1)} = \frac{f_2}{f_1}
\]

This therefore agrees with the Manley-Rowe relations.

**Condition for Gain:**

So far nothing has been said about what developed the voltages \( v_1 \) and \( v_2 \) across the tanks. In the case when both tanks oscillate with no external source connected at either frequency \( \omega \), the voltages are obviously produced by the pumping capacitor alone. If, however, a signal source is introduced in tank 1 and the negative admittance is not too high, then the result is an amplifier.

Let us redraw Fig. (2.5.1) to include a signal source \( i_g \), a generator conductance \( G_g \), a load \( G_L \) and for simplicity, let us assume that \( \omega_1 = v_1 \) and \( \omega_2 = v_2 \) therefore the tank 1 admittance becomes \( G_1 \) and the negative admittance becomes a pure conductance:

\[
Y(\omega_1) = -G = -\frac{\omega_1}{\omega_2} \frac{G_g^2}{4 G_{T2}}
\]

This is shown in Fig. (2.5.4).

---

**Fig. 2.5.4. Total conductance at frequency \( \omega_1 \).**
The gain of this circuit is defined as total power absorbed by load during amplification divided by total available power from signal generator; or power gain

\[ A = \frac{V_2^2 G_L}{1^2/4G_g} = \frac{4 G_g G_L}{(G_g G_L - G)^2} \]  

(2.5.8)

where

\[ G_T = G_g + G_g + G_L. \]

From this equation it can be seen that for high amplification \( G \) should be made large enough to approach \( G_T \). Although very high gain is possible, it must be realized that this can only be obtained at the cost of greater instability and a very small bandwidth. The product of (gain)-bandwidth can be shown \(^{(5)}\) to be approximately equal to

\[ \frac{2}{Q_2} \frac{V_2}{V_1} \sqrt{\frac{G_g G_L}{G^2}} \]  

(2.5.9)

and since for high gain \( G \approx G_T \), then this product is at most \( \frac{1}{Q_2 V_1} \). The bandwidth can therefore be increased by increasing the ratio of idling to signal frequency or decreasing the Q of the idling circuit.

**Optimum Load**

If one is interested in the load that would give the highest gain, then 3 cases have to be considered:

**Case 1**  \( G < G_g + G \)

The best load is given by \( G_L = G_g + G_g - G \)

(2.5.10)

in which case the amplification is

\[ A_{\text{max}} = \frac{G_g}{G_g + G_g - G} \]  

(2.5.11)

see Fig. (2.5.5a).
Case 2 \[ G < G_1 \]

No amplification is possible - this follows from case 1.

Case 3 \[ G > G_1 + G_g \]

\( G_L \) should be greater than \( G - (G_1 + G_g) \) or else the amplifier oscillates.

This is shown in Fig. (2.5.5b).

**Stability**

The variation of gain with pump power for a given load is important since it determines both the stability at high gain and the noise introduced by the pump. This can be obtained by differentiating gain with respect to \( G \).

\[
\frac{dA}{dG} = \sqrt{\frac{A^3}{L G_g}} \quad (2.5.12)
\]

This means that for a given amplification \( A \), the greater the product \( G_L G_g \), the better the stability. It can be shown, see Appendix (2), that for a given amplification case 3 \( G > G_1 + G_g \) requires a greater value for \( G_L \) than case 1 and should therefore be preferred for greater stability if enough negative conductance is available.

![Fig. 2.5.5 Gain as a function of load (G_L) for a constant pump power (in arbitrary units).](image)
2.5 A Class 2 Parametric Amplifier

Part II. More Accurate Theory

At this stage a more realistic model can be used to describe a class 2 amplifier. The variable capacitor $C_o$ in Fig. (2.5.1) is therefore replaced by the lumped circuit representation of an actual diode, where now

$$C_o = C_o + C_3 (\sin \omega_3 t + \theta_3) \quad (2.5.13)$$

The remainder of the circuit is left otherwise unchanged. The resulting diagram is shown in Fig. (2.5.6).

Fig. 2.5.6. A Two-Tank Parametric Amplifier.

Here

$$Y_{11} = G_1 + j(\omega_1 C_1 - \frac{1}{\omega_1 L_1}) + G_L + G_\xi$$

and

$$Y_{22} = G_2 + j(\omega_2 C_2 - \frac{1}{\omega_2 L_2}) + G_\xi$$

where the notation is the same as before. The additional $G_\xi$ term stands for any load that might be put into the tank 2 circuit.

Proceeding in much the same way as before (see Appendix 3), we arrive at the result that the admittance at terminals a-b, looking towards the diode is
where

\[ Y_{a-b} = Y_{11} + \frac{j\omega \omega_2 C_2^2}{R_s (j\omega \omega_2 C_2^2 / 4Y^*_2) + 1} = Y_{11} + Y \quad (2.5.14) \]

where

\[ Y^*_2 = -j\omega_3 C_3 + \frac{Y^*_2}{R_s Y^*_2 + 1} \cdot \]

The gain is now:

\[ \frac{1}{Y_{11} + Y} \cdot \]

A comparison with the previous negative admittance Equation (2.5.6) shows that the general effect of \( R_s \) and \( C_0 \) is to depreciate its negative magnitude. It is apparent that if \( R_s \) and \( C_0 \) are large enough, they will be able to turn \( Y \) positive with the result that no amplification will be possible. There is in fact a point where \( Y \) is exactly equal to zero. This can be found to occur for \( \omega_1 C_0 R_s = .5 \). In other words, unless \( \omega_1 C_0 R_s < .5 \), no negative admittance is present no matter how large the pump power (or \( C_3 \)) is. Because of the importance of \( \omega_1 C_0 R_s \) in determining the quality of the diode, and also its noise performance, its inverse ratio \( \frac{1}{\omega_1 C_0 R_s} \), usually known as the \( Q \) of a capacitance, has been redefined as the quality factor of a diode.

