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October, 2017

A thesis submitted to McGill University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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ACKNOWLEDGMENTS

I would like to extend my sincerest thanks and appreciation to those patient souls who helped me accomplish this study.

First of all, I would like to thank my supervisor Prof. Jagdish Handa for his continuous support and encouragement throughout my doctoral study. At many stages in the course of this research project I benefited from his advice, particularly so when exploring new ideas. His positive outlook and confidence in my research inspired me and gave me confidence. His careful editing contributed enormously to the production of this thesis.

My thanks are due to my co-supervisor, Prof. Francisco Ruge-Murcia, for his great insights, guidance, strictness, encouragements, and detailed comments during this study. He spent endless hours proofreading my research papers and giving me valuable comments which always resulted in improved versions of documents. Without his intellectual inspiration, technical advices, and constant quality requirements, this thesis would not have been as it is.

Besides my supervisors, I am deeply indebted to Prof. Markus Poschke. I have greatly benefited from all his comments and the time he spent with my ideas and thoughts. I am grateful to Prof. Francisco Alvarez-Cuadrado and Prof. Robert Cairns. They have been very kind enough to extend their help at various phases of this research, whenever I approached them. My sincere thanks go to Prof. Theodore Papageorgiou and Prof. Laura Lasio for their help with job market preparation. Special thanks are also due to Angela Fotopoulos, Lisa Stevenson, and Judy Dear for their administrative assistance. I would also like to thank the Department of Economics, McGill University, for generously providing me the scholarships.
Last but not least, I must express my profound gratitude to my mother Hongqin Cao and my father Naiqiang Zhao for their love and support throughout my life. Thank you both for giving me strength to reach for the stars and chase my dreams. My heartfelt thanks also go out to my friends, especially Cong Du, Yingtung Chan, and Tingting Wu, at McGill and those far away in China, who supported me through hard times and helped me in ways unknown to them.
CONTRIBUTION OF AUTHORS

This is a manuscript-based thesis with three essays.

Essays 1 and 2 (Chapters 2 and 3) are solely my own contributions.

The third essay (Chapter 4) is a joint work with Yingtung Chan, a Ph.D. candidate at the department of economics, McGill University. In this essay, his contribution included some of the model derivations, simulations, and estimations. I also contributed to model derivations and simulations and did the results analysis, as well as writing the manuscript.
ABSTRACT

This thesis addresses questions concerning international currency, trend inflation, and policy uncertainty. The first essay studies the impact of trend inflation on the share of the leading international currency in global payments. By constructing a three-country model with trend inflation in which the share of the leading international currency is endogenous, we find that the position of the leading international currency persists even if the inflation rate in its own country increases from 0 to 8 percent. Moreover, incorporation of trend inflation in the open-economy model reveals that domestic trend inflation amplifies the spillover effects of a domestic technology shock on foreign countries. Trend inflation in foreign countries reinforces these spillover effects through the effect of price dispersion.

To explore the costs and benefits of having an international currency, the second essay constructs a two-country model in which the home country provides the international currency and the foreign country uses it to purchase imports. Implications of this model are derived under two pricing assumptions. One, where the prices of all traded goods are set in the domestic currency and the other, where the prices of all traded goods are set in the international currency. We identify new sources of benefits and costs of having an international currency. The third essay investigates how monetary policy uncertainty affects macroeconomy under financial frictions. We apply a New-Keynesian model with a financial accelerator mechanism and introduce the monetary policy uncertainty as a mean preserving shock to the variance of the level interest rate shock. Results show a negative impact of policy uncertainty on macroeconomy. Financial accelerator mechanism amplifies this adverse effects. Furthermore, among the productivity uncertainty and risk shock, policy uncertainty induces the strongest output responses.
Cette thèse analyse des questions entourant le rapport entre les devises internationales, l'inflation tendancielle et l'incertitude des politiques. Le premier essai étudie l'impact de l'inflation tendancielle sur le rôle de la principale devise internationale dans les paiements mondiaux. En construisant un modèle à trois pays avec une inflation tendancielle dans laquelle la part de la principale devise internationale est endogène, nous constatons que la position de la principale devise internationale persiste même si le taux d'inflation dans son propre pays augmente de 0 à 8 pour cent. En outre, l'incorporation de l'inflation tendancielle dans le modèle de l'économie ouverte révèle que l'inflation tendancielle domestique amplifie les effets de retombées d'un choc technologique domestique sur les pays étrangers. La tendance de l'inflation dans les pays étrangers renforce ces retombées négatives par une dispersion des prix. Dans l'optique d'explorer les coûts et les avantages d'une devise internationale, le deuxième essai construit un modèle à deux pays, dans lequel le pays d'origine fournit la devise internationale et le pays étranger l'utilise pour acheter des importations. Les conséquences de ce modèle sont analysées grâce à deux hypothèses de tarification. Dans la première, les prix de tous les biens échangés sont fixés dans la monnaie nationale et dans la seconde, les prix de tous les biens échangés sont fixés par la devise internationale. Nous identifions ensuite les avantages et les coûts découlant de ce choix d'une devise internationale. Le troisième essai examine comment l'incertitude de la politique monétaire affecte la macroéconomie par des frictions financières. Nous appliquons un nouveau modèle keynésien avec un mécanisme d'accélération financière et introduisons l'incertitude de la politique monétaire comme un choc de conservation significatif pour la variation du choc du taux d'intérêt. Les résultats montrent un impact négatif de l'incertitude politique sur la macroéconomie. Le mécanisme d'accélération financière amplifie ces effets néfastes. Il apparaît ainsi qu’au sein de l’incertitude de productivité et
le choc de risque, l’incertitude des politiques induit les réponses les plus fortes.
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Chapter 1

Introduction

An international currency fulfills the three functions of money—medium of exchange, unit of account, and store of value—in the international context. Since the Second World War, the U.S. dollar has served as the world’s leading international currency. However, the dollar now faces several potential rivals for its role as a medium of exchange. The euro is the most credible of those alternatives. Moreover, as emerging economies, particularly China, account for an ever-growing share of the global economy and participate more actively in cross-border trade, their currencies will gain increasing importance in the international monetary system.

Chapter 2 and chapter 3 in this dissertation address a number of questions related to the role and challenges of international currencies. The literature on the use of international currencies has mainly been simply descriptive or based on partial equilibrium models in which relative prices or trades are exogenous. These two chapters provide a new perspective on the role and usage of international currencies by developing open-economy general equilibrium models to investigate these issues.

The second chapter, “International Currency, Trend Inflation, and Indeterminacy” investigates the impact of trend inflation on the share of the leading international currency
in global payments. We construct a three-country dynamic stochastic general equilibrium (DSGE) model with trend inflation in which the share of the leading international currency is endogenous. Results show that in the long run, the position of the leading international currency persists even if the inflation rate in its own country increases from 0 to 8 percent. In the short run, when both international currency countries start with the same inflation rate, an increase of up to 3 percent in trend inflation in the leading international currency country will not lead to a more volatile share of its currency compared with the zero inflation baseline case. Moreover, positive inflation has been the norm in most countries since the Second World War. Most studies on trend inflation work with closed economy models. Incorporation of trend inflation in the open-economy model reveals that domestic trend inflation amplifies the spillover effects of a domestic technology shock on foreign countries. Trend inflation in foreign countries reinforces these spillover effects through the price dispersion effect. When all three countries adopt an identical monetary policy, the determinacy region in the Taylor rule’s parameters space shrinks with trend inflation.

The third chapter, “Should the Yuan Become an International Currency?” is motivated by the fact that the evolution of the international role of the yuan is in the early stage of becoming a major pricing and settlement currency in trade and addresses the benefits and costs of having an international currency in its early stage. It constructs a two-country DSGE model in which the home country provides the international currency and the foreign country uses it to purchase imports. We derive the implications of this model under two pricing assumptions. One, where the prices of all traded goods are set in the domestic currency (PCP) and the other, where the prices of all traded goods are set in the international currency (PCP-LCP). Results show that, in all cases, technology shocks and monetary policy shocks in the home country have greater effects on consumption and output abroad than equivalent shocks in the foreign country. Monetary policy in the home country is thus more potent. Costs of having an international currency are: (i) the seigniorage collected by the home country decreases in response to a positive
technology shock in the home country; and (ii) the home country’s households experience losses because an asymmetric exchange rate pass-through leads to a sluggish adjustment of consumption in the home country. These costs are only incurred in the PCP-LCP model.

This dissertation also investigates the effects of monetary policy uncertainty on macroeconomy under financial frictions. Politicians and regulatory institutions frequently make decisions that alter the environment in which firms operate. Therefore, businesses and consumers alike often face a significant amount of uncertainty regarding the timing, content, and potential impact of policy decisions. This supposedly negative influence of policy uncertainty has recently received increased attention from academics and policy makers, with many commentators arguing that the policy uncertainty is one of the main reasons for the anemic recovery of the U.S. economy following the 2008–2009 financial crisis. More importantly, if the financial friction—information asymmetries between lenders and entrepreneurs—is a key source of macroeconomic fluctuations, the presence of this friction would amplify and propagate the policies uncertainty shocks significantly. It is thus important to investigate how monetary policy uncertainty would affect macroeconomy under financial friction.

Chapter 5, “Policy Uncertainty and Financial Frictions”, uses a New Keynesian model with a financial accelerator mechanism to answer the above question. While there has been a flourishing literature studying the role of uncertainty shocks in driving business cycle fluctuations, there is little research on the impact of uncertainty policy shocks on macroeconomy under financial friction. This chapter tries to fill this gap and contributes in the following ways. Our results show that a two-standard deviation shock to the interest rate volatility leads to approximately 0.01 percent drop in GDP. The financial accelerator mechanism amplifies the transmission through the effect on the net worth of entrepreneurs. By comparing with the TFP uncertainty shock and the risk shock, we find that the output effects of the monetary policy uncertainty shock is around eight
times larger than the effects of risk shock and is slightly larger than the effects of TFP shock. The monetary policy uncertainty has the largest effects among all three uncertainty shocks, although quantitatively the effects are limited.

This is a manuscript-based thesis with above-mentioned three chapters, followed by chapter 5, which provides general conclusions and suggested future work.
Chapter 2

International Currency, Trend Inflation, and Indeterminacy

2.1 Introduction

Since the Second World War the U.S. dollar has served as the world’s leading international currency, acting as a medium of exchange, as a unit of account, and as a component of official foreign exchange reserves. The dollar has maintained its top share in global payment currencies even during the high inflation period from the mid-‘70s to the early-‘80s and the subprime mortgage crisis of 2007–10. In terms of value, the share of the dollar in global payments has never fallen below 43.5%. At present, the dollar’s major alternative is the euro. The global share by value of payments made in the dollar was 51.9% and the euro ranked second with a 30.5% share in 2014.\(^1\) Looking back in history, early in the nineteenth century, the British pound was the leading international currency and continued in this role even after the UK’s economic dominance ended. This is explained by a particular property of an international currency: whatever currency has been used

\(^1\) See SWIFT (2015).
in the past will tend to be used in the future (see, e.g., Krugman (1984), Tavlas (1990), Chinn and Frankel (2007), and Goldberg (2010)). Currently, this property favours the continued role of the dollar as the leading international currency. To advance the internationalization of a currency, Tavlas (1990) summarizes three factors, among which low and stable inflation is the first requirement that needs to be fulfilled. Inflation debases the currency as a medium of exchange by eroding its purchasing power. This happens because international transactions often entail a lapse of time between the initiation and completion of a transaction.

How would inflation affect the share of the leading international currency? To what extent does the persistent use of the leading international currency favour its central role in short-run dynamics? Tackling these questions requires a multiple-country model with trend inflation where the share of the leading international currency is endogenous. Yet most New Keynesian dynamic stochastic general equilibrium (DSGE) models are derived under the assumption of zero steady-state inflation. Recent studies examine the implications of positive trend inflation in closed economy models (see, e.g., Amano, Ambler, and Rebei (2007), Ascari and Ropele (2007), Ascari and Ropele (2009), and Ascari and Sbordone (2014)). The motivation for these studies comes from the fact that inflation has systematically been positive since the Second World War. However, few works on open economy models allow for trend inflation. Fernandez-Corugedo (2007) considers the open economy implications of non-zero steady-state inflation rates both in the domestic and foreign economies, but under the restrictive assumption of a unit elasticity of substitution between domestic and foreign goods. This assumption can deliver tractable solutions but will insulate the economy from terms of trade movements. This paper relaxes this assumption and examines how trend inflation affects the share of the leading international currency in a three-country DSGE model where there are two international currencies.

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2 Trend inflation is defined as a non-zero steady-state inflation rate. This paper considers the situation when trend inflation is positive. The terms positive steady-state inflation and inflation target both refer to trend inflation.

3 As discussed in Imbs and Mejean (2015), with unitary elasticity, a marginal reduction in the utility value of output is accompanied by an exactly offsetting reduction in the utility value of consumption.
More specifically, the international currency A is the leading international currency held by all three countries for international payments, while the emerging international currency B is held only by its own country and country C. Therefore, households in the country C hold a portfolio of international currencies A and B with imperfect substitution between them.

The contributions of this paper are the following. The literature on the international use of currencies has mainly been simply descriptive or based on partial equilibrium models in which relative prices or trades are exogenous (Devereux and Shi (2013)). This paper provides a new perspective on the role and usage of international currencies by endogenizing the share of the leading international currency in international payments.

Results show that in the long run, the position of the leading international currency persists even if the inflation rate in its own country increases from 0 to 8 percent. In addition, the real balances of the leading international currency A held by the third country are relatively unaffected by levels of inflation in the emerging international currency country, whereas the real balances of the emerging international currency held by the third country are affected by inflation rates in both international currency countries. This in turn affects the seigniorage collected by each country.

In the short-run dynamics, when both international currency countries start with the same trend inflation, an increase of up to 3 percent in trend inflation in the leading international currency country will not lead to a more volatile share of its currency compared with the zero inflation baseline case. Sensitivity analysis shows that the combination of a weak substitution between the leading and the emerging international currencies and a high preference for the leading international currency of the third country generates these results.

Moreover, although positive inflation has been the norm in most countries since the Second
World War, most studies on trend inflation are confined to closed economies. Theoretically, it is of interest to examine the implications of trend inflation in open economies. This paper contributes to this literature by deriving an expression for the New Keynesian Phillips Curve (NKPC) with positive trend inflation in an open economy model with Calvo pricing and non-unit elasticity of substitution between domestic and foreign goods.

Results show that in an open economic setting, domestic trend inflation amplifies the spillover effects of a domestic technology shock on foreign countries through both the expenditure-switching effect and the intertemporal substitution effect. Trend inflation in foreign countries reinforces these spillover effects through the effect of price dispersion. However, in the case of low persistent monetary policy shocks, the impacts of trend inflation are limited.

Trend inflation will also greatly affect the determinacy regions in the Taylor rule’s parameter space of the model. More specifically, when all three countries adopt an identical monetary policy, the determinacy region shrinks rapidly with trend inflation. This indicates that the monetary policy should respond more aggressively to the deviation of inflation rate from its target when trend inflation is high.

This paper is related to several branches of literature. First, the definition of an international currency as formulated by Cohen (1971), Krugman (1984), and Kenen (1994), is a currency that fulfills the three functions of money (medium of exchange, unit of account, and store of value) in an international context. They also differentiate the uses of a currency between the private and the official sectors. A fully-fledged international currency should thus fulfill six functions. As a medium of exchange, it is used by the private sector to settle international economic transactions or by governments as a vehicle currency to intervene in foreign exchange markets. As a unit of account, it is used by market actors as a quotation currency for international trade and investment transactions, or by governments for either this purpose or as an anchor for pegging the national currency. Finally, as a store of value, it is held as an asset by either foreign private actors for investment
purposes or by governments in the form of their official foreign exchange reserves.

A currency may fulfill only some of these functions and still be considered an international currency (Cohen (2013)). Chinn and Frankel (2007) examine the determinants of international reserve currency shares over the period 1973–98. They conclude that the two main significant explanatory variables are the GDP share and the economy’s inflation rate relative to the world average. Krugman (1980) focuses on a vehicle currency as a medium of exchange between currencies instead of a medium of exchange between goods. He proves that only a currency that is important in world payments can serve as an international medium of exchange. One country’s economic dominance makes it more likely that all transactions between the other countries will use the currency of the dominant country. Devereux and Shi (2013) define a vehicle currency in the same way and build a monetary exchange economy model with multiple countries. One interesting finding of their paper is that if inflation in the vehicle currency country increases too much, then the use of this currency will collapse and countries may adopt another currency with lower inflation as the vehicle currency. Another related paper by Ogawa and Sasaki (1998) empirically analyzes to what extent, even when its inflation rate is high, the dollar continues to be predominant as an international currency due to its use as an international currency in the past. They show that the dollar will keep its position as the leading international currency even if a moderate inflation rate in the U.S. reduces the value of the dollar. An important paper by Canzoneri et al. (2013) examines the benefits and costs of being an international reserve currency country in a calibrated two-country DSGE model.

Second, since the publication of the seminal work of Obstfeld and Rogoff (1995), there has been an outpouring of research on open-economy dynamic general equilibrium models that incorporate imperfect competition and nominal rigidities (e.g., Clarida, Gali, and Gertler (2000), Corsetti, Dedola, and Leduc (2010), and Galí and Monacelli (2005), etc. to name but a few). These works usually assume a zero steady-state inflation rate.
At the same time, some authors started to exploit the implication of a positive steady-state inflation in a closed economy model. A pioneering contribution on this issue is King and Wolman (1996). Following this study, Ascari (2004) examines the effects of trend inflation on the dynamics of a standard closed economic model. He shows that when trend inflation is considered, the properties regard both the long run and the short run of a dynamic general equilibrium model with Calvo pricing. Amano, Ambler, and Rebei (2007) analyze how the business cycle characteristics of the model vary with trend inflation. Ascari and Ropele (2009) study how trend inflation alters the determinacy regions of the Taylor rule's coefficients space. Ascari and Sbordone (2014) offer a comprehensive discussion of trend inflation and the macroeconomic effects of conducting monetary policy with a higher inflation target. The key findings of these works are that trend inflation alters the dynamics of the model economy through the effect of price dispersion, and that positive trend inflation shrinks the determinacy region of basic New Keynesian DSGE models when monetary policy is conducted by a contemporaneous interest rate rule. Fernandez-Corugedo (2007) looks at the effects of trend inflation in an open economy model. He shows that higher trend inflation both in the domestic and foreign economies will reduce domestic welfare. Cogley and Sbordone (2008) derives a version of NKPC that incorporates trend inflation and estimates it. They show that once shifts in trend inflation are considered, a Calvo pricing model with constant parameters fits the data well with no need for indexation or a backward-looking component.

The remainder of the paper proceeds as follows. The following section presents the model. Section 2.3 contains the calibration. Section 2.4 discusses how trend inflation would change determinacy regions in the Taylor rule’s parameter spaces of the model. Section 2.5 analyzes the quantitative implications of the model. Section 2.6 considers how trend inflation would affect the share of the leading international currency in the short run as well as how trend inflation would alter the dynamics of the macroeconomy in responses to shocks. Section 2.7 concludes.
2.2 The Model

This section details a three-country DSGE model with nominal price rigidities and two international currencies. The world economy consists of three countries of equal size, countries A, B, and C, each populated with a continuum of households with population size normalized to unity. The leading international currency A issued by country A is the leading international currency held by all three countries. The emerging international currency B issued by country B is only held by countries B and C. Therefore, the households in country C hold a portfolio of international currencies A and B with imperfect substitution between them. Figure 2.1 illustrates the model structure.

![Figure 2.1: Model architecture.](image)

Furthermore, in the model a positive steady-state trend inflation is introduced along the lines of Ascari and Ropele (2009), Alves (2011), and Ascari and Rossi (2012). Agents in the three countries consume both home goods and foreign goods but have a symmetric home bias. Money is introduced in the form of money-in-the-utility. Households supply labor to firms within their own country in a competitive labor market. Firms maximize profits in a monopolistically competitive market using labor as the only input according to aggregate technology. We allow for tradable goods only and a complete asset market structure.
2.2.1 Households

The representative household’s intertemporal preferences over consumption, $C_{i,t}$, real money balances, $m_{i,t}$, and labor supply, $N_{i,t}$, are described by the following expected utility functions:

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^\tau U(C_{i,\tau}, m_{i,\tau}, N_{i,\tau}), \ i = A, B, C,$$

where the operator $\mathbb{E}_t$ represents the conditional expectation over all states of nature, $\beta \in (0, 1)$ is the time discount factor and $m_{i,t}$ are the real money balances which are different for the three countries under consideration.

In particular, the utility function of households in country A is:

$$U(C_{A,t}, M_{A,t}^A/P_{A,t}, N_{A,t}) = \frac{(C_{A,t}^\alpha (M_{A,t}^A/P_{A,t})^{1-\alpha} (1-\sigma))^{1-\sigma}}{1-\sigma} - \frac{N_{A,t}^{1+\phi}}{1+\phi},$$

where $M_{A,t}^A/P_{A,t}$ denotes holdings of both domestic and international real money balances, $P_{A,t}$ is the aggregate price level in country A, $\alpha$ represents the relative weight of consumption in utility, $\sigma$ and $\phi$ represent the inverse of intertemporal elasticity of substitution and the inverse of the elasticity of labor supply, respectively. Since currency A is the leading international currency, agents in country A only need to hold currency A. Whether consumption and real balances are the Edgeworth complements or substitutes depends on the sign of $(1 - \sigma)$. Since the role that money plays in the model is to facilitate trade, consumption and real balances should be complements. Similarly, the utility function of households in country B is:

$$U(C_{B,t}, M_{B,t}^B/P_{B,t}, M_{B,t}^A E_{B,t}^A/P_{B,t}, N_{B,t})$$

$$= \frac{(C_{B,t}^\alpha (M_{B,t}^B/P_{B,t})^\mu (M_{B,t}^A E_{B,t}^A/P_{B,t})^{1-\mu} [1-\alpha])^{1-\sigma}}{1-\sigma} - \frac{N_{B,t}^{1+\phi}}{1+\phi},$$

where $M_{B,t}^B/P_{B,t}$ denotes holdings of domestic real money balances, $M_{B,t}^A E_{B,t}^A/P_{B,t}$ denotes holdings of real balances of the international currency A, $E_{B,t}^A$ denotes the bilateral nominal
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The exchange rate between countries A and B (the price of country A’s currency in terms of the country B’s currency) and $P_{B,t}$ is the aggregate price level in country B. $\mu$ is the relative weight of domestic real balances in overall real money balances with a share of $(1 - \alpha)$. In contrast to the preferences of countries A and B, households in country C have to hold home currency C and an international currency composite:

$$U(C_{C,t}, M_{C,t}^C/P_{C,t}, X_t, N_{C,t}) = \frac{(\alpha C_{C,t})^\mu (1 - \mu)^{1 - \alpha} X_t^{1 - \mu}}{1 - \sigma} - \frac{N_{C,t}^{1 + \phi}}{1 + \phi},$$

where $X_t$ is a money aggregate defined as:

$$X_t = [\gamma (M_{C,t}^A E_{C,t}^{CA}/P_{C,t})^{\chi^{-1}} + (1 - \gamma) (M_{C,t}^B E_{C,t}^{CB}/P_{C,t})^{\chi^{-1}}]^{\frac{1}{\chi}}.$$

where $M_{C,t}^C/P_{C,t}$ denotes holdings of domestic real money balances, $M_{C,t}^A E_{C,t}^{CA}/P_{C,t}$ and $M_{C,t}^B E_{C,t}^{CB}/P_{C,t}$ determine holdings of the real balances of international currencies A and B, respectively, $P_{C,t}$ is the aggregate price level in country C. Imperfect substitution between international currencies is captured by using a CES function. The CES specification implies that households in country C will hold both international currencies even if inflation rates are different in the international currency countries. Therefore, it allows to endogenously pin down the composition ratio of international currencies. The parameter $\gamma \in [0, 1]$ determines the weight of the leading international currency A within the international currency composite; and $\chi > 0$ denotes the elasticity of substitution between the international currencies A and B. When $\gamma = 1$, the model collapses to an economy with a single international currency A.

Households in all countries have access to a complete asset of state contingent nominal claims denominated in the leading international currency A, which are traded domestically and internationally. Since financial markets are complete internationally, the currency denomination of the securities does not matter.
The budget constraint of households in country A is given by:

\[
C_{A,t}P_{A,t} + M_{A,t}^A + \sum_{h^{t+1}} Q_{t,t+1}^A B_{A,t+1} \leq W_{A,t}N_{A,t} + M_{A,t-1}^A + B_{A,t} + T_{A,t} + D_{A,t},
\]

(2.5)

where \(W_{A,t}\) represents the wage rate in the domestic competitive labor market, \(B_{A,t}\) is the nominal random payoff of the portfolios purchased in currency A by households in country A at time \(t\). Each asset in the portfolio \(B_{A,t+1}\) pays one unit of international currency A at time \(t+1\) and in state \(h^{t+1}\). \(Q_{t,t+1}^A \equiv Q^A(h^{t+1}|h^t)\) is the period-\(t\) price of one unit of international currency A in state \(h^{t+1}\) divided by the probability of occurrence of that state. \(D_{A,t}\) denotes the profits of monopolistically-competitive domestic producers and \(T_{A,t}\) is a lump-sum transfer paid by the government to households.

Households in country B face a budget constraint of the form:

\[
C_{B,t}P_{B,t} + M_{B,t}^A \mathcal{E}_{B,t}^A + M_{B,t}^B + \sum_{h^{t+1}} Q_{t,t+1}^A B_{B,t+1} \mathcal{E}_{B,t}^B \leq W_{B,t}N_{B,t} + M_{B,t-1}^B + M_{A,t-1}^A \mathcal{E}_{B,t}^A + B_{B,t} \mathcal{E}_{B,t}^A + T_{B,t} + D_{B,t}.
\]

(2.6)

The budget constraint of households in country C is given by:

\[
C_{C,t}P_{C,t} + M_{C,t}^A \mathcal{E}_{C,t}^A + M_{B,t}^B \mathcal{E}_{C,t}^B + M_{C,t}^C + \sum_{h^{t+1}} Q_{t,t+1}^A B_{C,t+1} \mathcal{E}_{C,t}^A \leq W_{C,t}N_{C,t} + M_{C,t-1}^C + M_{A,t-1}^A \mathcal{E}_{C,t}^A + M_{B,t-1}^B \mathcal{E}_{C,t}^B + B_{C,t} \mathcal{E}_{C,t}^A + T_{C,t} + D_{C,t},
\]

(2.7)

where \(B_{B,t}\) and \(B_{C,t}\) are the nominal random payoffs of the portfolios purchased in international currency A by households in countries B and C, respectively. By the same token, there are two currencies A and B appearing in the budget constraint of households in country B, yet the two international currencies and the domestic currency C appear in the budget constraint of households in country C.

The consumption aggregator \(C_{A,t}\), the aggregate consumption of locally produced goods
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$C_{A,t}^A$, and the aggregate consumption of imports $C_{j,A,t}$, $j = B, C$ in country A are given by:

\[
C_{A,t} \equiv \left[ \alpha_1 \left( C_{A,t}^A \right)^{\frac{\eta}{\eta - 1}} + \alpha_2 \left( C_{B,t}^A \right)^{\frac{\eta}{\eta - 1}} + \alpha_2 \left( C_{C,t}^A \right)^{\frac{\eta}{\eta - 1}} \right]^{\frac{\eta - 1}{\eta}},
\]  

(2.8)

\[
C_{A,t}^A \equiv \left[ \int_0^1 C_{A,t}^A(i)^{1 - \frac{1}{\eta}} di \right]^{\frac{1}{\eta - 1}},
\]  

(2.9)

\[
C_{j,A,t}^j \equiv \left[ \int_0^1 C_{j,A,t}^j(i)^{1 - \frac{1}{\eta}} di \right]^{\frac{1}{\eta - 1}},
\]

respectively. $\eta > 0$ is the elasticity of substitution between domestic produced goods and imports and $\alpha_1 \in (1/2, 1)$ indicates the home bias. $\epsilon$ denotes the elasticity of substitution among differentiated varieties within each country. $C_{A,t}^A(i)$ and $C_{j,A,t}^j(i^*)$ denote the domestic representative household’s consumption of the goods produced by the domestic firm $i$ and the foreign firm $i^*$, respectively.

Expenditure minimization associated with the consumption composite $C_{A,t}$ gives the consumer price index in country A as:

\[
P_{A,t} \equiv \left[ \alpha_1 (P_{A,t}^A)^{1 - \eta} + \alpha_2 (P_{B,t}^B)^{1 - \eta} + \alpha_2 (P_{C,t}^C)^{1 - \eta} \right]^{\frac{1}{1 - \eta}},
\]  

(2.10)

where $P_{j,t}^j, j = A, B, C$ denotes the price of a good $j$ in terms of the currency A. Thus the optimal consumption allocation of locally produced goods in country A is given by $C_{A,t}^A = \alpha_1 (P_{A,t}^A/P_{A,t})^{-\eta} C_{A,t}$. Demand for goods of the country $j, j = B, C$ by consumers in country A has the form of $C_{j,A,t}^j = \alpha_2 (P_{j,t}^j/P_{A,t})^{-\eta} C_{A,t}$. The consumption composites are defined analogously to the above equations in countries B and C.

