A numerical investigation of the flow around two staggered cylinders

by

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ABSTRACT

The cross-flow past a pair of staggered circular cylinders is investigated numerically at a Reynolds number of 800 using a 2-D random vortex method. The numerical model was validated at a Reynolds number of 200. A flow around two stationary cylinders was investigated at a Reynolds number of 800 for comparison. Besides staggered configurations, tandem and side-by-side arrangements were also studied for better understanding of a flow around two cylinders.

For staggered arrangements, four different cylinder configurations were discussed in the stationary cylinders case. For a small pitch ratio, P/D = 2, at $\alpha = 16^\circ$ the upstream cylinder’s inner shear-layer was deflected between the two cylinders and induced the separation of the gap shear layer of the downstream cylinder. One Strouhal number of 0.159 was observed in the present study. For a larger pitch ratio, P/D = 3.25, at $\alpha = 16^\circ$, the vortices shed from the upstream cylinder impinge on the downstream one. A single Kármán vortex street is formed behind the downstream cylinder, and therefore, the obtained Strouhal number is very close to that in a single stationary cylinder. At moderate incidence angles, $20^\circ \leq \alpha \leq 45^\circ$, a pair of vortices was shed via the inner shear-layers of the two cylinders, and then enveloped by the outer shear-layer from the upstream cylinder. This process yields a large combined vortex, which is followed by a vortex shed from the downstream cylinder’s outer shear-layer, and multiple Strouhal numbers with an integral relationship. In general, the mean drag coefficient of the downstream cylinder was smaller than that of the upstream cylinder. The negative mean lift coefficients were observed for the downstream cylinder for all configurations considered in this study.
RÉSUMÉ

L’écoulement transversal a passé une paire de cylindres circulaires quiconces est étudié numériquement au nombre Reynolds de 800 utilisant une méthode de tourbillonnement en 2 dimensions. Le modèle numérique a été validé au nombre Reynolds de 200. Un écoulement autour de 2 cylindres stationnaires a aussi été étudié au nombre Reynolds de 800 comme comparaison. À part les dispositions quiconces, les dispositions tandem et côte à côte ont aussi été examinées afin de mieux comprendre l’écoulement à l’entour des deux cylindres.

Pour les dispositions quiconces, quatre configurations différentes de cylindres ont été préparées dans le cas des cylindres stationnaires. Pour un petit degré de ratio, $P/D = 2$, à $\alpha = 16^\circ$, l’intérieur de la couche cisaillée du cylindre en amont a été dévié entre les deux cylindres et a provoqué la séparation de l’écart de la couche cisaillée du cylindre en aval. Un nombre Strouhal de 0.159 a été observé dans la présente étude. Pour un plus grand degré de ratio, $P/D = 3.25$, à $\alpha = 16^\circ$ les tourbillons se sont répandus du cylindre en amont pour ensuite empiéter sur celui en aval. Un seul tourbillon de Karman est formé derrière le cylindre en aval, et donc, le nombre de Strouhal obtenu est très proche de celui d’un seul cylindre stationnaire. Aux angles d’incidence modérée, $20^\circ \leq \alpha \leq 45^\circ$, une paire de tourbillons s’est répandue via le les couches cisaillées internes des deux cylindres, puis a été enveloppée par le la couche cisaillée externe du cylindre en amont. Cette procédure donne un grand tourbillonnement combiné, qui est suivi par un tourbillonnement répandu par le cylindre de la couche cisaillée externe en aval et les multiples nombres de Strouhal avec une relation intégrale. En général, le coefficient de traînée moyen du cylindre en aval a été plus petit que celui du cylindre en amont. Des coefficients de levage négatifs ont été observés pour le cylindre en aval pour toutes les configurations considérées dans cette étude.
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<td>radius of the cylinder</td>
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<td>Strouhal number of single cylinder</td>
</tr>
<tr>
<td>$F_x, F_y$</td>
<td>force components in the $x$ and $y$ direction</td>
</tr>
<tr>
<td>L</td>
<td>centre-to-centre longitudinal distance</td>
</tr>
<tr>
<td>m, n</td>
<td>number of grid nodes in the circumferential and radial directions</td>
</tr>
<tr>
<td>n</td>
<td>normal direction to the surface</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
</tr>
<tr>
<td>P</td>
<td>centre-to-centre distance between two cylinders</td>
</tr>
</tbody>
</table>
\[ r, \theta \] \quad \text{polar coordinates}

\text{Re} \quad \text{Reynolds number}

\text{St} \quad \text{Strouhal number}

\( t \) \quad \text{time}

\( T \) \quad \text{centre-to-centre transverse distance}

\vec{u} \quad \text{velocity vector}

\( u, v \) \quad \text{Cartesian velocity components}

\( u_\theta, u_r \) \quad \text{circumferential and radial components of velocity}

\( U \) \quad \text{free-stream velocity}

\( x, y \) \quad \text{Cartesian coordinates of a point}

\( X, Y \) \quad \text{Cartesian coordinates of a point}

\( \Delta t \) \quad \text{time step size}

\( \alpha \) \quad \text{incidence angle}

\( \Gamma \) \quad \text{circulation of a vortex}

\( \Gamma_{\text{max}} \) \quad \text{maximum circulation of a vortex}

\( \delta \) \quad \text{dirac delta function}

\( \nu \) \quad \text{kinematic viscosity}

\( \rho \) \quad \text{density}

\( \tau \) \quad \text{shear stress}

\( \psi \) \quad \text{stream function}

\( \omega \) \quad \text{vorticity}

\( \vec{\omega} \) \quad \text{vorticity vector}
Chapter 1

Introduction

1.1 Motivation for the Present Study

The study of viscous flow around bluff bodies has been pursued experimentally and numerically in the past decades. Such flows have been attractive to many fluid dynamicists because they embody a number of complex flow phenomena which have yet to be successfully modeled either analytically or computationally. They are easily found in many engineering applications such as: heat exchangers, offshore structures, tall buildings, bridges, pipelines, electrical cables, etc. A deep understanding of the flow field is therefore required from both academic and practical points of view.

As fluid flows over a bluff body, a boundary layer is formed on the surface of the body and separates from the surface. The separated flow forms the shear layers on both sides of the body. The shear layers roll up and two opposite rotating vortices are formed behind the body due to a velocity gradient across the shear layer. Insta-
bilities trigger one of the vortices to grow faster than the other. This vortex is then released from the body and is convected downstream. In the absence of the first vortex, the second vortex on the opposite side grows faster while the shear layer on the first side rolls up and forms a new vortex. This process continues, and a "street" of vortices appear in the wake of the bluff body. It is the classic Von Kármán vortex street and it encompasses the fundamental flow behaviours of separation, shear layer formation, interaction, and instabilities, mechanism for transition to turbulence, boundary layer growth, and the periodic formation and interaction of coherent vertical structures.