Another term which is often used to describe a diode is the cut-off frequency \( f_c = \frac{1}{2\pi C_0 R_s} \) or the frequency at which the \( Q \) of the diode is 1. Since no amplification is possible for \( \omega_1 C_0 R_s \geq .5 \), the maximum possible signal frequency at which a diode can be used is \( f \leq \frac{f_c}{2} \). This does not mean, of course, that at such high frequency the amplifier will behave properly. A more realistic frequency would be given by \( f_{max} \leq \frac{f_c}{10} \). It should be noted here that some authors and manufacturers define their cut-off frequency or \( Q \) at the maximum negative bias (just short of breakdown) at which point the capacitance is a minimum.
Their quoted values for $Q$ are therefore always larger. Since best performance of the amplifier is achieved at zero or at a very small negative bias, it seems therefore more logical to define the characteristics of the diode at zero bias.

**Effect of Pump Power on Negative Conductance**

Let us consider equation (2.5.14) again. To simplify it, assume that tank 1 circuit is at resonance at frequency $\omega_1$ and that tank 2 circuit is tuned in such a way as to make $Y_2 = G_{T2}$ a real number. After rationalization we obtain:

$$Y_{a-b} = Y_{11} + Y = G_e + G_1 + G_L + \frac{(1 - \frac{\omega_1 \omega_2 C_2^2 R_s}{4G_{T2}}) - \frac{\omega_1 \omega_2 C_2^2}{4G_{T2}} + \omega_1 C_0 R_s^2}{(1 - \frac{\omega_1 \omega_2 C_2^2 R_s}{4G_{T2}} + \omega_1 C_0 R_s^2)}$$

$$+ j \frac{\omega_1 C_0}{(1 - \frac{\omega_1 \omega_2 C_2^2 R_s}{4G_{T2}} + \omega_1 C_0 R_s^2)}$$

(2.5.15)

The real part is the only one of interest since it is assumed that the imaginary part can be cancelled out by proper tuning. The real part of $Y$ is of the form:

$$y = \frac{x(xa - 1) + k^2 a}{(1 - xa) + k^2 a^2}$$

where

$$x = \frac{\omega_1 \omega_2 C_2^2}{4G_{T2}}$$

$$ka = \omega_1 C_0 R_s$$

$$a = R_s$$
In Fig. (2.5.7) $y_a$ which is a normalized conductance is plotted against $x_a$, a normalized pumping factor (because $C_3$ depends on the pump power).

It can be observed from this graph that for smaller values of $x_a$, namely in Region I, $y_a$ can be either positive, negative, or zero. It has a minimum of

$$y_a = -(1 + 2a^2 k^2) + 2ak \left(2 + a^2 k^2\right)^{1/2}$$

when

$$x_a = 1 + a^2 k^2 - ak \left(2 + a^2 k^2\right)^{1/2}.$$ 

Note: if $ka = 0.5$ then the minimum value of $y_a = 0$ and therefore no negative conductance is possible as was mentioned before.

For greater values of $x_a$, in Region II, $y_a$ is always negative and has a minimum negative value of

$$- \left[ (1 + 2a^2 k^2) + 2ak \left(2 + a^2 k^2\right)^{1/2} \right]$$

for

$$x_a = 1 + a^2 k^2 + ak \left(2 + a^2 k^2\right)^{1/2}.$$ 

There is also a gap of impossible conductance which has a width of

$$4ak \sqrt{2 + a^2 k^2}.$$ 

In Region I, there is a best value for the pump power for a given $\omega_1 C_0 R_s$ which would give the greatest negative conductance. Region II is usually not discussed. One could argue that for the high pump power needed to get into that region the approximation (2.5.13) does not hold any more. Let us consider then the fractional increase in the pumping factor required to get from point A to point B, this is:

$$\frac{2ak \left(2 + a^2 k^2\right)^{1/2}}{1 + a^2 k^2 - ak \left(2 + a^2 k^2\right)^{1/2}}$$

(2.5.16)
Fig. 2.5.7. Normalized conductance of a diode (ya) as a function of the running factor (xa). The points marked "A" represent minima in Region I while those marked "B", maxima in Region II.
For any good diode, \( ak \) is small compared to 1. Typical values are 0.05 or smaller. Equation (2.5.16) then becomes

\[
\frac{2ak \sqrt{2}}{1 - \sqrt{2}ak} = \frac{2.8 ak}{1 - 1.4 ak}
\]

which is small. We can therefore presume that if approximation (2.4.13) is good enough to describe the behaviour of a good diode, then working into Region II might be possible. Because of the greater values of negative conductance that can be obtained in this region, it might prove interesting to investigate further.

2.6 Noise Consideration

One of the great interests in the parametric amplifier lies in the possibility of large gains at relatively low noise. It ranks next to the Maser in low noise performance and surpasses by far in quality most conventional amplifiers in the microwave region. Because of its relative simplicity and low cost, it becomes a competitor to the Maser if the ultimate in low noise is not required. It must be noted that it achieves low noise even at room temperature, although the Maser needs liquid helium temperatures for its operation.