### 2.2.1.1 Consumption and Savings Behaviour

The intertemporal optimization is solved by maximizing the lifetime utility function (2.1) subject to the budget constraint with respect to consumption, labor, bonds and liquidity
services in each country. Combining the first order conditions for consumption and labour determines the consumption labour trade-off:

\[ U^i_{C,t} = \frac{N^i_{t}}{w_{i,t}}, \quad (2.11) \]

where \( w_{i,t} \) is the real wage rate and \( U^i_{C,t} \) is the marginal utility of households in country \( i, i = A,B,C \). Combining the first order conditions for consumption and bonds leads to the Euler equation:

\[ \beta \frac{U^i_{C,t+1}P_{i,t}}{U^i_{C,t}P_{i,t+1}} = \frac{Q^A_t \varepsilon^A_{t}}{\varepsilon^A_{t+1}}. \quad (2.12) \]

Taking expectations conditional on the information available in period \( t \) and defining the gross nominal interest rate, \( R_{i,t} \), on a risk-free one-period bond in country \( i \) as:

\[ R_{i,t} \equiv \frac{1}{\mathbb{E}_t[Q^A_{t,t+1}\varepsilon^A_{t}/\varepsilon^A_{t+1}]} \]

the Euler equation is rewritten as:

\[ \beta \mathbb{E}_t \left[ \frac{U^i_{C,t+1}P_{i,t}}{U^i_{C,t}P_{i,t+1}} \right] = \frac{1}{R_{i,t}}. \quad (2.13) \]

The demand for domestic real balances is expressed as:

\[ \frac{M^i_{j,t}}{P_{i,t}} = \frac{(1 - \alpha)\mu}{\alpha} C_{i,t}(1 - \frac{1}{R_{i,t}})^{-1}. \quad (2.14) \]

The domestic real balances demand is increasing in domestic consumption and inversely related to the domestic nominal interest rate, as in conventional specifications. In addition to the demand for domestic real balances, the optimal real balances holdings of the international currencies A and B by country C are

\[ \left( \frac{M^j_{C,t}\varepsilon^C_{j,t}}{P^C_{C,t}} \right)^{\frac{1}{X}} = \lambda_j C_{C,t}(1 - \frac{1}{R_{j,t}})^{-1}X^{\frac{1}{X}}, \quad j = A,B, \quad (2.15) \]

where \( \lambda_j > 0 \) and where \( O^C_{j,t} \equiv \varepsilon^C_{j}P_{j,t}/P_{C,t} \), \( j = A,B \) denotes real exchange rates. Thus real money demands of the international currencies can be rewritten as \( m^j_{C,t}O^C_{j,t}, \quad j = \)}
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A, B. The Eq. (2.15) indicates that the demands for the international currencies are increasing in both international aggregate $X_t$ and consumption in country C and are decreasing in the interest rate of the international currency country under consideration. Similarly, the real balances demand of the international currency A by country B is $M_{B,t}^{tA}/P_{B,t} = \lambda C_{B,t}(1 - \frac{1}{R_{B,t}})^{-1}$.

### 2.2.1.2 The Share of the International Currency A

By combining Eq. (2.15) with $j = A$ and $B$, the share of the international currency A, i.e., $\Omega_t$, is derived as:

$$\Omega_t = \frac{m_{C,t}^{tA}O_{tA}^{tA} + m_{C,t}^{tB}O_{tB}^{tB}}{m_{C,t}^{tA}O_{tA}^{tA} + m_{C,t}^{tB}O_{tB}^{tB}} = \left[ 1 + \left( \frac{\gamma(1 - \frac{1}{R_{B,t}})}{1 - \frac{1}{R_{A,t}}} \right)^{-\chi} \right]^{-1}. \quad (2.16)$$

Note that $\partial \Omega_t / \partial R_{A,t} < 0$, which implies that if the opportunity cost of holding the international currency A increases, or a rise occurs in the inflation expectation of the international currency A, the relative demand for the international currency A decreases. Moreover, $\partial \Omega_t / \partial \gamma > 0$, which indicates that increases in the parameter $\gamma$, the preference of the international currency A in currency composite $X_t$, lead to increases in the holding of the international currency A in country C. The sign of $\partial \Omega_t / \partial \chi$ depends on the relative size of nominal interest rates in both countries A and B and also the relative size of the parameter $\gamma$.

In general, higher interest rate (or a higher inflation expectation) in country A combined with a relatively low value of $\gamma$ will give rise to a negative sign. Intuitively, the idea is that if households in country C do not have a strong preference for currency A and the opportunity cost of holding currency A is also high, the larger elasticity of substitution between the two international currencies, the higher the shift from international currency A to currency B. On the other hand, a strong preference for international currency A
and a low opportunity cost of holding it will produce a positive sign of $\partial \Omega_t / \partial \chi$. In this situation, the larger elasticity of substitution between the two international currencies would lead to the shift from currency B to currency A in global payments.

### 2.2.2 Firms

A unit measure of monopolistically competitive domestic firms indexed by $i$, produces differentiated intermediate goods, which are then aggregated into a final domestic good using a standard Dixit-Stiglitz aggregator. Since this paper focuses on the function of international currencies as a medium of exchange, neither international currency is used as a unit of account in invoicing trade. To this end, we assume that firms in each country set their price following a producer currency pricing (PCP), which means the prices of all traded goods are set in the domestic currency instead of in the international currencies. This section only presents the necessary equation for country A. Those for the country B and C are analogous and can be found in the equation summary in Appendix A.

The production function for each of the final good producers is:

$$Y_{A,t} = \left[ \int_0^1 Y_{A,i,t}^{\frac{-\epsilon}{\epsilon-1}} di \right]^{\frac{-\epsilon}{\epsilon-1}}. \quad (2.17)$$

Their demand for intermediate inputs is

$$Y_{A,i,t+j} = \left( \frac{P_{A,i,t}}{P_{A,i,t+j}} \right)^{-\epsilon} Y_{A,t+j}. \quad (2.18)$$

The intermediate goods producers use a linear technology as follows:

$$Y_{A,i,t} = Z_t^A N_{A,i,t},$$

where $Z_t^A$ is a country specific technology shock, which evolves as an $AR(1)$ process. Firms set their optimal prices in a staggered manner à la Calvo (1983) rule. Each time, only with probability $1 - \theta$, can they re-optimize their prices, i.e., $P_{A,i,t}^{A*}$. Furthermore, $\Pi_A$
is defined as the central bank’s long-run inflation target or the level of trend inflation in
country A. Note that in steady state, the CPI inflation rate \( \Pi_A \) equals the PPI inflation
rate \( \Pi^A \) (see section 2.2.3). The price setting problem is:

\[
\max_{P_{A,i,t}^*} \sum_{j=0}^{\infty} Q_{A,i,t+j} \theta^j [P_{A,i,t+j} - \frac{W_{A,t+j}^A}{A_{A,t+j}^A} P_{A,i,t+j} Y_{A,i,t+j} + \theta_{A,t+j} Y_{A,i,t+j}]
\]

s.t. \( Y_{A,i,t+j} = \left( \frac{P_{A,i,t}}{P_{A,i,t+j}} \right)^{-\epsilon} Y_{A,i,t+j} \).

A positive value of \( P_{A,i,t}^* \) requires \( 1 \leq \Pi^A < (1/\theta \beta)^{1/\epsilon} \), which is consistent with the kind
of equation derived by Ascari and Ropele (2009). The first order condition with respect
to price yields the New Keynesian Phillips Curve (NKPC), which, after log-linearization
around a positive steady-state inflation, can be written as:

\[
\hat{\Pi}_t^A = \frac{(1 - \theta \beta (\Pi^A)^{\epsilon})(1 - \theta (\Pi^A)^{\epsilon-1})}{\theta (\Pi^A)^{\epsilon}} M_{A,t} - \beta [1 + \epsilon (\Pi^A - 1)(1 - \theta (\Pi^A)^{\epsilon-1})] \mathbb{E}_t[\hat{\Pi}_{t+1}^A]
\]

\[
+ \beta (1 - \Pi^A) (1 - \theta (\Pi^A)^{\epsilon-1}) (\hat{U}_{C,t}^A + \hat{Y}_{A,t} - \mathbb{E}_t \hat{\psi}_t^A)
\]

\[
+ \beta (1 - \Pi^A) (1 - \theta (\Pi^A)^{\epsilon-1}) \left( \frac{C_{11}^1 T_{BA}^t + C_{12}^1 T_{CA}^t}{C_1^1} \right) + \beta \mathbb{E}_t[\hat{\Pi}_{t+1}^A],
\]

(2.19)

where a circumflex denotes log-deviations from the steady state, \( \hat{U}_{C,t}^A \) is the marginal
utility of households in country A, \( \hat{T}_{BA}^t \) and \( \hat{T}_{CA}^t \) represent the terms of trade between
country A and B and the terms of trade between country A and C, respectively, and
\( C_{ii} > 0, C_i > 0, i = 1, 2 \) are parameters. The conventional NKPC emerges as a special
case when steady-state inflation is zero, which is obtained by substituting \( \Pi^A = 1 \) into
Eq. (2.19):

\[
\hat{\Pi}_t^A = \frac{(1 - \theta \beta)(1 - \theta)}{\theta} M_{A,t}^A + \beta \mathbb{E}_t[\hat{\Pi}_{t+1}^A],
\]

where \( M_{A,t}^A \) denotes the log deviation of marginal cost of firms in country A. Eq. (2.19)
differs from the standard NKPC in several aspects. Note that Ascari and Sbordone (2014)
discussed the differences between the NKPC with positive steady-state trend inflation and
the standard NKPC in a closed economy setting and showed that all the coefficients are non-linear functions of trend inflation and parameters of the pricing model; higher trend inflation implies a lower weight on current marginal cost and a higher weight on expected future inflation; price dispersion enters the NKPC and domestic output enters the NKPC through two channels. These properties are preserved in the open economy with trend inflation setting, but there are several new features. Unlike the closed economy with trend inflation setting, the terms of trade enters the NKPC: one is through the $\hat{MC}_t^A$ channel and the other is the last item in the Eq. (2.19). This captures the interdependence: economic activities abroad affect the domestic economy via international relative prices. In contrast with the open economy without trend inflation setting, the additional terms of trade item has the property that higher trend inflation indicates a higher weight on terms of trade.

With trend inflation, the log deviation of marginal cost in Eq. (2.19) is expressed as:

$$\hat{MC}_t^A = \phi \hat{s}_t^A + \phi \hat{Y}_t^A - \hat{U}_{C,t}^A - (\phi + 1) \hat{Z}_t^A - \left( \frac{C_{11}}{C_1} \hat{T}_{BA}^A + \frac{C_{12}}{C_1} \hat{T}_{CA}^A \right),$$

where $\hat{s}_t^A$ denotes the price dispersion in country A.\(^4\) Note that in a neighbourhood of the zero inflation steady state, price dispersion equals zero up to a first-order approximation. However, with positive steady-state inflation, price dispersion is first-order indifferent from zero and increases in trend inflation. Increase in price dispersion will increase the marginal cost. This is because increase in price dispersion will allocate labour from ‘high-price’ firms to ‘low-price’ firms. Average labour productivity is lower due to diminishing returns. In a sense, at the aggregate level the economy uses too much labour to produce a given level of output. Upward pressure on real wages incurs higher marginal costs of given production (see Damjanovic and Nolan (2010)).

\(^4\) Price dispersion is defined as $\int_0^1 \left( \frac{p_{A,i,t}}{p_{A,t}} \right)^{-\epsilon} di$.\)
### 2.2.3 International Linkages

The international linkages include trading of international bonds, i.e., the associated international risk sharing and trading of consumption goods. Given that state contingent nominal bonds denominated in the international currency A are tradable internationally, the marginal rates of substitution between any two states are equalized for any two countries even *ex post*. Combining with the symmetric initial conditions of wealth, the risk sharing condition is obtained as follows:\(^5\)

\[
U^A_{C,t} = O^{BA}_{t} U^B_{C,t}
\]

\[
U^A_{C,t} = O^{CA}_{t} U^C_{C,t}.
\]  

(2.20)

Notice that real exchange rates are time-varying because of the home preference bias.

We define the bilateral terms of trade between country \(i\) and country \(j\) from the point of view of country \(i\) as the price of imported goods from country \(j\) relative to the price of the exported goods abroad, such that \(T^{ij}_t = \frac{P^j_{i,t}}{(E^{ij}_t P^i_{j,t})} = \frac{P^j_{i,t}}{P^i_{i,t}}, \ i,j = A,B,C\). Take \(T^{BA}_t\) as an example, a increase in \(T^{BA}_t\) indicates a deterioration in the terms of trade of country B but an improvement in the terms of trade of country A. Given the previous definitions, the real exchange rate and the CPI inflation rate can be expressed as a function of the terms of trade. For the sake of brevity, we only present these equations for country A as follows:

\[
O^{BA}_t = \left[ \frac{\alpha_1 + \alpha_2(T^{BA}_t)^{\eta-1} + \alpha_2(T^{CA}_t)^{\eta-1}}{\alpha_1(T^{BA}_t)^{\eta-1} + \alpha_2 + \alpha_2(T^{CA}_t)^{\eta-1}} \right]^{\frac{1}{1-\eta}},
\]  

(2.21)

\(^5\)Note that by employing Euler equations and Eq. (2.20), one derives the uncovered interest rate parity condition (UIRP). However, as mentioned by Felices and Tuesta (2013), UIRP does not represent an additional equilibrium condition in the complete financial market setting.
where a increase in $O_t^{BA}$ indicates real exchange rate appreciation for currency A.

\[
\Pi_{A,t} \equiv \frac{P_{A,t}}{P_{A,t-1}} = \left[ \frac{\alpha_1 (\Pi_t^A)^{1-\eta} + \alpha_2 (\Pi_t^A/T_t^{BA})^{1-\eta} + \alpha_2 (\Pi_t^A/T_t^{CA})^{1-\eta}}{\alpha_1 + \alpha_2 (T_t^{BA})^{\eta-1} + \alpha_2 (T_t^{CA})^{\eta-1}} \right]^{1/\eta}. \tag{2.22}
\]

When evaluating $\Pi_{A,t}$ at its steady state, the CPI inflation rate is equivalent to the PPI inflation rate, i.e., $\Pi_A = \Pi^A$.

### 2.2.4 Governments and Central Banks

The government in each country injects money into the economy through nominal transfers. Therefore, seigniorage revenue in nominal terms $\Xi_{i,t}$ of country $i$ can be defined as:

\[
\Xi_{i,t} = M_i^t - M_{i,t-1}, \quad i = A, B, C, \tag{2.23}
\]

where $M_i^t$ is the aggregate nominal money supply in country $i$. More importantly, we assume that money supply evolves according to the following rule: $M_i^t = \mu_{i,t} M_{i,t-1}$ where $\mu_{i,t}$ is the gross growth rate of the nominal money supply. In the steady state, the money growth rate is simply equal to the positive trend inflation rate.

Central banks across all countries target deviations of domestic CPI inflation and GDP from their steady-state values using a Taylor rule:

\[
\frac{R_{i,t}}{R_i} = \left( \frac{R_{i,t-1}}{R_i} \right)^{\phi_R} \left[ \left( \frac{\pi_{i,t}}{\pi_i} \right)^{\phi_{\pi}} \left( \frac{Y_{i,t}}{Y_i} \right)^{\phi_Y} \right]^{1-\phi_R} e^{v_{i,t}}, \quad i = A, B, C, \tag{2.24}
\]

where $R_i$ and $Y_i$ are the steady-state values of the nominal interest rate and output, respectively, and $\phi_R$ denotes the policy inertia. Parameters $\phi_{\pi}$ and $\phi_Y$ govern the importance of domestic inflation and output gap in the Taylor rule, respectively. $v_{i,t}$ is an AR (1) policy shock.
2.2.5 Market Clearing

Most of the market equilibrium conditions are obvious. Here, we document those which may create some confusion. For the money market, the clearing conditions are given as:

\[ M_t^A = M_{A,t}^A + M_{B,t}^A + M_{C,t}^A \]
\[ M_t^B = M_{B,t}^B + M_{C,t}^B \]
\[ M_t^C = M_{C,t}^C. \]  

(2.25)

For the labor market, taking country A as an example, aggregating over \( i \) from the individual firm’s production function (2.18), one derives the aggregate labor demand as:

\[ N_{A,t} = \frac{Y_{A,t}}{Z_{A,t}^A} s_t^A, \]

(2.26)

where according to the definition of \( s_t^A, s_t^A \geq 1 \), it increases the amount of labor required to produce a given level of output (see discussion in section 2.2.2).

2.3 Calibration

The model is calibrated at a quarterly frequency. We calibrate the leading international currency country A to U.S. data and the emerging international currency country B to euro area data. Since most parameter values for the euro area are in the same range as the parameter values for the U.S., we choose the identical parameter value for both countries except for the case when the difference between the two values of a certain parameter is significant. Instead of calibrating the non-international currency country to the “rest of world” data, we calibrate it in a symmetric fashion to U.S. data. This approach allows one to examine the impact of asymmetric trend inflation in both international currency countries without considering the impact of asymmetric parameter values.
The discount factor $\beta$ is set equal to 0.99, which implies an annual real interest rate of 4 percent. In the utility function, we set the relative weight of consumption to 0.9. For the United States, estimates of this parameter vary from 0.93 to 0.99 depending on the method of estimation and the utility function specification (see Holman (1998)). Estimates for the share of domestic real balances in liquidity aggregate are not available. We set it to 0.8, indicating a "non-dollarization" economy.

The inverse of elasticity of intertemporal substitution is $\sigma = 0.9$. De Walque, Smets, and Wouters (2005) estimate a two-country DSGE model for the euro area and the U.S. economy. They find that the estimation of the $\sigma$ is around 0.92 for the U.S. economy and varies from 0.84 to 1.07 for the euro area. We set it at 0.9 because a number smaller than 1 will generate the positive cross derivative of $U_{CM}$. This positive derivative indicates the consumption and real money balances of international currencies are Edgeworth complements. Since international currencies act as a medium of exchange in the model, consumption and real balances of the international currencies should be complements.

The Frisch labor elasticity $\phi$ is set to 1. Elasticity of substitution between imports and local goods is assumed to be $\eta = 2$. In the literature there is no consensus on this elasticity. The estimation results in De Walque, Smets, and Wouters (2005) show that in the model without uncovered interest rate parity (UIRP), it is 1.97 for the euro area and 1.16 for the U.S., but in the model with UIRP, it is 3.01 for the euro area and 1.74 for the U.S.. The elasticity of substitution between domestic varieties $\epsilon$ is set to 6, reflecting a 20% price markup. In each country the expenditure share of domestic goods, $\alpha_1$, is set to 0.8, slightly lower than the values assumed in Coenen et al. (2007). They calibrate their two-country DSGE model to represent the euro area and U.S. economies and set this value at 0.85 for the euro area and at 0.9 for the U.S..

We assume each country imports equally from the two foreign countries, i.e., $\alpha_2 = 0.1$. The price duration parameter is set to 0.25, implying that the average duration of price is around 4 quarters. In the quantitative analysis section, the steady-state annualized trend
inflation rates in countries A, B, and C are all assumed to be 2% in order to compare the model simulated data to the real data. In the dynamic analysis section, steady-state annualized trend inflation rates in countries A and B are assumed to take different values, i.e., 0%, 2%, 4%, 8%, whereas the steady-state inflation rate in country C is fixed at 0%.

The interest rate smoothing parameter in the Taylor rule, $\rho_r$, is set to 0.8, and coefficients $\phi_\pi$ and $\phi_y$ take the classic values of 1.5 and 0.5/4, respectively, as in Taylor (1993). The calibration of the productivity and monetary shock process follows Coenen et al. (2007).

We follow Ogawa and Sasaki (1998) and use Eq. (2.16) in section 2.2.1.2 to estimate $\gamma$, the relative weight of real balances of the leading international currency A and $\chi$, the elasticity of substitution between two international currencies. The data used to estimate are the share of the dollar in the composite of the dollar and euro in foreign currency liabilities, the CPI inflation rates in the U.S. and the euro area for the period from 2007Q2–2015Q4. Results show that the relative weight of real balances of the dollar in the international currency aggregate is 0.79 and the elasticity of substitution between the euro and the dollar is 1.24. By assuming a unit elasticity of substitution between two international currencies, the estimation result in Ogawa and Sasaki (1998) shows that the mean of this $\gamma$ is in the range of 0.69 to 0.82.\footnote{The two international currencies considered in their work are the dollar and an international currency bundle, including the Japanese Yen and the German Mark.}
## Table 2.1: Structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Utility function</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The relative weight of consumption in utility</td>
<td>0.9</td>
</tr>
<tr>
<td>$\mu$</td>
<td>The relative weight of domestic real balances in liquidity</td>
<td>0.8</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The relative weight of real balances of international currency A</td>
<td>0.79</td>
</tr>
<tr>
<td>$\chi$</td>
<td>The elasticity of substitution between two international currencies</td>
<td>1.24</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>The inverse of elasticity of consumption substitution</td>
<td>0.9</td>
</tr>
<tr>
<td>$\phi$</td>
<td>The inverse of elasticity of labor supply to real wage</td>
<td>1</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Substitutability between domestic varieties</td>
<td>6</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Substitutability between domestic and foreign goods</td>
<td>2</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Degree of home bias</td>
<td>0.8</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Degree of relative weight on imports</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Production function</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability of non-price adjustment</td>
<td>0.75</td>
</tr>
<tr>
<td>$\Pi_A, \Pi_B$</td>
<td>Inflation rate in countries A and B</td>
<td>$1, 1.02^{\frac{3}{4}}, 1.04^{\frac{3}{4}}, 1.06^{\frac{3}{4}}, 1.08^{\frac{3}{4}}$</td>
</tr>
<tr>
<td>$\Pi_C$</td>
<td>Inflation rate in country C</td>
<td>$1, 1.02^{\frac{3}{4}}$</td>
</tr>
<tr>
<td><strong>Taylor Rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>Coef. on inflation gap</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_Y$</td>
<td>Coef. on output gap</td>
<td>$0.5/4$</td>
</tr>
<tr>
<td>$\phi_R$</td>
<td>Interest rate smoothing parameter</td>
<td>0.8</td>
</tr>
<tr>
<td><strong>Shock processes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Persistence of TFP shocks in three countries</td>
<td>0.91</td>
</tr>
<tr>
<td>$\rho_{ra}$</td>
<td>Persistence of MP shocks in countries A and C</td>
<td>0.04</td>
</tr>
<tr>
<td>$\rho_{rb}$</td>
<td>Persistence of MP shock in country B</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_a}$</td>
<td>Standard deviation of TFP shocks in three countries</td>
<td>0.55%</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_r}$</td>
<td>Standard deviation of interest rate shocks in three countries</td>
<td>0.26%</td>
</tr>
</tbody>
</table>

*Notes: The structural parameters for all three countries are the same except otherwise noted in the table.*
Chapter 2. *International Currency, Trend Inflation, and Indeterminacy*

2.4 Determinacy

This section discusses how trend inflations in countries A and B affect the rational expectations equilibrium (REE) determinacy properties of the model when the Taylor rule contains a interest rate smoothing (IRS).\(^7\) The determinacy of the model is related to the question because higher trend inflation may result in an indeterminate REE of the model for the classical values of the Taylor rule parameters as shown in Ascari and Sbordone (2014).

As discussed in Woodford (2003), in a standard closed economy model with a zero steady-state inflation, if one considers a quarterly calibrated Taylor rule in the form of

\[ \hat{i}_t = \rho \hat{i}_{t-1} + \phi_{\pi} \hat{\pi} + \frac{\phi_y}{4} \hat{y} \]

where the output response coefficient \( \phi_y \), the inflation response coefficient \( \phi_{\pi} \), and the interest rate inertia \( \rho \) are assumed to be no less than zero, the equilibrium is determinate if and only if the response coefficients satisfy \( \phi_{\pi} + \phi_y \frac{\partial \hat{Y}}{\partial \hat{\pi}}_{LR} > 1 - \rho \) (where LR is short for long run). Moreover, \( \frac{\partial \hat{Y}}{\partial \hat{\pi}}_{LR} \) is given by \( \frac{1-\beta}{4\kappa} \). Since the calibration of \( \beta \) is normally close to 1, as long as \( \phi_{\pi} > 1 - \rho \), the determinacy is guaranteed. According to Ascari and Ropele (2009), in a closed economy with trend inflation, interest rate inertia makes the Taylor principle, i.e., \( \phi_{\pi} > 1 - \rho \), insignificant. Instead, the value of \( \phi_y \) matters for REE determinacy. This is because the derivative \( \frac{\partial \hat{Y}}{\partial \hat{\pi}} \) depends on trend inflation and turns negative as soon as trend inflation is positive. Consequently, the determinacy region is affected by the particular value of trend inflation. By applying some simplifying assumptions, Ascari and Ropele (2009) also derive a determinacy condition analytically. In contrast to a closed economy model, in an open economy setting, the addition of trend inflation will greatly

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\(^7\) According to Woodford (2003), determinacy is defined as: the system has a unique bounded solution in the case of bounded disturbances. The Blanchard-Kahn condition for determinacy is: the solution of the rational expectations model exists and is unique if and only if the number of eigenvalues outside the unit circle is exactly equal to he number of non-predetermined variables.
increase the dimensions of the system and makes analytical results impossible. We adopt a simulation approach and provide numerical findings.

As stressed by Bullard and Schaling (2006), in a multiple country world, each country must separately meet conditions for determinacy. That is, if one country fails, then worldwide equilibrium is indeterminate and one country cannot fix the failure of a second country to meet appropriate conditions. Therefore, we assume an identical policy in all three countries, in the sense of identical values for the Taylor rule coefficients, to explore how the increasing of trend inflation in countries A and B affects REE determinacy regions. In particular, we let the trend inflation rate in country A varies from $0\% \sim 8\%$ while keeping the trend inflation rate in country B at $0\%, 2\%, 6\%, \text{and} 8\%, \text{respectively}$.

Figure 2.2 illustrates the determinacy regions in the space of coefficients $(\phi_\pi, \phi_y)$. In general, the upper bound of the coefficient $\phi_y$ is around 3 and shaded areas with different hues represent different levels of trend inflation in country A. All 4 figures show that with a given level of trend inflation in country B, a rise in trend inflation in country A indicates a clockwise move of the determinacy region. This finding is consistent with Ascari and Sbordone (2014). They show that in a closed economy model, the determinacy region shrinks rapidly with trend inflation. The underlying reason, as discussed in Ascari and Sbordone (2014) is that trend inflation flattens the NKPC, and thus monetary policy should respond more aggressively to inflation to induce a certain reduction in output. Moreover, higher trend inflation in country B will shrink determinacy regions as expected and make distinctions between adjacent areas in each graph less prominent. The reason for this decrease in prominence is that since all three countries have to adopt an identical policy, and trend inflation in country B is relatively high, monetary policy should respond more to deviations of inflation from its target and less to output gaps. The white dot marks when $\phi_\pi$ and $\phi_y$ take standard values of $1.5$ and $0.5/4$, respectively. In panel (d)
when trend inflation in both countries A and B takes 8%, the white dot is at the edge of the black triangle area. Figure 2.2 thus reports that a classical value set of the Taylor rule chosen by all three countries will still simultaneously ensure determinacy.

Figure 2.2: Determinacy regions for different levels of trend inflation. The white dot denotes when $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. Shaded areas represent determinacy regions and different hues indicate different values of trend inflation of country A. The Darker area overlaps the lighter area.
2.5 Quantitative Analysis

This section analyzes the quantitative implications of the model. First, we show how positive trend inflation affects the steady-state relationship between selected variables and trend inflation. Second, we compare some of the moments generated by the model with the data for the euro area and the United States.

2.5.1 The Cost of Trend Inflation in the Long Run

Allowing for positive trend inflation increases the state space of the model and makes analytical solutions infeasible. As discussed in section 2.2.2, with trend inflation, price dispersion is an important first-order variable and increases in trend inflation. It affects the long-run behaviour and short-run dynamics of the model through its effect on marginal cost of firms. We start by analyzing how trend inflation affects the long-run properties of the model and the share of the leading international currency.

As in and Sbordone (2014) provide an analysis on the impact of trend inflation on the steady-state output in a closed economy environment. They find that aggregate productivity is rapidly decreasing with trend inflation. This is simply because price dispersion increases the amount of labor required to produce a given level of output, according to

\[ Y_t = \frac{Z_t}{s_t} N_t. \]

A loss in productivity in turn translates into an output loss. As a result, long-run superneutrality breaks down and a negative long-run relationship between inflation and output emerges. Following the same reasoning, in our model, steady-state domestic output \( Y_A \) decreases as domestic inflation \( \Pi_A \) increases.

Besides the domestic effect, trend inflation in country A will also affect the steady-state

\[ Y_A = \frac{Y_{A,0}}{s_t}, \]

where

\[ s_t = \frac{\Pi_{A,0}}{\Pi_{A,0} + \Pi_{B,0}}. \]

Following Damjanovic and Nolan (2010), they treat price dispersion as a negative shift in productivity and map a percentage increase in price dispersion into an equivalent percentage decrease in aggregate productivity.
output in countries B and C. Panel (a) of Figure 2.3 displays the steady-state output of three countries as a function of trend inflation in countries A and B, respectively, while keeping the inflation rate at zero in country C. There are new features in the open economy environment. As inflation in country A increases, real exchange rates $O^{BA}$ and $O^{CA}$ appreciate (see Panel (b) of Figure 2.3). The expenditure-switching effect leads to increases in outputs in countries B and C.

Figure 2.3: The effects of trend inflation on the steady state of variables. The values of all x-axis are expressed in percentage.

The left graph of Panel (c) depicts the steady-state share of the international currency A in country C as a function of trend inflation in country A, with the solid line and the
dashed line representing a 0 trend inflation and an 8 percent trend inflation in country B, respectively. The steady-state share of international currency A is around 65 percent when trend inflation takes 2 percent in country A and country B has zero trend inflation. This value is roughly the average share of the dollar from 2007 to 2015 in the Eurocurrency market when the U.S. had around 2 percent trend inflation and the euro area had zero trend inflation.

The curve shows that a higher trend inflation in country A undermines the leading position of the international currency A. In particular, when trend inflation in country A is around 8 percent the share of international currency A is on a par with the share of international currency B in country C. That is, when the inflation rate in country A is 8%, it starts to lose its leading international currency status. Sensitivity analysis implies that a higher degree of preference for international currency A allows for a higher trend inflation in country A given the same share of the currency A. The intuition is, a higher preference for the leading international currency in country C will tolerate a higher inflation rate of currency A. Furthermore, a higher degree of substitution between currency A and B will steepen the share curve. More intuitively, with higher elasticity of substitution between currency A and B, country C can more easily substitute away the currency that depreciates more.

Panel (d) of Figure 2.3 shows a relationship between the steady-state balances of international currencies and trend inflation. It conveys two main implications. First, real balances of currency A held by country C are relatively unaffected by different levels of trend inflation in country B. In addition, they are considerably stable at different levels of trend inflation in country A. Second, in contrast, the real balances of currency B held by country C are affected not only by their own country inflation but also by inflation in country A. A decrease in inflation rate in country A will substantially reduce the values of real balances of currency B at each level of its own country’s inflation rate. The combination of high preference for international currency A and a weak substitution between
these two currencies explains this result. Sensitivity analysis shows that the combination of a lower preference \((\gamma = 0.55)\) and a higher elasticity substitution \((\chi = 3)\) generates more responsive real balances of currency A held by country C to different values of trend inflation in country B.