One particular type of bluff body of considerable interest is the circular cylinder. It is due to its simple geometry making it easy for analysis and simulation. The simple case of the cylinder configurations is a single circular cylinder and it shows the Reynolds number effect on the flow separation and the vortex shedding process. The Reynolds number, Re, is based on the cylinder diameter for fluid density, \( \rho \), and absolute viscosity, \( \mu \),

\[
\text{Re} = \frac{\rho UD}{\mu}
\]

At very low Reynolds numbers, \( \text{Re} < 5 \), no flow separation occurs. The separated shear layers meet at some distance downstream of the rear of the cylinder and form a stable recirculation zone which is comprised of two vortices of equal strength and opposite rotation. As the Reynolds number increases, \( 5 \leq \text{Re} \leq 40 \), the flow separates from the rear of the cylinder and two vortices form immediately behind the cylinder. At \( 40 \leq \text{Re} \leq 150 \), the boundary layers separate from each side of the cylinder surface and form two shear layers that bind the wake. In this region, the wake becomes unstable. As each vortex is shed, the flow pattern around the cylinder becomes asymmetrical and, thus, generates oscillating lift and drag coefficients. The Kármán vortex sheet is laminar when the Reynolds number is up to 150. For \( 150 \leq \text{Re} \leq 300 \), the transition of the vortex sheet from laminar to turbulent occurs.
1.1. Motivation for the Present Study

The transition mechanism which consists of the appearance of instabilities in the separated shear layer, the formation of a secondary streamwise vortex structure superimposed on the primary vortices, increased transverse interaction between the primary vortices on the opposite side of the vortex street, three dimensional effects, and boundary layer separation, increases the three dimensional character of the cylinder wake and generates an enhanced level of mixing associated with turbulent flow. In the range of the Reynolds number of $300 \leq \text{Re} \leq 3 \times 10^5$, the vortex shedding is strong and periodic.

In addition to these complex flow phenomena, the circular cylinder in cross flow is a well known configuration associated with flow induced vibrations. The alternating vortex shedding results in an alternating force component on the body which, in turn, causes vibration of an elastic body. One such example is the Aeolian tone, where the oscillation of a wire sheds its vortex at its natural frequency. The aeolian tone has been known since the Ancient Greeks. Strouhal measured the frequency of the audible tone produced by wires moving in the air in 1878 and proposed the dimensionless parameter,

$$\text{St} = \frac{f D}{U},$$

where $f$, $D$, and $U$ are the frequency of the tone, the wire diameter, and the free stream velocity, respectively. The Strouhal number is primarily a function of the Reynolds number for a rigid circular cylinder and remains constant at 0.2 for $300 < \text{Re} < 2 \times 10^5$. For an oscillating cylinder or an elastically mounted cylinder, the Strouhal number changes drastically near the natural frequency of the system or the oscillating frequency. This phenomenon is known as "lock-in". Severe problems such as vibrations of structure or structural failure can be caused by flow induced vibrations besides sound emissions.

The flow field is modified when two or more circular cylinders are placed in close
proximity to one another and exhibits complex interactions between the shear layers, vortices and Kármán vortex street, and the structures themselves. Sumner (1999) experimentally investigated the flow around two circular cylinders. He classified nine flow patterns for various cylinder arrangements and discussed the wake structures in detail. Despite its importance in academic and practical points of view, this subject has rarely been studied experimentally and numerically. In the present study, we re-examine Sumner’s (1999) experiments in the aspect of numerical simulation. The complex flow physics are better understood through in-depth investigations of the vortex dynamics and the shedding frequencies. The present study aims to provide more general and robust information for flow around two circular cylinders by comparing and summarizing the results.

1.2 Numerical Method

Incompressible, viscous flow is mathematically modelled by the Navier-Stokes and continuity equations. No analytical solution is available to these model equations yet, but numerical solutions have become practical since the advent of the modern computer. The most widely used numerical methods are the finite difference (FDM) and finite element methods (FEM), which are based on an Eulerian frame. In finite difference methods, the derivative terms are approximated as a difference quotient. The space and time domains are discretized, and the solution is obtained only at these discrete points. At a high Reynolds number, the velocity gradient near the surface of the body tends to be sharp, and therefore, very fine mesh sizes are required to provide sufficient accuracy in the boundary layer. Such mesh sizes make finite difference methods less efficient computationally. In finite element methods, a spatial discretization and a weighted residual formulation are used to transform the governing partial differential equation (PDE) into an integral equation (weak form). The primary challenge in finite element methods is to
create the numerically stable equation set that approximates the governing PDE. In finite element methods, a large number of algebraic equations need to be solved to approximate the solution; thus, the computational cost is also relatively high.

In a Lagrangian reference frame, the nonlinear advection terms are removed from the governing equations, and thus, the stability issue is relieved, as compared to Eulerian based methods. This characteristic allows a larger time step as compared to Eulerian methods. This can be beneficial when a large amount of calculation is needed. A well known method based on a Lagrangian reference frame is the vortex method. Vortex methods were originally developed as a tool to simulate unsteady, incompressible, high Reynolds number flows. Vortex methods are very convenient for considering inviscid fluid in terms of parcels of vorticity because it characterizes incompressible flow by regions of concentrated vorticity embedded in irrotational fluid. In vortex methods, the vorticity concentrated regions are discretized and tracked in a Lagrangian reference frame. Vortex methods consider, successively in substeps, the inviscid and the viscous parts of the equations. In the first substep, vortices move with the local velocity to satisfy the inviscid part of the equation. Diffusion effects are taken into account in the following substeps by the random walk method.

Recently, owing to high performance computing, higher order and higher dimensional calculations such as direct numerical simulations (DNS), large eddy simulations (LES), simulations based on Reynolds averaged Navier-Stokes equations (RANS), and detached eddy simulations (DES) have become feasible. In direct numerical simulations, small-scale turbulences are directly resolved in the computational domains, but required grid sizes in the range of several hundred million points, and therefore the computational cost is astronomically high. In LES, which eliminates small-scale turbulences instead of solving them through filtering procedure, the computational cost is relatively lower than that in direct numerical simulations; however, the computational cost is still very high for complex geome-
tries. In simulations based on RANS, the equations are decomposed into the time averaged and fluctuating portions. This method requires an additional turbulence model for the Reynolds stress, and therefore, the implementation of this method might be difficult. Finer grid sizes are required in order to account for the turbulence. DES is another approach to solve the governing equations, where the nearfield flow is resolved using RANS but the LES approach is employed in the wake region. The approach is computationally more efficient than LES or DNS however it is still substantially more expensive than the RANS approach.

This study aims to better understand the flow physics of the vortex shedding process behind circular cylinders at various Reynolds numbers, pitch ratios between the cylinders, and incidence angles. A large number of simulations need to be performed for the various configurations and oscillation frequencies. Vortex methods provide meaningful results with acceptable accuracy at a relatively low computational cost. Therefore, a vortex method is used to simulate flow in the present study.

### 1.3 Objectives of the Present Study

The main objective of this research is to investigate numerically the flow around a pair of circular cylinders. The Reynolds number is set to 800, the pitch ratio between cylinders is varied from 1.0 to 5.0 and the incidence angle is incremented from 0° to 90°. These parameters are chosen in order to reproduce Sumner’s (1999) experiments. In the present study, it is assumed that three-dimensional effects will not be dominant at this range of Reynolds numbers, and therefore a two-dimensional model is used to simulate the flow. The primary focus is on the physical process such as the near wake structures, vortex shedding frequencies, and the shear layer interaction. The objectives are as follows. The accomplishment of these objectives
1.3. Objectives of the Present Study

will help us better understand complex flow phenomena and obtain more reliable information for flow around two circular cylinders.