The most important noise sources are:

1. Thermal noise due to the series resistance of the diode \( R_s \) at frequency \( \omega_1 \).

2. Thermal noise due to the series resistance of the diode \( R_s \) at frequency \( \omega_2 \) which is converted into \( \omega_1 \) while being amplified.

3. Thermal noise from the antenna at \( \omega_1 \).

4. Thermal noise from the antenna at \( \omega_2 \).
(5) Thermal noise from the circuitry at \( \omega_1 \).

(6) Thermal noise from the circuitry at \( \omega_2 \).

(7) Shot noise.

(8) Pump noise, affecting the gain.

The first 6 of these are the most important, the 7th one is shown by Uhlir(11) to be negligible at microwave frequencies. The last one can be made very small by proper adjustment of the circuitry.

Comparing the parametric amplifier with the conventional type we note that the former has for added burden to accept extra noise generated at the idling frequency to which it must be tuned. This is then converted by the mixing action of the amplifier into a signal frequency noise over and above noise usually associated with the signal band.

If the signal is received only at the signal frequency, the receiver is then said to give "single-sideband reception". If full advantage is taken of the idling frequency band, by generating and receiving a special signal with proper phase at both signal and idling frequency, then the amplifier gives "double-sideband reception" at a considerably lower noise.

**Noise Figure**

One way to define the quality of a receiver is through its noise figure which is defined as:

\[
\frac{S_1}{N_1} \frac{S_0}{N_0} = \frac{1}{S_0} x \frac{N_0}{N_1} = \frac{1}{\text{Power gain}} x \frac{N_0}{K T_o B}
\]
where \( \frac{S_i}{N_i} \) = Signal-to-noise ratio available at input

\( \frac{S_o}{N_o} \) = Signal-to-noise ratio available at output

\( T_0 \) = Standard temperature of 290°K

\( B \) = Bandwidth of amplifier

\( K \) = Boltzmann's constant

Experimental results with parametric amplifiers show remarkable low noise: Uenohara, for instance, reports \(^{(14)}\) noise figure as low as 0.9 db. for double-sideband operation (equivalent to about 3.9 db. single-sideband) at room temperature, using special Gallium-Arsenide point contact diode of very high cut-off frequency (100 KMc or higher). This was obtained for a signal at 6 KMc with 16 db. gain.

![Diagram](https://via.placeholder.com/150)

**Fig.2.5.8.** A parametric amplifier with a circulator.
When very low noise is required, use is made of a circulator to isolate output from input thus avoiding noise originating in the output to be amplified and sent back to it. The circulator also allows better matching of generator, amplifier, and load, giving better stability to the system as a whole. A typical arrangement using a circulator is shown in Fig. (2.5.8).
III EXPERIMENTAL SET UP

3.1 The Parametric Amplifier

The parametric amplifier by itself is essentially a rectangular resonant cavity which must resonate at 3 required frequencies. It contains a parametric diode which is so positioned that the field surrounding it must be as large as possible for each of the three frequencies. This results in strong cavity-to-diode coupling. The cavity must also contain means to couple power in and out and to adjust its resonant frequencies.

Fig. (3.1.1) shows the physical features of the outside of the parametric amplifier used in this project. The cavity has been designed to resonate at around 3000, 7000, and 10,000 Mc. in the 3 modes $\text{TE}_{101}$, $\text{TE}_{103}$ and $\text{TE}_{501}$ respectively. With the diode in position, it was found to work best at the frequencies 2975, 7000 and 9975 Mc. These were the frequencies at which most measurements were made. They can be seen to obey the Manley-Rowe relation $f_p - f_s = f_s$. We note on the photograph the waveguide input to the cavity for the pump power (at 9975 Mc.). The signal input to the cavity (at 2975 Mc) is a loop and can be seen to terminate at the rear of the cavity as a type-N female connector. The output coupling is also a loop and is terminated by another type-N connector. It can be seen right in front of the picture. On the left side of the cavity is a large stub which helped considerably in adjusting the resonant frequencies. It could be interchanged for another loop serving as an auxiliary input or output. On top of the cavity there are 6 large screws which are used to adjust the resonant frequencies and
FIG. 3·1·1 - THE PARAMETRIC AMPLIFIER

FIG. 3·1·2 - THREE COUPLING LOOPS
cavity-to-diode coupling. They were designed so that a screw would penetrate at a point of high electric field for one frequency but low electric field for the other two frequencies. The small BNC female socket at the center forms part of the bias circuit of the parametric diode which is situated directly below it. Fig. (3.1.3) is a simplified cross-section through the center of the cavity, exposing the parametric diode and its position inside the cavity.

![Diagram](image)

Fig. 3.1.3. Cross-section through the parametric amplifier.

We note that the diode sits, in a small tube at the bottom of the cavity, on top of a spring which pushes it upwards against a small rod extending down from the center pin of the BNC socket. Both the height and vertical attitude of the diode can be adjusted by means of screws at the base of the BNC socket which is spring-loaded. A special joint allows free sidewise motion at the point of contact between rod and diode. A small brass plate surrounds the rod while making electrical contact with it. This plate is insulated from the top of the cavity.
It thus serves as a by-pass for the microwave power which would otherwise tend to escape into the biasing circuit.

The coupling into or out of the cavity at the signal frequency can be controlled at will by varying the insertion of the coupling loops, rotating the loops or changing their size. Fig. (3.1.2) shows 3 types of loops which were tried at the output. The smallest one was the easiest to adjust although the largest one offered the most stability.