Finally, the Bailey curve for the economy is shown in Figure 2.4.\(^\text{11}\) It depicts the relationship between seigniorage and different steady-state inflation rates. The solid lines represent the seigniorage of international currency A as a function of different levels of trend inflation in country B, and the lines with circles are the seigniorage of international currency B as a function of different levels of trend inflation in country A. Owing to its leading position as an international currency, the seigniorage of international currency A is much higher than the seigniorage of international currency B. Moreover, the thickness of the lines represents the range of seigniorage of one international currency as the inflation rate of the other country varies at any given level of domestic inflation rate. Take the line with circles as an example, when keeping the inflation rate in country B at a certain level, a rise in the inflation rate of country A, which leads to substitution toward the international currency B, will increase seigniorage of currency B. Since currency A is less prone to be affected by inflation in country B, the solid lines are thinner than the lines with circles.

\(^\text{11}\) The Bailey curve, was named for Martin Bailey, who first applied this kind of curve to seigniorage in Bailey (1956).
2.5.2 Data Moments vs. Model Moments

The data used to compare the statistic properties are quarterly observations of real GDP per capita, real consumption per capita, consumer price inflation rates in the euro area and the United States, and the share of the liabilities in U.S. dollars in Eurocurrency markets. The sample period is 1999Q1–2015Q4. It starts at 1999Q1 because the euro was introduced to world financial markets as an accounting currency on 1 Jan 1999. All variables are logged and detrended by HP filter. In Table 2.2, under each column labeled “Data”, we summarize a series of statistics that describe characteristics of the euro area and the United States. These characteristics refer to the second moments, first-order
autocorrelation of each series, and the cross-correlation between theses series.

Table 2.2: Second moments (orthogonal shocks)

<table>
<thead>
<tr>
<th>Standard Deviation(%)</th>
<th>U.S.</th>
<th>Euro Area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>GDP</td>
<td>1.61</td>
<td>1.18</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.15</td>
<td>0.96</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.50</td>
<td>0.51</td>
</tr>
<tr>
<td>Share of the U.S. dollar</td>
<td>4.74</td>
<td>2.30</td>
</tr>
</tbody>
</table>

Autocorrelation

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Euro Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.78</td>
<td>0.88</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.80</td>
<td>0.92</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.44</td>
<td>0.18</td>
</tr>
<tr>
<td>Share of the U.S. dollar</td>
<td>0.90</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Cross Correlation within Countries

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Euro Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP-Consumption</td>
<td>0.96</td>
<td>0.87</td>
</tr>
<tr>
<td>GDP-Inflation</td>
<td>0.10</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Cross Correlation over Countries

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Euro Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>−0.05</td>
<td>0.56</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.37</td>
<td>0.41</td>
</tr>
<tr>
<td>Inflation</td>
<td>−0.11</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Cross Correlation with the share of the U.S. dollar

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Euro Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.73</td>
<td>0.19</td>
</tr>
<tr>
<td>Inflation</td>
<td>−0.06</td>
<td>−0.10</td>
</tr>
</tbody>
</table>

Notes: The trend inflation is assumed to be 2% in all three countries in the model.

The structural shocks considered in the model are technology shocks and monetary policy shocks, which are orthogonal to each other in all three countries. Since both the target inflation rates of the European Central Bank and of the Federal Reserve are 2%, we assume a 2% steady-state inflation rate of all the countries in the model. The calibrated model captures the standard deviations of real consumption and inflation in the U.S.. However, it generates relatively the same results for the euro area and the U.S.. This is because of the symmetric assumption of the structural parameters. In the model, the
standard deviation of consumption is higher than the standard deviation of inflation in the euro area, while the opposite holds in the data. The standard deviation of the share of the U.S. dollar in the model is also higher than in the data. This is because the share of the U.S. dollar is affected by shocks from all three countries.

The assumption of orthogonal shocks across countries implies that foreign countries are affected by shocks from their home country only through the spillover effect. But the spillover effect generated by the trade flows and the terms-of-trade effect are not strong enough to generate the observed synchronization across countries. This synchronization will reduce the volatility of the share of the U.S. dollar. The traditional solution for creating a positive correlation between the endogenous variables is to allow for a correlation between the innovations. By assuming a correlation of 0.2 between technology shocks in the euro area and the U.S., and a correlation of 0.29 between monetary policy shocks in the euro area and the U.S., the model generates a lower standard deviation of 3.95% of the share of the U.S. dollar.\footnote{Yu (2012) shows that the cross correlation between technology shocks in the U.S. and a group of 9 OECD countries is 0.26. The estimation results in De Walque, Smets, and Wouters (2005) show that the cross correlation between TFP shocks in the euro area and the U.S. is 0.07 and the cross correlation between monetary policy shocks is 0.29.} Moreover, the model reproduces positive correlation across countries (see Table 2.3 for comparison). However, it still fails to generate a positive correlation between the share of the U.S. dollar and the real GDP or a negative correlation between the share of the U.S. dollar and the inflation in the euro area.
Table 2.3: Second moments (correlated shocks)

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th></th>
<th>Euro Area</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model Data</td>
<td>Model Data</td>
<td>Model Data</td>
<td>Model Data</td>
</tr>
<tr>
<td>Standard Deviation(%)</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>GDP</td>
<td>1.60 1.18</td>
<td>1.59 1.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>1.19 0.96</td>
<td>1.18 0.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.49 0.51</td>
<td>0.51 1.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of the U.S. dollar</td>
<td>3.95 2.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autocorrelation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.78 0.88</td>
<td>0.81 0.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.78 0.92</td>
<td>0.81 0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.47 0.18</td>
<td>0.51 0.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of the U.S. dollar</td>
<td>0.89 0.73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross Correlation within Countries</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP-Consumption</td>
<td>0.96 0.87</td>
<td>0.96 0.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP-Inflation</td>
<td>0.08 0.32</td>
<td>0.12 0.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross Correlation over Countries</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.19 0.56</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.56 0.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.14 0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross Correlation with the share of the U.S. dollar</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.68 0.19</td>
<td>−0.51 0.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>−0.05 −0.10</td>
<td>0.20 −0.22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The trend inflation is assumed to be 2% in all three countries in the model.

2.6 Dynamic Analysis

Changes in trend inflation affect the steady state, which in turn affect the log-linear dynamics of the macroeconomy in response to shocks, simply because they change the point around which the model is log-linearly approximated. More specifically, they affect the parameters of the log-linearized model; as shown in section 2.2, trend inflation not only flattens the NKPC, but also adds an additional channel to the NKPC, taking in
more effects from international relative prices. To analyze these effects, in particular how trend inflation rates in countries A and B affect the dynamics of the share of international currency A in country C, we consider two kinds of structural shocks, technology shocks and monetary policy shocks. We set the 0% inflation environment as the baseline case and compare it with cases when trend inflations take 2%, 4% and 8% in country A while keeping the trend inflation rate in country B at 2%. At the end of section 2.6.2, we will briefly report the results of these shocks in the cases of keeping trend inflation in country B at 8% while trend inflation in country A is 2%, 4%, and 8%. In all experiments, we set the steady-state inflation rate in country C at 0%.

2.6.1 Technology Shocks

This section examines the impulse responses to one standard deviation of uncorrelated positive technology shocks emanating from each of the three countries. Figures 2.5–2.6 report impulse responses of key macroeconomic variables to a positive technology shock originating from country A. The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.

On the whole, in the baseline case, as in the standard open NK model, when countries produce substitutes, a positive technology shock increases domestic output but reduces foreign outputs through the expenditure-switching effect. According to our definition of terms of trade, the declines in $T_{BA}$ and $T_{CA}$ indicate that terms of trade deteriorate for country A. The expenditure-switching effect will negatively affect outputs of countries B and C, whereas the intertemporal consumption substitution effect will increase the import demand of country A. According to the calibration of the model, the first effect dominates
the second. Outputs in countries B and C thus decrease.

Moreover, in the model, by introducing real money balances through the Cobb-Douglas form, a positive cross derivative $U_{cm}$ implies a stronger increase of consumption in country A. The high persistence of the technology shock will increase the foreign outputs at first. Inflation falls in all countries. Central banks thus cut interest rates. The decreased interest rate will in turn increase consumption in each country. Because the reduction in interest rate is larger in country A than in country B, country C will increase its holding of international currency A and deplete its holding of international currency B. The share of international currency A is therefore increased.

Comparing the cases with trend inflation, a decreased inflation rate in country A decreases price dispersion because $\partial s^A_t / \partial \hat{\pi}^A_t > 0$ and hence decreases marginal cost in country A. Output increases more than in the baseline case, i.e., there is a outward shift of $Y_{A,t}$. With a more flattened NKPC, the higher the trend inflation rate, the less firms react to a raise in output in country A. Therefore, compared to output responses, inflation reacts less. According to the price dispersion equation $s^A_t = \epsilon (\Pi^A)^{\epsilon-1} \left[ \frac{\Pi^A - 1}{1 - \theta (\Pi^A)^{\epsilon-1}} \right] \hat{\pi}^A_t + \theta (\Pi^A)^\epsilon s^A_{t-1}$, price dispersion will feed back to inflation, hence output, making responses more persistent. Higher trend inflation leads to a higher price dispersion response. The more responsive inflation rate in country A will reflect in the terms of trade between countries A and B and in the terms of trade between countries A and C as well. This will in turn decrease the outputs in countries B and C during the adjustment process to their steady-state values.

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13 The first effect is governed by the elasticity of substitution across countries, $\eta$ and the second is governed by the parameter $1/\sigma$, in our calibration, $\eta > 1/\sigma$.

14 This is because the anticipation of a nominal rate increase (hence, of a decline in real balances) lowers the expected one period ahead level of the marginal utility of consumption (for any expected consumption next period), which induces an increase in current consumption (see Galí (2009)).
Figure 2.5: IRFs to a positive technology shock originating from country A. The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.
Figure 2.6: IRFs to a positive technology shock originating from country A (continued). The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.

With trend inflation, the interest rate in country A is further reduced. In the case of 8% trend inflation in country A, the increase of $C_{A,t}$ (due to the decreased $R_{A,t}$ and the anticipation of the increase in $R_{A,t+1}$) brings a major increase in $Y_{B,t}$ and $Y_{C,t}$, which leads
Chapter 2. *International Currency, Trend Inflation, and Indeterminacy*

to an initial increase in interest rates in countries B and C. Because of the a weak substitution between the international currencies A and B, the effects of change in $M_{BC,t}$ and change in $\hat{R}_{At}$ on $M_{AC,t}$ are limited.\textsuperscript{15} Thus less response of the share of the international currency A compared to the baseline case is observed in Figure 2.5.

Figures 2.7–2.8 draw impulse responses of the selected variables when a positive technology shock takes place in country B. The transmission mechanism is the same as in the case above. Note that since the trend inflation in country B is only 2%, the difference of responses of country B’s variables between the baseline case and all other cases is quite small. Notice that different levels of trend inflation in country A affect the responses of its variables in different magnitude mainly through the price dispersion effect. In particular, when the trend inflation is around 8% in country A, a larger reduction in price dispersion will further increase the output in all countries, i.e., will lead to an upward shift of $Y_A$. Inflation, interest rate and consumption in country A will have a relatively larger adjustment accordingly. Because $\partial \hat{M}_{BC,t} / \partial \hat{R}_{B,t}$ decreases in $\Pi_B$, the increase in the holding of international currency B is less than in the baseline case. The share of international currency A decreases. However, due to the a weak substitution between international currencies A and B and a large value of preference for the international currency A, $\gamma$, this decline is no larger than in the baseline case when country A has 2% trend inflation and country B has trend inflation less than 4%.\textsuperscript{16} Figures 2.9–2.10 display impulse responses of the selected variables to the positive technology shock originating from country C. This positive shock will increase the holdings of both international currencies. Due to the spillover effect, the interest rate reduction in countries A and B are quite small. The change in the share of the international currency A is also small.

\textsuperscript{15} More specifically, when $\chi = 1.24$, $\partial M_{AC,t} / \partial M_{BC,t} = \frac{(1-\chi)b_2}{1-(1-\chi)b_1} \in (-0.04, -0.02)$, and $\partial M_{AC,t} / \partial \hat{R}_{At} = \frac{\chi}{(\Pi_A - 1)(1-(1-\chi)b_2)} \in (34.73, 102.21)$, where $b_1, b_2 \in (0, 1)$, $b_1 + b_2 = 1$, and $b_1 > b_2$. $\partial M_{AC,t} / \partial \hat{R}_{At}$ is a decreasing function of $\Pi_A$ and $\gamma$ and a increasing function of $\chi$. $\partial M_{AC,t} / \partial \hat{R}_{At}$ decreasing in $\Pi_A$ also explains the overlapping of the line with plus signs (4% inflation in country A) and the line with dots (8% inflation in country B) in panel ‘Real money balances $M_{AC}$’ in Figure 2.5.

\textsuperscript{16} According to our experiment, when country A has trend inflation around 5% while keeping the inflation rate of country A at 2%, the decline in the share will be approximately the same as in the baseline case.
Figure 2.7: IRFs to a positive technology shock originating from country B. The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.
Figure 2.8: IRFs to a positive technology shock originating from country B (continued). The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.
Figure 2.9: IRFs to a positive technology shock originating from country C. The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.
Figure 2.10: IRFs to a positive technology shock originating from country C (continued). The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.
2.6.2 Monetary Policy Shocks

Figures 2.11–2.12 illustrate the responses of selected variables to one standard deviation of uncorrelated expansionary monetary policy shock in country A. In the baseline case, an expansionary monetary policy shock increases both output and inflation in the domestic country and contracts outputs and inflation rates in foreign economies due to the expenditure-switching effect. Consumption increases in all three countries because of the intertemporal substitution effect. Although the qualitative pattern with trend inflation is similar to the baseline case, some key differences are worth stressing.

First, since the change in inflation is quite small and short-lived, a higher trend inflation does not make an observable difference in dynamics of inflations or terms of trade. Higher trend inflation will bring a larger increase in price dispersion in country A and thus reduce output in country A.

Second, the central bank in country A targeting both output gap and inflation rate raises the interest rate in reaction to the increases in domestic inflation and output. However, since the shock is not persistent ($\rho_{ra} = 0.04$), this increase in the interest rate does not outweigh the decrease in the interest rate caused by the exogenous monetary shock. The domestic interest rate declines as a result. This increases the share of international currency A. However, compared with the baseline case, higher trend inflation indicates a smaller increase.
Figure 2.11: IRFs to an expansionary MP shock originating from country A. The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.
Figure 2.12: IRFs to an expansionary MP shock originating from country A (continued). The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.

Figures 2.13–2.14 show that an expansionary monetary policy shock from country B will reduce the share of international currency A. When country A has trend inflation of less than 4%, the decline is no larger than in the baseline case. When country A has trend inflation of 8%, a major reduction is observed.
Figures 2.15–2.16 depict the case with an expansionary monetary policy shock from country C. The solid line and the dashed line coincide because in the baseline case and in the case with $\Pi_A = \Pi_B = 2\%$, a monetary policy shock in country C will bring about relatively the same size of change in $R_{A,t}$ and $R_{B,t}$. A trend inflation of 4\% or more in country A will reduce this share.

As a comparison, we investigate the cases with high persistence expansionary monetary policy shocks ($\rho_r = 0.8$). As expected, the variables respond by a larger magnitude. An anti-inflationary monetary policy will raise the domestic interest rate. In this scenario, the impact of the endogenous policy adjustment dominates the impact of the exogenous negative shock on the domestic interest rate. Consequently, the domestic interest rate increases instead of decreasing. This leads to the inverse response of the share of the international currency A compared with the cases with a low persistence of 0.04, i.e., the share decreases in response to a negative monetary policy shock from country A and increases in response to a negative monetary policy shock from country B. Likewise, the higher the trend inflation in country A, the larger the increment (the smaller the reduction). A moderate trend inflation (2\%–4\%) indicates a less volatile response compared with the zero trend inflation case.

\(^{17}\) As a matter of fact, it is an order of magnitude larger than cases without persistence.
Figure 2.13: IRFs to an expansionary MP shock originating from country B. The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.
Figure 2.14: IRFs to an expansionary MP shock originating from country B (continued). The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.
Figure 2.15: IRFs to an expansionary MP shock originating from country C. The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.
Figure 2.16: IRFs to an expansionary MP shock originating from country C (continued). The solid line depicts the responses to shocks when trend inflation rates in all three countries are zero. The dashed line, the dashed line with plus signs, and the solid line with dots represent the responses to shocks when country A has 2%, 4%, and 8% trend inflation, respectively, while keeping the trend inflation in country B at 2%.

We also examine the impulse responses to the above two kinds of shocks when trend inflation in country B is set at 8% and trend inflation in country A takes 2%, 4%, and 8% in turn. Although the qualitative patterns are similar, they differ in several ways. First, in the case of a positive technology shock taking place in country A, a higher trend inflation
rate in country A will shift the responses of domestic output and inflation outwards further. Moreover, higher trend inflation increases the persistence of responses. Second, in contrast to the cases with technology shocks, in the cases with monetary policy shocks, a higher trend inflation in country B does not generate too much difference compared with the cases when country B has a 2 percent trend inflation. Third, in all cases, a higher trend inflation in country B will allow country A to have a higher trend inflation, which keeps the changes of share less volatile than in the baseline case.

2.6.3 Sensitivity Analysis

This section checks the robustness of our findings to changes in the value of elasticity of substitution between international currencies and the preference for the international currency A in the currency composite. As expected, a higher degree of substitution between international currencies A and B and a lower value of $\gamma$ in the currency portfolio will undermine the leading role of international currency A. We explore the cases when parameter $\chi$ takes a larger value, i.e., from 5 to 20 and $\gamma$ takes a lower value, i.e., 0.52. The general conclusion is that the key results found in the previous analysis persist in the cases when $\chi$ is smaller than 12 while keeping $\gamma$ above 0.52.

Figure 2.17 illustrates the responses of the share of international currency A to all shocks when $\gamma$ is 0.52 and $\chi$ is 12. The solid line with dots represents the responses to shocks in the zero trend inflation baseline case. The solid line with circles and the dashed line with triangles depict responses to shocks when trend inflation in country A is 2% and 4%, respectively while holding the trend inflation rate in country B at 2%. The case when $\chi$ is 12 and $\gamma$ is 0.52 is special because the responses of the share in the scenario of $\Pi_A = \Pi_B = 2\%$ coincide with the responses of the share in the baseline case. Therefore, any larger value of $\Pi_A$ than $\Pi_B = 2\%$ indicates a more volatile response of the share of the international currency A compared with the baseline case.
Figure 2.17: IRFs of the share of the international currency A to all shocks. The solid line with dots represents the responses to shocks in the zero trend inflation baseline case. The solid line with circles and the dashed line with triangles depict responses to shocks when trend inflation in country A is 2% and 4%, respectively while holding the trend inflation rate in country B at 2%. In all cases γ takes 0.52 and χ is 12.

2.7 Concluding Remarks

This paper sets out to study how trend inflation affects the share of the leading international currency in global payments. We construct a three-country sticky-price DSGE model for the theoretical analysis where the share of the leading international currency is endogenous and trend inflation is embedded in the linearization of the NKPC.
The main results are as follows. First, in the long run, the position of the leading international currency persists even if the inflation rate in its own country increases from 0 to 8 percent. In addition, real balances of the leading international currency A held by the third country are relatively unaffected by the level of inflation in the emerging international currency country, whereas the real balances of the emerging international currency held by the third country are affected not only by inflation in its own country but also by inflation in the leading international currency country. This in turn affects the seigniorage collected by each country.

Second, in the short run, when both international currency countries start with the same trend inflation, an increase of up to 3 percent in trend inflation in the leading international currency country will not lead to a more volatile share of its currency compared with the zero inflation baseline case. Sensitivity analysis shows that the combination of a weak substitution between the leading and the emerging international currencies and a high preference for the leading international currency of the third country generates these results.

Third, incorporation of trend inflation in an open economic environment reveals new dynamics. Domestic trend inflation amplifies the spillover effects of a domestic technology shock on foreign countries through both the expenditure-switching effect and the intertemporal substitution effect. Trend inflation in foreign countries reinforces these spillover effects through the effect of price dispersion. But in the case of low persistent monetary policy shocks, trend inflation effects are quite limited. Finally, trend inflation will greatly affect the determinacy regions in the Taylor rule’s parameters space of the model. More specifically, when all three countries adopt an identical monetary policy, the determinacy region shrinks rapidly with trend inflation. This indicates that the monetary policy should respond more aggressively to the deviation of inflation rate from its target.
Chapter 3

Should the Yuan Become an International Currency?

3.1 Introduction

As the world’s largest trading nation and second largest economy, China needs a currency with international status that can match its economic status. Over the past three years, the Chinese authorities have taken a number of steps to internationalize the yuan. According to the People’s Bank of China statistics, by May 2015, China had signed currency swap deals with 31 countries and regions for a total of 3 trillion yuan. In the meantime, the demand for the international usage of yuan is expanding. According to Society for Worldwide Interbank Financial Telecommunication statistics, since November 2014, the Chinese yuan has entered the top five world payment currencies. By October 2013, the market share of the yuan in the global traditional trade finance, that is, letters of credit and payments, had reached 8.66 percent. It is the first time that yuan has surpassed the
euro to become the world’s second largest trade settlement currency.\textsuperscript{18} Overall, global yuan payments increased in value by 20.3\% in December 2014, while the growth for payments across all currencies was 14.9\%. Over the past two years, the yuan has had a consistent three digit growth over the past two years with an increase in value of payments by 321\%. The evolution of the international role of the yuan raises a number of important questions, especially what are the costs and benefits of the yuan becoming an international currency? This paper constructs a two-country DSGE model in which the home country provides the international currency and the foreign country uses it to purchase imports to answer this question.

There is no well-established framework to define what is meant by internationalization of a currency. Kenen (1994) gave an early specification of the functions of the dollar as an international currency. Chinn and Frankel (2007) summarizes the functions of international currency in a broad definition that: an international currency is a currency that fulfills the three functions of money (as a medium of exchange, a unit of account, and a store of value) in an international context. A fully-fledged international currency should thus fulfill six functions. The private sector uses the international currency as a medium of exchange for trade in goods, services and assets. They also use it as a way to cheaply exchange two currencies, i.e., by carrying out two separate transactions against the international vehicle and as an invoice currency for goods and assets. A meaningful distinction made by Truman (2007) is between a "reserve currency" and an "international currency" with the former referring to official transactions and the latter to transactions involving foreign private agents. According to Prasad and Ye (2013), the internationalization of the yuan refers to: (1) internationalization, corresponding to the medium of exchange role; (2) capital account convertibility; and (3) reserve currency, in which role the yuan is held by foreign central banks as international reserves. Therefore, for the moment, without full convertibility and capital account liberalization, yuan internationalization is in the

\textsuperscript{18}U.S. dollar still holds the first place, which accounts for 81.08\% of market share in international trade settlement. The portion of yuan is still small. The top five regions and countries for yuan trade settlement business are Mainland China, Hong Kong, Singapore, Australia, and Germany.
first stage: of becoming a major settlement and pricing currency in trade.

This paper analyzes the costs and benefits of yuan internationalization in its role as the medium of payments in international exchanges. We investigate this issue by developing a two-country DSGE model with asymmetric cash-in-advance (CIA) constraints in which the home country provides the international currency whereas the foreign country has to accumulate it to purchase imports. The asymmetric CIA constraints characterize the medium of payments function. Moreover, we examine the implications of the model under two different pricing behaviours of the firms. One is the symmetric producer currency pricing (PCP). The other is the asymmetric producer currency pricing–local currency pricing (PCP-LCP). We use the PCP-LCP assumption to characterize the function of unit of account of the international currency.

Results show that, seigniorage is a major source of benefit to the home country even in the first stage of internationalization of its currency. More importantly, in all cases, technology shocks and monetary policy shocks in the home country have greater effects on consumption and output abroad than equivalent shocks in the foreign country. Monetary policy in the home country is thus more potent. This result is consistent with Canzoneri et al. (2013), although they assumed that international currency bonds to play the role of facilitating the trade and serving as international reserves.

Costs of having an international currency are: (i) the seigniorage collected by the home country decreases in response to a positive technology shock in the home country; and (ii) the home country’s households experience losses because an asymmetric exchange rate pass-through leads to a sluggish adjustment of consumption in the home country. These costs are only incurred in the PCP-LCP case. The finding that the home country losses from PCP-LCP behaviour, though a surprising one, accords with Devereux, Shi, and Xu (2004). According to Obstfeld and Rogoff (1995), under PCP setting, international risk sharing is symmetric. Devereux, Shi, and Xu (2004) show that PCP-LCP behaviour, which they call "dollar standard", generates the asymmetry in risk sharing.
that the exchange rate pass-through is zero for the home country but positive for the
foreign country. We show that, in the PCP-LCP model, shocks emanating from the home
country can only be absorbed by the over-adjustment of domestic prices' and that the
movement of variables to steady state is relatively gradual compared to the responses in
the PCP model.

This paper relates to different strands of the literature. Firstly, the advantages and dis-
advantages of having an international currency (including the function of store of value)
have long been discussed. Chinn and Frankel (2007) provides a summary on this topic.
Advantages of running an international currency include: convenience for the country’s
residents, political power, reputation, and international seigniorage which is most impor-
tant and can be directly quantifiable. Costs include accumulated foreign liabilities and the
burden of responsibility. Eichengreen (2011) discusses the exorbitant privilege of the dol-
lar and the challenge of the euro and the yuan to the dollar’s international currency role.
More recently, Canzoneri et al. (2013) construct a two-country DSGE model calibrated
to the US data, which focuses on the transaction services and international reserves that
international currency bonds provide. They conclude that the exorbitant privilege accru-
ing to the international currency comes from the bond seigniorage, asymmetric responses
to exogenous shocks and a macroeconomic hegemony in monetary and fiscal policy.

This paper is also related to another fast-growing strand of the literature on pricing to
market (PTM). Many international macro models assume that prices are either all rigid
in the producer’s currency or in the local currency, and this assumption is symmetric
across countries. The PCP assumption is straightforward that firms set prices in domestic
currency for goods sold both domestically and abroad. Obstfeld and Rogoff (1995) adopt
the PCP assumption in their two-country model. However, this assumption is challenged
by the phenomenon that foreign firms maintain or even increase their export prices in the
U.S. when the dollar rises.
Krugman (1986) first examines this PTM phenomenon. Betts and Devereux (1996) pioneer in developing a general equilibrium model with symmetric PTM strategy adopted by both countries firms. They assume that firms set prices in the domestic currency for the goods sold domestically and in the foreign currency for the foreign market. This price setting behaviour is thus called LCP. Okano et al. (2012) analyze optimal monetary policy in a symmetric LCP model by comparing it with a symmetric PCP model. However, in the case of the U.S. economy, contrary to the symmetric LCP behaviour, evidence from micro data shows that 90% of imports are priced in US dollars while 97% of exports are priced in US dollars, which suggests an LCP in imports and PCP in exports for the U.S. (see Gopinath and Rigobon (2008)). According to Gopinath, Itskhoki, and Rigobon (2010), the fraction of non-dollar priced imports from most of countries is small. This is the case of “dollar pricing” (DP). Devereux, Shi, and Xu (2004) provide an early model with dollar pricing. Corsetti et al. (2007) also discuss the asymmetric transmission of DP.

The remainder of this paper is organized as follows: Section 3.2 outlines the models with both the symmetric PCP setting and the asymmetric PCP-LCP setting. Section 3.3 solves the model and discusses the choice of parameters in the model. Finally, Section 3.4 presents the main results while Section 3.5 draws some final conclusions.

3.2 The Model

We construct a two-country model belonging to the class of DSGE models with nominal rigidities and imperfect competition. The economy consists of two asymmetric countries, $H$ and $F$. The asymmetry lies in that country $H$ provides the international currency, while country $F$ acquires the $H$ country’s currency to facilitate trade. Apart from this international currency, they share identical preferences and the labor market is competitive. In addition, we derive two models; one of them adopts PCP, while the other adopts PCP-LCP. The way we introduce the deviations from the law of one price (LOOP) follows
Corsetti, Dedola, and Leduc (2010) and Okano et al. (2012).

The economy of the home and the economy of the foreign each is populated by a representative household, a monetary authority, and firms. Each firm produces a differentiated good and sets nominal prices in a staggered fashion à la Calvo (1985). This nominal rigidity allows a monetary policy to have an influence on real activities in the short run.

Next we describe in detail the problems faced by households and firms located in such economy. Before doing so, a brief remark on notation is in order. Each country produces a continuum of differentiated goods, represented by the unit interval, with \( i \in [0, 1] \). Preferences over these goods are of the Dixit-Stiglitz type, and all goods are internationally tradable. Finally, variables with a star superscript correspond to the foreign economy.

### 3.2.1 PCP Model

Under PCP assumption, firms in both countries only set prices in the domestic currency for goods sold both domestically and abroad. The LOOP holds, which is given by \( P_{H,t}(i) = Z_t P_{H,t}^*(i) \) and \( P_{F,t}(i) = Z_t P_{F,t}^*(i) \) (\( Z_t \) denotes the bilateral nominal exchange rate). Since we assume that all goods are tradable, PPP holds: \( P_{H,t} = Z_t P_{H,t}^* \) and \( P_{F,t} = Z_t P_{F,t}^* \).

#### 3.2.1.1 Households

A representative household in country \( H \) derives utility from consumption \( C_t \), and disutility from hours of work \( N_t \). Its preferences are described by the following expected utility function:

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t),
\]

\[ (3.1) \]
where $E_t$ denotes the expectation, conditional on the information set at period $t$ and $\beta \in (0, 1)$ denotes the discount factor. The household’s preference has the form of:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi},$$

where $\sigma$ denotes the degree of relative risk aversion and $\phi$ denotes the inverse of the labor supply elasticity. The preferences of the representative household in country $F$ are defined analogously. More precisely, $N_t \equiv \int_0^1 N_t(i) di$ and $C_t$ is a composite consumption index defined by:

$$C_t \equiv \left[ (1 - \alpha)^\frac{1}{\epsilon} (C_{H,t})^\frac{\eta-1}{\eta} + \alpha^\frac{1}{\epsilon} (C_{F,t})^\frac{\eta-1}{\eta} \right]^\frac{\eta}{\eta-1}, \tag{3.2}$$

where $C_{H,t}$ is an index of consumption of domestic goods, defined as $C_{H,t} \equiv \left( \int_0^1 C_{H,t}(i)^\frac{1}{1-\epsilon} di \right)^{1-\epsilon}$. Similarly $C_{F,t}$ is an index of imported goods, given by and $C_{F,t} \equiv \left( \int_0^1 C_{F,t}(i)^\frac{1}{1-\epsilon} di \right)^{1-\epsilon}$. Parameter $\epsilon$ denotes the elasticity of substitution between varieties produced within any given country. Parameter $\alpha \in [0, 1]$ is related to the degree of home bias in preferences and is taken to provide an index of openness. Parameter $\eta > 0$ measures the substitutability between domestic and foreign goods, from the viewpoint of the domestic consumer. $C_{t}^*$ is defined analogously to $C_t$ in Eq. (3.2).