1. Sumner et al. (2000) suggest that the vortex shedding frequencies are more properly associated with the individual shear layers than with the individual cylinder. He showed that the vortex shedding from the inner shear layer of the downstream cylinder is often synchronized with the alternate shedding from the two free shear layers of the upstream cylinder. Consequently, two different vortex shedding frequencies are obtained. This has been proved in recent experimental studies by Price et al. (2007) and Hayder (2008). A close observation will be made on the interaction of the individual shear layers and the effect on vortex shedding frequencies. This will provide a better understanding on the shear layer interaction.

2. Two Strouhal numbers are reported for some of the staggered configurations of two circular cylinders. This is especially true for some Reynolds numbers at low subcritical range where the two Strouhal numbers show an integral relationship. This relationship is explained by the shear layer interaction.

3. One of the disadvantages using the vortex method is the difficulty of estimating the pressure field. In the vortex method, the pressure field needs to be recovered from the given velocity and vorticity field. As the Reynolds number increases, in particular, Re > 200, the numerical values are contrary to experimental results. An attempt is made to improve the pressure calculations when the Reynolds number is larger than 200. For a pair of staggered cylinders, the pressure field of each cylinder is estimated as there exist limited experimental data. A comparison is made with the results from the present study.
1.4 Outline of the thesis

This thesis is organized as follows: in Chapter 2, a brief survey of the literature pertaining to the fluid dynamics of circular cylinders is presented. This review is focused on those directly related to the present study, since there are hundreds of studies concerning flow around circular cylinders. This chapter is divided into two sections: (i) review of a single cylinder; (ii) review of two stationary cylinders.

In Chapter 3, the random vortex method is studied in detail. The basic concepts and governing equations are presented. Then the splitting algorithm, vortex-in-cell method, and random walk method are introduced as the solutions of the governing equations. Lastly, the applications to the present study are described.

In Chapter 4, the numerical model is validated at a Reynolds number of 200. The near wake structure, force coefficients, and the Strouhal number are obtained for a single stationary cylinder. For an oscillating cylinder, the "lock-in" boundary is obtained. Additional study is carried out for an oscillating cylinder at a Reynolds number of 800. The flow patterns are predicted for various oscillation frequencies.

Numerical results of two stationary cylinders are discussed in Chapter 5. Three different configurations, tandem, side-by-side, and staggered, are investigated by means of a random vortex method. The near wake structure, shedding frequencies, and the force coefficients are obtained for each case.

Finally, conclusions from this study and some recommendations for future work are presented in Chapter 6.
Chapter 2

Literature Review

This chapter presents a review of literatures of the flow around circular cylinders. First, this review introduces studies on a single cylinder problems, a stationary case and a transversely oscillating case. Then, two stationary cylinder problems are reviewed for various cylinder arrangements.

2.1 A single cylinder

The cross flow over a single stationary cylinder is a classic problem in fluid mechanics, and considerable processes have been made in understanding the problem based on laboratory experiments and numerical studies. A large number of numerical studies were conducted and confirmed previous experimental results. In this section, we start reviewing numerical studies conducted by using vortex methods, which are often used for a circular cylinder problem and also adopted in the present study.

The algorithm, known as viscous splitting, was first formulated by Chorin (1973).
In the first substep, vortices are convected at the local velocity to satisfy the inviscid part of the equations. Diffusion effects are taken into account at the following substeps by the random walk method, in which vortices undergo a Brownian like method. Chorin (1973) presented the use of discrete vortices with finite cores (blobs) and calculated the velocities for the blobs using the Biot-Savart law, which has a computational expense of order, $N^2$. Thus, the method is computationally expensive for a larger number of vortices. Christiansen (1973) studied the motion of a two dimensional incompressible inviscid flow. Instead of using the vortex blobs method, he introduced the cloud in cell (CIC) approach. The vorticity distribution in the flow is replaced by a finite set of point vortices interacting through the streamfunction which satisfies the Poisson equation. The velocity field is calculated by solving the Poisson equation, and the velocities of the point vortices are obtained by an area weighed interpolation scheme. The CIC method reduces the computational cost considerably.

Vortex methods were originally developed for high Reynolds number flows, and thus many numerical studies were conducted at higher subcritical Reynolds numbers. Stansby and Dixon (1983) simulated the steady and oscillating flows around a circular cylinder at Reynolds numbers of up to $10^4$ using a Lagrangian scheme. The diffusion part was simulated by the random walk method, and the velocity field was obtained by solving the Poisson equation. They tried to calculate the forces, however, their values overestimated known experimental values. Smith and Stansby (1988) studied an impulsively started flow around a circular cylinder in the range of Reynolds numbers $2.5 \times 10^2$ to $10 \times 10^5$. The flow was simulated by a Lagrangian vortex solution using the random walk method for diffusion and the vortex in cell (VIC) method for convection. They found good agreement when there are sufficiently large number of vortices. They suggested that a Lagrangian vortex method is more stable than a traditional Eulerian scheme. The unsteady wake behind an impulsively started cylinder in viscous fluid was investigated by
Cheer (1989) using a random vortex method. She used the vortex sheet method inside the boundary layer and the vortex blob method outside the boundary layer. The numerical simulations were done at Reynolds numbers of 3000, 9500, and $10^4$, and the results were in good agreement with experiments. Cheng et al. (1997) studied flow around a circular cylinder at $Re = 1000, 3000,$ and $9500$. They employed a hybrid vortex method, which is a combination of the diffusion vortex method and the VIC method, by dividing the flow into two regions. The diffusion-vortex method is used in the region near the surface, while the VIC method is used in the exterior domain. The results agreed well with experiments. Flow around a rotating cylinder is also investigated in his study, and the presented results were also in good agreement.

Applications of vortex methods were also made for low Reynolds number flows ($Re < 1000$). Smith and Stansby (1988) and Chang and Chern (1991) simulated the flow around a single cylinder at low Reynolds numbers of $250 \sim 300$ along with their high Reynolds number simulations. The results agreed well with experiments. Akbari (1999) carried out a numerical study of the flow around a circular cylinder using a random vortex method at $Re = 200$. The vorticity structure obtained from his study agreed well with experimental results. He also calculated the force coefficient for Reynolds numbers up to 1000. The present study adopts Akbari’s (1999) numerical algorithm and extends it for the flow around two circular cylinders at $Re = 800$. The pressure calculation is also re-examined in this study.