3.2 The Parametric Diodes

The most important item of the amplifier is the parametric diode. It is also the most difficult to procure. Many diodes are now on the market but only a few can be used at the high frequencies which are used here. Most of these special high-Q diodes were still in the experimental stage at the time the experiment was performed.

3 diodes have been tried, these are:

(1) A diffused p-n junction diode made by Microwave Associates No. MA 460A - the cut-off frequency is quoted at 20 KMc for maximum bias.

(2) A point contact p-n junction diode - Hughes HPA 2810 (20 KMc cut-off frequency at zero bias).

(3) A diffused p-n junction diode - Microwave Associates No. 460E. (70 KMc cut-off frequency at maximum bias).

These parametric diodes are similar in external appearance to a microwave cartridge type mixer diode such as a 1N23 crystal.

Of the 3 diodes only the third one worked satisfactorily. Unfortunately, the first two were the only ones available at the beginning of the experiment. Consequently, much time and effort was expended in vain trying to get the amplifier working properly until finally, a written
confirmation of the inadequacy of each of these diodes was received from the manufacturers.

3.3 Test Equipment

A block diagram describing the basic test equipment is shown in Fig. (3.3.1) while Fig. (3.3.2) displays the physical arrangement of the equipment. The 4 basic circuits are: (1) the pump power circuit, (2) the signal generator circuit, (3) the signal output circuit, (4) the bias circuit. A description of each circuit follows.

(1) Pump Power Circuit

The pump power is furnished by a 2K39 klystron which is powered by a P.R.D. type-801 power supply. The output of the klystron then passes through a uniline to decrease the effect of a change in the impedance as seen by the klystron. Next, the pump power goes through an absorption wavemeter, a precision attenuator to control the power level, a directional coupler to monitor it, and finally through a slide screw tuner before entering the amplifier.

(2) Signal Input Circuit

The signal source (at 2975 Mc) is supplied by a RK707B Raytheon klystron powered by an h.p.-type 715 power supply. The signal is fed through a calibrated attenuator, an absorption wavemeter, a coaxial cable, and a double stub tuner into the parametric amplifier. When measurements had to be made at 7 KMc, (the idling frequency), then an X-26B Varian klystron with a P.R.D. 801 power supply was used instead.

(3) Signal Output Circuit

The signal output circuit starting from the amplifier consists of a double stub tuner, a low pass filter with a 4100 Mc cut-off frequency
FIG. 331  BLOCK DIAGRAM OF TEST SET UP
to remove both idling and pump frequency from the output, a coaxial type wavemeter, a crystal detector inside a crystal mount, and finally an indicating instrument which could be either a C.R.O., an electrometer, a microammeter, a standing wave amplifier or any other reading instrument.

(4) **Bias Circuit**

The bias circuit consists simply of a 45, 22-1/2 V. battery, a rheostat box, a voltmeter, a microammeter and a connecting cable which fits into a BNC socket on top of the parametric amplifier.

Most of these components can be seen in Fig. (3.3.2).
IV EXPERIMENTAL TECHNIQUE

4.1 Basic Requirements

There are certain basic requirements which must be satisfied before amplification can be produced. The most important ones are as follows:

(1, 2, and 3) The cavity (with diode in) should resonate at the pump, idling and signal frequency simultaneously.

(4, 5, and 6) There should be good coupling between cavity and diode at the pump, idling and signal frequency simultaneously.

(7) There should be enough pump power reaching the diode; and if high gain is also required we further have, that

(8) The Q of the idling circuit should be high enough,

(9) The load at the signal should not be too large.

The first six requirements are very difficult to achieve simultaneously. The difficulty arises primarily from the interdependence of the adjustments. In other words, adjustment made at a particular frequency affects those made at another frequency. Success can be achieved, however, if a methodical approach is used together with a fair amount of patience.

The different techniques that were used in this experiment to obtain the basic requirements and then amplification are described in the remainder of this chapter.
4.2 Cavity Resonance

The resonant frequencies of a cavity may be obtained by any one of a great number of well established methods which are described in most microwave handbooks. It seems, therefore, unnecessary to dwell too long on these.

Two of the methods that were used are:

1. To observe the frequency that gives a minimum Voltage Standing Wave Ratio in the coaxial line (or waveguide) which has the cavity as its termination.

2. To sweep the frequency of the klystron around the resonant frequency of the cavity in the same coaxial line or waveguide and to observe the resultant absorption of power on a cathode ray oscilloscope.

The cavity with the parametric diode in was then adjusted to resonate at the 3 required frequencies, namely 9975, 7000 and 2975 Mc respectively.

4.3 Cavity Resonance and Coupling (Static Adjustment)

The cavity had then to be adjusted to allow good coupling between the diode and itself at the above 3 frequencies.

The technique that gave best results made use of the fact that the small d.c. current that flows through the diode is a measure of the field existing near the diode. The highest current thus occurs when the field inside the cavity is greatest. This implies:

1. That the cavity resonates at this associated frequency;
2. That the coupling between cavity and diode is good at this same frequency.
The cavity screws and the position of the crystal were therefore adjusted until a maximum d.c. current was recorded at the crystal. This was done for the 3 frequencies involved. If the d.c. current was too small to be detected directly, a squarewave modulation was applied to the signal which was then easily amplified and detected.

4.4 Power Requirements

It is very important that enough pump power reach the diode. The amount needed varies with the diode being used but it is usually between 50 and 500 mw.