The optimal allocation of any given expenditure within each category of goods yields the demand functions as follows:

$$C_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} C_{H,t}^*$$

$$C_{F,t}(i) = \left( \frac{P_{F,t}(i)}{P_{F,t}^*} \right)^{-\epsilon} C_{F,t}^*$$

$$C_{H,t}^* = \left( \frac{P_{H,t}(i)}{P_{H,t}^*} \right)^{-\epsilon} C_{H,t}^*$$

$$C_{F,t}^* = \left( \frac{P_{F,t}(i)}{P_{F,t}^*} \right)^{-\epsilon} C_{F,t}^*$$

where $P_{H,t} \equiv \left( \int_0^1 P_{H,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ is the domestic price index and $P_{F,t} \equiv \left( \int_0^1 P_{F,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ is a price index for goods imported from country $F$ (expressed in domestic currency). In the same sense, the optimal allocation of expenditures between domestic and foreign goods
is given by:

\[
C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t,
\]

\[
C_{H,t}^* = (1 - \alpha) \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \quad ; \quad C_{F,t}^* = \alpha \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\eta} C_t^*.
\]

The consumer price index in the country \(H\) is derived from the above optimal allocations and is expressed as a combination of domestic price index and price index of imports:

\[
P_t \equiv [(1 - \alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}}.
\]

\(P_t^*\) is defined analogously to this equation. Combining expressions \( \int_0^1 P_{H,t}(i)C_{H,t}(i)di = P_{H,t}C_{H,t} \) and \( \int_0^1 P_{F,t}(i)C_{F,t}(i)di = P_{F,t}C_{F,t} \) with Eq. (3.4), we obtain,

\[
C_{H,t}P_{H,t} + C_{F,t}P_{F,t} = C_tP_t.
\]

The timing is as follows, in each period the representative agent must first go to the money market and obtain cash to facilitate transactions in the consumption goods markets. The asymmetry lies in the CIA constraints that households faces. \(H\) country provides the international currency. Therefore, for both domestic goods and imported goods \(H\) households only need \(H\) currency while \(F\) households have to acquire domestic currency to buy domestic goods and \(H\) currency to buy imports.

The maximization of Eq. (3.1) for households in country \(H\) is subject to the following constraints:

\[
C_{H,t}P_{H,t} + C_{F,t}P_{F,t} \leq M_{H,t},
\]

\[
C_{H,t}P_{H,t} + C_{F,t}P_{F,t} + M_{H,t} + P_tB_{H,t} + P_t^*B_{F,t}Z_t + \frac{\tau}{2}P_tB_{H,t}^2 + \frac{\tau}{2}P_t^*B_{F,t}^2Z_t \\
\leq W_{t}N_{t} + M_{H,t-1} + P_{t-1}B_{H,t-1}R_{t-1} + P_{t-1}^*B_{F,t-1}R_{t-1}^*Z_{t} + D_{t} + T_{t} + T_{t}^f.
\]
In each country, firms are assumed to be owned by domestic residents and profits are distributed across households denoted as \( D_t \), in a lump-sum form: 
\[
D_t = Y_t P_{H,t} - W_t N_t.
\]
Households in each country hold two kinds of bonds: domestic and foreign risk-free bonds. In country \( H \), the holdings of \( H \) and \( F \) bonds carried over from preceding period: \( B_{H,t-1} \) and \( B_{F,t-1} \), in units of consumption, earn gross nominal returns \( R_t \) and \( R^*_t \), respectively. Following Schmitt-Grohé and Uribe (2003), we introduce a quadratic adjustment cost of holding bonds, \( \tau_t P_t B_{H,t}^2 \) and \( \tau_t P_t B_{F,t}^2 Z_t \). Doing so aims at avoiding the problem of indeterminacy of the steady state in the two-country incomplete financial market model. \( \tau \) denotes the scale parameter for the bond adjustment cost. We follow Ghironi and Melitz (2005) in setting it at 0.0025. Such a low value is sufficient to generate stationarity in response to transitory shocks but is small enough to avoid overstating the role of this friction in determining the dynamics of the model. \( T_f \) is the fee rebate, taken as given by households, and equals \( \frac{\tau_t P_t B_{H,t}^2}{2} + \frac{\tau_t P_t B_{F,t}^2 Z_t}{2} \) in equilibrium.

The maximization of Eq. (3.1) for households in country \( F \) is subject to the following constraints:

\[
C^{*}_{F,t} P_{F,t}^* \leq M_{F,t}^*, \tag{3.9}
\]

\[
C^{*}_{H,t} P_{H,t}^* \leq M_{H,t}^*/Z_t, \tag{3.10}
\]

\[
C^{*}_{F,t} P_{F,t}^* + C^{*}_{H,t} P_{H,t}^* + M^*_{H,t} + M^*_{H,t}/Z_t + P^*_t B^*_{F,t} + P^*_t B^*_{H,t}/Z_t + \frac{\tau}{2} P^*_t B^*_{F,t}^2 + \frac{\tau}{2} P^*_t B^*_{H,t}^2/Z_t \leq W^*_t N^*_t + M^*_{F,t-1} + M^*_{H,t-1}/Z_t + P^*_t B^*_{F,t-1} R^*_t + P^*_t B^*_{H,t-1} R^*_t - 1/Z_t + D^*_t + T^*_t + T^*_{f}. \tag{3.11}
\]

The interpretation of the regular resource constraint Eq. (3.11) is similar to that for the \( H \) country. The \( F \) country has two CIA constraints instead of one for the \( H \) country: Eq. (3.9) indicates that to consume domestic products, \( F \) households need domestic currency \( M^*_{F,t} \); the other is Eq. (3.10), which indicates that imports consumption requires the international currency \( M^*_{H,t} \).
The optimal conditions for $H$ households are given by:

$$\frac{C_t - \sigma}{P_t} = \left[ 2 - \frac{1}{R_t} (1 + \tau B_{H,t}) \right] \frac{N_t^\phi}{W_t},$$

(3.12)

which is an intratemporal allocation of $C_t$ and $N_t$ and

$$E_t \left[ \frac{N_t^\phi}{W_{t+1}} \right] = \frac{N_t^\phi}{W_t} (1 + \tau B_{H,t}),$$

(3.13)

which is an Euler equation, and:

$$\frac{R_t}{R_t^*} = E_t(Z_{t+1}) \frac{1 + \tau B_{H,t}}{1 + \tau B_{F,t}},$$

(3.14)

which is an uncovered interest rate parity (UIP) relationship for the gross nominal interest rate between countries $H$ and $F$.

The somewhat similar optimal conditions for $F$ households are:

$$\frac{C_{t}^* - \sigma}{P_{t}^*} = \left[ 2 - (1 - \alpha) \frac{1 + \tau B_{F,t}^*}{R_t^*} \left( \frac{P_{F,t}^*}{P_t^*} \right)^{1-\eta} - \alpha \frac{1 + \tau B_{H,t}^*}{R_t^*} \left( \frac{P_{H,t}^*}{P_t^*} \right)^{1-\eta} \right] \frac{N_t^\phi}{W_t^*},$$

(3.15)

$$E_t \left[ \frac{N_t^{\phi^*}}{W_{t+1}^*} \right] = \frac{N_t^{\phi^*}}{W_t^*} (1 + \tau B_{F,t}^*),$$

(3.16)

$$\frac{R_t}{R_t^*} = E_t(Z_{t+1}) \frac{1 + \tau B_{H,t}^*}{1 + \tau B_{F,t}^*}. $$

(3.17)

Notice that the level of bond holding by each country enters into the FOCNs, which helps pin down the value of bond holdings, solving the problem of indeterminacy of the steady state.
3.2.1.2 Law of One Price and Terms of Trade

Before proceeding with analysis of the equilibrium we introduce several assumptions and definitions, and derive a number of identities that are extensively used later.

We start by defining the terms of trade (TOT) as:

\[ S_t = \frac{P_{F,t}}{Z_t P_{H,t}^*}, \]  

(3.18)

where \( S_t \) is the foreign TOT, or the inverse of the home TOT. The numerator is the export price of goods produced in the \( F \) country in terms of the \( H \) country’s currency and the denominator is the import price of goods produced in the \( H \) country in terms of the \( F \) country’s currency.

Since country \( H \)’s firms set prices in terms of the international currency, for goods sold at home and abroad, the LOOP holds; so that \( Z_t P_{H,t}^* = P_{H,t} \). However, this does not necessarily hold for country \( F \)’s firms. Following Monacelli (2003), define the LOOP gap as:

\[ \Psi_t = \frac{Z_t P_{F,t}^*}{P_{F,t}}, \]  

(3.19)

where \( \Psi_t \in (0, 1] \). \( \Psi_t = 1 \) indicates no price setting power granted to country \( F \)’s firms in the PCP part. Therefore, TOT can be simplified to \( S_t = P_{F,t}/P_{H,t} \), or, equivalently, \( S_t = P_{F,t}^*/P_{H,t}^* \). Terms of trade moves one-to-one with the exchange rate, as well as with the domestic relative price of imports. We further discuss the case of violation of the LOOP in Section 3.2.2.
Lastly, the real exchange rate is expressed as \( Q_t = Z_t P_t^* / P_t \). Substituting Eq. (3.5) and Eq. (3.19) into this relationship, we obtain:

\[
Q_t = \left[ \frac{(1 - \alpha) S_t^{1-\eta} + \alpha}{\alpha S_t^{1-\eta} + (1 - \alpha)} \right]^{\frac{1}{1-\eta}}.
\]  

(3.20)

This expression for the real exchange rate would be different from the one we derive later for the PCP-LCP model.

### 3.2.1.3 Firms

Assume a continuum of firms indexed by \( i \in [0, 1] \). Each firm produces a differentiated good, but they all use an identical technology, represented by the production function:

\[
Y_t(i) = A_t N_t(i),
\]

(3.21)

where \( A_t \) denotes the level of technology, assumed to be common to all firms and follows a stationary AR(1) process. \( P_{P,t} \) denotes the PPI in country \( H \), and is defined as:

\[
P_{P,t} = \frac{P_{H,t} C_{H,t} + Z_t P_{H,t}^* C_{H,t}^*}{C_{H,t} + C_{H,t}^*},
\]

which can be rewritten as \( P_{P,t} = P_{H,t} \) when the LOOP holds. The PPI for country \( F \) is defined analogously.

All firms face an identical isoelastic demand schedule given by:

\[
Y_t = \left[ \int_0^1 Y_t(i) \frac{\epsilon - 1}{\epsilon} di \right]^{\frac{\epsilon}{\epsilon - 1}}.
\]

Following Calvo (1985), each firm may reset its price only with probability \( 1 - \theta \) in each period, independently of the time elapsed since its last price adjustment. Combining with the PCP assumption, \( H \) firms only need to set the price \( P_{H,t}(i) \) (or equivalently \( Z_t P_{H,t}^*(i) \)) and \( F \) firms only need to set price \( P_{F,t}(i) \) (or equivalently \( P_{F,t}/Z_t \)). This indicates that under PCP, firms let the foreign currency price of their product vary with the exchange rate. Let \( P_{H,t}(i) \) denote the price set by a firm \( i \), which adjusts its price in period \( t \).
Thus $P_{H,t+k}(i) = P_{H,t}(i)$ with probability $\theta^k$ for $k = 0, 1, 2, \ldots$ Since all firms reset prices in any given period will choose the same price, we can drop the subscript $i$. The firm’s maximization problem would be:

$$\max_{P_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \prod_{k'=0}^{k} \frac{1}{R_{t+k'}} [Y_{t+k}(i)(\bar{P}_{H,t} - MC_{t+k})],$$

s.t. $Y_{t+k}(i) = \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k}$.

This implies the optimal price-setting strategy:

$$\bar{P}_{H,t} = \epsilon \left[ \sum_{k=0}^{\infty} Y_{t+k}(i)(\beta \theta)^k C_{t+k}^{t,\sigma} \frac{P_{H,t-1}}{P_{t+k}} \Pi_{H,t-1,t+k} MC_{t+k} \right]^{-1} \left[ \sum_{k=0}^{\infty} Y_{t+k}(i)(\beta \theta)^k C_{t+k}^{t,\sigma} \frac{P_{H,t-1}}{P_{t+k}} \Pi_{H,t-1} \right]^{\epsilon - 1}.$$  

(3.23)

This corresponds to the one derived by Galí (2009). We can obtain a similar price setting rule for $F$ country’s firms under the PCP assumption.

### 3.2.1.4 Government

Monetary policy is represented by a Taylor rule, which is subject to stochastic financial shocks. Following the literature, the Taylor rule takes a form of:

$$\log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + (1 - \rho_r) \phi_\pi (\Pi_t / \bar{\Pi}) + \epsilon_{r,t},$$

(3.24)

where $\rho_r$ characterizes the persistence of nominal interest rate and $\phi_\pi$ represents the degree of response of interest to inflation. $P_{it}$ denotes the gross inflation rate and $\bar{\Pi}$ denotes the target inflation rate by the Central Bank. This simple monetary policy rule is introduced to reflect the basic principle of monetary policy of leaning against the wind of inflation movements. When monetary authorities implement an interest rate rule, the quantity of money circulated in the financial market is endogenously determined. The Taylor rule in
country $F$ is defined analogously. The Home government’s flow budget constraint takes the following form:

$$T_t = M_t - M_{t-1},$$

where $M_t$ is country $H$’s money supply. Government uses newly issued money to pay lump-sum transfers, or, in other words, the seigniorage from issuing fiat money is redistributed in a lump-sum fashion to the domestic households in each period. Symmetrically, we have the similar representation for country $F$:

$$T^*_t = M^*_t - M^*_{t-1},$$

where $M^*_t$ is country $F$’s money supply.

### 3.2.1.5 Market Clearing

Market clearing conditions for the home and the foreign goods market can be written as:

$$Y_t = C_{H,t} + C^*_{H,t},$$

$$Y^*_t = C_{F,t} + C^*_{F,t}. \quad (3.25)$$

The clearing conditions for international private bonds and money markets are as follows:

$$M_{H,t} + M^*_{H,t} = M_t;$$

$$M^*_{F,t} = M^*_t;$$

$$B_{H,t} + B^*_{H,t} = 0;$$

$$B_{F,t} + B^*_{F,t} = 0, \quad (3.27)$$

where international currency supply $M_t$ must equal the sum of both country $H$ and $F$’s demands ($M_{H,t} + M^*_{H,t}$). However, country $F$’s money supply $M^*_t$ just needs to be met by
its domestic money demand $M_{F,t}^*$. Equilibrium also requires that, in countries $H$ and $F$, the holdings of each type of bond must add up to zero because each country is populated by a unitary mass of identical households that make identical equilibrium choices, and only the country $H$ ($F$) issues bonds denominated in currency $H$ ($F$). Substitute $T_t$ into Eq. (3.8), by combining Eq. (3.27), we obtain:

$$C_tP_t = (M_{H,t}^* - M_{H,t-1}^*) + Y_tP_{H,t} + P_{t-1}B_{H,t-1}R_{t-1}$$
$$+ P_{t-1}B_{F,t-1}R_{t-1}Z_t - P_tB_{H,t} - P_tB_{F,t}Z_t.$$

Note that for country $H$, the seigniorage $(M_{H,t}^* - M_{H,t-1}^*)$, earned abroad by issuing the international currency, rises with total consumption expenditures abroad. On the other hand, $M_{H,t}^*$ denotes the international currency held by country $F$ and equals country $F$’s imports consumption according to Eq. (3.10), one of the CIA constraints for households in country $F$. Therefore, we define the seigniorage revenue obtained by country $H$ from country $F$ as $\Xi_t \equiv M_{H,t}^* - M_{H,t-1}^*$ which can also be written as $\Xi_t = C_{H,t}^*P_{H,t}^*Z_t - C_{H,t-1}^*P_{H,t-1}^*Z_t$. Performing the similar operation on Eq. (3.11) as it is done on Eq. (3.8), yields:

$$C_t^*P_t^* = - (M_{H,t}^* - M_{H,t-1}^*)/Z_t + Y_tP_{F,t}^* + P_{t-1}B_{F,t-1}^*R_{t-1}^*$$
$$+ P_{t-1}B_{H,t-1}^*R_{t-1}/Z_t - P_t^*B_{H,t}^* - P_t^*B_{F,t}^*/Z_t.$$

In contrast, country $F$’s total consumption is decreased by the amount of $(M_{H,t}^* - M_{H,t-1}^*)$. The seigniorage revenue can be considered as a resources transfer from country $F$ to the international currency country $H$. Combining these two budget constraints, we can derive the Balance of Payments for country $H$ as:

$$Q_t(B_{F,t} - \frac{R_{t-1}B_{F,t-1}}{\Pi_t^*}) - (B_{H,t}^* - \frac{B_{H,t-1}^*R_{t-1}}{\Pi_t})$$
$$= \frac{1}{2} \left[ \frac{P_{H,t}}{P_t}Y_t - C_t - Q_t(P_{F,t}^*Y_t^* - C_t^*) \right] + (m_{H,t}^* - \frac{m_{H,t-1}^*}{\Pi_t}),$$

(3.28)

where $m_{H,t}^*$ denotes real $H$ currency holding by the $F$ country, $\Pi$ and $\Pi^*$ denote domestic
and foreign inflation, respectively. The left hand side of the preceding equation is country $H$’s net foreign assets; the first bracket on right hand side is the current account of country $H$, and the second is seigniorage collected by country $H$. Writing the equation in a more intuitive way, we have:

$$Q_t \Delta \text{Total foreign assets} - \Delta \text{Total foreign liabilities} = \Delta \text{Net foreign assets}$$

$$= CA_t + \text{Seigniorage}_t.$$

For country $F$, this relationship can be expressed as: $\Delta \text{Net foreign assets}^* = Q_t CA_t^* + \text{Seigniorage}_t$. Notice that country $H$ can use seigniorage to finance a current account deficit, while this seigniorage that country $F$ pays to country $H$ is an extra burden on country $F$’s current account. Here, by issuing the international currency, country $H$ can finance an equal amount of its balance of payments deficit. In this sense, seigniorage can be seen as a benefit for the international currency country. The next section presents the PCP-LCP model.

### 3.2.2 PCP-LCP Model

Under LCP assumption, the LOOP does not necessarily hold because firms can price-discriminate across countries. We assume that for the $H$ country $P_{H,t}(i) = Z_t P_{H,t}^*(i)$, while for the $F$ country $P_{F,t}(i) = Z_t P_{F,t}^*(i)$. Hence $P_{F,t} = Z_t P_{F,t}^*$ does not necessarily hold, in which case $\Psi_t \neq 1$, so that PPP also does not necessarily hold.

The PCP-LCP setting extends Corsetti, Dedola, and Leduc (2010) and Okano et al. (2012), whose symmetric model assumes equally-sized country, with each adopting LCP, to an asymmetric one that only the $F$ country adopts LCP. This setting also differs from Betts and Devereux (1996), who assume that fractional tradable goods of each country are pricing to market goods. Although Betts and Devereux (1996) allow violations of the LOOP, PPP still holds.
Since the main difference between these two pricing strategies is the intermediate producers’ maximization problems, market clearing conditions and the evolvement of prices are kept the same as in the PCP case. In the following analysis, except for the currency pricing assumption, the environment considered in this section is exactly the same as in the PCP model. We outline only the equations that differ from those in the previous section.

3.2.2.1 Law of One Price and Terms of Trade

Substitute Eq. (3.18) and Eq. (3.19) into the definition equation of real exchange rate and noting that \( \Psi \neq 1 \), we have:

\[
Q_t = \left[ \frac{(1 - \alpha)(S_t \Psi_t)^{1-\eta} + \alpha}{\alpha S_t^{1-\eta} + (1 - \alpha)} \right]^{\frac{1}{1-\eta}}.
\]  

Comparing this equation with Eq. (3.20), the LOOP gap \( \Psi_t \) enters into the numerator of Eq. (3.29). Under the PCP assumption, \( Z_t \) and \( P^*_F,t \) move proportionally, since the exchange rate pass-through on import prices is complete. But with the LCP assumption, if the firm does not reoptimize its price, the exchange rate pass-through is zero.

3.2.2.2 Firms

In addition to the Calvo price setting, we assume that firms in country \( F \) set prices in terms of the domestic currency for the domestic market and in terms of country \( H \)'s currency for the exports to country \( H \). Under this hypothesis, the maximization problems
faced by producers in $F$ country are as follows:

$$\max_{\tilde{P}_{F,t}, \tilde{P}_{F,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left[ \tilde{P}_{F,t} \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} \right] \right\} C_{F,t+k}^* + \tilde{P}_{F,t} Z_{t+k} \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k}^* - MC_{t+k}^* \left( \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k}^* + \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k}^* \right) \right\},$$

(3.30)

where $\tilde{P}_{F,t}$ and $\tilde{P}_{F,t}$ are the prices chosen by firms when they change the prices of goods produced and sold in country $F$ and goods produced in country $F$ and sold in country $H$, respectively. $Q_{t,t+k}$ denotes the stochastic discount factor. $MC_{t+k}^* \equiv P_{F,t}^* M C_t^*$ denotes nominal marginal costs in $F$ country, $MC_t^* \equiv (W_t^*/A_t P_{F,t}^*)$ and $P_{F,t}$ denotes the PPI of $F$ country:

$$P_{F,t}^* = \frac{P_{F,t}^* C_{F,t}^* + P_{F,t}/Z_t C_{F,t}^*}{C_{F,t}^* + C_{F,t}^*}.$$

Combining market clearing condition Eq. (3.26) and the LOOP gap Eq. (3.19), the preceding equation can be rewritten as:

$$P_{F,t}^* = P_{F,t}^* \frac{C_{F,t}^*}{Y_t^*} + \left( 1 - \frac{C_{F,t}^*}{Y_t^*} \right) \frac{P_{F,t}^*}{\Psi_{F,t}}.$$

(3.31)

Under the LCP assumption, we have multiple FONCs because of the separate selection of prices. Appendix B provides the linearized PCP-LCP model and details in deriving the following two equations:

$$\tilde{p}_{F,t}^* = p_{F,t}^* \left( 1 - \beta \right) \sum_{k=0}^{\infty} \theta^k E_t \left( \pi_{F,t+k}^* \right) + \left( 1 - \beta \right) \sum_{k=0}^{\infty} \theta^k E_t \left( \pi_{F,t+k}^* \right)$$

$$- \left( 1 - \gamma \right) \left( 1 - \beta \right) \sum_{k=0}^{\infty} \theta^k E_t \psi_{t+k},$$

(3.32)

$$\tilde{p}_{F,t} = p_{F,t-1} + \sum_{k=0}^{\infty} \theta^k E_t \left( \pi_{F,t+k} \right) + \left( 1 - \beta \right) \sum_{k=0}^{\infty} \theta^k E_t \left( \pi_{F,t+k} \right)$$

$$+ \gamma \left( 1 - \beta \right) \sum_{k=0}^{\infty} \theta^k E_t \psi_{t+k},$$
where $\gamma \equiv \frac{C_F^*}{Y^*}$ denotes the domestic imports consumption–foreign GDP ratio. These equations are consistent with that derived by Okano et al. (2012) and Monacelli (2003) though with different weights before the LOOP gap. In what follows we will examine the log-linearized form of the price setting equation Eq. (3.32) and compare it with the PCP assumption result.

In the PCP model, the coincidence of PPI and CPI implies that firms’ sales and payments are both measured by PPI or CPI, no matter whether the goods are sold domestically or exported. However, with the LCP assumption, foreign firms pay costs based on $P_{F,t}^*$, which is measured by the real marginal cost term, but make sales at $P_{F,t}^*$. The gap is $p_{F,t}^* - p_{F,t} = -(1 - \gamma)\psi_t$, which is the LOOP gap weighted by imports–GDP ratio. Even though domestic selling has no currency disparity, LCP behaviour still drive a wedge between PPI and CPI. This is why we have a weighted average of the expected future LOOP gap (see Eq. (3.32)). Obviously, the LCP strategy will generate a gap between the export price $P_{F,t}$ in the $H$ country and $F$ firms’ PPI, specifically, $p_{F,t}^* - (p_{F,t} - z_t) = \gamma\psi_t$. $F$ firms obtain their sales of goods exported in terms of country $H$’s currency and make payments in terms of their domestic currency. Therefore, their sales in terms of $H$’s currency should be divided by the nominal exchange rate, as in bracket $(p_{F,t} - z_t)$. The discrepancy of sales and payments equals the weighted LOOP gap. A weighted average of the expected future LOOP gap also shows up in the second equation of Eq. (3.32). This term has a weight of domestic imports–GDP ratio and a negative sign.

The LCP setting not only affects the nature of New Keynesian Philips Curve (NKPC), but also affects the real marginal cost and the balance of payments equations. The derivation of these equations is put in Appendix B. Apart from that, the households’ problems, market clearing conditions, and the monetary policy for the PCP-LCP model would be identical to that of the PCP model. The competitive equilibrium of PCP-LCP model has two more endogenous variables need to be decided: price function $\{\pi_{F,t}, \psi_{F,t}\}$ and two more equations are added into the system.
3.3 Steady State and Calibration

The way to solve for the steady state is assuming that country $F$ is symmetric to country $H$. The virtue of this approach is that whenever there arise any asymmetries in the results, they are due to the international currency assumptions. Let a variable with a upper bar and without a time subscript denote its deterministic steady state, and a variable with a tilde, or in some cases a corresponding lower-case letter, stands for its log deviation from its steady state, i.e., $\tilde{X}_t \equiv \log(X_t/X)$ or $v_t \equiv \log(V_t/V)$. Appendix B provides the full log-linearized PCP model. Combining Eq. (3.14), Eq. (3.17) and Eq. (3.27), we obtain $B_{H,t} = B_{F,t}$ and $B_{H,t}^* = B_{F,t}^* = -B_{H,t}$ which are consistent with Ghironi and Melitz (2005). Since households face quadratic costs of adjusting bond holdings with identical scale parameters across bonds, it is optimal to adjust holdings of different bonds equally so as to spread the costs evenly. An competitive equilibrium is a set of home household’s decision rules, $\{C_t, C_{H,t}, C_{F,t}, N_t, m_t\}$; portfolio choice $\{B_{H,t}\}$; a set of foreign households’ decision rules $\{C^*_t, C^*_{H,t}, C^*_{F,t}, N^*_t, m^*_H, m^*_t\}$; portfolio choice $\{B^*_{H,t}\}$; a set of home firms’ decision rules $\{Y_t, MC_t\}$; a set of foreign firms’ decision rules, $\{Y^*_t, MC^*_t\}$; a set of price functions $\{q_t, r_t, r^*_t, s_t, w_t, w^*_t, \pi_t, \pi^*_t, \pi_{H,t}, \pi^*_{F,t}\}$; such that: (1) household’s decision rules solve the households’ optimization problem; (2) firms’ decision rules solve the firms’ optimization problem; and (3) the appropriate market clearing conditions hold.

We use the PCP model as the baseline model to be calibrated. The parameters of the model are chosen to be close to the standard choice for the values of parameters in the literature. Parameters and their values are listed in Table 3.1. We assume log-utility function, which implies that $\sigma = 1$. The discount factor $\beta$ is set at 0.96, which implies a riskless annual return of about 4% in the steady state.

Since consumption exhibits home bias in developed economies, following the international business cycle literature, we choose consumption home bias to be $1-\alpha = 0.8$, which implies
that the steady-state imports–GDP ratio is 20%, slightly higher than in Corsetti, Dedola, and Leduc (2008). They assume that the share of home-traded goods in the tradable consumption basket is 72% and that of non-traded goods is 45%, indicating that the import-GDP ratio is 15.4%, so that the home bias is 0.85.

Following Christiano, Eichenbaum, and Evans (2005), the labor supply elasticity is set to be unity. The estimate of the elasticity of substitution between domestic and foreign products varies in the literature. Corsetti, Dedola, and Leduc (2008) calibrate their two-country model to the U.S. relative to a set of OECD countries on annual data and estimated that $\eta = 0.95$. For European countries most empirical studies suggest values of $\eta$ below 1. Bernard et al. (2010) find that based on trade data $\eta$ equals 4. This paper assumes $\eta = 0.9$ as in Heathcote and Perri (2002). The elasticity of substitution between the domestic goods varieties is set at $\epsilon = 6$, implying a 20% price markup, as in Galí and Monacelli (2005).

Table 3.1: Structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.96</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Degree of openness</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>The inverse of elasticity of consumption substitution</td>
<td>1</td>
</tr>
<tr>
<td>$\phi$</td>
<td>The inverse of elasticity of labor supply to real wage</td>
<td>1</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Substitutability between domestic varieties</td>
<td>6</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Substitutability between domestic and foreign goods</td>
<td>0.9</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Bond adjustment cost parameter</td>
<td>0.0025</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability of non-price adjustment</td>
<td>$1 - 0.75^4$</td>
</tr>
</tbody>
</table>

Parameters in production

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{\pi}$</td>
<td>Taylor Rule Coef. on inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Persistence of TFP shocks</td>
<td>0.905</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>Persistence of interest rate shocks</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_a}$</td>
<td>Standard deviation of TFP shocks</td>
<td>0.52%</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_r}$</td>
<td>Standard deviation of interest rate shocks</td>
<td>0.2%</td>
</tr>
</tbody>
</table>
The price duration parameter $\theta = 1 - 0.75^4$ implies an average price duration of $1/(1 - 0.75^4) = 1.5$ years, which is in the reasonable range of price stickiness. Parameter $\lambda$ is defined as $[(1-\theta)(1-\beta\theta)/\theta]$ in the log-linearised price setting equation. According to Taylor and Williams (2010), the coefficient $\phi_\pi$ of inflation should be larger than unity to avoid indeterminacy of equilibrium. We set it at 1.5 which is standard in the literature.

The calibration of the productivity and monetary shock process follows Coenen et al. (2007). The interest rate smoothing parameter in the Taylor rule, $\rho_r$, is set to 0.8. The persistence of TFP shock is set to 0.905. The standard deviation of TFP shock and the standard deviation of the interest rate shock is 0.55% and 0.26% in Coenen et al. (2007). We set $\sigma_a$ and $\sigma_r$ at 0.52% and 0.20%, respectively, slightly lower than the values in their paper. These standard deviations of shocks are determined by matching the standard deviations of GDP, consumption, and inflation in the quarterly U.S. data from 2007Q2-2015Q4. GDP is measured by real GDP excluding government expenditure. Consumption is measured by real personal expenditure. The inflation rate is measured by changes in the GDP implicit price deflator. The raw data are seasonally adjusted at the source. All series are HP-filtered, excluding inflation. The numerical results of both PCP and PCP-LCP models will be reported in section 3.4.