Besides vortex methods, finite volume, finite element, and LES methods have been used to study the flow around a circular cylinder. Braza et al. (1986) examined the flow around a single cylinder using a finite volume method at low Reynolds numbers, $Re < 1000$. The Navier-Stokes equations in the velocity-pressure form were solved to obtain the velocity and pressure fields in the wake. Their results were in good agreement with experimental data. A finite difference scheme was used by Sengupta and Sengupta (1994) to study the initial stage of an unsteady

Although a two-dimensional numerical solution of the flow around circular cylinders is still considered as a meaningful results to many fluid dynamicists, the problem is in general a three-dimensional and turbulent problem at $\text{Re} > 200$. Bloor (1963) investigated the region of transition to turbulence for the flow of a circular cylinder. Bloor (1963) observed that the turbulent motion is developed for Reynolds numbers greater than 200. Near $\text{Re} = 350$, Kármán vortices became increasingly unstable with the formation of transition waves and shear layer instability vortices. Bloor (1963) suggested that it is a result of distortion due to large scale three-dimensional effects. Williamson (1996) studied three-dimensional instabilities with wind tunnel experiments. Laminar vortex shedding appears up to $\text{Re} = 190$ depending on experimental conditions. As the Reynolds number increases from 190 to 250, the wake transition occurs with the appearance of three-dimensional instabilities. In this range of Reynolds numbers, large scale instabilities (mode A) were observed, while small scale instabilities (mode B) are observed at $\text{Re} > 250$ in Williamson (1996). Prasad and Williamson (1997) studied three-dimensional effects in the turbulent wake by manipulating the cylinder’s end condition. They found that the imposition of oblique shedding could extend the transition regimes from mode A to mode B. The mode transition also exists at $\text{Re} \approx 5000$ under a parallel shedding condition.

Numerically, high resolution and dimensional simulations have recently become approachable due to high performance computing. Higher resolution and dimensional simulations of the flow around an impulsively started cylinder were conducted by Koumoutsakos and Leonard (1995). Based on a vortex method, they implemented a novel technique to resolve diffusion effects and enforce the no-slip boundary condition. Henderson (1997) investigated the transition of the wake
2.1. A single cylinder

states from two-dimensional to three-dimensional using direct numerical simulations. The onsets of secondary instabilities were studied in the Reynolds number range from 10 to 1000. Zhang and Dalton (1998) investigated the unsteady flow of a circular cylinder using finite difference and spectral element methods. Reynolds numbers of 100 and 200 were considered in their study. The force coefficients and the Strouhal number were estimated in this study. At Re = 200, three dimensional effects were observed. Persillon and Braza (1998) also investigated the transition to turbulence in the wake by three-dimensional Navier-Stokes simulations for 100 ≤ Re ≤ 300. In their study, the frequency modulation and the formation of a discontinuity region in the Re-St relationship were discussed with the birth of streamwise vorticity and the kinetic energy distribution in the near wake.

The flow around a circular cylinder with forced oscillation has also been examined to understand a flow induced vibration. An alternating vortex shedding from a cylinder causes an unsteady force to the structure. This force is sometimes a source of flow induced vibration of the cylinder, especially, when the structural natural frequency equals the vortex shedding frequency. This fluid elastic problem is widely understood by studying the flow around a circular cylinder with forced oscillation. When a cylinder body is subject to forced oscillation, the vortex formation frequency is affected by the driving frequency of the cylinder. The shedding frequency is often controlled by the oscillation frequency, and this is called "lock-in". Koopman (1967) studied the wake geometry of a transversely oscillating cylinder at low Reynolds numbers using hot-wire wind tunnel experiments. The lock-in diagram was reported up to Re = 300. Stansby (1975) also presented the lock-in diagram in his study, but at higher Reynolds numbers of 3600 to 9200. In addition to "lock-in", subharmonic synchronization, where the frequency of oscillation is half of the natural shedding frequency, was reported by Ongoren and Rockwell (1988). Experiments using a free surface water channel were conducted at Reynolds numbers of 600 - 1300 in their study. The flow structures were investigated for various
shapes of cylinders, and the results were presented by a hydrogen bubble visualization technique. In their experiments, it was found that the vortex shedding phase is synchronized to the cylinder motion in subharmonic synchronization but is not synchronized in "lock-in".

The "lock-in" is also observed in numerical studies. The following studies were conducted at low Reynolds numbers and the obtained lock-in diagrams were in comparable ranges with Koopman’s (1967) results. Meneghini and Bearman (1995) obtained the "lock-in" diagram at Re = 200 using the vortex method. The results agreed well with published experimental data. Akbari and Price (1997 a, b) investigated the flow behind a transversely oscillating cylinder at Re = 200 and 500 using a random vortex method and calculated lock-in diagram for Re = 200. Good agreements with Koopman (1967) were shown for \( \frac{f_c}{f_s} < 1 \). Zhang and Dalton (1997) numerically studied the flow over a circular cylinder forced to oscillate transversely using a semi-implicit finite difference scheme. The Reynolds numbers of the consideration are 200 and 885. Fundamental lock-in was observed in this study, and it was also found that the drag and lift coefficients increase at lock-in. Anagnostopoulos (2000) calculated the lock-in diagram using a finite element technique at Re = 106. He reported that the PSDs of the lift coefficient and the velocity yield different values due to the "beating effect" for \( \frac{f_c}{f_s} > 1 \). He used the PSD of the velocity to calculate the lock-in boundary for \( \frac{f_c}{f_s} > 1 \).

The forces acting on the cylinder surfaces also undergoes a considerable modification when a cylinder is subject to a transversely forced oscillation. Tanida et al. (1973) conducted experiments for the forces on an oscillating cylinder at Reynolds numbers of 4 to \( 10^4 \) and measured the lift and drag with various frequencies of oscillation. Lock-in was reported near the natural vortex shedding frequency, and the force coefficients were observed to increase at the lock-in region. Chilukuri (1987) investigated the flow structures past a transversely vibrating cylinder by an implicit finite difference scheme for a laminar flow. The numerical results from a
stationary cylinder agreed well with experiments. For a transversely oscillating cylinder, the vortex shedding wake agreed with experiments at small oscillating amplitudes. It was reported here that the amplification of mean drag occurs at lock-in. Blackburn and Melbourne (1992, 1997) measured sectional lift forces over the Reynolds number range $1.1 \times 10^5$ to $5.5 \times 10^5$. Experiments were conducted in a wind tunnel, and sectional lifts were measured at six different positions. Their investigation was focused on motion related lift forces at a low oscillation amplitude. Morse and Williamson (2009) measured fluid forces of the controlled oscillating cylinder at $Re = 4000$. In their study, remarkable agreements between the shapes of the regimes obtained from force measurements and the regimes of vortex formation mode identified from flow visualization were found. New distinct mode of vortex formation was observed, and it was termed 2Po. 2Po is similar to the 2P mode, two pair of vortices shed per cycle, but the secondary vortex is much weaker. The switches between the 2Po and the 2P modes occurred intermittently. It was found that the body may be able to vibrate due to the existence of a component of fluid forcing at the body vibration frequency.

Another interesting feature is various wake patterns obtained at different oscillating frequencies when a cylinder is oscillating transversely. Bearman (1984) investigated the vortex shedding from oscillating bluff bodies. In this investigation, oscillating bluff bodies of various forms were considered in order to identify common features. Williamson and Roshko (1988) studied the vortex shedding patterns of an oscillating cylinder at $Re = 300 - 1000$. The oscillation amplitude up to 5 and the frequency up to 10 were investigated in this study. Distinct wake patterns were observed depending on the amplitude and the frequency. The abrupt change from the 2S (two single vortex) mode to the 2P (two paired vortices) mode was observed as the wavelength ratio, $U_\infty/f_eD$, was increased. This abrupt change, which caused the jump in the fluctuating fluid forces and phase, was also found by Bishop and Hassan (1964). When $U_\infty/f_eD$ increased further, the 2P + 2S wake
pattern was observed. This pattern induced a 1/3-subharmonic synchronization.