In order to estimate the amount of power that was available for our diode, the rectified current it produced was calibrated against the incident power it absorbed. This calibration was made using a bolometer and power bridge. If the power level was too small, the coupling between diode and cavity, or cavity and waveguide, had to be changed.

4.5 Cavity Resonance and Coupling (Dynamic Adjustment)

The static adjustment (see 4.3) by itself is not accurate enough for amplification. The reason is that, with the pump power on, a d.c. voltage is produced across the diode which changes its effective capacitance (sec. 2.3). This change in turn affects the resonant frequencies of the cavity. For high pump power level, this variation could be as much as a few megacycles.

To avoid this, once the static adjustment had been made, the following dynamic adjustment procedure was used:
The pump and idling frequency klystrons were turned on. A small amount of power at the signal frequency was now detected at the output of the amplifier due to the mixing action of the diode. The cavity screws and stubs, and the diode position were then adjusted until maximum power was obtained. If a good low pass filter was used at the output, the signal frequency was found to obey the relation
\[ f_s = f_p - f_. \]

With the adjustments for maximum signal power completed, requirements 1 to 6 (sec. 4.1) were met since they are necessary for a mixing action. At this point the first 7 basic requirements were satisfied. Amplification was then the next goal.

4.6 Amplification Technique

After many trials, this method was found to be the most rewarding:

The pump power (squarewave modulated) and the signal source (not modulated) were both turned on. The output was then detected by an a.c. amplifier. This amplified output was the increase in signal output due to the pumping action alone. Very careful adjustments increased this output until the amplifier started to oscillate, i.e. power was detected at the output with signal input turned off. This oscillation frequency was then measured. If it was found to be monocromatic and equal to the required signal frequency, it was then possible to proceed. If not, then a different screw-stub configuration for the cavity had to be tried and all procedures described in this chapter had to be repeated. The reason for this is explained in App. (4).
The modulation was then reversed, i.e. the pump power was not modulated while the signal power was squarewave modulated. The output was then the actual amplified signal. With final adjustments made for greatest power output, gains greater than unity were required to ascertain that real amplification was produced. Here is the reason: It was seen before that the pump power produces a d.c. bias voltage in the diode which changes the resonant frequency of the cavity. For certain adjustments the pump power will tune or untune the cavity with respect to the signal frequency as it is turned on, thus producing switching rather than amplification.

Leaving the pump power on (with no modulation), the signal power was then swept in frequency by applying a sawtooth modulation to the reflector voltage of the signal klystron. The resulting pattern at the output of the amplifier, as observed on the C.R.O. was as in Fig. (4.6.1).

![Figure 4.6.1](image)

*Fig. 4.6.1. Power output vs. frequency as seen on the C.R.O.*

The signal was therefore amplified most strongly at the frequency corresponding to point A in this figure. The final adjustments were
then made and the correct bias applied to the diode until the amplification was increased to the required level.

4.7 Gain Measurements

We here define gain as:

\[
\text{gain} = \frac{\text{power output under amplification}}{\text{maximum power available from signal source}}
\]

In order to obtain the maximum power available, a crystal detector or bolometer was made to terminate the coaxial line of the signal source instead of the amplifier, and the crystal mount stub was adjusted for maximum signal. For a given power level (attenuator setting), this signal was recorded as a voltage on the C.R.O., on the electrometer or directly as power on a power bridge. To obtain the power output under amplification corresponding to the same signal level, the amplifier was reconnected into the signal circuit and the crystal (or bolometer) which had been used before in the same mount now formed the load at the output of the amplifier. The amplified signal was then detected on the same instrument as before.

When the crystal was used as a detector, the ratio of these two voltages gave the gain directly since at the very low power level that was used, the crystal d.c. voltage is proportional to the incident microwave power.

4.8 Bandwidth Measurement

The simplest method used to measure the bandwidth of the amplifier was to sweep the signal frequency by applying a sawtooth voltage to the
reflector voltage of the klystron and to observe the pattern at the output of the amplifier on a C.R.O. An absorption wavemeter which was inserted in the circuit was used to trace a frequency reference point along the pattern. The bandwidth was obtained by recording the frequencies which corresponded to the half power points.
5.1 Amplification

Amplification has been obtained and then measured for many different combinations of pump power, signal level, loops, loop position, diode attitude and cavity screw-stub geometric configuration.

5.1.1 Maximum Gain

The highest stable gain recorded (as defined in sec. 4.7) was approximately 100 (or 20 db). More gain could be achieved by increasing the pump power, but oscillation would set in at random intervals. This was probably caused by inadequate pump power stability.

5.1.2 Pump Power Requirement

The pump power required for a given gain was controlled by many factors such as load, bias voltage, cavity screw adjustment etc... For the particular diode used (MA-460E) a power level as high as 350 mw was sometimes needed. This high demand is predicted by the manufacturer and is produced by the high series inductance which characterizes this type of diode. Other series such as the MA-450 - now in production - have very low series inductance and require therefore less pump power. These diodes were, however, not available due to their very high costs and long delivery time.

5.1.3 Bias Requirement

The bias required for best performance of the amplifier was usually within the range from -.35 to -.65 V. As was mentioned earlier, the pump power produces a d.c. voltage across the diode which changes the
mean capacitance of the diode and hence the resonant frequencies of the cavity. If the highest gain is wanted for a given pump power, this change in capacitance has to be offset by an increase in external negative bias voltage. This is illustrated in Fig. (5.1.3) where the external bias voltage necessary for maximum gain is plotted against pump power.