### 3.4 Results

This section analyzes the quantitative and qualitative implications of both models. First, we compare the standard deviations and relative volatilities of GDP, consumption, and inflation in the actual data with those in the model simulated data. Second, we analyze the effects of TFP shocks and monetary policy shocks on main macroeconomic variables.

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19 Because of the unavailability of some Chinese macroeconomic data, we choose to match the standard deviations of model data to the standard deviations of U.S. data.
using impulse response functions. Third, we decompose and quantify the contribution of different shocks to the macroeconomic effects.

3.4.1 General Results

Table 3.2 reports the standard deviations and relative volatilities of GDP, consumption, and inflation for the two simulated models: PCP and PCP-LCP models. Since we use the PCP model as the baseline model to be calibrated, the table also shows the PCP model’s ability to successfully match some statistics features of the U.S. macroeconomic data. The standard deviations are expressed in percentage terms. The model-implied standard deviations and relative volatilities are calculated for different combinations of the shocks. For the sake of brevity, in the table we use “LCP” to stand for PCP-LCP model.

Table 3.2: Standard deviations and relative volatilities

<table>
<thead>
<tr>
<th>Var.</th>
<th>Data</th>
<th>All shocks</th>
<th>TFP shocks</th>
<th>MP shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>PCP</td>
<td>LCP</td>
<td>PCP</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Standard Deviations</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Y_H$</td>
<td>1.18</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_H$</td>
<td>0.96</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Pi_H$</td>
<td>0.51</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Y_F$</td>
<td>1.18</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_F$</td>
<td>0.96</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Pi_F$</td>
<td>0.51</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Relative Volatilities</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Y_H$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_H$</td>
<td>0.81</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Pi_H$</td>
<td>0.43</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Y_F$</td>
<td>1</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_F$</td>
<td>0.81</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Pi_F$</td>
<td>0.43</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Notes: The standard deviations are expressed in percentage terms.

Column two, in Table 3.2, displays standard deviations and relative volatilities of the
actual data. Apart from the key currency features, we calibrate the foreign country in a symmetric way to the home country. The data used for comparison is identical for both countries. Columns 3-4 report standard deviations and relative volatilities simulated when all of the model’s shocks are active. Columns 5-6 and columns 7-8 display the model-implied standard deviations and relative volatilities when considering only TFP shocks in both countries and monetary policy shocks in both countries, respectively. In general, the volatility of all three variables in the foreign country is larger than the volatility of these variables in the home country, except the consumptions in the PCP-LCP model. These asymmetries between the home and the foreign country are due to the international currency assumptions.

In the data, consumption is less volatile than output: the standard deviation of output is 1.18 and consumption is 0.96. The simulation results with all of the shocks show that, in the PCP model, the relative volatilities are close to those in the data. However, in the PCP-LCP model, the domestic consumption volatility is 1.89, about 1.7 times as volatile as output, and about 2 times as volatile as consumption in the PCP model. When the models are simulated with only technology shocks or monetary policy shocks, the domestic consumption volatility in the PCP-LCP model is also larger than that in the PCP model. We will detail this amplification mechanism in the next section.

3.4.2 Impulse Response Analysis

The PCP and PCP-LCP models share the same deterministic steady state. This section examines the impulse responses to the orthogonal technology shocks and orthogonal monetary policy shocks in both countries and compares the results.
3.4.2.1 PCP Model

We start by describing the dynamic effects of independent productivity shocks to Home and Foreign countries on a number of macroeconomic variables. In each case, the shock is assumed to be highly persistent ($\rho_a = 0.905$), with a standard deviation of 1%.

The first two rows of Figure 3.1 display the impulse responses of both country $H$ and country $F$ for technology shocks emanating from the $H$ country. Solid lines depict IRFs for $H$ variables and dashed lines depict IRFs for $F$ variables. In the fourth panel, the dashed line depicts IRFs of country $H$’s terms of trade and the solid line denotes that of real exchange rate.

On the third row, the graphs with “cross” in the title compare the spillover effects for consumption and output when the equivalent technology shock originates in the other country. For example: the solid line denotes country $H$’s responses to the productivity shock from country $F$ and dashed lines line denotes country $F$’s responses to the productivity shock from country $H$. In this way, if there arises any asymmetry, it is because of the international currency assumption.

In the PCP model, the reaction of the economy to the positive technology shock is analogous to that implied in the real business cycle literature. An innovation in country $H$ causes real output to rise. This reduces inflation, implying that the monetary authority that follows a standard Taylor rule would lower the nominal interest rate. However, because of home bias, the foreign price level is hardly affected. This leads to a depreciation of the real exchange rate and improvement of country $H$’s terms of trade. The expenditure switching effect due to improved domestic terms of trade encourages exports of country $H$, which has a negative effect on country $F$’s output. The inter-temporal substitution effect of the lower interest rate in both countries increases both countries’ consumption, which has a positive effect on $F$’s output. The total effect on $F$’s output depends on the relative strength of these various effects. Since substitutability between domestic and
foreign goods specified by $\eta = 0.9$ is smaller than the elasticity of consumption substitution $1/\sigma$ which is equal to 1, the first effect outweighs the second. The $F$ economy falls behind the booming $H$ economy with greater expenditure on imports. This leads to the increase in seigniorage, which is received by country $H$.

![Graphs showing IRFs to productivity shocks under PCP assumption. Solid lines depict the responses of variables in the home country and dashed lines depict the responses of variables in the foreign country.]

Figure 3.1: IRFs to productivity shocks under PCP assumption. Solid lines depict the responses of variables in the home country and dashed lines depict the responses of variables in the foreign country.

It is interesting to contrast the behaviour of variables in the two countries when the size of the productivity shock originates in each of the two countries is the same. The
spillover effects in country $F$ are stronger than that in country $H$. For any given shock, no matter in which country it originates, while the impact is shared by both countries because of the international currency assumption, the variables in the $F$ economy prove to be more volatile than those of the $H$ country (this can be observed in table 3.2). From the last graph, $H$ collects some seigniorage, or in other words receives resources transfers from $F$ when the productivity shock originates in country $H$. However, the amount of seigniorage hardly changes when the shock emanates from country $F$. Therefore, the international currency country benefits from having an internationa currency in the case of productivity shocks.

Figure 3.2 illustrates the effects of an expansionary monetary policy shock, with the persis-  

tence of $\rho_r = 0.8$. Notice that because of price rigidity, monetary policy is no longer neutral in the short run. The positive monetary policy shock in country $H$ has similar effects on consumption and output as country $H$’s positive productivity shock. As noted earlier, the movements of variables to steady states are quite fast in terms of magnitude, the monetary policy shock has more powerful effects. According to real business cycle literature, whether the expansionary monetary policy is "beggar-thy-neighbour" or "benefit both sides", taking output movement as the criterion, depends on the relative size of $\eta$ and $1/\sigma$. In the case when $\eta < 1/\sigma$, expansionary domestic monetary policy expands foreign output.

A decrease in the interest rate in country $H$ raises its inflation so that its terms of trade deteriorates. The resulting depreciation of country $H$’ real exchange rate implying that $F$ households spend more on imports from the $H$ country, indicating an increase in the seigniorage collected. Again, we omit the responses of variables to positive monetary policy shocks emanating from country $F$. Comparing the spillover effect, it is apparent that the $F$’s economy has a stronger response to the $H$ shock than the $H$ economy to that of the $F$ shock. In both cases, a rise in seigniorage is observed. Note that in Canzoneri et al. (2013), their international currency bonds facilitating international trade model also
gives rise to the result that a monetary policy shock originating in the $H$ country has a stronger effect abroad than an equivalent policy actions emanating from the $F$ country.

Figure 3.2: IRFs to monetary policy shocks under PCP assumption. Solid lines depict the responses of variables in the home country and dashed lines depict the responses of variables in the foreign country.
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3.4.2.2 PCP-LCP Model

This section examines the reaction of two economies to shocks under the PCP-LCP assumption. Apart from the international currency assumption, asymmetric LCP assumption implies that the exchange rate pass-through into $H$ country’s CPI is zero while that into country $F$’s CPI will be positive. Therefore, the responses of $H$ and $F$ economy to equivalent shocks originating in the other country should be different, as can be seen in Figure 3.3a and Figure 3.3b.

In Figure 3.4, the first row with "own" shows the effect on variables in the country in which the shock originates, the second row with "cross" shows the spillover effects to the other country. In the third row, the first box shows the responses of $\pi_{H,t}$ (the solid line), $\pi_{F,t}^*$ (the dashed line) and $\pi_{F,t}$ (the dash-dot line) to a productivity shock emanating from the $H$ country; the second box shows these variables responses to the equivalent shock emanating from country $F$.

Figure 3.3b shows that, following a $F$ positive productivity shock, the qualitative effects on consumption, output, inflation, the interest rate, and imports in both countries are basically the same as those in Figure 3.1 which shows the impact of the shock occurring in country $H$. Note that the responses of real exchange rate and the terms of trade for country $H$ are the inverse images of the responses that occur when this sort of shock takes place in country $H$. This indicates that, under the PCP-LCP assumption, the efficient operation of the exchange rate pass-through for country $F$ allows for the expenditure switching effect to improve households’ welfare in $F$. But in Figure 3.3a, the terms of trade for $H$ first jumps above the steady state value (the value of the first point is positive), then sluggishly goes all the way back to the steady state because exchange rate pass-through is zero, the domestic shock can only be absorbed by the initial over-adjustment of domestic

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20 The home productivity shock is denoted as $A_{H,t}$, the foreign productivity shock is denoted as $A_{F,t}$. The home monetary policy shock is denoted as $v_{H,t}$ in Figure 5, that of foreign shock is denoted as $v_{F,t}$. More details can be found in appendices.
prices. The movement back to the steady state is relatively gradual. In this process, exports of country $H$ fall instead of increasing, which reduces seigniorage.

Figure 3.3: IRFs to productivity shocks under PCP-LCP assumption. Solid lines depict the responses of variables in the home country and dashed lines depict the responses of variables in the foreign country. Dash-dot lines depict the responses of real exchange rate and dash lines with circles depict the responses of the law of one price gap.
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Figure 3.4 compares own and spillover effects, the finding that the spillover effect for country $F$ is lagger than the spillover effect for country $F$ still holds. In contrast to Figure 3.1, because of the over-adjusted price, the movement of variables to the steady state is more gradual. Furthermore, due to the asymmetric exchange rate pass-through, the PCP-LCP assumption proves to be costly for the international currency country.

![Figure 3.4](image)

Figure 3.4: Comparison of IRFs to productivity shocks under PCP-LCP assumption. Solid lines depict the responses of variables in the home country and dashed lines depict the responses of variables in the foreign country. Dash-dot line depicts the response of PPI inflation of goods produced in country $F$ and sold in country $H$. 
Figure 3.5 shows the effects of a positive monetary policy shock. The qualitative effects of this shock on consumption and output are very similar to those of a positive productivity shock.

In part a of Figure 3.5, the real exchange rate depreciates, but due to the zero pass-through, terms of trade for $H$ deteriorate instead of improving, which makes domestic imports rise. Quantitatively, the increase in the imports is larger than the increase in exports. Moreover, the increase in exports is less than that in the PCP case under the same monetary policy shock. The rise in seigniorage is thus smaller. In part b of Figure 5, the adjustment process of real exchange rate and terms of trade is faster than that in part a. However, the magnitude of the increase in $F$’s imports is equivalent to that in the PCP case under the same monetary policy shock. It is hard to tell whether the international currency country’s households are worse off without going into welfare analysis. In the view of the longer adjustment process and abnormal expenditure switching effects due to the zero exchange rate pass-through, country $H$’s households lose from the PCP-LCP assumption.

Figure 3.6 leads to a similar conclusion that an interest rate shock emanating from the international currency country has a larger effect on variables abroad than the effect of an equivalent foreign country shock on variables in the home country; in each case seigniorage rises.
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Figure 3.5: IRFs to monetary policy shocks under PCP-LCP assumption. Solid lines depict the responses of variables in the home country and dashed lines depict the responses of variables in the foreign country. Dash-dot lines depict the responses of real exchange rate and dash lines with circles depict the responses of the law of one price gap.
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To sum up, the following four results stem from the international currency asymmetric assumption in both PCP and PCP-LCP models. First, as in the literature whether the expansionary monetary policy is "beggar-thy-neighbour" or "benefit both sides", the impact on output depends on the extent of substitutability between domestic and foreign goods. $\eta$, relative to the elasticity of consumption substitution, $1/\sigma$. In calibration, the latter dominates so that negative interest rate shocks benefit both countries. Second,
comparing with a productivity shock, a monetary policy shock, with the same standard
deviation, has more powerful effects on consumption, output, and inflation. More impor-
tantly, an interest rate cut initiated by the $H$ country has stronger spillovers effects on
the $F$ economy than if the equivalent monetary policy shock originates in the $F$ country
in both models.

Third, comparing the PCP model and the PCP-LCP model, apart from the risk sharing
asymmetries due to the international currency assumption, there is another source of
asymmetry: the exchange rate pass-through is zero for country $H$ but positive for country
$F$ because of the PCP-LCP assumption. Therefore the shocks that emanate from country
$H$ can only be absorbed by the over-adjustment of domestic prices. Furthermore, the
movement of variables to steady state is relatively gradual compared to responses in PCP
model. In this sense, with all export prices set in the international currency, $H$ country’s
consumers are actually worse off instead of better off, given volatility of consumption is
an important indicator for welfare analysis.

Finally, one merit of using the CIA constraints to introduce the international currency
asymmetry is that it allows to explicitly examine the responses of seigniorage. Except
the case when the productivity shock originates in country $H$ under the PCP-LCP as-
sumption, seigniorage will increase under positive shocks that take places in both coun-
tries.

### 3.4.3 Variance Decomposition

This section considers the forecast-error variance decompositions for output, consumption,
and inflation of both countries in two models. Table 3.3 and Table 3.4 show the forecast-
error variance decompositions of the variables attributed to each of the four shocks for
one-quarter-ahead and the infinity horizon, respectively.\(^{21}\)

\(^{21}\) The decomposition at horizon $\infty$ corresponds to the unconditional variance decomposition
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On the whole, in the PCP model, domestic shocks are the major source of fluctuations in domestic variables. Since the shock is assumed to be orthogonal, the spillover effects of shocks in the $H$ economy towards the $F$ economy or the other way around are weak. Specifically, domestic technology shock (labeled “TFP”) takes a predominant role in explaining the domestic output and consumption fluctuations. Variations in inflation are largely explained by domestic monetary policy shocks (labelled “MP”). Jointly, in the home country, these two shocks account for about 98% of fluctuations in domestic output and 91% of fluctuations in domestic consumption, whereas, in the foreign country, these two shocks account for about 99% of fluctuations in domestic output and 87% of fluctuations in domestic consumption.

Table 3.3: One-quarter-ahead variance decomposition

<table>
<thead>
<tr>
<th></th>
<th>PCP Model</th>
<th>PCP-LCP model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$TFP_H$</td>
<td>$TFP_F$</td>
</tr>
<tr>
<td><strong>Home</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>29.07</td>
<td>0.65</td>
</tr>
<tr>
<td>Consumption</td>
<td>24.84</td>
<td>3.95</td>
</tr>
<tr>
<td>Inflation</td>
<td>3.93</td>
<td>1.02</td>
</tr>
<tr>
<td><strong>Foreign</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.79</td>
<td>29.03</td>
</tr>
<tr>
<td>Consumption</td>
<td>4.28</td>
<td>23.50</td>
</tr>
<tr>
<td>Inflation</td>
<td>1.01</td>
<td>3.45</td>
</tr>
</tbody>
</table>

*Notes: Numbers are expressed in percentages.*

In the PCP-LCP model, $F$ shocks play an even smaller role in explaining fluctuations in variables of the $H$ country. However, $H$ shocks attributes more to the fluctuations in variables of the $F$ country. This is because that the incomplete exchange rate pass-through in the home country isolates the effect of foreign shocks. In general, monetary policy shocks account for a small fraction of the fluctuations in output and consumption in the two models, which is consistent with the findings in Christiano, Eichenbaum, and
Evans (2005). Moreover, monetary policy shocks have effects on output and consumption fluctuations at longer horizons in the presence of the price rigidity.

### Table 3.4: Unconditional variance decomposition

<table>
<thead>
<tr>
<th></th>
<th>PCP Model</th>
<th>PCP-LCP Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$TFP_H$</td>
<td>$MP_H$</td>
</tr>
<tr>
<td>Home</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>78.98</td>
<td>0.31</td>
</tr>
<tr>
<td>Consumption</td>
<td>70.23</td>
<td>6.74</td>
</tr>
<tr>
<td>Inflation</td>
<td>6.46</td>
<td>0.96</td>
</tr>
<tr>
<td>Foreign</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.59</td>
<td>78.39</td>
</tr>
<tr>
<td>Consumption</td>
<td>9.13</td>
<td>67.14</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.92</td>
<td>6.63</td>
</tr>
</tbody>
</table>

*Notes: Numbers are expressed in percentages.*

Furthermore, $H$ shocks account for a larger fluctuation of $F$’s variables than the equivalent $F$ shocks’ contribution to the fluctuation of $H$’s variables. For example, In Table 3.4, monetary policy shock (labelled “MP”) from the $H$ country has a greater effect on consumption in the $F$ economy (3.01%) compared with the effect of the monetary policy shock from the $F$ country on consumption in the $H$ economy (1.38%). This is due to the assumption that the foreign country has to hold the international currency issued by the home country to purchase imports.

### 3.5 Concluding Remarks

The evolution of the international role of the yuan is in the early stage of becoming a major pricing and settlement currency in trade. This paper is motivated by this fact and addresses the benefits and costs of having an international currency in its early stage. To this end, we construct a two-country DSGE model with asymmetric CIA constraints in
which the home country provides the international currency while the foreign country has to accumulate it to purchase imports. Besides, the model studies two pricing assumption, one is symmetric PCP and the other is asymmetric PCP-LCP.

Results show that in both models, shifts in monetary policy and technology innovation originating in the home country have greater spillover effects than equivalent shocks originating in the foreign country. The monetary policy in the home country is thus more potent. This result is consist with Canzoneri et al. (2013) who assume that international currency bonds play the role of facilitating the trade and act as international reserves.

However, under the PCP-LCP assumption, the home country makes a loss. The PCP-LCP assumption generates the asymmetry that the exchange rate pass-through is zero for the home country but positive for the foreign country. Therefore, shocks emanate from the home country can only be absorbed by the over-adjustment of domestic prices. Furthermore, the PCP-LCP model implies that the movement of variables to the steady state is relatively gradual compared with the movement of variables to their steady state upon the impact in the PCP model. In this sense, the home country’s consumers make a loss, as showed by Devereux, Shi, and Xu (2004).

Finally, the seigniorage, which the home country can use to mitigate the further deteriorate of net foreign assets, is another major source of benefits even in the first stage of internationalization. One advantage of using the CIA constraints is that the change in the amount of seigniorage can be directly observed and assessed in the model simulation. Under the PCP-LCP assumption, except for the case of productivity shock originating in the home country, seigniorage increases in reaction to positive shocks in both countries.
Chapter 4

Policy Uncertainty and Financial Frictions

4.1 Introduction

The anemic recovery of U.S. economy from the “Great Recession” has raised questions about the importance of macroeconomic and policy uncertainty\textsuperscript{22} in curtailing economic activity. Businesses and consumers alike have been uncertain about the monetary and regulatory environment, and this fear of an unknowable future could lead them to postpone investment and consumption. More importantly, if the financial friction—information asymmetries between lenders and entrepreneurs—is a key source of macroeconomic fluctuations, the presence of this friction would amplify and propagate the policies uncertainty shocks significantly. How would monetary policy uncertainty affect macroeconomy under

\textsuperscript{22}Policy uncertainty refers to a class of economic risk where the future path of government policy is uncertain (Baker, Bloom, and Davis (2016)).
financial friction? Specifically, what is the mechanism by which monetary policy uncertainty affects key variables and is this monetary policy uncertainty quantitatively important? This paper uses a New Keynesian model with a financial accelerator mechanism similar to Bernanke, Gertler, and Gilchrist (1999) to answer the above questions.

While there has been a flourishing literature studying the role of uncertainty shocks in driving business cycle fluctuations, led by Bloom (2009), there is little research on the effects of uncertainty policy shocks under financial friction. This paper tries to fill this gap and contributes in the following ways. We investigate the effects of monetary policy uncertainty shock under financial frictions. Monetary policy uncertainty shock is introduced as a mean preserving shock to the variance of monetary policy level shock, as is commonly assumed in the uncertainty literature. Our results show that an increase in the interest rate volatility by increasing the costs of external debt of entrepreneurs reduces their investment demand, which then translates into a decrease in GDP. Specifically, the financial accelerator mechanism amplifies the transmission through the effect on the net worth of entrepreneurs. A fall in entrepreneur’s net worth increases their costs of obtaining external funds, which further depresses investment. Households consume less because of precautionary saving. However, upon the impact of interest rate uncertainty shock, households tend to save more than under the TFP uncertainty shock. This lies in the fact that the uncertainty about the interest rate directly relates to households’ intertemporal consumption decision. Reductions in both investment and consumption demand exert a downward pressure on GDP.

In our model, which is calibrated to the U.S. quarterly data from 1948Q1—2017Q1, a two-standard deviation shock to the interest rate volatility leads to approximately 0.01 percent drop in GDP. By comparing with the TFP uncertainty shock and the risk shock, we find that the output effects of the monetary policy uncertainty shock is around eight times larger than the effects of risk shock and is slightly larger than the effects of TFP shock. The monetary policy uncertainty has the largest effects among all three uncertainty
shocks, although quantitatively the effects are limited. The possible reason is that the parameterization of uncertainty shocks considered in the model is relatively small.

This paper is at the intersection of two strands of the literature: the uncertainty literature that studies the impact of uncertainty shocks on macroeconomy; and the financial accelerator literature that studies the inefficiencies in financial markets created by information asymmetries between lenders and entrepreneurs.

In the uncertainty literature, the studies tend to agree on the contractionary effects of uncertainty on economic activity, yet differ in their views on the size of such a recession. On one hand, by using a partial equilibrium model, Bloom (2009) reports a one-standard deviation positive volatility shock causes a decrease in output by about 1 percent. Bloom et al. (2012) find that when general equilibrium effects are shut off, the decrease in output is around 3 percent to a one-standard deviation volatility shock. On the other hand, the effect of uncertainty shock is reported to be little or none, especially when the study takes the general equilibrium approach. Fernández-Villaverde et al. (2015) report that output, consumption, and investment drops by around 0.15 percent, 0.02 percent, and 0.6 percent, respectively in response to a two-standard deviation shock to the fiscal policy uncertainty. Similarly, Born and Pfeifer (2014) use an estimated New Keynesian model and show that a two-standard deviation increase in the policy uncertainty reduces the output and consumption by 0.065 percent and 0.03 percent, respectively. Our results that a drop of 0.01 percent and a drop of 0.038 percent in output and consumption respectively, to a two-standard deviation shock to monetary policy uncertainty are of the same order of magnitude but lower than the contractionary effects in above-mentioned studies. The reason is that we only consider the effect of interest rate uncertainty shock while these studies consider the effects of several joint fiscal policy uncertainties. Basu and Bundick (2012) investigate the effects of TFP uncertainty shock and preference uncertainty shock. In their study, the output, consumption, and investment decrease by 0.04 percent, 0.06 percent, and 0.01 percent respectively upon the impact of a TFP uncertainty shock. The
three aggregates decrease by 0.17 percent, 0.16 percent, and 0.2 percent respectively upon the impact of a preference uncertainty shock. However, by using a DSGE model Bidder and Smith (2012) show that a stochastic volatility shock reduces output, consumption, and investment by 2 percent, 1.5 percent, and 2.5 percent respectively.

The financial accelerator literature, pioneered by Bernanke, Gertler, and Gilchrist (1999) (henceforth BGG), shows that asymmetric information in credit markets give the net worth of entrepreneurs a role to play in the business cycle through their impact on the cost of external finance (see Christensen and Dib (2008)). These financial frictions may significantly amplify the magnitude of fluctuations of the economy (see, among others, Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), Christiano, Motto, and Rostagno (2010), and Christiano, Motto, and Rostagno (2014)). In particular, Christensen and Dib (2008) estimate the standard BGG model for the U.S. economy using maximum likelihood and find the data is in favour of the model with financial accelerator. Christiano, Motto, and Rostagno (2014) investigate the time series properties of the shock to the variance of entrepreneurs’ idiosyncratic productivity (which they label a “risk shock”) using financial aggregate data through the estimation of a richer BGG model. They emphasize the importance of risk shock in driving business cycles, i.e. over 60 percent of the business cycle variance in output is accounted for by the risk shock. Therefore, we incorporate the risk shock in our model and compare the relative strength of it with the monetary policy uncertainty shock and the TFP uncertainty shock.

The remainder of this paper proceeds as follows. Section 4.2 presents the model with financial accelerator, including the sources of uncertainty. Section 4.3 discusses the choice of parameters in the model and the solution method employed. Section 4.4 presents the main results while Section 4.5 concludes.
4.2 The Model

This section outlines the DSGE model with financial accelerator that we use in our analysis. It closely resembles the original BGG model and is modified to incorporate three uncertainty shocks. The model economy is populated by a continuum of households, financial intermediaries, entrepreneurs, capital producers, final good producers, intermediate good producers, and a central bank.

The financial accelerator mechanism works in the following manner: entrepreneurs use their own funds and funds borrowed from financial intermediaries to purchase capital at the end of each period. After the purchase of capital, entrepreneurs face an idiosyncratic productivity shock that each entrepreneur costlessly observes, but the financial intermediary must pay a fee to observe the entrepreneur’s productivity. This asymmetric information between entrepreneurs and financial intermediaries affects the supply of credit and amplifies the magnitude of fluctuations in economic activity.

4.2.1 Households

There is a continuum of households, each indexed by \( i \in (0, 1) \). They consume a composite final good, invest in safe bank deposits, supply labor, and own shares of a monopolistic competitive firm that produces differentiated varieties of intermediate goods. The preference of the representative household is as follows:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ C_t^{1-\sigma_c} \left( 1 - \sigma_c \right) - \eta L_t^{1+\sigma_l} \left( 1 + \sigma_l \right) \right\},
\]

where the operator \( E_t \) represents the conditional expectation over all states of nature in period \( t \), \( \beta \in (0, 1) \) is the time discount factor, and \( C_t, L_t \) denotes per capita consumption and labor supply, respectively. \( \sigma_c \) and \( \sigma_l \) represent the inverse of intertemporal elasticity
of substitution and the inverse of the elasticity of labor supply, respectively. $\eta$ is the weight of labor supply in the utility function.

The budget constraint of the representative household is:

$$C_t P_t + D_{t+1} = W_t L_t + R_t D_t + \Pi_t,$$

where $P_t$ is the aggregate price level. $D_t$ represents the nominal safe bank deposits held by a financial intermediary, which pays the gross nominal interest rate, $R_t$ between $t$ and $t+1$ per unit of deposit to the representative household. $W_t$ is the nominal wage for household labor. $\Pi_t$ represents profits received from ownership of retail firms distributed in lump-sum.

The consumption aggregator $C_t$ is defined as:

$$C_t \equiv \left[ \int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right]^\frac{1}{1-\epsilon},$$

where $\epsilon$ denotes the elasticity of substitution among differentiated varieties. $C_t(i)$ denote the household’s consumption of goods produced by firm $i$. Expenditure minimization associated with the consumption composite $C_t$ gives the consumer price index as:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} di \right]^\frac{1}{1-\epsilon}.$$

The representative household chooses $C_t$, $D_{t+1}$, and $L_t$ to maximize its expected lifetime utility Eq. (4.1) subject to budget constraint Eq. (4.2). Solving the household’s problem yields standard first-order conditions for consumption, labor supply, and deposits:

$$\frac{1}{R_t} = \mathbb{E}_t \left\{ \frac{\beta C_{t+1}^{-\sigma_e} P_t}{C_t^{-\sigma_e} P_{t+1}} \right\}$$

$$\eta C_t^{\sigma_e} L_t^{\sigma_l} = w_t,$$
where \( w_t = W_t / P_t \) is real wage.

### 4.2.2 Entrepreneurs

#### 4.2.2.1 Optimal Contract

As in Bernanke, Gertler, and Gilchrist (1999), entrepreneurs are risk neutral. Each entrepreneur has a constant probability \( \gamma \) of surviving to the next period. This assumption of finite horizons for entrepreneurs precludes the possibility that the entrepreneurial sector will ultimately accumulate enough wealth to be fully self-financing. In each period \( t \), an entrepreneur acquire physical capital \( K^j_{t+1} \). The price paid per unit of capital in period \( t \) is \( Q_t \). Acquisitions of capital are financed by entrepreneurial wealth \( N^j_{t+1} \) and borrowing \( B^j_{t+1} \) from a financial intermediary. This intermediary obtains its funds from household deposits and faces an opportunity cost of funds equal to \( R_t \). This can be represented by the equation:

\[
K^j_{t+1}Q_t = N^j_{t+1} + B^j_{t+1}. \tag{4.5}
\]

After purchasing the capital, the entrepreneur experiences an idiosyncratic shock \( \omega^j \) to its return \( R^k_{t+1} \). Following Bernanke, Gertler, and Gilchrist (1999), we assume that \( \omega^j \) is i.i.d. across time and across entrepreneurs, with a unit-mean log normal distribution function, \( F(\omega) \), and \( E[\omega^j] = 1 \). \( \sigma_t \), which represents the standard deviation of \( \log(\omega_t) \), is assumed to vary stochastically over time. This shock, denoted by Christiano, Motto, and Rostagno (2014) as risk shock, characterizes the extent of cross-sectional dispersion in \( \omega \). They use risk to refer exclusively to variations in idiosyncratic uncertainty, which is in essence similar to Bloom (2009)'s concept of uncertainty. We will discuss its law of motion in Section 4.2.6.