Several numerical studies followed Williamson and Roshko’s (1988) original work. Ponta and Aref (2006) numerically investigated the flow over a transversely oscillating cylinder using a kinematic laplacian equation method. They found that some vortex patterns are sensitive to the Reynolds number. Ponta and Aref (2006) did not observe the 2P mode at Re = 140. Atluri et al. (2009) studied the near wake structure at Re = 500 to 8000 using a two-dimensional large eddy simulation. They discussed the effects of the frequency modulation and the Reynolds number on the vortex pattern.

The transition between two states, low and high frequency states were studied by Carberry et al. (2001). It was found that the change in the mode of vortex shedding is related to the change in the timing of vortex formation which is in turn related to the shift in the lift phase. A self excited transition at a constant frequency of oscillation was observed for a narrow band of frequency ratios. In this case, the self excited transition depends on the relative stability of three wake states: the low and high frequency wake states and the wake state of the stationary cylinder before start up. Carberry et al. (2003) extended their study to investigate the transition between low and high frequency states. The instantaneous vortex lift phases and the phase-referenced quantitative wake structure were used to explore this state. The wake state from the controlled oscillating cylinder were studied by Carberry et al. (2005). Experiments were conducted in a free-surface water channel at Reynolds numbers between 2300 and 9100. The transition, between low and high frequency states, occurred as the oscillating frequency passed through the natural frequency. The structure of the near wake changed during this transition. The low-frequency wake state generated long attached shear layers which resulted in the 2P mode of vortex shedding while the high frequency wake state had a much shorter wake length and vortices were shed in the 2S or Kármán model. It was characterized by a jump in the phase and amplitude of both the total and vortex lift. The wake states
for the forced oscillations and the elastically mounted cylinder were very similar. It was proven using concepts of energy in their study.

2.2 Two Circular Cylinders

A complex fluid behaviour occurs when two or more circular cylinders are situated in the flow. The existence of the neighbouring cylinders affects on the interaction of shear layers, Kármán vortex shedding, near wake structure, and the vortex patterns. Multiple cylinder configurations are often found in many engineering applications, and therefore, experimental and numerical studies have been performed in order to understand the complexity of the flow around two or more cylinders. In particular, the flow around two circular cylinders is intensively examined in tandem, side-by-side, and staggered configurations. These are reviewed in the following subsections.

2.2.1 Tandem Arrangements

In tandem arrangements, the centre-to-centre longitudinal pitch ratio, \( L/D \), is an important parameter. In general, no vortex shedding from the upstream cylinder is observed for smaller longitudinal pitch ratios. The longitudinal distance where the upstream cylinder starts to shed vortices is defined as the critical spacing. These are shown in figure 2.1. The previous studies have been mostly focused on the flow near the critical spacing, and some of the experimental studies are briefly reviewed here. Experimental investigations were carried out by Igarashi (1981) for \( 8.7 \times 10^3 < \text{Re} < 5.2 \times 10^4 \). Six different flow regimes were classified by Igarashi (1981) in detail. For \( L/D < 3.09 \), no vortex is shed from the upstream cylinder. At \( L/D = 3.97 \), he observed vortices shed in the gap. The critical spacing, where the
vortex shedding from the upstream cylinder appears, was found at \( \text{L/D} = 3.53 \) in his study. Sumner (1999) conducted two different experiments at low Reynolds numbers. One was performed in the water tunnel at \( \text{Re} = 850 \sim 1350 \), and \( \text{L/D} \) was varied from 1.0 to 5.0. The other was performed in a towing tank at \( \text{Re} = 1900 \) for \( 1.0 < \text{L/D} < 4 \). For \( \text{L/D} < 1.5 \), he observed a single Kármán vortex street behind the downstream cylinder at a higher shedding frequency than that in the single cylinder. At intermediate \( \text{L/D} \), three flow patterns were found in Sumner (1999). The first flow pattern occurred where shear layers shed from the upstream cylinder reattached alternately to the downstream cylinder. In the second flow pattern, shear layers shed from the upstream cylinder simultaneously reattached to the downstream cylinder and form stable pair of eddies in the gap between the cylinders. Finally, he observed the above two flow patterns with intermittent shedding of the gap vortices. The critical spacings were obtained at \( \text{L/D} = 3.5 \) at \( \text{Re} = 1900 \) in the towing tank and \( \text{L/D} = 5.0 \) at \( \text{Re} = 1320 \) in the water tunnel. Experiments were conducted in a low-speed, closed circuit wind tunnel at \( \text{Re} = 6.5 \times 10^4 \) by Alam et al. (2003). The critical spacing was found at \( \text{L/D} = 3.0 \) where a bistable flow was observed. They found that the fluctuating force acting on the upstream cylinder is directly related to the phase of the flow pattern of the downstream cylinder.

In tandem arrangements, the vortex shedding frequency has a distinct pattern for different \( \text{L/D} \). When the cylinders are in contact, the shedding frequency is similar to that in a single cylinder case. As \( \text{L/D} \) increases, the shedding frequency decreases, and the vortex shedding frequency becomes the smallest at the critical spacing. When \( \text{L/D} \) is larger than the critical spacing, the shedding frequency increases. Xu and Zhou (2004) measured the shedding frequencies by two hot wires in closed circuit wind tunnel experiments. Experiments were conducted over \( \text{Re} = 800 \sim 4.2 \times 10^4 \) for the cylinder centre to centre spacing of \( \text{L/D} = 1 \sim 15 \). Four flow regimes were categorized in their study: (i) shear layer roll up behind the downstream cylinder.
Figure 2.1: Near wake structures behind two tandem cylinders (a) when L/D is less than the critical spacing (b) when L/D is larger than the critical spacing (adopted from Xu and Zhou (2004))

for 1 \leq L/D \leq 2; (ii) shear layer reattachment on the downstream cylinder for 2 \leq L/D \leq 3; (iii) shear layer reattachment and vortex shedding from the upstream cylinder co-exist for 3 \leq L/D \leq 5; (iv) the upstream cylinder sheds vortices for L/D > 5. Their study also showed that the shedding frequencies depend on the Reynolds numbers. As the Reynolds number decreased, the critical spacing was observed at larger L/D. Kuo et al. (2008) investigated the self-sustained oscillatory flow characteristics between two tandem circular cylinders. Experiments were conducted in the water tunnel at Re = 1000. No vortex was shed from the upstream cylinder for L/D \leq 4.5. For L/D \geq 6, they observed the vortex shedding in the gap between the two cylinders. At the transitional range of 4.5 \leq L/D \leq 5.5, intermittent vortex shedding was observed.