5.1.4 Stability Versus Load

In order to verify the theory that stability should increase with load (2.5 Part I "Stability"), gain was measured for different pump powers. This was done for 2 insertions of the output loop. This is equivalent to 2 output loads. The result is plotted in Fig. (5.1.4). It can be observed that:

1. A greater load allows a higher maximum gain;
2. For a given amplification, the greater of the two loads produces a smaller slope and therefore greater stability up to a certain point. This can be seen by drawing a horizontal line corresponding to a given gain, and comparing the 2 slopes at the intersections of the two curves with that line.

Both these results are predicted by the theory. These results therefore seem to point out that greater load indicates greater stability and, therefore, probably lower noise. It has to be noted, however, that this has to be paid for by a higher pump power required for a given gain. The magnitude of the load permitted will also be limited by the Q of the diode especially when high gains are required.

5.1.5 Saturation Effect

Saturation effect which is present in most types of amplifier can also be expected in the parametric amplifier. In other words, if the
Fig. 3.1.3. External bias necessary for maximum gain as a function of pump power.
Fig. 5.1.4. Gain as a function of pump power for two different loads.
signal to be amplified approaches the level of the pump power, the gain will go down. This is probably caused by the coupling from the signal back into the pump. The analogy in mechanical terms would be that the driver is being affected by the driven mechanism.

To study this saturation effect, gain was obtained for different signal power levels using a pump power level that would amplify just short of oscillation. The result is plotted in Fig. (5.1.5). It is seen that amplification is dependent to a large extent on signal power level, and that for high amplification the signal level should be kept below 0.1 μwatt.

It might seem surprising at first that the signal has to be kept at a level so much lower than the pump power (≈ 150 mw). But it must be realized that only a very small fraction of this power is involved in the parametric amplification process. For example, a large part of the power is used up as $I^2 R$ loss in the semi-conductor itself and in the cartridge, and another part is used up in the formation of higher harmonics which can exist in the cavity.

5.2 Bandwidth

The bandwidth, as theory predicts, was very small. In fact, for high gains, the bandwidth was so small as to be very difficult to measure. A qualitative trend rather than exact measurements was therefore obtained. The bandwidth was estimated on the C.R.O. (sec. 5.3) and it was observed that it increased continuously as the gain was decreased (by lowering the pump power) until it reached a maximum of 2 MHz when the gain approached a minimum value. A much higher bandwidth should be
expected for a better (higher Q) parametric diode as is discussed in
the conclusion.

5.3 Observation on the Cathode Ray Oscilloscope

The adjustment of the amplifier to give best performance has been
greatly facilitated by arranging to obtain visual observation of the
amplification. Fig. (5.3) represents four stages of amplification
as seen on the C.R.O. The set up was arranged as follows:

In Fig. (5.3a, b, and c), the pump power was squarewave modulated
at about 1000 c.p.s. while the reflector voltage of the signal klystron
was sawtooth modulated at about 500 c.p.s. The C.R.O. input was taken
from the output of the amplifier. By adjusting the two modulation
frequencies, it was possible to obtain two curves on the same display.
The lower one represents the output of the amplifier for the pump power
off condition, in other words for no amplification; the upper one re-
presents the amplifier output under amplification (pump power on). We
can therefore learn at a glance:

(1) The effect the pump power has on the amplifier,

(2) the difference in output power between the pump power off
and on condition,

(3) the bandwidth of the amplifier under these same conditions.

Fig. (5.3a) represents the amplifier under very small amplifica-
tion. We note that signal out with pump on is about five times the
signal out with pump off at their maximum amplitudes. It may seem
strange at first to observe that the two resonance peaks do not occur
at exactly the same frequency, but on second thought it will be realized that when the pump power is on, it will produce a d.c. voltage in the parametric diode which will change its effective capacitance and hence the resonant frequency of the cavity.

Fig. (5.3b and c) are of the same type as Fig. (5.3a) except that now we have a much higher amplification; in fact, the output for the pump power off condition appears almost as a straight line when compared to the amplification curve. The amplitude scales of these pictures are not the same. For instance, the scale of Fig. (5.3b) is approximately 25 times that of Fig. (5.3a).

Fig. (5.3d) represents the amplifier output under high gain (around 17 db). Here the pump power was given no modulation and hence only the amplified signal appears.

Since the frequency scale is approximately the same on each of these four photographs, they demonstrate the progressive decrease in bandwidth as the amplification is increased.

5.4 Frequency Converter

The parametric amplifier could be used advantageously as a frequency converter. For example, when the frequency of the signal was 7 KMc and that of the pump 9975, the output was at 2975 KMc with a gain of about 16 db, if we define gain as

\[
\text{gain} = \frac{\text{output power at 2975 KMc}}{\text{maximum input power available at 7 KMc}}
\]

Here again the bandwidth was very small at such gains.
5.5 Oscillator

The amplifier was also used as an oscillator. By pumping power at 9975 Mc it was possible to generate power at two other frequencies, one at 2975 the other at 7000 KMc.

5.6 Motorboating

Under certain adjustments, a strong pump power induced motorboating. This was probably caused by the fact that a strong pump power produced a d.c. voltage across the diode strong enough to detune the cavity thus stopping the amplification process.
VI CONCLUSION

The feasibility of a high gain S-Band parametric amplifier has been demonstrated by the laboratory work that has been carried out for the present thesis.