Given \( K^j_{t+1}, B^j_{t+1}, \) and \( R^k_{t+1} \), the optimal contract is characterized by a gross non-default loan rate, \( Z^j_{t+1} \), and \( \overline{\omega}^j \), a threshold value of the idiosyncratic shock \( \omega^j \). For values of
the idiosyncratic shock greater than or equal to \( \omega^j \), the entrepreneur is able to repay the loan at the contractual rate \( Z^j_{t+1} \). The entrepreneur who can not repay will declare bankruptcy. In this case, the financial intermediary pay a monitoring cost to collect whatever left. That is, the intermediary receives \( (1 - \mu) \omega^j R^k_{t+1} Q_t K^j_{t+1} \). The entrepreneur with the realized shock \( \omega^j \) will earn zero profit, i.e., its earning \( \omega^j R^k_{t+1} Q_t K^j_{t+1} \) equals its cost of borrowing \( Z^j_{t+1} B^j_{t+1} \). Hence, the cutoff value of \( \omega^j \) is defined by:

\[
\omega^j R^k_{t+1} Q_t K^j_{t+1} = Z^j_{t+1} B^j_{t+1}.
\]

(4.6)

A risk averse financial intermediary has the utility function in the form of \( x^{1-\phi} \), where \( x \) denotes the share of total returns to capital or deposits that go to the financial intermediary. The values of \( \omega^j \) and \( Z^j_{t+1} \) under the optimal contract are determined by the requirement that the financial intermediary receive an expected return equal to the opportunity cost of its funds. Therefore, the loan contract must satisfy:

\[
\int_{\omega^j}^{\infty} \frac{(\omega^j R^k_{t+1} Q_t K^j_{t+1})^{1-\phi}}{1-\phi} dF(\omega) + \int_{0}^{\omega^j} \frac{((1 - \mu) \omega^j R^k_{t+1} Q_t K^j_{t+1})^{1-\phi}}{1-\phi} dF(\omega) = \frac{(R^k_{t+1} B^j_{t+1})^{1-\phi}}{1-\phi}.
\]

(4.7)

Combining Eq. (4.6) with Eq. (4.7) yields the following expression for \( \omega^j \):

\[
\int_{\omega^j}^{\infty} (Z^j_{t+1} B^j_{t+1})^{1-\phi} dF(\omega) + \int_{0}^{\omega^j} ((1 - \mu) \omega^j R^k_{t+1} Q_t K^j_{t+1})^{1-\phi} dF(\omega) = (R^k_{t+1} B^j_{t+1})^{1-\phi}.
\]

(4.8)

We introduce the concept of leverage, \( \kappa^j_{t+1} \), defined as the value of the entrepreneur’s capital divided by net worth: \( \kappa^j_{t+1} = \frac{Q_t K^j_{t+1}}{N^j_{t+1}} \). From Eq. (4.5) and Eq. (4.8), leverage is expressed as:

\[
\kappa^j_{t+1} = \left[ 1 - \frac{R^k_{t+1}}{R^k_{t+1}} \left( [\omega^j]^{1-\phi} (1 - F(\omega)) + (1 - \mu)^{1-\phi} \int_{0}^{\omega^j} (\omega^j)^{1-\phi} dF(\omega) \right)^{1-\phi} \right]^{-1}.
\]

(4.9)
Define the risk spread as $s_t = \frac{R_{t+1}^k}{R_{t+1}}$. Entrepreneur maximizes its utility given the above combination of $(\overline{\omega}, \kappa_t)$ and risk spread. Since entrepreneur’s expected returns to capital should be at least as the same amount as returns of alternatively depositing its net worth in financial intermediary. That is to say,

$$\int_0^\infty [\omega R_{t+1}^k Q_t K_{t+1}^j - Z_{t+1}^i B_{t+1}^j] dF(\omega) \geq N_{t+1}^j R_{t+1}.$$

Combining the above equation, Eq. (4.6), and the definition of the leverage, the maximization problem for the entrepreneur is as follows:

$$\max_{\omega} \int_0^\infty [\omega - \overline{\omega}^j] dF(\omega) \frac{R_{t+1}^k}{R_{t+1}} \kappa_{t+1}. \quad (4.10)$$

The solution of this problem gives rise to:

$$\frac{1 - F(\overline{\omega})}{1 - \Gamma(\overline{\omega})} = \frac{1}{1 - \phi} \frac{\Phi(\overline{\omega})^{-1}(\overline{\omega}^j)^{-\phi}(1 - \phi)(1 - F(\overline{\omega})) - (1 - (1 - \mu)^{1-\phi}) \overline{\omega}^j F'(\overline{\omega})}{s_t^{-1} \Phi(\overline{\omega})^{1-\phi} - 1}, \quad (4.11)$$

where $\Gamma(\overline{\omega}^j) \equiv \overline{\omega}(1 - F(\overline{\omega})) + \int_0^{\overline{\omega}^j} \omega dF(\omega)$ denotes the share of overall return to the capital that goes to the financial intermediary including the monitoring cost and $\Phi(\overline{\omega}) \equiv (\overline{\omega}^j)^{1-\phi}(1 - F(\overline{\omega})) + (1 - \mu)^{1-\phi} \int_0^{\overline{\omega}^j} \omega^{1-\phi} dF(\omega)$.

### 4.2.2.2 Net Worth

To ensure that entrepreneurial net worth will never be enough to fully finance the new capital acquisitions, BGG assumed that they have a finite lifetime. In particular, each entrepreneur is assumed to survive until the next period with probability $\gamma$. The aggregate profits of entrepreneurs at the end of period $t$ is $[1 - \Gamma(\overline{\omega})] R_{t-1}^k Q_t K_t$. Then aggregate entrepreneurial net worth at the end of period $t$, $N_{t+1}$, is given by

$$N_{t+1} = \gamma [1 - \Gamma(\overline{\omega})] R_{t-1}^k Q_t K_t, \quad (4.12)$$
where \( \gamma \) denotes a constant surviving rate to the next period of each entrepreneur (implying an expected lifetime of \( \frac{1}{1-\gamma} \)). Net worth is positively related to the capital price and the stock of capital. However, the impact of capital return on net worth is ambiguous. An increase in capital return indicates a higher value of capital owned by entrepreneurs, while an increase in capital return also raises the risk premium and reduces the net worth (see Faia and Monacelli (2007)).

### 4.2.3 Firms

There are two types of firms. Wholesale intermediate goods are produced by perfectly competitive wholesale firms and then sold to monopolistically competitive retail firms who costlessly differentiate them.

#### 4.2.3.1 Wholesale Firms

Wholesalers hire labor from households in a competitive labor market at real wage \( w_t \) and rent capital from entrepreneurs at rental rate \( r_t \). The production function of the representative wholesale firms is given by:

\[
Y_t = A_t K_t^\alpha L_t^{1-\alpha},
\]

where \( A_t \) is a TFP shock, which follows a AR(1) process with time-varying volatility:

\[
A_t = (1 - \rho_A) A + \rho_A A_{t-1} + e_{A,t}^{\sigma_{A,t} \epsilon_{A,t}}, \text{ where } \epsilon_{A,t} \overset{i.i.d.}{\sim} \mathcal{N}(0,1),
\]

where coefficient \( \rho_A \in (0,1) \) determines the persistence of the TFP level shock, \( \epsilon_{A,t} \) follows an \( i.i.d. \) standard normal process and \( A \) denotes the steady state level of TFP.
We will specify the process of time-varying innovation $\sigma_{A,t}$ in the volatility shock process section.

The maximization problem of the wholesaler is:

$$\max_{K_t, L_t} P^w_t A_t K_t^\alpha L_t^{1-\alpha} - P_t w_t L_t - P_t r_t K_t.$$  \hspace{1cm} (4.15)

The first order conditions with respect to capital and labor are as follows:

$$\alpha \frac{1}{\chi_t} \frac{Y_t}{K_t} = r_t$$

$$(1 - \alpha) \frac{1}{\chi_t} \frac{Y_t}{L_t} = w_t.$$  \hspace{1cm} (4.16)

where $\chi_t \equiv \frac{P_t}{P^w_t}$ denotes the gross markup of retail goods over wholesale goods.

**4.2.3.2 Retail Firms**

A unit measure of monopolistically competitive firms indexed by $i$, purchases intermediate wholesale goods at $P^w_t$ and differentiate them, which are then aggregated into a final good using a standard Dixit-Stiglitz aggregator. The production function for each of the final good producers is:

$$Y_t = \left[ \int_0^{1} Y_{i,t}^{\frac{\epsilon}{\epsilon - 1}} \, di \right]^{\frac{\epsilon - 1}{\epsilon}}.$$  \hspace{1cm} (4.17)

Their demand for intermediate inputs is $Y_{i,t+j} = \left( \frac{P_{i,t}}{P_{t+j}} \right)^{-\epsilon} Y_{t+j}$.

The intermediate goods producers set their optimal prices in a staggered manner à la Calvo (1983) rule. Each time, only with probability $1 - \theta$, can they re-optimize their
prices, i.e., $P_{i,t}^*$. The price setting problem is:

$$\max_{P_{i,t}^*} \mathbb{E}_t \sum_{j=0}^{\infty} Q_{t,t+j}^f \theta^j [P_{i,t}^* Y_{i,t+j} - P_{t+j}^w Y_{i,t+j}]$$

s.t. $Y_{i,t+j} = \left( \frac{P_{i,t}^*}{P_{t+j}} \right)^{-\epsilon} Y_{t+j}$.

The first order condition with respect to the retailer’s price $P_{i,t}^*$ is:

$$P_{i,t}^* = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \left[ \sum_{j=0}^{\infty} Q_{t,t+j}^f (\theta \beta)^j (P_{t+j})^{\epsilon-1} P_{t+j}^w Y_{i,t+j} \right]}{\mathbb{E}_t \left[ \sum_{j=0}^{\infty} Q_{t,t+j}^f (\theta \beta)^j (P_{t+j})^{\epsilon-1} Y_{t+j} \right]}.$$  \hspace{1cm} (4.18)

where $Q_{t,t+j}^f$ is the discount factor. The price level evolves according to:

$$P_t = \left[ \theta P_{t-1}^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon} \right]^{1/(\epsilon - 1)}.$$  \hspace{1cm} (4.19)

### 4.2.4 Capital Producers

The perfectly competitive capital producer purchases an amount of capital $K_t$ from entrepreneurs at the beginning of each period. During the period, they buy $I_t$ of general output from the final goods producers, and transform it into new capital via the technology:

$$K_t = I_t + (1 - \delta)K_{t-1} - \frac{\phi_K}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1},$$  \hspace{1cm} (4.20)

where $\delta$ is the rate of depreciation, $\phi_K$ is a parameter that governs the magnitude of the adjustment cost. The capital producer’s objective function is:

$$\max_{K_t} K_{t+1} Q_t - I_t.$$
The first order condition of the capital producer’s optimization problem is:

\[
\frac{1}{Q_t} = 1 - \phi_K \left( \frac{I_t}{K_t} - \delta \right) .
\] (4.21)

### 4.2.5 Monetary Policy and Resource Constraint

The central bank targets the nominal interest rate through a conventional Taylor rule:

\[
\frac{R_t}{R} = \left( \frac{1 + \Pi_t}{1 + \Pi} \right)^{\phi_{\Pi}} \left( \frac{Y_t}{Y} \right)^{\phi_Y} v_t ,
\] (4.22)

where \( \phi_{\Pi}, \phi_Y \) represent the coefficients the central bank changes interest rates in response to inflation and output deviations, respectively. \( R \) denotes the steady-state value of nominal interest rate. \( \Pi \) and \( Y \) are the target inflation rate and target output level set by the central bank, respectively. Finally, the nominal interest rate shock \( v_t \) follows an AR(1) process:

\[
v_t = (1 - \rho_v)v + \rho_v v_{t-1} + \epsilon_{v,t} + \sqrt{\sigma_{v,t}} \epsilon_{v,t}, \text{ where } \epsilon_{v,t} \overset{i.i.d}{\sim} \mathcal{N}(0, 1).
\] (4.23)

Here, \( \sigma_{v,t} \) allows for time-varying volatility and will be discussed in detail in Section 4.2.6.

The model is closed by the following aggregate resource constraint:

\[
Y_t = C_t + I_t + D_t ,
\] (4.24)

where \( D_t \) is the aggregate resource used for monitoring:

\[
D_t \equiv \mu \int_0^\infty f(\omega) \omega R_t^K Q_{t-1} K_t / P_t .
\]
4.2.6 Volatility Shock Processes

Our model includes three uncertainty shocks: risk shock, $\sigma_t$; the volatility shock to TFP, $\sigma_{A,t}$; and monetary policy volatility shock, $\sigma_{v,t}$. An uncertainty shock is defined as an unexpected exogenous variation in the volatility of a shock process. An increase in this variation implies a widening of tails of the level shock’s distribution while keeping the mean of the level shock unchanged. Whereas an increase in the level shock indicates a temporary increase in its mean, there is no change in its distribution shape. We follow Christiano, Motto, and Rostagno (2014) to model the risk shock as the log-deviation of $\sigma_t$ from its steady state value as in Eq. (4.25).

$$\log(\sigma_t) = (1 - \rho_\sigma)\sigma + \rho_\sigma \log(\sigma_{t-1}) + \eta_\sigma \epsilon_t^{\sigma},$$

where \( \rho_\sigma \) determines the persistence of the risk shock. \( \sigma \) is the unconditional mean of $\sigma_t$. \( \eta_\sigma \) is the standard deviation of $\epsilon_t^{\sigma}$. Therefore, as $\sigma_t$ increases, the dispersion of the outcomes of entrepreneurs increases too. The higher dispersion of realization of outcomes, the higher probability of the bankruptcy rate. The financial intermediaries will increase the lending rate and thus depress the capital demand of entrepreneurs.

In order to model monetary policy uncertainty shocks and the TFP uncertainty shock, we use the stochastic volatility approach as proposed by Fernández-Villaverde et al. (2011). The main feature of uncertainty shock processes is that, $\sigma_{A,t}$ and $\sigma_{v,t}$ are not constant, but each follow an AR(1) process:

$$\sigma_{A,t} = (1 - \rho_{\sigma_A})\sigma_A + \rho_{\sigma_A}\sigma_{A,t-1} + \eta_{\sigma_A}\epsilon_t^{\sigma_A},$$

where $\epsilon_t^{\sigma_A} \sim \mathcal{N}(0, 1)$, \( \epsilon_t^{\sigma_A} \) is i.i.d.

$$\sigma_{v,t} = (1 - \rho_{\sigma_v})\sigma_v + \rho_{\sigma_v}\sigma_{v,t-1} + \eta_{\sigma_v}\epsilon_t^{\sigma_v},$$

where $\epsilon_t^{\sigma_v} \sim \mathcal{N}(0, 1)$, \( \epsilon_t^{\sigma_v} \) is i.i.d.

where $\rho_{\sigma_A}$, and $\rho_{\sigma_v}$ determine the persistence of the uncertainty shock. $\sigma_A$ and $\sigma_v$ is the unconditional mean of $\sigma_{A,t}$ and $\sigma_{v,t}$, respectively. $\eta_{\sigma_A}$ and $\eta_{\sigma_v}$ control the standard
deviation of $\epsilon_t^A$ and $\epsilon_t^v$, respectively. The exogenous shocks $\epsilon_t^a$, $\epsilon_t^A$, and $\epsilon_t^v$ are assumed to be independent of the level shock $\epsilon_{A,t}$ and $\epsilon_{v,t}$.

### 4.3 Calibration and Solution of the Model

We calibrate the model to the U.S. economy largely following Bernanke, Gertler, and Gilchrist (1999), Christensen and Dib (2008), and Christiano, Motto, and Rostagno (2014). A subset of the structural parameters are calibrated to match several steady-state observations and standard deviations using quarterly data from 1948Q1–2017Q1.

Table 4.1: Structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-financial Sector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>$1.03^{-1/4}$</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>The Frisch elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Weight on labor</td>
<td>5.8</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Substitutability between domestic varieties</td>
<td>6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>$1/3$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability of non-price adjustment</td>
<td>0.75</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Capital adjustment cost</td>
<td>0.5882</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Gross steady-state inflation rate</td>
<td>1</td>
</tr>
<tr>
<td><strong>Financial Sector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>The monitoring cost parameter</td>
<td>0.21</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The survival rate of entrepreneurs</td>
<td>0.9728</td>
</tr>
<tr>
<td>$K/N$</td>
<td>The steady-state ratio of capital to net worth</td>
<td>2</td>
</tr>
<tr>
<td><strong>Taylor Rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Coef. on inflation gap</td>
<td>3</td>
</tr>
<tr>
<td>$\phi_Y$</td>
<td>Coef. on output gap</td>
<td>$1/4$</td>
</tr>
</tbody>
</table>

The calibrated structural parameters of the model are reported in Table 4.1. The discount factor $\beta = 1.03^{-1/4}$ generates an annual interest rate of about 3 percent. The risk aversion
parameter $\sigma_c = 1$ so that utility is logarithmic in consumption and the inverse of the Frisch labor supply elasticity $\sigma_l$ is set to 1, in line with Christiano, Motto, and Rostagno (2014). The parameter $\eta$ denotes the weight on working hours in the utility function. We set $\eta$ so that hours worked is around one third in the model. The parameter $\epsilon$ measures the elasticity of substitution among domestic varieties and is assigned the commonly used value of 6, implying a steady-state price markup of 20%. As is also within convention, the capital share, $\alpha$, is $1/3$. We set the depreciation rate of capital $\delta$ to 0.025, a value frequently used in the literature. For price-setting, we assume the Calvo parameter $\theta = 0.75$. Following Christensen and Dib (2008), the capital adjustment cost $\chi$ is set to 0.5882. The steady-state gross inflation rate is set equal to 1.

Following Bernanke, Gertler, and Gilchrist (1999), we fix the monitoring cost ratio $\mu$ at 0.21, indicating the monitoring cost is a proportion 0.21 of the realized gross payoff to the firm’s capital. We also fix the survival rate of entrepreneurs, $\gamma$, at 0.9728, yielding an annualized business failure rate of about three percent. Finally, we also use Bernanke, Gertler, and Gilchrist (1999) value of 2 for the steady-state ratio of capital to net worth, $K/N$. For the Taylor rule, we set $\phi_{\pi} = 3$ and $\phi_y = 0.25$, which are common values in the literature.

Table 4.2 displays the persistence and standard deviation of the volatility processes in our model. The persistence of TFP shock, $\rho_A$ is chosen at 0.63 as in Christensen and Dib (2008). The other parameters related to the TFP volatility shock process are calibrated following Basu and Bundick (2012). The TFP volatility shock persistence $\rho_A$ is set equal to 0.83 and the steady state value of TFP volatility shock $\sigma_A$ is set to $-5.6$. 

Table 4.2: Parameter values of volatility shock processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TFP Volatility Shock Process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_A )</td>
<td>Persistence of TFP shock</td>
<td>0.63</td>
</tr>
<tr>
<td>( \sigma_A )</td>
<td>Steady-state of TFP volatility shock</td>
<td>-5.6</td>
</tr>
<tr>
<td>( \rho_{\sigma_A} )</td>
<td>Persistence of TFP volatility shock</td>
<td>0.81</td>
</tr>
<tr>
<td>( \epsilon_{A,t} )</td>
<td>Standard deviation of TFP shock</td>
<td>1</td>
</tr>
<tr>
<td>( \eta_{\sigma_{A,t}} )</td>
<td>Standard deviation of the TFP volatility shock</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Risk Shock Process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_\sigma )</td>
<td>Persistence of risk shock</td>
<td>0.94</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Steady-state of risk shock</td>
<td>0.2588</td>
</tr>
<tr>
<td>( \eta_{\sigma_t} )</td>
<td>Standard deviation of risk shock</td>
<td>0.015</td>
</tr>
<tr>
<td><strong>Monetary Policy Volatility Shock Process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_\sigma_v )</td>
<td>Persistence of MP volatility shock</td>
<td>0.92</td>
</tr>
<tr>
<td>( \sigma_v )</td>
<td>Steady-state of MP volatility shock</td>
<td>-6.8</td>
</tr>
<tr>
<td>( \epsilon_{v,t} )</td>
<td>Standard deviation of MP shock</td>
<td>1</td>
</tr>
<tr>
<td>( \eta_{\sigma_{v,t}} )</td>
<td>Standard deviation of the MP volatility shock</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Born and Pfeifer (2014) estimate the TFP uncertainty shock process using a quarterly U.S. data set of TFP from 1970Q1–2012Q2 and find the steady state value of TFP volatility shock \( \sigma_A \) is \( -5.3 \). We set the standard deviation of the TFP uncertainty shock at 0.3, slightly lower than 0.312, the value set by Born and Pfeifer (2014).

Since we calibrate \( K/N = 2 \) to match its data correspondence, the steady-state of risk shock, \( \sigma \), is calculated equal to 0.2588. This value is consistence with Christiano, Motto, and Rostagno (2014). By following the estimation procedure in Cesa-Bianchi and Fernandez-Corugedo (2015), we estimate the risk shock process Eq. (4.25) using annual data from the Census panel of manufacturing establishments over the sample period of 1972–2010 and find the persistence of the risk shock, \( \rho_\sigma \), and the standard deviation of the risk shock, \( \eta_{\sigma_t} \), are 0.94 and 0.015, respectively.\(^{23}\) Cesa-Bianchi and Fernandez-Corugedo (2015) estimate a different risk shock process and their results show that the persistence of

\(^{23}\)This data set is constructed by Bloom et al. (2012) and is available at the website: http://www.stanford.edu/~nbloom/index files/Page315.htm.
the risk shock and the standard deviation of the risk shock are 0.79 and 0.025, respectively. Chugh (2016) find that the standard deviation of the risk shock is 0.037.

By following Born and Pfeifer (2014), we set the persistence of the Taylor rule volatility shock process is set to 0.92. The steady state value of the Taylor rule volatility shock process is set to −6.8, slightly lower than −6.5, which is estimated by Born and Pfeifer (2014). The standard deviation of the monetary policy volatility shock is chosen at 0.3.

The model is solved using a third-order perturbation around the deterministic steady states since there is not a closed-form solution due to the highly nonlinear nature of the model. Our primary focus is examining the effects of increases in the second moment of monetary policy shock process. As noted by Schmitt-Grohé and Uribe (2004), shocks only enter with their first moments if taking a first order approximation. This first moments of future shocks drop out when taking expectations of the linearized equations. Similarly, as noted by Fernández-Villaverde et al. (2011), second moment shocks enter only as cross-products with the other state variables. Therefore we could not study the effects of shocks to the second moments alone. In a third-order approximation, second moment shocks enter independently in the Taylor approximated policy functions. Thus a third-order approximation allows us to compute an impulse response to an increase in the uncertainty shocks, while the level shocks remain constant. Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010), Ruge-Murcia (2012), and Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2013) show how to apply third-order approximation to estimate DSGE models. We use the pruning algorithm suggested by Kim et al. (2008) (see also Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2013)) in our simulations to avoid the explosive sample paths that higher-order approximations usually generate.
4.4 Results

In this section we analyze both quantitative and qualitative implications of the model. First, we show the model’s ability to successfully match some statistics features of the U.S. macroeconomic data. Second, we analyze the effects of an uncertainty shock to monetary policy on main macroeconomic variables using impulse response functions. Third, we examine the variance decomposition of the amplification effects and risk adjustment effects.

4.4.1 Numerical Results

Table 4.2 compares the standard deviations and relative volatilities of GDP, consumption, investment, interest rate, and labor in the actual data with those in the model simulated data. The table also reports the steady-state properties in the data and the corresponding objects of the model.

The data we have used are 277 quarterly observations of the GDP, the consumption, the investment, the labor force, the inflation rate, and the 3-month Treasury-Bill rates. The sample period is from 1948Q1–2017Q1. GDP is measured by real GDP excluding government expenditure. Consumption is measured by real personal expenditure. Investment is measured by real gross private domestic investment. The labor force is measured by the civilian population aged 16 and over. GDP, consumption, and investment are expressed in per capita terms using the labor force. The inflation rate is measured by changes in the GDP implicit price deflator. The raw data on labor force and interest rates are monthly and were converted to a quarterly frequency by averaging the observations for the three months in each quarter. Except for the interest rates and the labor force, the raw data

\[^{24}\text{The raw data were taken from the FRED database available at the Web site of the Federal Reserve Bank of St. Louis (www.stls.frb.org).}\]
are seasonally adjusted at the source. All series are HP-filtered, excluding inflation and interest rates.

Table 4.3: Moments comparison

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard Deviation</th>
<th>Relative Volatilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>GDP</td>
<td>1.04</td>
<td>1.03</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td>Investment</td>
<td>1.32</td>
<td>3.16</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.10</td>
<td>0.26</td>
</tr>
<tr>
<td>Labor</td>
<td>0.92</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Model Average | Sample Average
C/GDP          | 0.85 | 0.85 |
I/GDP          | 0.14 | 0.19 |

Notes: The standard deviations are expressed in percentage terms.

Overall the model does a good job matching targeted standard deviation of GDP and consumption. In addition, the model can successfully replicate the non-targeted average steady-state ratio of consumption to output and average steady-state ratio of investment to output. However, in the data the standard deviation of investment is about 3 times as volatile as GDP whereas in the model the standard deviation of investment is only about 1.57 times as volatile as GDP. The model under-predicts the interest rate volatility as well. This is attributable, in part, to the more aggressive response of central bank to the deviations of inflation target and output target. The volatility of labor force is about 0.63 times as volatile as output, whereas in the data it is only 0.18 as volatile as output.
4.4.2 The Effects of Uncertainty Shocks

4.4.2.1 The Effects of the Monetary Policy Uncertainty Shock

We now turn to studying the dynamic effects of monetary policy uncertainty. Since an uncertainty shock is a second-moment shock that widens the distribution of the level shock leaving its mean unchanged. Figure 4.1 shows the effect of the Taylor rule uncertainty shock hitting at $t = 1$ on the distribution of GDP instead of on the distribution of level Taylor rule shock. The green solid line denotes the distribution of GDP without any shock. The red dashed line and the blue dot-dash line represents the response of the distribution of GDP to the Taylor rule uncertainty shock in the succeeding period 2 and 3, respectively. With a positive uncertainty shock, the distribution of GDP becomes wider as the variance of the Taylor rule shock increases, without affecting its mean. As the effect of the shock dies out, the distribution of GDP is returning to its original shape, i.e. it moves from the dashed line to the dot-dash line.

Figure 4.1: The Effect of the Taylor Rule Uncertainty Shock on the GDP distributions in the First Three Periods.
As discussed in Fernández-Villaverde et al. (2011), time-varying volatilities move the ergodic distributions of the model endogenous variables away from their deterministic steady states. They suggest computing impulse responses deviations from the ergodic mean of model simulated data instead of the deterministic steady-state values. We follow their approach. In Appendix A, we detail the procedure. In the uncertainty literature, an uncertainty shock is commonly defined as a two-standard deviation increase in the respective shock’s volatility.

Figure 4.2 plots the impulse responses of consumption, investment, output, employment, interest rates, net worth, capital price, and bankruptcy to a two-standard deviation monetary policy uncertainty shock. Upon impact all variables displayed here decrease, except for the bankruptcy. Output does so by around 0.01 percent, due to fall in both consumption and investment. Investment decrease by also around 0.01 percent, while consumption’s reaction is slightly weaker at 0.004 percent from its ergodic steady state value.

The increase in the future interest rate volatility has two direct effects. First, it increases the expected borrowing cost for entrepreneurs, reducing their incentive to invest. Investment, and thus capital stock decrease over time. According to equation Eq. (4.11), the rise in the borrowing cost also makes the entrepreneur hard to survive. Entrepreneurs with a lower realization of $\omega$ would be unable to cover the cost of borrowing and are forced to leave the market. The threshold $\bar{\omega}$ and hence the bankruptcy rate increases.

Another direct impact of interest rate uncertainty is on the household’s consumption decision. At the beginning, a higher uncertainty in the return of the bond makes the risk-averse households switch their saving into consumption, so the consumption increases. However, this effect is soon dominated by the effect brought out by precautionary savings. Households begin to consume less and work more in the subsequent period. The increased labor supply lowers wage and firms’ marginal costs.
However, due to the price stickiness in the short run, the decreased marginal costs are not accompanied by decreased prices, leading to an increase in markups (see Basu and Bundick (2012)). On the other hand, firms now face increased uncertainty about future
marginal costs. Due to the convexity of the marginal profit curve, firms will increase their price in order to avoid a too low price setting. (see Born and Pfeifer (2014)). These two forces combined together contribute to the increase in inflation. In the end, firms’ profits rise. Investment is further reduced. Output decreases. This observed negative co-movement of consumption, investment, and output is generated under the assumption of sticky price, whereas it is not the case under the flexible price setting (see Basu and Bundick (2012)).

Furthermore, due to the concavity of the adjustment cost, the decrease in investment demand induce a high capital adjustment cost, the capital producer is willing to reduce the price of capital to curb the capital loss. The gross return to capital, which is the sum of return from the production and the price gain of selling capital in the next period, would decrease in the first period due to the lower price of capital. While later on, with a lower level of capital, the marginal product of capital increases and outweighs the impact from the decrease in capital price, hence raises the response of capital return to a positive level.

More importantly, the financial accelerator in assistance to the above-mentioned co-movement, i.e., entrepreneurs’ net worth falls in response to the lower price of capital, leading to a further depressed investment. Because a decreased capital price indicates a decreased net worth, this then translates to a lower leverage and a larger risk premium, investment is depressed further. Generally speaking, through the effect of the net worth of entrepreneurs, the financial accelerator mechanism amplifies the impact of the monetary policy uncertainty shock. Finally, in response to the depressed output, the central bank reduces the interest rate, which encourages the consumption.

Overall, with a higher interest rate policy uncertainty, households consume less because of precautionary saving with or without the financial accelerator mechanism. With financial accelerator mechanism, it is expected that the investment would decrease more dramatically, which is more in line with what we observed during the recent crisis. Hence, an
uncertain policy (for example, a frequently changing interest rate targeting) during the recession could slow down the recovery speed of the economy.

### 4.4.2.2 Comparison of All Three Uncertainty Shocks

In general, the effects monetary policy uncertainty shock are small. Even a two-standard deviation shock to monetary policy decreases output, consumption, and investment by 0.01%, 0.004%, and 0.004%. Born and Pfeifer (2014) report that a joint policy risk shock (including uncertainty shocks to capital tax rates, labor tax rates, investment specific technology, government spending, and monetary policy shock) decreases output by a mere 0.065% and a TFP risk shock decreases output by about 0.009%. They suggest the reason comes from that the aggregate policy risk shocks are too small and not sufficiently amplified. However, if one assumes a larger parameter to the risk shock, the parameter changes not amplify the risk shocks, but also the level shocks. This will generate too large second moments in the model. In the following section, we compare the quantitative effects of all the three uncertainty shocks (monetary policy uncertainty shock, TFP uncertainty shock, and risk shock) in the model to gain more insight on the impact of the monetary policy uncertainty shock qualitatively.