In addition to these experimental studies, many numerical studies have been performed over the years. Slaouti and Stansby (1992) studied the flow around two cylinders in the tandem arrangement using a two-dimensional random vortex method. The simulations were conducted at a Reynolds number of 200, and L/D was varied from 1.2 to 10. The obtained critical spacing was at 3.4 \leq L/D \leq 3.8.
They did not observe the vortex shedding from the upstream cylinder when $L/D < 3.4$. The force coefficients were also calculated in their study. A sudden increase in the amplitude of the lift fluctuation for the upstream cylinder was observed for $L/D > 3.8$ owing to the vortex shedding from the upstream cylinder. Mittal et al. (1997) investigated numerically the near wake structures for $L/D = 2.5$ and 5.5 at $Re = 100$ and 1000. The flow was simulated by using a stabilized finite element method. At $L/D = 2.5$, two different flow patterns were observed. In this spacing, no vortex was shed from the upstream cylinder at $Re = 100$, while vortex shedding from the upstream cylinder was observed at $Re = 1000$. At $Re = 1000$, they reported the critical spacing in the range of $2 < L/D < 2.5$. Meneghini et al. (2001) carried out a two-dimensional numerical study for the flow around two tandem cylinders. The flow was solved using a finite element method at Reynolds numbers of 100 and 200. They observed the negative mean drag coefficient for the downstream cylinder when the longitudinal spacing was less than 3D (critical spacing). As the gap increases, the mean drag coefficient increased and became less negative in their study. In Meneghini et al. (2001), the amplitude of the lift coefficient on the downstream cylinder was higher than that of a single stationary cylinder for $L/D > 3.0$ due to the impingement of the vortex shed from the upstream cylinder. Jester and Kallinderis (2003) performed a two-dimensional numerical study at Reynolds numbers of 80 and 1000 using a second order Streamline Upwind Petrov-Galerkin projection scheme. Their results for the tandem arrangement obtained at $Re = 1000$ were similar to the results in Mittal et al. (1997). The obtained critical spacings in these 2-dimensional numerical studies (Mittal et al. 1997; Jester and Kallinderis 2003) are smaller than those reported in experiments (Sumner 1999; Xu and Zhou 2004).

Three-dimensional numerical investigations were also done recently. Deng et al. (2006) performed simulation for tandem arrangements using a virtual boundary method at $Re = 200$. They compared their three-dimensional results with the two-
dimensional results. The critical spacing was obtained at $3.5 < L/D < 4$. They found that the three-dimensional phenomenon occurred at $L/D > 3.5$. Due to this three-dimensional effect, the Strouhal numbers obtained using two-dimensional simulations were overestimated compared to those of a three-dimensional study for $L/D > 3.5$. Carmo and Meneghini (2006) investigated the flow around a pair of circular cylinders at $60 < \text{Re} < 320$. This study considered the flow around the tandem arrangements and differentiated six different flow regimes. The shedding frequencies are obtained from both two and three dimensional simulations. It was found that two-dimensional simulation overestimates the vortex shedding frequency at $\text{Re} > 200$. Kitagawa and Ohta (2008) also performed a three-dimensional numerical investigation for the flow in the tandem arrangement. The simulations were conducted at $\text{Re} = 2.2 \times 10^4$, and $L/D$ was varied from 2.0 to 5.0. At $L/D = 2.0$ and 2.5, the shear layers shed from the upstream cylinder reattached alternately to the downstream cylinder. At $L/D = 3.0$, they observed two different flow patterns. One of them showed the symmetrical reattachment of the upstream shear layers to the downstream cylinder which was followed by the absence of a Kármán vortex street behind the downstream cylinder. In the other flow pattern, small vortices were observed between the gap of the cylinders. They found the critical spacing at $L/D \approx 3.25$. The force coefficients and the Strouhal numbers presented in their study agreed well with the published data.

### 2.2.2 Side-by-side Arrangements

The side-by-side arrangement is defined with the centre-to-centre transverse pitch ratio, $T/D$. For the side-by-side configuration, the interesting feature is the biased flow pattern which exists for intermediate transverse spacings. In the biased flow pattern, the gap flow is deflected to one of the cylinders forming a wide wake behind the other cylinder (see figure 2.2). Two different shedding frequencies are
observed; the higher frequency is from the narrow wake, and the lower frequency is from the wide wake. Many experimental studies were carried out in the biased flow regime. When the gap increases, independent vortex streets are formed behind the cylinders. Bearman and Wadcock (1973) performed the wind tunnel experiments at a Reynolds number of $2.5 \times 10^4$. They measured the base pressures for the gap spacings, $T/D = 1.1 \sim 2.0$ and found that one cylinder experiences higher base pressure than the other due to the biased flow pattern. In their study, the base pressure was very high when the gap is small. They explained it with the effect of the base bleed that the entrainment of the fluid within the shear layer induces the base pressure to increase. They also observed at $T/D = 1.1 \sim 2.0$ that the Strouhal frequency was weak and intermittent. Kim and Durbin (1988) carried out experiments in a low speed, open return suction wind tunnel at $Re = 3.3 \times 10^3$. As reported in Bearman and Wadcock (1973), they also observed the higher base pressure for the cylinder with the wider wake and the lower base pressure for the cylinder with the narrower wake. Also, two different frequencies were observed from the biased flow pattern in their study. They investigated the transition of the flow direction in the biased flow pattern. They found that the transition occurs randomly in time and is very sharp at low Reynolds numbers. Alam et al. (2003) investigated the aerodynamic characteristics for the side-by-side arrangement at $Re = 5.5 \times 10^4$. The biased flow pattern was observed in $T/D$ range of 1.2 - 2.2. In this regime, the gap flow switches from one side to the other spontaneously. For $T/D > 2.2$, synchronized vortex shedding was observed with the dominant anti-phase shedding. The force coefficients and the Strouhal numbers were also measured. The flow structure in the biased flow regime were thoroughly investigated by Alam et al. (2007). Experiments were conducted at $Re = 4.7 \times 10^4$ for $T/D = 1.1 - 2.0$. At $T/D = 1.1$, a separation bubble is observed in the base region of one cylinder. At $T/D = 1.2$, no separation bubble is observed. It was suggested in their study that the formation of a separation bubble is strongly related to the lift force acting on the cylinder surface.
2.2. Two Circular Cylinders

Figure 2.2: Biased flow pattern behind two side-by-side cylinders (adopted from Sumner (1999)).

Besides the biased flow pattern, different flow patterns are observed in the flow around two side-by-side cylinders as L/D increases. Williamson (1985) investigated the wake structure in the range of 2.0 < T/D < 6.0. He performed two experiments: (i) wind tunnel experiment at Re = 50 ~ 150; (ii) water tunnel experiment at Re = 200. Williamson (1985) reported that, in this spacing range, the anti-phase shedding is the predominant feature, but there is possibility for an in-phase shedding. The flow pattern changes from the anti-phase shedding to in-phase shedding intermittently in his study. Figure 2.3 shows the schematics of the anti- and in-phase flow patterns. When the in-phase shedding occurred, Williamson (1985) observed a large scale single wake further downstream produced by the paring and enveloping process. Sumner (1999) studied the flow patterns for the side-by-side configuration at low Reynolds numbers of Re = 500 ~ 3000. When T/D < 1.25, he observed a single vortex street behind the cylinders. For 1.25 ≤ T/D ≤ 2.5, the biased flow pattern was observed in his study. In this region, two different frequencies, the higher frequency from the narrow wake and the lower frequency from the wide wake, were observed in Sumner (1999). For larger T/D, two independent vortex streets, either anti-phase or in-phase shedding, were observed. The predominant shedding was the anti-phase shedding in Sumner (1999) as reported in Williamson (1985).
The shedding frequency obtained in the anti-phase vortex shedding was close to that in the single cylinder case. An impulsively started flow around two side-by-side cylinders were also performed by Sumner (1999). Unlike the results in the mean steady flow, he found that there were only two types of flow. One was the flow pattern without a gap flow which occurred when the cylinders were in contact, T/D = 1.0, and the other was the flow pattern when there was a gap between the cylinders. Sumner (1999) described in his study that, for the impulsively started two side-by-side cylinder, the starting flow field remained symmetric over the complete range of transverse pitch ratios, and there is no evidence of the biased flow pattern or the deflection of the gap flow at 1.2 < T/D < 2.0. Xu et al. (2003) experimentally investigated the Reynolds number effect on the flow patterns for the side-by-side arrangement. This investigation was conducted at the Reynolds number range of 150 - 1.43 × 10^4 and in the T/D range of 1.2 - 1.6. For 1.25 ≤ T/D ≤ 1.6, two different flow patterns were observed depending on the Reynolds numbers. At higher Reynolds numbers, the transition from single street to two streets occurred within this T/D range.