The parametric amplifier which has been designed and built for this project has shown high gain possibilities: stable amplification as high as 20 db has been produced.

At high gain, the bandwidth was very small. This is predicted by theory, e.g. for a gain of 20 db using an idling circuit with a Q of 1000, theory predicts a bandwidth of about 0.02% (see sec. 2.5, Part I). At 3000 Mc this would give a bandwidth of at most 600 Kc which is indeed very small. It is believed, however, that a higher bandwidth can be obtained by using a better diode with a higher Q. This would permit use of a lower Q for the idling circuit while keeping the same gain; this in turn would increase the bandwidth.

Noise figure measurements were not obtained. These are some of the reasons: First, noise measurements usually require either an exact knowledge of the bandwidth of the amplifier, or else the use of a suitable noise generator. Noise figure determination for parametric amplifiers, however, makes use of noise sources almost invariably because it is difficult in this case to dispense with them. For example, many technical difficulties are encountered in measuring the very small bandwidth involved accurately enough to be used in noise figure calculations. Unfortunately, no suitable noise generator was available for this project.
Second, no circulator was available, consequently the possibilities of the amplifier would not have been fully exploited, and the noise figure would have been unnecessarily high. Third, since noise figure is very dependent on resonant cavity adjustments, coupling, matching etc., direct-reading-noise-figure-meters are commonly used today to minimize noise figure by observing the meter continuously while making the adjustments. Here again, with no means to optimize the noise performance, a noise figure compared to that obtained by other workers would not have done justice to the amplifier.

Some theoretical investigations on optimum load condition, the stability of the amplifier, and the effect of pump power on the negative conductance of parametric diodes, were carried out, and some interesting results were obtained: It was found, for instance, that to obtain a maximum gain, there exists an optimum load at the signal frequency only if the negative conductance is smaller than the sum of all the positive conductances at the signal frequency (the load conductance excluded); that for a given amplification, the greater the product between load and generator conductance, the greater the stability of the amplifier with respect to variation in pump power - this is particularly important at high gains where adjustments are usually critical; that there is a theoretical possibility to obtain higher negative conductance than that usually expected out of a diode.

Although the amplifier as built was working satisfactorily, four improvements might be suggested:

(1) The use of higher quality diodes with cut-off frequencies of 100 KMc or better, which are now available.

(2) The use of a circulator to isolate output from input thus increasing the stability and noise performance as predicted by theory.
(3) The design of a cavity of a different type allowing better independent adjustment at each frequency.

(4) The elimination of the mixing cavity and in its place the use of a special mixer mount to do the mixing job of the cavity; and, the use of 3 separate cavities, each tuned to only one frequency, to serve as the 3 resonant circuits.
APPENDIX 1

A Variable Capacitance Diode as a Circuit Element

Let us take the general relation between the capacitance of the diode and the applied bias voltage to be

\[ C_v = \frac{C_o}{\sqrt{1 - \frac{v}{\phi}}} = C_o \left(1 - \frac{v}{\phi}\right)^{-1/n} \]

Suppose now that we apply a small incremental voltage \( \Delta v \);

therefore \( C_{v+\Delta v} = C_o \left(1 - \frac{v + \Delta v}{\phi}\right)^{-1/n} = C_o \left(1 - \frac{v}{\phi} - \frac{\Delta v}{\phi}\right)^{-1/n} \)

\[ = C_o \left(1 - \frac{\Delta v}{\phi - v}\right)^{-1/n} \left[1 - \frac{\Delta v}{\phi + v}\right]^{-1/n} \]

For a negative voltage replace \( v \) by \( -v \) so that

\[ C_{-v+\Delta v} = C_o \left(1 + \frac{\Delta v}{\phi - v}\right)^{-1/n} \left[1 - \frac{\Delta v}{\phi + v}\right]^{-1/n} \]

if \( \Delta v \ll \phi + v \)

we can make the approximation

\[ C_{-v+\Delta v} = C_o \left(1 + \frac{\Delta v}{\phi + v}\right)^{-1/n} \left[1 + \frac{\Delta v}{n(\phi + v)} + \cdots \right] \]

and so \( C_{-v+\Delta v} \approx C_o' + C_o' \frac{\Delta v}{n(\phi + v)} \)

where \( C_o' = C_o \left(1 + \frac{\Delta v}{\phi + v}\right)^{-1/n} \)

which brings out the linearity in \( \Delta v \).
APPENDIX 2

To Show That for Equal Amplification, Case 3 Requires a Greater Value of $G_r$ than Case 1

Assume for case 1 the value of $G_L$ to give maximum amplification.

Let the amplification be denoted by $A$, and the subscript 1 and 3 refer to cases 1 and 3 respectively.