Figure 4.3 plots impulse responses to a two-standard deviation monetary policy uncertainty shock (blue solid line), to a two-standard deviation TFP shock (green dashed line), and to a two-standard deviation risk shock (red dot-dash line). As discussed in Christiano, Motto, and Rostagno (2014), the transmission of risk shock operates through the increased cost of entrepreneurs’ external debt. More specifically, higher dispersion of the risk shock indicates larger realization returns for some entrepreneurs (larger $\omega$) while larger losses for others. This implies a higher bankruptcy rate. The higher bankruptcy rate raises the expected monitoring costs for banks, resulting in a higher lending rate. As the increased borrowing costs reduce the capital demand of entrepreneurs, investment eventually falls. Capital price decreases as well as output. In response of the decreased
output, central bank reduces the interest rate. Consumption increases as a result. This has been noted in Christiano, Motto, and Rostagno (2014), the risk shock resembles an increase in the tax rate on the investment return. This kind of shock discourages savings but encourages consumption.

![Graphs showing IRF to various shocks](image)

Figure 4.3: IRFs to a two-standard deviation monetary policy uncertainty shock (blue solid line), to a two-standard deviation TFP uncertainty shock (green dashed line), and to a two-standard deviation risk shock (red dot-dash line). Level shocks are held constant. Horizontal axes represent quarters. All responses are in percent, except for interest rate and inflation, which are in percentage points.

Our main concern here is the magnitude of the response of variables considered to the three uncertainty shocks. Based on our calibration, in general, the monetary policy uncertainty shock has the largest effect, followed by the TFP uncertainty shock and then the risk...
shock. More specifically, the reductions in GDP, investment, and labor induced by the TFP uncertainty shock and the monetary policy shock are relatively the same. Both the TFP uncertainty shock and the monetary policy uncertainty shock reduce the GDP, the investment, and the labor by around 0.01 percent, 0.0105 percent, and 0.006 percent, respectively. However, the monetary policy uncertainty shock has more pronounced effect on consumption, net worth and bankruptcy rate. The monetary policy uncertainty shock reduces the consumption by about 0.0038 percent while the TFP uncertainty shock only reduces the consumption only about a half of 0.0038 percent. The decrease in the net worth under the monetary policy shock is also around two times larger than the decrease in the net worth under the TFP uncertainty shock. The quantitative effects on bankruptcy rate under the two shocks display the same pattern. This is because the uncertainty shock to the interest rate has direct effect on the costs of entrepreneurs’ external debt as well as on the households’ intertemporal consumption decision. More specifically, with a more pronounced increase in both the external debt costs and uncertainty about it under the interest rate uncertainty shock, entrepreneur will cut its capital investment more than in the case of under TFP uncertainty shock. Households also save more upon the impact of interest rate uncertainty shock.

Although the risk shock has the largest effect on the bankruptcy rate among all three shocks, due to the propagation of this effect to economy is weak, the responses of rest variables are quite limited compared with the responses of variables to the other two uncertainty shocks. In the main text, we set the standard deviation of the risk shock to 0.015, a bit lower than 0.025, which is estimated by Cesa-Bianchi and Fernandez-Corugedo (2015). The relative small responses may come from the relative small standard deviation of the risk shock. Appendix C reports another set of impulse responses, by setting $\epsilon_{\sigma_t}$ to 0.025 while keeping other parameters unchanged to check if the small responses origin from the small deviation of risk shock. However, the results suggest that our main conclusion holds, although slightly larger responses of variables are observed.
4.4.3 Amplification Effects v.s. Risk Adjustment Effects

In this section, we examine the variance decomposition of the amplification effects (the impact of the realized shocks) and risk adjustment effects (the anticipation effect of future shocks). To this end, we solve the model with the nonlinear moving average perturbation algorithm developed by Lan and Meyer-Gohde (2013) and decompose variances as in Lan and Meyer-Gohde (2014). Rewrite our nonlinear DSGE model into the following matrix general form:

$$0 = E_t [f(y_{t+1}, y_t, y_{t-1}, \epsilon_t)],$$

where \( y_t \) is the vector of the endogenous variables, and \( \epsilon_t \) is the vector of the exogenous shocks. The solution of the model, the policy function, takes the form as:

$$y_t = f(\sigma, \epsilon_t, \epsilon_{t-1}, \ldots),$$

where the parameter \( \sigma \in [0, 1] \) scales the distribution of the stochastic shocks \( \epsilon_t \) with \( \sigma = 0 \) denoting the non-stochastic model. To approximate the policy function Eq. (4.29) to the third order, Lan and Meyer-Gohde (2013) take a Volterra expansion around the deterministic steady state and get the following expression:

$$y_t^{(3)} = \bar{y} + \frac{1}{2} y^2 + \frac{1}{2} \sum_{i=0}^{\infty} (y_i + y_{i+1}) \epsilon_{t-i} + \frac{1}{2} \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} y_{j,i} (\epsilon_{t-j} \otimes \epsilon_{t-i})$$

$$+ \frac{1}{6} \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} y_{k,j,i} (\epsilon_{t-k} \otimes \epsilon_{t-j} \otimes \epsilon_{t-i}),$$

where \( \bar{y} \) denotes the deterministic steady state of the model, at which all partial derivatives \( y^2, y_i, y_{i+1}, y_{i,j}, \) and \( y_{k,j,i} \) are evaluated. \( y_i, y_{i,j}, \) and \( y_{k,j,i} \) capture the amplification effects of the realized shocks. More specifically, the amplification effects capture the effect of changes in the dispersion of the shock distribution on the magnitude of shocks realized...
from this distribution. \( y_{\sigma^2} \) and \( y_{\sigma^2,i} \) control the approximation for risk. That is, the risk adjustment effects capture the effect of changes in the dispersion of the shock distribution on the evaluation of expectations. Accordingly, Eq. (4.30) can be rewrite as this way:

\[
y^{(3)}_t \equiv y^{(3)\text{risk}}_t + y^{(3)\text{amp}}_t.
\]

Therefore, the variance decomposition that analyzes the composition of the volatility of endogenous variables can be derived and expressed as follows:

\[
\Xi^{y(3)} = \Xi^{y(3)\text{risk}}_0 + \Xi^{y(3)\text{amp}}_0 + \Xi^{y(3)\text{risk,amp}}_0,
\]

where \( \Xi^{y(3)}_0 \) is the variance of the endogenous variables. \( \Xi^{y(3)\text{risk}}_0 \) contains the variations from the time-varying risk adjustment and \( \Xi^{y(3)\text{amp}}_0 \) contains the variations from the amplification effects. \( \Xi^{y(3)\text{risk,amp}}_0 \) contains the variations from the interaction between the two types of effects.

In Table 4.4, column two and three report the percentage contributions of the amplification effects and the time-varying risk adjustment effects for each volatility shock, respectively. The fourth column reports the percentage contribution of the interaction between the two types of effects. Table 4.5 compares the decomposition results of when all shocks are active and of when volatility shocks are shut off. On the whole, when the volatility shocks are shut off, the contribution of the risk adjustment effects is trivial. Adding volatility shocks alters the composition of variance, i.e., although the increase in the contribution of risk adjustment effects is limited, the contribution of interaction component increases. For example, when the TFP shock is the only source of uncertainty, for investment, the risk adjustment effects explain only 1.94% of the variance and the interaction component explains 18.94 percent.

By only adding the monetary policy uncertainty shock, the contribution of risk adjustment effects for consumption increases around 2 percent and the contribution of amplification
### Table 4.4: Variance decomposition (amplification effects v.s. risk adjustment effects)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Total Risk Adj. (%)</th>
<th>Total Amp. (%)</th>
<th>Total Risk-Amp. Interplay (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Only Risk Shock</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>3.72</td>
<td>85.26</td>
<td>12.03</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.16</td>
<td>90.09</td>
<td>8.76</td>
</tr>
<tr>
<td>Investment</td>
<td>1.33</td>
<td>97.5</td>
<td>1.17</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>1.12</td>
<td>93.5</td>
<td>5.38</td>
</tr>
<tr>
<td>Bankruptcy</td>
<td>0.11</td>
<td>102.44</td>
<td>-2.55</td>
</tr>
<tr>
<td>Inflation</td>
<td>6.63</td>
<td>64.78</td>
<td>28.59</td>
</tr>
<tr>
<td><strong>Only TFP Uncertainty Shock</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>1.88</td>
<td>81.66</td>
<td>16.46</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.81</td>
<td>88.56</td>
<td>10.64</td>
</tr>
<tr>
<td>Investment</td>
<td>1.94</td>
<td>79.12</td>
<td>18.94</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.45</td>
<td>93.8</td>
<td>5.75</td>
</tr>
<tr>
<td>Bankruptcy</td>
<td>0.77</td>
<td>90.92</td>
<td>8.31</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.79</td>
<td>92.17</td>
<td>7.04</td>
</tr>
<tr>
<td><strong>Only MP Uncertainty Shock</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.22</td>
<td>98.25</td>
<td>1.53</td>
</tr>
<tr>
<td>Consumption</td>
<td>2.43</td>
<td>81.67</td>
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</tr>
<tr>
<td>Investment</td>
<td>0.1</td>
<td>99.07</td>
<td>0.83</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>2.71</td>
<td>79.4</td>
<td>17.89</td>
</tr>
<tr>
<td>Bankruptcy</td>
<td>2.74</td>
<td>79.2</td>
<td>18.06</td>
</tr>
<tr>
<td>Inflation</td>
<td>2.35</td>
<td>82.6</td>
<td>15.05</td>
</tr>
</tbody>
</table>

**Notes:** The variance decompositions are expressed in percentage terms.

effects falls to about 81 percent. An increase in the contribution of risk adjustment effects is usually explained by the increase in the risk-averse household’s precautionary saving. The same pattern of precautionary behaviour of entrepreneurs lead to an increase in the contribution of risk adjustment effects for the bankruptcy rate as well. Entrepreneurs in face of higher uncertainty of future borrowing costs also postpone their investment leading to a decrease in capital price. This translates to a fall in entrepreneurs’ net worth and hence induces the varies in the bankruptcy rate.
Table 4.5: Variance decomposition (volatility shock v.s. level shock)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Total Risk Adj. (%)</th>
<th>Total Amp. (%)</th>
<th>Total Risk-Amp. Interplay (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Shocks</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>4.1</td>
<td>84.17</td>
<td>11.73</td>
</tr>
<tr>
<td>Consumption</td>
<td>2.1</td>
<td>83.77</td>
<td>14.12</td>
</tr>
<tr>
<td>Investment</td>
<td>2.35</td>
<td>94.56</td>
<td>3.09</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>1.77</td>
<td>89.35</td>
<td>8.88</td>
</tr>
<tr>
<td>Bankruptcy</td>
<td>0.24</td>
<td>103.49</td>
<td>-3.73</td>
</tr>
<tr>
<td>Inflation</td>
<td>8.75</td>
<td>61.39</td>
<td>29.86</td>
</tr>
<tr>
<td>Only TFP Level Shock and MP Level Shock</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.18</td>
<td>96.74</td>
<td>3.08</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.15</td>
<td>94.72</td>
<td>5.13</td>
</tr>
<tr>
<td>Investment</td>
<td>0.19</td>
<td>96.54</td>
<td>3.27</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.06</td>
<td>97.68</td>
<td>2.27</td>
</tr>
<tr>
<td>Bankruptcy</td>
<td>0.14</td>
<td>101.78</td>
<td>-1.92</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.53</td>
<td>94.61</td>
<td>4.86</td>
</tr>
</tbody>
</table>

Notes: The variance decompositions are expressed in percentage terms.

However, the contribution of risk adjustment effects are quite limited in all three cases in Table 4.4. This is because the uncertainty shocks are too small and not sufficiently amplified. Although, intuitively, a larger uncertainty shocks will increase the contribution of amplified effects as well, our sensitivity analysis shows that, by increasing the steady state value of the monetary policy uncertainty shock from $-6.8$ to $-5$, the contribution of risk adjustment effects will increase by about 10 percent.

### 4.5 Concluding Remarks

This paper uses a DSGE model with financial accelerator mechanism to investigate the effects of monetary policy uncertainty shock under financial frictions. Monetary policy uncertainty shock is introduced as a mean preserving shock to the variance of monetary policy level shock.
Our results show that the increase in the interest rate volatility by increasing the costs of external debt of entrepreneurs reduces their investment demand, which then translates into the decrease in GDP. More importantly, the financial accelerator mechanism amplifies the transmission through the effect on the net worth of entrepreneurs. Specifically, a fall in entrepreneurs’ net worth increases their costs of obtaining external funds, which further depresses investment. Households consume less because of precautionary saving. However, upon the impact of interest rate uncertainty shock, households tend to save more than under the TFP uncertainty shock. This lies in the fact that the uncertainty about the interest rate directly relates to households’ intertemporal consumption decision.

In our model, which is calibrated to the U.S. quarterly data from 1948Q1–2017Q1, a two-standard deviation shock to the interest rate volatility leads to only a proximately 0.01 percent drop in GDP. By comparing with the TFP uncertainty shock and the risk shock, we find that the output effects of the monetary policy uncertainty shock is around eight times larger than the effects of risk shock and is slightly larger than the effects of TFP shock. The monetary policy uncertainty has the largest effects among all three uncertainty shocks, although quantitatively the effects are limited. The possible reason is that the parameterization of uncertainty shocks considered in the model is quite small.
Chapter 5

Conclusions and Future Work

5.1 Conclusions

This dissertation sets out to study how trend inflation affects the share of the leading international currency in global payments, the benefits and costs of having an international currency in the first stage of internationalization, and how monetary policy uncertainty would affect macroeconomy under financial friction. Chapter 2 provides a new perspective on the role and usage of international currencies. The results in Chapter 3 contribute to knowledge of the benefits and costs of having an international currency. The findings of Chapter 4 may help understand the role played by monetary policy uncertainty in holding back the recovery in the U.S. after the Great Recession.

The study of the persistence of the dominant international currency position (Chapter 2) reveals that, in the long run, the position of the leading international currency persists even if the inflation rate in its own country increases from 0 to 8 percent. In addition, real balances of the leading international currency held by the third country are relatively unaffected by the level of inflation in the emerging international currency country, whereas
the real balances of the emerging international currency held by the third country are affected not only by inflation in its own country but also by inflation in the leading international currency country. This in turn affects the seigniorage collected by each country. In the short run, when both international currency countries start with the same trend inflation, an increase of up to 3 percent in trend inflation in the leading international currency country will not lead to a more volatile share of its currency compared with the zero inflation baseline case. Equally importantly, incorporation of trend inflation in the open-economy model reveals that domestic trend inflation amplifies the spillover effects of a domestic technology shock on foreign countries. Trend inflation in foreign countries reinforces these spillover effects through the price dispersion effect.

Investigation of the benefits and costs of having an international currency (Chapter 3) shows that, the benefits of having an international currency are: monetary policy in the international currency country is more potent. The costs of having an international currency are: the seigniorage collected by the international currency country decreases in response to a positive technology shock in its own country; and the international currency country’s households experience losses because an asymmetric exchange rate pass-through leads to a sluggish adjustment of consumption in its own country.

From the study of the impact of policy uncertainty on macroeconomy (Chapter 4), we show that when the economy is subject to policy uncertainty, the costs of obtaining external funds of entrepreneurs increase, which reduces their investment demand and then translates into the decrease in GDP. The financial accelerator mechanism amplifies this negative effects of policy uncertainty on economic activity through reducing the net worth of entrepreneurs and further depressing investment demand. A two-standard deviation shock to the interest rate volatility leads to approximately 0.01 percent drop in GDP. By comparing with the TFP uncertainty shock and the risk shock, the output effects of the monetary policy uncertainty shock is around eight times larger than the effects of risk
shock and is slightly larger than the effects of TFP shock. The monetary policy uncertainty has the largest effects among all three uncertainty shocks, although quantitatively the effects are limited.

5.2 Future Work

There are several important aspects untouched in this paper. First, the welfare implications for the third country when the global economy evolves from a system centered on one international currency to a two-international-currency system is an important issue and can also be investigated in the framework of Chapter 2. Second, for the event that the yuan is promoted to be an international currency, we analyzed the problem theoretically. We need to estimate the structural parameters based on trade data of China and its partners. Third, it is documented that policy uncertainty shocks have strong asymmetric effects on economy activity. To quantify the impact of policy uncertainty on macroeconomy in booms and busts under financial frictions is important for enhancing the understanding of the role played by policy uncertainty in driving business cycles. Exploring these issues in great detail will be left for future research.
Appendices

A Appendix to Chapter 2

A.1 Derivation of the Open Economy NKPC with Trend Inflation

This section contains a detailed derivation of the open economy NKPC (Eq. (2.19)) listed in Section 2.2. Firms in each country can only change prices à la Calvo. Here taking firms in country A as an example, in each period, only a random fraction \((1 - \theta)\) of firms are able to re-optimize their prices, i.e., \(P_{A,i,t}^{A^*}\). The price setting problem becomes:

\[
\max_{P_{A,i,t}^{A^*}} \mathbb{E}_t \sum_{j=0}^{\infty} Q_{t,t+j} \theta^j \left[ P_{A,i,t}^{A^*} Y_{A,i,t+j} + \frac{W_{t+j}^A}{Z_{t+j}^A} Y_{A,i,t+j} \right] \tag{A-1}
\]

subject to the sequence of demand constraints

\[
Y_{A,i,t+j} = \left( \frac{P_{A,i,t}^{A^*}}{P_{A,i,t+j}^A} \right)^{-\epsilon} Y_{A,i,t+j}, \tag{A-2}
\]
where $Y_{A,t}$ denotes aggregate output in country $A$ and $Y_{A,i,t}$ denotes the input produced by an intermediate goods firms, $i$. Equivalently, Eq. (A-1) can be expressed as:

$$\max_{P_{A,i,t}^A} \mathbb{E}_t \sum_{j=0}^{\infty} Q_{t,t+j} \theta^j \left[ P_{A,i,t}^A \left( \frac{P_{A,i,t}^A}{P_{A,t+j}^A} \right)^{-\epsilon} Y_{A,t+j} - MC_{A,t+j} P_{A,t+j}^A \left( \frac{P_{A,i,t}^A}{P_{A,t+j}^A} \right)^{-\epsilon} Y_{A,t+j} \right],$$

(A-3)

where $MC_{A,t+j}$ denotes the real marginal cost. Therefore, $P_{A,i,t}^A$ must satisfy the first order condition as follows:

$$\mathbb{E}_t \sum_{j=0}^{\infty} Q_{t,t+j} \theta^j \left[ (1-\epsilon) (P_{A,i,t}^A)^{-\epsilon} (P_{A,t+j}^A)^{\epsilon} Y_{A,t+j} + \epsilon MC_{t+j}^A P_{A,t+j}^A (P_{A,i,t}^A)^{-\epsilon} (P_{A,t+j}^A)^{\epsilon} Y_{A,t+j} \right],$$

from which we get the expression for $P_{A,i,t}^A$:

$$P_{A,i,t}^A = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \left[ \sum_{j=0}^{\infty} Q_{t,t+j} \theta^j (P_{A,t+j}^A)^{\epsilon} MC_{t+j}^A Y_{A,t+j} P_{A,t+j}^A \right]}{\mathbb{E}_t \left[ \sum_{j=0}^{\infty} Q_{t,t+j} \theta^j (P_{A,t+j}^A)^{\epsilon} Y_{A,t+j} \right]}.$$  

(A-4)

In real terms (dividing both sides by $P_{A,t}^A$), Eq. (A-4) becomes:

$$\frac{P_{A,i,t}^A}{P_{A,t}^A} = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \left[ \sum_{j=0}^{\infty} Q_{t,t+j} \theta^j (P_{A,t+j}^A)^{\epsilon} MC_{t+j}^A Y_{A,t+j} P_{A,t+j}^A \right]}{\mathbb{E}_t \left[ \sum_{j=0}^{\infty} Q_{t,t+j} \theta^j (P_{A,t+j}^A)^{\epsilon} Y_{A,t+j} P_{A,t+j}^A \right]}.$$ 

(A-5)

We then follow Ascari and Sbordone (2014), defining $p_{A,i,t}^A \equiv \frac{P_{A,i,t}^A}{P_{A,t}^A}$ and rewriting Eq. (A-5) as:

$$p_{A,i,t}^A = \frac{\epsilon}{\epsilon - 1} \frac{\psi_t^A}{\phi_t^A},$$

(A-6)

where the auxiliary variables $\psi_t^A$ and $\phi_t^A$ are defined, respectively, as:

$$\psi_t^A = \mathbb{E}_t \left[ \sum_{j=0}^{\infty} U_{C,t+j} (\beta \theta)^j (\Pi_{t,t+j})^\epsilon MC_{t+j}^A Y_{A,t+j} \frac{P_{A,t+j}^A}{P_{A,t}^A} \right],$$

(A-7)
\[ \phi_t^A \equiv \mathbb{E}_t \left[ \sum_{j=0}^{\infty} U_{C,t+j}^A (\beta \theta)^j (\Pi_{t,t+j}^A)^{\epsilon - 1} Y_{A,t+j}^A \frac{P_{A,t+j}^A}{P_{A,t}^A} \right]. \] (A-8)

The infinite summations in Eqs. (A-7) and (A-8) can be written recursively as:

\[ \psi_t^A = U_{C,t}^A MC_t^A Y_{A,t}^A \frac{P_{A,t}^A}{P_{A,t}} + \theta \mathbb{E}_t [\beta (\Pi_{t,t+1}^A)^{\epsilon} \psi_{t+1}^A], \] (A-9)

and

\[ \phi_t^A = U_{C,t}^A Y_{A,t}^A \frac{P_{A,t}^A}{P_{A,t}} + \theta \mathbb{E}_t [\beta (\Pi_{t,t+1}^A)^{\epsilon - 1} \phi_{t+1}^A]. \] (A-10)

Moreover, the dynamics of the domestic price index are described by the equation:

\[ P_{A,t}^A = [\theta(P_{A,t-1}^A)^{1-\epsilon} + (1-\theta)(P_{A,t}^{A*})^{1-\epsilon}]^{\frac{1}{1-\epsilon}}, \] (A-11)

which can be written as:

\[ P_{A,t}^{A*} = \left( \frac{1-\theta(\Pi_{t}^{A})^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}}. \] (A-12)

Log-linearizing Eqs. (A-6), (A-7), (A-8), and (A-12), around the positive inflation steady state \( \Pi^A \) and after some manipulation, gives:

\[ \hat{\Pi}_t^A = \beta(1-\Pi^A)(1-\theta(\Pi^A)^{\epsilon-1}) (\hat{U}_{C,t}^A + \hat{Y}_{A,t} - \mathbb{E}_t [\hat{\psi}_{t+1}^A]) \]

\[ + \frac{(1-\theta(\Pi^A)^{\epsilon}) (1-\theta(\Pi^A)^{\epsilon-1})}{\theta(\Pi^A)^{\epsilon-1}} MC_t^A + \beta [1 + \epsilon(\Pi^A - 1)(1-\theta(\Pi^A)^{\epsilon-1})] \mathbb{E}_t [\hat{\psi}_{t+1}^A] \]

\[ + \beta(1-\Pi^A)(1-\theta(\Pi^A)^{\epsilon-1}) \left( \frac{C_{11}}{C_1} \hat{T}_{t}^{BA} + \frac{C_{12}}{C_1} \hat{T}_{t}^{CA} \right), \] (A-13)

where \( C_1 = \alpha_1 + \alpha_2 (T^{BA})^{\eta-1} + \alpha_3 (T^{CA})^{\eta-1} \), \( C_{11} = \alpha_2 (T^{BA})^{\eta-1} \), and \( C_{12} = \alpha_3 (T^{CA})^{\eta-1} \).
A.2 The Complete Log-linearized Model

The complete log-linearized model are defined in this section. Below, $\Delta$ denotes the change over two consecutive time periods. Variables without time subscripts denote its steady-state values and the circumflex denotes log deviations from the deterministic steady state.

The first-order conditions for households in country A are:

$$-\sigma \mathbb{E}_t[\Delta \hat{C}_{A,t+1}] - \frac{(1-\alpha)(1-\sigma)}{R_A-1} \mathbb{E}_t[\Delta \hat{R}_{A,t+1}] = -(\hat{R}_{A,t} - \mathbb{E}_t[\hat{\Pi}_{A,t+1}]), \quad (A-14)$$

$$\hat{m}_{A,t}^A + \frac{1}{R_A-1} \hat{R}_{A,t} = \hat{C}_{A,t}. \quad (A-15)$$

The first-order conditions for households in country B are:

$$-\sigma \mathbb{E}_t[\Delta \hat{C}_{B,t+1}] - \frac{(1-\mu)(1-\alpha)(1-\sigma)}{R_A-1} \mathbb{E}_t[\Delta \hat{R}_{A,t+1}]$$

$$- \frac{\mu(1-\alpha)(1-\sigma)}{R_B-1} \mathbb{E}_t[\Delta \hat{R}_{B,t+1}] = -(\hat{R}_{B,t} - \mathbb{E}_t[\hat{\Pi}_{B,t+1}]);$$

$$\hat{m}_{B,t}^B + \frac{1}{R_B-1} \hat{R}_{B,t} = \hat{C}_{B,t}; \quad (A-16)$$

$$\hat{m}_{B,t}^A + \hat{O}_{BA,t} + \frac{1}{R_A-1} \hat{R}_{A,t} = \hat{C}_{B,t}. \quad (A-17)$$

The first-order conditions for households in country C are:

$$-\sigma \mathbb{E}_t[\Delta \hat{C}_{C,t+1}] - \frac{\mu(1-\alpha)(1-\sigma)}{R_C-1} \mathbb{E}_t[\Delta \hat{R}_{C,t+1}] - \frac{(1-\mu)(1-\alpha)(1-\sigma)b_1}{R_A-1} \mathbb{E}_t[\Delta \hat{R}_{A,t+1}]$$

$$- \frac{(1-\mu)(1-\alpha)(1-\sigma)b_2}{R_B-1} \mathbb{E}_t[\Delta \hat{R}_{B,t+1}] = -(\hat{R}_{C,t} - \mathbb{E}_t[\hat{\Pi}_{C,t+1}]);$$

$$\hat{m}_{C,t}^C + \frac{1}{R_C-1} \hat{R}_{C,t} = \hat{C}_{C,t}; \quad (A-19)$$

$$\hat{m}_{C,t}^A + \hat{O}_{BA,t} + \frac{1}{R_A-1} \hat{R}_{A,t} = \hat{C}_{C,t}. \quad (A-20)$$
Price dispersion, present and future marginal revenues, and NKPC in country A are:

\[
\hat{s}_t^A = -\epsilon(1 - \theta(P^A)^\epsilon)\hat{p}_{A,t}^A + \theta(P^A)^\epsilon(\epsilon\hat{\Pi}_t^A + \hat{s}_{t-1}^A); \tag{A-24}
\]

\[
\hat{\psi}_t^A = (1 - \theta\beta(P^A)^\epsilon)(\hat{U}_t^A - \hat{Y}_A,t + \hat{Y}_A,t + (\hat{P}_{A,t} - \hat{P}_{A,t}))
+ \theta\beta(P^A)^\epsilon(\epsilon\hat{E}_t[\hat{\Pi}_{t+1}^A] + \hat{E}_t[\hat{s}_{t+1}^A]); \tag{A-25}
\]

\[
\hat{\Pi}_t^A = \beta(1 - P^A)(1 - \theta(P^A)^\epsilon^{-1})(\hat{U}_t^A - \hat{Y}_A,t + \hat{Y}_A,t + (\hat{P}_{A,t} - \hat{P}_{A,t}))
+ \frac{1 - \theta\beta(P^A)^\epsilon(1 - \theta(P^A)^\epsilon^{-1})}{1 - \theta(P^A)^\epsilon^{-1}}\hat{M}_t^A
+ \beta[1 + \epsilon(P^A - 1)(1 - \theta(P^A)^\epsilon^{-1})\hat{E}_t[\hat{\Pi}_{t+1}^A]]
+ \beta(1 - P^A)(1 - \theta(P^A)^\epsilon^{-1})(\hat{P}_{A,t} - \hat{P}_{A,t}). \tag{A-26}
\]

Price dispersion, present and future marginal revenues, and NKPC in country B are:

\[
\hat{s}_t^B = -\epsilon(1 - \theta(P^B)^\epsilon)\hat{p}_{B,t}^B + \theta(P^B)^\epsilon(\epsilon\hat{\Pi}_t^B + \hat{s}_{t-1}^B); \tag{A-27}
\]

\[
\hat{\psi}_t^B = (1 - \theta\beta(P^B)^\epsilon)(\hat{U}_t^B - \hat{Y}_B,t + \hat{Y}_B,t + (\hat{P}_{B,t} - \hat{P}_{B,t}))
+ \theta\beta(P^B)^\epsilon(\epsilon\hat{E}_t[\hat{\Pi}_{t+1}^B] + \hat{E}_t[\hat{s}_{t+1}^B]); \tag{A-28}
\]

\[
\hat{\Pi}_t^B = \beta(1 - P^B)(1 - \theta(P^B)^\epsilon^{-1})(\hat{U}_t^B - \hat{Y}_B,t + \hat{Y}_B,t + (\hat{P}_{B,t} - \hat{P}_{B,t}))
+ \frac{1 - \theta\beta(P^B)^\epsilon(1 - \theta(P^B)^\epsilon^{-1})}{1 - \theta(P^B)^\epsilon^{-1}}\hat{M}_t^B
+ \beta[1 + \epsilon(P^B - 1)(1 - \theta(P^B)^\epsilon^{-1})\hat{E}_t[\hat{\Pi}_{t+1}^B]]
+ \beta(1 - P^B)(1 - \theta(P^B)^\epsilon^{-1})(\hat{P}_{B,t} - \hat{P}_{B,t}). \tag{A-29}
\]
Price dispersion, present and future marginal revenues, and NKPC in country C are:

\[ s_t^C = - \epsilon(1 - \theta(\Pi^C)^\gamma)\hat{P}_{C,t} + \theta(\Pi^C)^\gamma(\epsilon\tilde{\Pi}_t^C + s_{t-1}^C); \]  
(A-30)

\[ \hat{\psi}_t^C = (1 - \theta \beta(\Pi^C)^\gamma)(\hat{U}_{C,t} + \hat{Y}_{C,t} + (\hat{P}_{C,t} - \hat{P}_{C,t})); \]  
(A-31)

\[ \hat{\Pi}_t^C = \beta(1 - \Pi^C)(1 - \theta(\Pi^C)^\gamma - 1)(\hat{U}_{C,t} + \hat{Y}_{C,t} - \epsilon\tilde{\Pi}_t^C) \]  
(A-32)