![Anti-phase and In-phase flow patterns](image)

Figure 2.3: Anti-phase and in-phase flow patterns (adopted from Williamson (1985)).

Numerical studies have also been conducted for the flow around the side-by-side cylinders. Slaouti and Stansby (1992) conducted a two-dimensional numerical study at Re = 200 and classified four different flow patterns by the T/D spacing.
They also observed the biased flow pattern for $1.2 < T/D < 2.0$. Meneghini et al. (2001) extended their study to the side-by-side arrangement. Two dimensional simulations were performed at Reynolds numbers of 100 and 200 for $1.5 \leq T/D \leq 4$. At $T/D = 1.5$ and 2.0, they observed the deflected flow pattern with the flopping phenomenon. At $T/D = 3.0$, the anti-phase vortex shedding was observed in Meneghini et al. (2001). They observed the repulsive forces between the cylinders for all $T/D$ considered in their study. Jester and Kallinderis (2003) performed a two-dimensional numerical study using a second order streamline upwind Petrov-Galerkin projection scheme. The flow was simulated at $Re = 80$ and 1000 for $T/D = 1.5$ and 3.0. They observed the deflected flow pattern at $T/D = 1.5$ and the anti-phase vortex shedding at $T/D = 3.0$. Large eddy simulations were performed for two side-by-side cylinders by Chen et al. (2003). This investigation was done at $Re = 750$ for $T/D = 1.7$ and 3.0. At $T/D = 3.0$, the anti-phase vortex shedding was observed, and the deflected flow pattern was observed at $T/D = 1.7$ in their study. Kang (2003) used an immersed boundary method to study numerically the flow around two side-by-side cylinders at low Reynolds numbers, $40 < Re < 160$. Six different flow patterns were classified in detail. The force coefficients and the Strouhal numbers were also calculated. Lian et al. (2009) carried out a numerical investigation for the side-by-side cylinders at $Re = 100 \sim 200$ using a spectral difference method. At $T/D = 1.1$, the two cylinders behaved like a single bluff body. A biased flow pattern was observed for $T/D = 1.4 - 2.0$. For $T/D > 2.0$, synchronized flow patterns were observed in Liang et al. (2009).

### 2.2.3 Staggered Arrangements

The staggered arrangement of circular cylinders is the configuration most commonly found in engineering applications and is defined by the combination of the centre to centre longitudinal and transverse pitch ratios. A number of experimental
studies revealed the complexity of the flow around the staggered configuration. As an experimental work, Kiya et al. (1980) studied the fluid behaviour by measuring vortex shedding frequencies. In their study, vortex shedding frequencies were measured for different arrangements of two cylinders: tandem, side-by-side, and staggered arrangements. The existence of two kármán vortex shedding processes was inferred from two shedding frequencies obtained in Kiya et al. (1980). Sumner (1999) also measured the vortex shedding frequency for the staggered circular cylinders. He suggested that vortex shedding frequencies are more properly associated with individual shear layers than with individual cylinders.

Considerable efforts have been made to categorize the flow patterns obtained from the different staggered configurations. Investigation on interference between two staggered circular cylinders at high subcritical Reynolds number was done by Gu and Sun (1999) (see figure 2.4). Based on pressure measurements and flow visualization, three flow regimes were obtained: (i) wake interference, where the downstream cylinder is partially or completely submerged in the wake of the upstream cylinder; (ii) shear layer interference, where the shear layer reattachment onto the downstream cylinder occurs; (iii) shear layer interference, where the individual wake regions of each cylinder are established. Sumner et al. (1999) investigated the flow around two staggered circular cylinders in cross flow at $850 < \text{Re} < 1900$. This study revealed the complexity of the flow in detail and classified nine flow regimes which fall into the three regimes which are defined in Gu and Sun (1999). Sumner and Richards (2003) measured vortex shedding frequencies of circular cylinders arranged in the staggered configuration with pitch ratios of the cylinder centres, $P/D = 2.0$ and 2.5. The Strouhal number data were unreliable where the downstream cylinder experiences a maximum inward-directed lift force, and this indicated the influence of the extreme aerodynamics forces on the vortex shedding activity. Another wind tunnel investigation for the Strouhal frequency of two staggered circular cylinders was conducted over a wide range of configu-
2.2. Two Circular Cylinders

Rations at Re = 5.5 × 10⁴ by Alam and Sakamoto (2005). The wavelet analysis was used to identify multi-stable flow patterns and lock-in phenomena. It was reported that multi-stable flow patterns are caused by intermittent mutual lock-in of the two frequencies of two cylinders.