To avoid confusion let us redefine all our terms as follows:

- $G_L \rightarrow x$
- $G_1 \rightarrow a$
- $G_1 \rightarrow b$
- $G \rightarrow G_1$ where $i = 1$ or 3

Then let

$$A = \frac{a}{a + b - G_1} = \frac{4ax_3}{(a + b + x_3 - G_3)^2}$$

(see Eq. 2.5.8)

and

$$x_3 = a + b - G_1$$

(see Eq. 2.5.11)

Solving for $x_3$ we get:

$$x_3 = \left[ a + b + G_3 - 2G_1 \right] \pm \left\{ \left[ 2G_1 - G_3 - (a + b) \right]^2 - (a + b - G_3)^2 \right\}^{1/2}$$

and

$$x_1 = a + b - G_1$$

(see Eq. 2.5.10)

Therefore

$$\frac{x_3}{x_1} = 1 + \frac{G_3 - G_1}{a + b - G_1} + \left\{ \left[ 1 + \frac{G_3 - G_1}{a + b - G_1} \right]^2 - \left( \frac{a + b - G_3}{a + b - G_1} \right)^2 \right\}^{1/2}$$

and since

$$G_3 > G_1$$

therefore

$$x_3 > x_1$$

**Note:** Only the positive sign is permitted for $x_3$ or else the amplifier would oscillate.
APPENDIX 3

Negative Admittance and Gain of a Parametric Diode

Referring to Fig. (2.5.6) the total voltage seen by \( C_3(t) \) is:

\[ V_{c3} = V_1 \sin (\omega_1 t + \phi_1) + V_2 \sin (\omega_2 t + \phi_2) = \text{Real of } (-jV_1 e^{j(\omega_1 t + \phi_1)} - jV_2 e^{j(\omega_2 t + \phi_2)}) \]

Using the same definition for current as before, i.e. taking the positive current flowing from \( d \) to \( c \) as positive, then:

\[ i_{c3} = \frac{-d}{dt} (c_3 \ V_{c3}) \]

The two components are therefore:

\[ i_{c3}(\omega_1) = -j \frac{\omega_1 C_3}{2} V_2 e^{j(\phi_3 - \phi_2)} e^{j\omega_1 t} \]

and

\[ i_{c3}(\omega_2) = -j \frac{\omega_2 C_3}{2} V_1 e^{j(\phi_3 - \phi_2)} e^{j\omega_2 t} \]

The negative conductance at \( \omega_1 \) due to \( C_3 \) alone is therefore:

\[ Y(\omega_1) = \frac{\text{complex current } i_{c3}(\omega_1) - \omega_1 C_3 V_2 e^{j(\phi_3 - \phi_2 - \phi_1)}}{-\text{complex voltage } V(\omega_1) = \frac{\omega_2}{2} V_1 e^{j(\phi_3 - \phi_2 - \phi_1)}} \quad (A.3.1) \]

In order to find \( \frac{V_2}{V_1} \) in terms of tank 2 parameters: the circuit at frequency \( \omega_2 \) can now be redrawn as in Fig. (A.3).
From this Fig. we have:

\[ i_{22} = \frac{V_2}{R_s + \frac{1}{Y_{22}}} = \frac{V_2 Y_{22}}{R_s Y_{22} + 1} \]

\[ i_{c_0} = V_2 (j \omega_2 C_0) \]

but

\[ i_{c_3} = i_{22} + i_{c_0} = \frac{V_2 Y_{22}}{R_s Y_{22} + 1} + V_2 j\omega_2 C_0 \]

and so

\[ \frac{i_{c_3}}{V_2} = j\omega_2 C_0 + \frac{Y_{22}}{R_s Y_{22} + 1} = Y_2 \]

where \( Y_2 \) is now the admittance looking from c-d towards \( Y_{22} \)

\[ \text{but} \quad \frac{i_{c_3}}{V_2} = \frac{-j\omega_2 c_3}{2} \frac{V_1 e^{j(\phi_3-\phi_1)} e^{j\omega_2 t}}{-j V_2 e^{j\phi_2} e^{j\omega_2 t}} = \frac{V_1}{V_2} \frac{\omega_2 c_3}{2} e^{j(\phi_3-\phi_2-\phi_1)} = Y_2. \quad (A.3.2) \]

Taking the complex conjugate of Eq. (A.3.2) and substituting in Eq. A.3.1 we get:

\[ Y(\omega_1) = \frac{-\omega_2 c_3^2}{4Y_2^*} \quad (A.3.3) \]

where

\[ Y_2^* = -j\omega_2 C_0 + \frac{Y_{22}}{R_s Y_{22} + 1}. \]

Referring again to Fig. (2.5.6), the total admittance looking from a-b towards the diode is therefore:

\[ Y_{a-b} = Y_{11} + \frac{j\omega_1 c_0 - \frac{\omega_1^2 c_3^2}{4Y_2^*}}{R_s (j\omega_1 c_0 - \frac{\omega_1^2 c_3^2}{4Y_2^*}) + 1} = Y_{11} + Y \]

To obtain the gain-maximum power available from load is:

\[ \frac{1/2}{4G_g} \]
Total power absorbed by load under amplification is:

\[ v^2 G_L = \left| \frac{i_L}{y_{11} + y} \right|^2 G_L \]

The gain is therefore:

\[ \frac{4 G_L G_e}{|y_{11} + y|^2} \]
APPENDIX 4

Need for the Amplifier to Fall into Oscillation at the Proper Signal Frequency

Suppose that for a sufficiently large pump power the amplifier, besides breaking into oscillation at the correct idling and signal frequency, will also oscillate at a spurious frequency close enough to that of the signal so as to pass through the output filter. If the Q of the circuit associated with that frequency is higher than that of the signal frequency, then the amplifier will break into oscillation before high amplification can be achieved. This can be seen from Fig. (A.4.1).

Here the vertical line A represents the point at which oscillation would normally set in at the signal frequency. B is the point at which the spurious oscillation sets in. As a result, the maximum amplification that can be obtained before oscillation sets in is represented by amplitude CB. This condition occurred in the experiment. The only effective way found to remove this unwanted frequency was through a complete change in configuration of the adjusting screws and stub.
REFERENCES