International linkages and risk sharing conditions are:

\[ (O^{BA})^{1-\eta}C_2 \hat{O}_{t}^{BA} = C_{21} \left( \frac{\alpha_1}{\alpha_2} - (O^{BA})^{1-\eta} \hat{T}_{t}^{BA} - C_{22}(1 - (O^{BA})^{1-\eta}) \hat{T}_{t}^{CB}; \]  
(A-33)

\[ (O^{CA})^{1-\eta}C_3 \hat{C}_{t}^{CB} = C_{31} \left( \frac{\alpha_1}{\alpha_2} - (O^{CA})^{1-\eta} \hat{T}_{t}^{CB} + C_{32}(1 - (O^{CA})^{1-\eta}) \hat{T}_{t}^{CA}; \]  
(A-34)

\[ (O^{CB})^{1-\eta}C_3 \hat{T}_{t}^{CB} = C_{32}(1 - (O^{CB})^{1-\eta}) \hat{T}_{t}^{CB}; \]  
(A-35)

\[ \hat{P}_{A,t} - \hat{P}_{A,t}^A = - \frac{C_{11}}{C_1} \hat{T}_{t}^{BA} - \frac{C_{12}}{C_1} \hat{T}_{t}^{CA}; \]  
(A-36)

\[ \hat{P}_{B,t} - \hat{P}_{B,t}^B = \frac{C_{21}}{C_2} \hat{T}_{t}^{BA} - \frac{C_{22}}{C_2} \hat{T}_{t}^{CB}; \]  
(A-37)

\[ \hat{P}_{C,t} - \hat{P}_{C,t}^C = \frac{C_{31}}{C_3} \hat{T}_{t}^{CA} + \frac{C_{32}}{C_3} \hat{T}_{t}^{CB}; \]  
(A-38)

\[ \hat{\Pi}_{A,t} = \hat{\Pi}_{t}^A - \frac{C_{11}}{C_1} \Delta \hat{T}_{t}^{BA} - \frac{C_{12}}{C_1} \Delta \hat{T}_{t}^{CA}; \]  
(A-39)

\[ \hat{\Pi}_{B,t} = \hat{\Pi}_{t}^B + \frac{C_{21}}{C_2} \Delta \hat{T}_{t}^{BA} - \frac{C_{22}}{C_2} \Delta \hat{T}_{t}^{CB}; \]  
(A-40)

\[ \hat{\Pi}_{C,t} = \hat{\Pi}_{t}^C + \frac{C_{31}}{C_3} \Delta \hat{T}_{t}^{CA} + \frac{C_{32}}{C_3} \Delta \hat{T}_{t}^{CB}; \]  
(A-41)
The semi-structural parameters are defined as follows:

\[ \hat{O}_t^{BA} = \hat{U}^A_{C,t} - \hat{U}^B_{C,t}; \]  
\[ \hat{O}_t^{CA} = \hat{U}^A_{C,t} - \hat{U}^C_{C,t}; \]  
\[ \hat{O}_t^{BA} = \hat{O}_t^{CA} - \hat{O}_t^{CB}. \]  

Market clearing conditions are:

\[ \hat{Y}_{A,t} = \alpha_1 C_1^{\frac{m}{\gamma}} C_A \frac{Y_A}{Y_B} \left[ \eta \left( \frac{C_{11}}{C_1} \hat{T}^{BA}_t - \frac{C_{11}}{C_1} \hat{T}^{CA}_t \right) + \hat{C}_{A,t} \right] \]
\[ + \alpha_2 (T^{BA})^\eta C_2^{\frac{m}{\gamma}} C_B \frac{Y_B}{Y_A} \left[ \eta \left( \frac{C_{21}}{C_2} \hat{T}^{BA}_t - \frac{C_{22}}{C_2} \hat{T}^{CB}_t \right) + \hat{C}_{B,t} \right] \]  
\[ + \alpha_2 (T^{CA})^\eta C_3^{\frac{m}{\gamma}} C_C \frac{Y_C}{Y_A} \left[ \eta \left( \frac{C_{31}}{C_3} \hat{T}^{CA}_t + C_{32} \hat{T}^{CB}_t \right) + \hat{C}_{C,t} \right]; \]  
\[ \hat{Y}_{B,t} = \alpha_1 C_2^{\frac{m}{\gamma}} C_B \frac{Y_B}{Y_B} \left[ \eta \left( \frac{C_{11}}{C_1} \hat{T}^{BA}_t - \frac{C_{11}}{C_1} \hat{T}^{CB}_t \right) + \hat{C}_{B,t} \right] \]
\[ + \alpha_2 (T^{BA})^\eta C_1^{\frac{m}{\gamma}} C_A \frac{Y_A}{Y_B} \left[ \eta \left( \frac{C_{11}}{C_1} \hat{T}^{BA}_t - \frac{C_{12}}{C_1} \hat{T}^{CA}_t + \hat{C}_{A,t} \right) \right] \]
\[ + \alpha_3 (T^{CB})^\eta C_3^{\frac{m}{\gamma}} C_C \frac{Y_C}{Y_A} \left[ \eta \left( \frac{C_{31}}{C_3} \hat{T}^{CA}_t + C_{32} \hat{T}^{CB}_t \right) + \hat{C}_{C,t} \right]; \]  
\[ \hat{Y}_{C,t} = \alpha_1 C_3^{\frac{m}{\gamma}} C_C \frac{Y_C}{Y_C} \left[ \eta \left( \frac{C_{31}}{C_3} \hat{T}^{CA}_t + C_{32} \hat{T}^{CB}_t \right) + \hat{C}_{C,t} \right] \]
\[ + \alpha_3 (T^{CA})^\eta C_3^{\frac{m}{\gamma}} C_A \frac{Y_A}{Y_B} \left[ \eta \left( \frac{C_{11}}{C_1} \hat{T}^{BA}_t + \hat{C}_{A,t} \right) \right] \]
\[ + \alpha_3 (T^{CB})^\eta C_2^{\frac{m}{\gamma}} C_B \frac{Y_B}{Y_A} \left[ \eta \left( \frac{C_{21}}{C_2} \hat{T}^{BA}_t + (1 - \frac{C_{22}}{C_2}) \hat{T}^{CB}_t \right) + \hat{C}_{B,t} \right]. \]  

Monetary policies in three countries are:

\[ \hat{R}_{i,t} = \phi_R \hat{R}_{i,t-1} + (1 - \phi_R)(\phi_A \hat{Y}_{i,t} + \phi_Y \hat{Y}_{i,t}) + v_{i,t}, \]  
\[ v_{i,t} = \rho_{v_i} v_{i,t-1} + \epsilon_{v_{i,t}}, \quad i = A, B, C. \]  

The semi-structural parameters are defined as follows:

\[ b_1 = \frac{1}{1 + \frac{1}{1 - \gamma} \omega^{\frac{1}{\gamma}}} \quad b_2 = \frac{1}{1 + \frac{1}{1 - \gamma} \omega^{\frac{1}{\gamma}}}, \]
where

\[ \omega = \frac{m_B^C O^{CB}}{m_C^A O^{CA}} = \left( \frac{1 - \gamma \left( 1 - 1/R_A \right)}{\gamma \left( 1 - 1/R_B \right)} \right)^\chi. \]

\[ C_1 = \alpha_1 + \alpha_2 (T_{BA})^{\eta-1} + \alpha_2 (T_{CA})^{\eta-1}; \quad C_{11} = \alpha_2 (T_{BA})^{\eta-1}; \quad C_{12} = \alpha_2 (T_{CA})^{\eta-1}, \]

\[ C_2 = \alpha_1 + \alpha_2 (T_{BA})^{1-\eta} + \alpha_2 (T_{CB})^{\eta-1}; \quad C_{21} = \alpha_2 (T_{BA})^{1-\eta}; \quad C_{22} = \alpha_2 (T_{CB})^{\eta-1}, \]

\[ C_3 = \alpha_1 + \alpha_2 (T_{CA})^{1-\eta} + \alpha_2 (T_{CB})^{1-\eta}; \quad C_{31} = \alpha_2 (T_{CA})^{1-\eta}; \quad C_{32} = \alpha_2 (T_{CB})^{1-\eta}. \]
A.3 Variance Decomposition

Table A-1 summarizes the results for the unconditional variance decomposition of the forecast errors for output, consumption, inflation, the share of the U.S. dollar, the real exchange rates and the seigniorage based on the simulation of the model in section 2.5.2.

Table A-1: Variance decomposition

<table>
<thead>
<tr>
<th></th>
<th>$TFP_{US}$</th>
<th>$TFP_{EA}$</th>
<th>$TFP_{RW}$</th>
<th>$MP_{US}$</th>
<th>$MP_{EA}$</th>
<th>$MP_{RW}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U.S.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>54.29</td>
<td>0.06</td>
<td>0.06</td>
<td>45.40</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>Consumption</td>
<td>52.79</td>
<td>1.65</td>
<td>1.69</td>
<td>41.29</td>
<td>1.35</td>
<td>1.23</td>
</tr>
<tr>
<td>Inflation</td>
<td>39.04</td>
<td>0.04</td>
<td>0.05</td>
<td>57.10</td>
<td>1.82</td>
<td>1.95</td>
</tr>
<tr>
<td><strong>EA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.04</td>
<td>53.66</td>
<td>0.06</td>
<td>0.05</td>
<td>46.10</td>
<td>0.09</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.88</td>
<td>53.08</td>
<td>1.65</td>
<td>1.36</td>
<td>40.85</td>
<td>1.19</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.04</td>
<td>36.62</td>
<td>0.04</td>
<td>1.49</td>
<td>60.10</td>
<td>1.71</td>
</tr>
<tr>
<td><strong>Share of the U.S. dollar</strong></td>
<td>40.02</td>
<td>40.16</td>
<td>0.00</td>
<td>14.42</td>
<td>5.40</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>$REX_{US-EA}$</strong></td>
<td>25.42</td>
<td>25.69</td>
<td>0.00</td>
<td>23.43</td>
<td>25.46</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>$REX_{US-RW}$</strong></td>
<td>25.79</td>
<td>0.00</td>
<td>26.21</td>
<td>23.83</td>
<td>0.00</td>
<td>24.17</td>
</tr>
<tr>
<td><strong>$REX_{EA-RW}$</strong></td>
<td>0.00</td>
<td>25.60</td>
<td>25.54</td>
<td>0.00</td>
<td>25.30</td>
<td>23.56</td>
</tr>
<tr>
<td><strong>Seigniorage_{US}</strong></td>
<td>11.45</td>
<td>0.03</td>
<td>0.03</td>
<td>84.67</td>
<td>1.86</td>
<td>1.96</td>
</tr>
<tr>
<td><strong>Seigniorage_{EA}</strong></td>
<td>0.06</td>
<td>28.73</td>
<td>0.07</td>
<td>4.03</td>
<td>62.52</td>
<td>4.59</td>
</tr>
</tbody>
</table>

Notes: Numbers are expressed in percentages.

Domestic shocks are the major source for forecast errors in domestic variables. Since the shock is assumed to be orthogonal, the spillover effects of shocks in the U.S. economy towards the euro area or the other way around are too weak. Domestic technology shocks (labelled “TFP”) take a predominant role in explaining the domestic output and consumption fluctuations. Notice that TFP shock from the U.S. has a greater effect on consumption in the euro area (1.88%) compared with the effect of the technology shock from EA on consumption in the U.S. (1.65%). This is because in the model, we assume the emerging international country also holds the leading international currency. The intertemporal substitution effect thus is stronger for the emerging international currency.
country. Domestic monetary policy shocks (labelled “MP”) are the major source for the forecast error in domestic inflation and seigniorage. The share of the U.S. dollar is affected equally by the TFP shocks from the U.S. and the euro area (around 40%). But the contribution of MP shocks from the U.S. is stronger (around 14%) than the contribution of MP shocks from the euro area (5.4%). This is due to the fact that in the model the change of the real balances of the emerging international currency held by the third country is very sensitive to the change in the real balances of the leading international currency held by the third country.
A.4 Data Description

The seven quarterly series we use to compare the second moments and correlations are real GDP per capita, real personal consumption in both the U.S. and the euro area, CPI in U.S. and HICP for the euro area and the share of the U.S. dollar (USD) during the period of 1999Q1–2015Q4. For the United States, real GDP per capita and real personal consumption expenditure are taken from the Federal Reserve Bank of St.Louis. For the euro area, real GDP per capita and real personal consumption come from the “Main Economic Indicators-complete database” of the Organization for Economic Co-operation and Development (OECD). The CPI for the U.S. and HICP for the euro area are from the International Financial Statistics, IMF. We follow Ogawa and Sasaki (1998) using data on liabilities in foreign currencies for Eurocurrency markets as a proxy for the balances of international currencies. The share of the USD is calculated by the USD in foreign currency liabilities over the sum of the USD and the Euro in foreign currency liabilities. All series are logged and HP-filtered.
A.5 Supplementary Figures for Chapter 2

Figure A.1: USD and GBP acceptances in trade, 1927-1937 (millions of GBP). The red solid line depicts the total U.S. dollar acceptance in trade and the star dots represent the British Pound acceptance in trade. *Source: Eichengreen and Flandreau (2012)*
Figure A.2: The effects of trend inflation on the steady-state real balances of international currency A in country B. The values of all x-axis are expressed in percentage.
Figure A.3: The effects of trend inflation on the steady-state money supply of each country. The values of all x-axis are expressed in percentage.
Appendix B. Appendix to Chapter 3

B.1 The Log-linearized Equilibrium System with PCP

The complete set of log-linearized equilibrium system for the PCP model is presented in this section. Below, $\Delta$ denotes the change over two consecutive time periods. We use variables with overbars and without time subscripts to denote its steady-state values. The circumflex denotes log deviations from the deterministic steady state.

The first-order conditions for households are:

\[ B \]  
\[ \begin{align*}  
  \left[ -\sigma \tilde{C}_t + \tilde{W}_t - \phi \tilde{N}_t \right] \frac{C^{\sigma} W}{N^\phi} = \frac{\tau B^*_H \tilde{B}_{H,t}}{R} + \frac{1 - \tau B^*_H}{R} \tilde{R}_t; \\
  \left[ -\sigma \tilde{C}_t^* + \tilde{W}_t^* - \phi \tilde{N}_t^* \right] \frac{C^{\sigma} W^*}{N^{\phi}} = \frac{-2\tau B^*_H \tilde{B}_{H,t}}{R^*} + \frac{1 + \tau B^*_H}{R^*} \tilde{R}_t^* + \frac{1 + \tau B^*_H}{R} \tilde{R}_t; \\
  \phi E_t \tilde{N}_{t+1}^* - \phi \tilde{N}_t^* + \tilde{W}_t^* - E_t \tilde{W}_{t+1} + \tilde{R}_t^* = \frac{\tau B^*_H}{\tau B^*_H - 1} \tilde{B}_{H,t}; \\
  \phi E_t \tilde{N}_{t+1} + \tilde{W}_t - E_t \tilde{W}_{t+1} + \tilde{R}_t = \frac{\tau B^*_H}{\tau B^*_H + 1} \tilde{B}_{H,t}; \\
  \tilde{R}_t - \tilde{R}_t^* = E_t \Delta q_{t+1} - E_t \tilde{\pi}_{t+1} + E_t \tilde{\pi}_{t+1}; \\
  \tilde{M}_C_t = \left[ \frac{\tau B^*_H}{R} \tilde{B}_{H,t} + \frac{1 - \tau B^*_H}{R} \tilde{R}_t \right] \frac{N^\phi}{C^{\sigma} W} + \phi \tilde{Y}_t + \sigma \tilde{C}_t + \alpha s_t - (1 + \phi)a_t; \\
  \tilde{M}_C_t^* = \left[ \frac{-2\tau B^*_H}{R^*} \tilde{B}_{H,t} + \frac{1 + \tau B^*_H}{R^*} \tilde{R}_t^* + \frac{1 + \tau B^*_H}{R} \tilde{R}_t \right] \frac{N^{\phi}}{C^{\sigma} W^*} + \phi \tilde{Y}_t^* + \sigma \tilde{C}_t^* + \alpha s_t^* - (1 + \phi)a_t^*; \\
  q_t = (1 - \alpha)s_t + \alpha s_t^*; \\
  \tilde{\pi}_{H,t} = \beta E_t \tilde{\pi}_{H,t+1} + \lambda \tilde{M}_C_t 
\end{align*} \]
\[ \pi_{F,t}^* = \beta E_t \pi_{F,t+1}^* + \lambda \tilde{C}^*_t, \quad (B-10) \]

where \( \lambda \equiv \frac{(1-\beta \theta)(1-\theta)}{\theta} \).

\[ \pi_t = \pi_{H,t} + \alpha \Delta s_t; \quad (B-11) \]

\[ \pi_t^* = \pi_{F,t}^* + \alpha \Delta s_t^*; \quad (B-12) \]

\[ s_t = -s_t^*; \quad (B-13) \]

\[ y_t = a_t + n_t; \quad (B-14) \]

\[ y_t^* = a_t^* + n_t^*; \quad (B-15) \]

\[ \tilde{Y}_t^* = \frac{\tilde{C}_F^*}{\tilde{Y}} \tilde{C}_{F,t}^* + \frac{\tilde{C}_F^*}{\tilde{Y}} \tilde{C}_{F,t}; \quad (B-16) \]

\[ \tilde{Y}_t = \frac{\tilde{C}_H}{\tilde{Y}} \tilde{C}_{H,t} + \frac{\tilde{C}_H^*}{\tilde{Y}} \tilde{C}_{H,t}^*; \quad (B-17) \]

where \( \tilde{C}_{H,t}, \tilde{C}_{H,t}^*, \tilde{C}_{F,t}, \) and \( \tilde{C}_{F,t}^* \) are expressed as follows:

\[ \tilde{C}_{H,t} = \alpha \eta s_t + \tilde{\varsigma}; \quad (B-18) \]

\[ \tilde{C}_{F,t} = \tilde{\varsigma} - \eta (1 - \alpha) s_t; \quad (B-19) \]

\[ \tilde{C}_{F,t}^* = \tilde{\varsigma}^* + \alpha \eta s_t^*; \quad (B-20) \]

\[ \tilde{C}_{H,t}^* = \tilde{\varsigma}^* - \eta (1 - \alpha) s_t^*. \quad (B-21) \]

The Balance of Payments for country \( H \) is expressed as:

\[
\begin{align*}
B_F(q_t + \tilde{B}_{F,t}) - R^* B_F(q_t + \tilde{R}_{F,t-1} + \tilde{B}_{F,t-1} - \pi_{t}^*) - B_{H}^* \tilde{B}_{H}^* \\
= \frac{1}{2} \tilde{Y}_t - \alpha s_t - \frac{1}{2} \tilde{C} \tilde{\varsigma} - \frac{1}{2} \tilde{Y}^* (q_t + \tilde{Y}_{t}^* - \alpha s_{t}^*) + \frac{1}{2} \tilde{C}^* (q_t + \tilde{C}_{t}^*) \\
+ m_t^* \tilde{m}_{t,H}^* - m_t^* (\tilde{m}_{H,t-1} - \pi_{t}) - B_{H}^* \tilde{R} (\tilde{B}_{H,t-1} + \tilde{R}_{t-1} - \pi_{t}),
\end{align*}
\]

\quad (B-22)
where $\tilde{m}_{H,t}$ and $\tilde{m}^*_{H,t}$ are:

$$\bar{m}_H \tilde{m}_{H,t} = C\tilde{C}_t + \bar{C}_H^*(-\alpha s_t + \tilde{C}_{H,t}^*);$$

(B-23)

$$\bar{m}^*_H \tilde{m}^*_{H,t} = C^* F,t (-\alpha s^*_t + \tilde{C}^*_{F,t}).$$

(B-24)

Home Monetary Policy;

(B-25)

Foreign Monetary Policy;

(B-26)

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t};$$

(B-27)

$$a^*_t = \rho_a a^*_{t-1} + \epsilon_{a,t}. $$

(B-28)
B.2 The Alternative Currency Pricing: PCP-LCP

Alternatively, if we assume PCP-LCP pricing for foreign country, in a monopolistically competitive market, the optimization problem of foreign firm is as follows:

\[
\begin{align*}
\text{max}_{\tilde{P}_{F,t}, \tilde{P}_{F,t}} & \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k}^* + \frac{\tilde{P}_{F,t}}{Z_{t+k}} \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k} \
& - M C_{t+k}^{*n} \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k}^* + \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k} \right\} \n\end{align*}
\]

where \(\tilde{P}_{F,t}\) and \(\tilde{P}_{F,t}\) are the prices set by firms when they choose to change the prices of goods sold domestically and abroad, respectively.

Taking derivative with respect to \(\tilde{P}_{F,t}\), we have the following condition:

\[
\sum_{k=0}^{\infty} \theta^k E_t \left\{ \left[ \tilde{P}_{F,t}^* - \frac{\epsilon}{\epsilon - 1} M C_{t+k}^{*n} \right] \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k} \right\} = 0. \tag{B-30}
\]

Taking derivative with respect to \(\tilde{P}_{F,t}\) gives rise to:

\[
\sum_{k=0}^{\infty} \theta^k E_t \left\{ \left[ \frac{\tilde{P}_{F,t}}{Z_{t+k}} \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k} - \frac{\epsilon}{\epsilon - 1} M C_{t+k}^{*n} \left( \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t+k}} \right)^{-\epsilon} C_{F,t+k} \right] \right\} = 0. \tag{B-31}
\]

Solve for \(\tilde{P}_{F,t}\) and \(\tilde{P}_{F,t}\) and take log-linearization around their steady state values will give us Eq. (3.32).

For the sake of brevity, we only list the equations that are different from the PCP model for the log-linearized equilibrium conditions of the PCP-LCP model. The equations (A-1)–(A-6), (A-9), (A-11), (A-13)–(A-19), (A-23), and (A-25)–(A-28) in the PCP-LCP model are kept and equations (A-7), (A-8), (A-10), (A-12), (A-20)–(A-22), and (A-24) are replaced by the following equations:
\[ \pi^*_F, t = \beta_{E} \pi^*_{F, t+1} - (1 - \gamma) \lambda \psi_{F, t} + \lambda \tilde{MC}^*_t; \]  
(B-32)

\[ \pi_{F, t} = \beta_{E} \pi_{F, t+1} + \gamma \lambda \psi_{F, t} + \lambda \tilde{MC}^*_t; \]  
(B-33)

\[ q_t = \alpha s^*_t + (1 - \alpha) s_t + (1 - \alpha) \psi_{F, t}; \]  
(B-34)

\[ \pi^*_t = \alpha \Delta (s^*_t - \psi_{F, t}) + \pi^*_F, t; \]  
(B-35)

\[ \pi_t = (1 - \alpha) \pi_{H, t} + \alpha \pi_{F, t}; \]  
(B-36)

\[ \tilde{C}^*_H, t = \tilde{C}^*_t - (1 - \alpha) \eta (s^*_t - \psi_{F, t}); \]  
(B-37)

\[ \tilde{C}^*_F, t = \tilde{C}^*_t + \alpha \eta (s^*_t - \psi_{F, t}); \]  
(B-38)

\[ \tilde{MC}^*_t = \left[ \frac{-2 \tau B^*_H \tilde{B}^*_H, t}{R^*} + \frac{1 + \tau B^*_H \tilde{R}^*_t}{R^*} + \frac{1 + \tau B^*_H \tilde{R}_t}{R^*} \right] \frac{N^{\phi}}{\tilde{C}^* - \sigma W^*} + \phi \tilde{Y}^*_t + \sigma \tilde{C}^*_t \]  
(B-39)

where \( \gamma = \frac{C^*_F}{\tilde{F}} \) is the ratio of domestic imports to output in the foreign country.
C  Appendix to Chapter 4

C.1 Impulse Response Construction

We follow Fernández-Villaverde et al. (2011) and generate impulse responses as the response to a two standard deviation shock to uncertainty shocks at the ergodic mean in the absence of shocks steady state. We generate the impulse responses in the following manner (see technical appendix to Fernández-Villaverde et al. (2011) as well as technical appendix to Born and Pfeifer (2014)). To assure non-explosive behaviour of the simulations, we use the pruning algorithm of Kim et al. (2008).

1. We first simulate the model, starting from its steady state, for 2096 periods. We drop the first 2000 periods as a burn-in.

2. Based on the last 96 periods, we compute the mean of the ergodic distribution for each variable in our model. Adding more periods has essentially no impact on the mean (Figure C.1 shows that the distribution of GDP does not change too much after around 2000 periods of simulation).

3. Starting from the ergodic mean and in the absence of shocks, we hit the model with a two-standard-deviation shock to the volatility process.

4. We report the resulting impulse responses as deviations from the variables’ ergodic means.
Figure C.1: Density of Simulated GDP Series.
C.2 The Complete Model

The complete model are defined in this section. Variables without time subscripts denote its steady state values.

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \eta \frac{L_t^{1+\sigma_l}}{1+\sigma_l} \right\}; \quad (C-1)
\]

\[
C_t P_t + D_{t+1} = W_t L_t + R_tD_t + \Pi_t; \quad (C-2)
\]

\[
\frac{1}{R_t} = \mathbb{E}_t \left\{ \frac{\beta C_{t+1}^{1-\sigma_c}}{C_t^{1-\sigma_c}} \frac{P_t}{P_{t+1}} \right\}; \quad (C-3)
\]

\[
\eta C_t^{\sigma_c} L_t^{\sigma_l} = w_t; \quad (C-4)
\]

\[
K_{t+1}J_t = N_{t+1}J_t + B_{t+1}^J; \quad (C-5)
\]

\[
\varpi^J R_{t+1}^J Q_t K_{t+1}^J = Z_{t+1}^J B_{t+1}^J; \quad (C-6)
\]

\[
\kappa_{t+1}^J = \left[ 1 - \frac{R_{t+1}^J}{R_t} \left( \frac{(\varpi^J)^{1-\phi}}{(1-F(\varpi))} + (1-\mu)^{1-\phi} \int_0^{\varpi^J} (\omega^{J})^{1-\phi} dF(\omega) \right) \right]^{-1}; \quad (C-7)
\]

\[
\frac{1 - F(\varpi)}{1 - \Gamma(\varpi)} = \frac{1}{1-\phi} \Phi(\varpi)^{-\phi} \left( (1-\phi)(1-F(\varpi)) - (1-\mu)^{1-\phi} \varpi^J F'(\varpi) \right); \quad (C-8)
\]

\[
N_{t+1} = \gamma \left[ 1 - \Gamma_{t-1}(\varpi_t) \right] R_t^J Q_{t-1} K_t; \quad (C-9)
\]

\[
Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}; \quad (C-10)
\]

\[
A_t = (1-\rho_A)A + \rho_A A_{t-1} + e^{\sigma_{A,t}} \epsilon_{A,t}, \text{ where } \epsilon_{A,t} \overset{i.i.d.}{\sim} \mathcal{N}(0, 1); \quad (C-11)
\]

\[
\frac{1}{\chi_t} \frac{Y_t}{K_t} = r_t; \quad (C-12)
\]

\[
(1-\alpha) \frac{1}{\chi_t} \frac{Y_t}{L_t} = w_t; \quad (C-13)
\]

\[
Y_t = \left[ \int_0^1 Y_{t,i}^{\leq 1} di \right]^{\frac{1}{\phi}}. \quad (C-13)
\]
\[ P_{t,t}^* = \frac{\epsilon}{\epsilon - 1} \mathbb{E}_t \left[ \sum_{j=0}^{\infty} Q_{t,t+j}^f (\theta \beta)^j (P_{t+j}^1)^{\epsilon - 1} P_{t+j}^w Y_{t+j} \right] ; \]  
\[ (C-14) \]

\[ P_t = \left[ \theta P_{t-1}^{1-\epsilon} + (1 - \theta) (P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} ; \]  
\[ (C-15) \]

\[ K_t = I_t + (1 - \delta) K_{t-1} - \frac{\phi_K}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} ; \]  
\[ (C-16) \]

\[ \frac{1}{Q_t} = 1 - \phi_K \left( \frac{I_t}{K_t} - \delta \right) ; \]  
\[ (C-17) \]

\[ R_t = \left( \frac{1 + \Pi_t}{1 + \Pi} \right)^{\phi_m} \left( \frac{Y_t}{Y} \right)^{\phi_Y} v_t ; \]  
\[ (C-18) \]

\[ v_t = (1 - \rho_v) v + \rho_v v_{t-1} + \epsilon^{\sigma_{v,t}} \epsilon_{v,t}, \text{ where } \epsilon_{v,t} \overset{i.i.d.}{\sim} \mathcal{N}(0, 1) ; \]  
\[ (C-19) \]

\[ Y_t = C_t + I_t + D_t ; \]  
\[ (C-20) \]

\[ \log(\sigma_t) = (1 - \rho_\sigma) \sigma + \rho_\sigma \log(\sigma_{t-1}) + \eta_\sigma \epsilon_t^\sigma, \text{ where } \epsilon_t^\sigma \overset{i.i.d.}{\sim} \mathcal{N}(0, 1) ; \]  
\[ (C-21) \]

\[ \sigma_{A,t} = (1 - \rho_\sigma^A) \sigma_A + \rho_\sigma^A \sigma_{A,t-1} + \eta_\sigma^A \epsilon_t^{\sigma_A}, \text{ where } \epsilon_t^{\sigma_A} \overset{i.i.d.}{\sim} \mathcal{N}(0, 1) ; \]  
\[ (C-22) \]

\[ \sigma_{v,t} = (1 - \rho_\sigma^v) \sigma_v + \rho_\sigma^v \sigma_{v,t-1} + \eta_\sigma^v \epsilon_t^{\sigma_v}, \text{ where } \epsilon_t^{\sigma_v} \overset{i.i.d.}{\sim} \mathcal{N}(0, 1) . \]  
\[ (C-23) \]
C.3 Impulse Responses with Different Parameterization of $\eta_\sigma$

This set of impulse responses is plotted with $\eta_\sigma$ equal to 0.025, while keeping other parameters the same as in the main text.

Figure C.2: IRFs to a two-standard deviation monetary policy uncertainty shock (blue solid line), to a two-standard deviation TFP uncertainty shock (green dashed line), and to a two-standard deviation risk shock (red dot-dash line). Level shocks are held constant. Horizontal axes represent quarters. All responses are in percent, except for interest rate and inflation, which are in percentage points.
References