Sumner et al. (2005) measured the mean aerodynamic forces and vortex shedding frequencies for two staggered cylinders at Re = 3.2 × 10⁴ and Re = 7.4 × 10⁴. Their study showed that the outer lift peak of the downstream cylinder is associated with the proximity or impingement of the shed Kármán vortices from the upstream cylinder. Alam et al. (2005) carried out experiments to determine the characteristics of flow and fluid forces acting on a pair of staggered circular cylinders at Re = 5.5 × 10⁴. A closed circuit wind tunnel was used for this study and a wide range of configurations were investigated. It was found that the bistable flow is caused by the intermittent formation and burst of a separation bubble on the upstream or downstream cylinder. Four different bistable flows were reported, and their distinct characteristics were introduced. The first kind of bistable flow appeared at α = 10°, P/D = 3.1 ∼ 3.4 due to intermittent reattachment and detachment of only the inner shear layer of the upstream cylinder onto the downstream cylinder. At α = 25°, P/D = 2.9 - 3.1, the intermittent formation of the narrow wake and a fully developed Kármán wake behind the upstream cylinder were observed and resulted in the second kind of bistable flow. The third kind of bistable was due to the formation and burst of a separation bubble and was shown at α = 10°, P/D = 1.1 ∼ 2.3. The fourth bistable flow occurred at very close spacings where the separation bubble was formed or burst on the inside surface of the upstream cylinder. It was reported that the maximum drag force of the downstream cylinder occurs where the inner shear layer of the upstream cylinder rolls just ahead of the front surface of the downstream cylinder. Also, an interaction of the incident vortices from the upstream cylinder with the downstream cylinder and the occurrence of synchronized vortex shedding caused a high fluctuating lift force.
There exist limited number of numerical studies for the staggered arrangements due to its complexity. Akbari (2005) had carried out the study for staggered cylinder pairs in cross flow using a random vortex method. In order to include the effect of the other cylinder in the flow, the source panel method was used on the cylinder surface. The simulations were conducted at a Reynolds number of 800, and he identified five distinct flow regimes: (i) base bleed flow for small pitch ratios, \( P/D \leq 1.0 \), and large angles of incidence, \( \alpha \geq 30^\circ \); (ii) shear layer reattachment for moderate pitch ratios, \( 1.1 \leq P/D \leq 2.0 \), and small angles of incidence, \( \alpha < 10^\circ \); (iii) vortex pairing and enveloping for moderate pitch ratios, \( 1.1 \leq P/D \leq 2.0 \), and high incidence angles, \( \alpha \geq 30^\circ \); (iv) vortex impingement for large pitch ratios, \( P/D > 3.0 \), and small angles of incidence, \( \alpha \leq 10^\circ \); and (v) complete vortex shedding for \( P/D \geq 2.5 \) and \( \alpha \geq 30^\circ \). Direct numerical simulations and Floquet stability analyses of the flow around the staggered cylinder pairs were conducted by Carmo et al. (2008). They studied the flow at \( \text{Re} = 200, 250, 300, \) and \( 350 \) in staggered arrangement with a fixed streamwise separation of \( 5D \) and cross-stream separations varying from \( 0 \) to \( 3D \). Three instability modes, mode A (elliptic instability), mode B (hyperbolic instability), and mode C (nonlinear behaviour) were characterized. The basic modes in the wake transition in the flow around a single circular cylinder, modes A and B, were found in the near wake of the upstream cylinder and for all arrangements investigated in their study indicating that the physical mechanisms responsible for the manifestation of mode A and B should be the same as in the single cylinder case. In addition to mode A and mode B, the nonlinear behaviour of mode C was found in the near wake of the downstream cylinder. They found that mode C bifurcation was subcritical for \( T/D = 1.0 \), supercritical for \( T/D = 2.0 \), and \( 2.5 \) and again subcritical for \( T/D = 3.0 \).
2.2. Two Circular Cylinders

Figure 2.4: Three distinct flow patterns in staggered configuration (adopted from Gu and Sun (1999)).
Chapter 3

Numerical Method

The random vortex method is described in this chapter. First, we will introduce basic theoretical concepts of the numerical model and then derive the governing equations. Next, the random vortex method used in the present study will be discussed in detail. Finally, we will describe the calculations used in this study, such as the force coefficients and Strouhal number.

3.1 Theoretical Concepts

The vortex method starts with the idea that the fluid flow is more economically described in terms of the vorticity field. The vorticity carrying regions can be followed in Lagrangian frame work and the velocity field can be obtained over the vorticity field. The vorticity vector is the measurement of the rate of angular rotation of a fluid element and defined as

\[ \vec{\omega} = \nabla \times \vec{u} \]  

(3.1)
The circulation contained within any closed contour in a body of fluid is defined as

$$\Gamma = \oint \mathbf{u} \cdot d\mathbf{l}$$

(3.2)

The circulation can be related to vorticity as follows by using Stoke’s theorem.

$$\oint_{c} \mathbf{u} \cdot d\mathbf{l} = \int_{S} (\nabla \times \mathbf{u}) \cdot \hat{n} ds = \int_{S} \mathbf{\omega} \cdot \hat{n} ds.$$  

(3.3)

This states that the circulation around a curve is equal to the flux of vorticity through an open surface \(S\) bounded by the curve \(C\). The equation 3.1 is reversible and determines \(\mathbf{u}\) uniquely under some general conditions. This is called the ‘Biot-Savart law’, and the velocity is defined as

$$\mathbf{u} = \frac{1}{4\pi} \int \frac{\mathbf{\omega}(x', t) \times (x - x')}{|\vec{x} - x'|} dx'.$$

(3.4)

### 3.2 The Governing Equations

A flow is generally described by the conservation laws of the momentum, mass, and energy. Assuming the incompressibility and no external force, the flow around bluff bodies is expressed by the Navier-Stokes equation and the continuity equation.

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u},$$

$$\nabla \cdot \mathbf{u} = 0$$

(3.5)

(3.6)

where \(\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\) is the material derivative, \(\mathbf{u} = (u, v)\) is the velocity vector, \(\rho\) is the density, \(p\) is the pressure, and \(\nu\) is the kinematic viscosity. The boundary condition must be specified on a solid boundary in order to have a solution and is written as

$$\mathbf{u} = 0.$$  

(3.7)
This boundary condition is divided into two parts. First is the no flow boundary condition which is

\[ \vec{u} \cdot \hat{n} = 0 \quad \text{on a solid boundary at rest.} \quad (3.8) \]

\( \hat{n} \) is a unit vector normal to the boundary. This states that the flow can not penetrate the solid boundary. The second is the no-slip boundary condition. In general, it is expressed as

\[ \hat{t} \times \vec{u} = 0, \quad \text{on a solid boundary at rest}, \quad (3.9) \]

This simply tells us that the flow at the solid boundary has no motion relative to it. In a viscous flow, the solid boundary is a distributed source of vorticity, and this boundary condition is the mechanism by which the vorticity are generated in the flow. The detailed description will be given later in this chapter.

3.3 The Model Equations

In a vortex method, fluid motion is discretized in terms of vorticity. The vorticity field is defined in equation 3.1. Taking the curl of both sides of the equation 3.5, we obtain the following equation

\[ \frac{D\omega}{Dt} = \omega (\nabla \cdot \vec{u}) + \nu \nabla^2 \omega. \quad (3.10) \]

The equation is now given in a velocity-vorticity formulation, \((\vec{u}, \omega)\), which gives much easier access to a solution, instead of the primitive variables (i.e. velocity and pressure). This is called the evolution equation of the vorticity field.

We now have the governing equations of the flow around bluff bodies in vortex
3.4 The Viscous Splitting Algorithm

The evolution of the flow is considered in discrete time steps. In each time step, the vorticity field is convected and diffused. The algorithm of viscous splitting considers the convection and diffusion processes successively. This is based on fractional step methods. Considering a differential equation of the form

\[ \frac{\partial \omega}{\partial t} = \text{CONV}(\omega) + \text{DIFF}(\omega) \]  

with the boundary condition, \( \omega(0) = \omega_0 \). CONV and DIFF are the differential operators. Then, the solution for equation 3.15 can be written as

\[ \omega(t) = A_{\text{CONV+DIFF}}(t)\omega_0. \]  

Another way to solve equation 3.15 is to divide the equation into two parts by the operators and solve them separately.

\[ \frac{\partial \omega}{\partial t} = \text{CONV}(\omega), \omega(0) = \omega_0, \]  

\[ \frac{\partial \omega}{\partial t} = \text{DIFF}(\omega), \omega(0) = \omega_0. \]
3.4. The Viscous Splitting Algorithm

The solutions are

\[
\omega_{\text{CONV}}(t) = A_{\text{CONV}}(t) \omega_0, \quad (3.19)
\]

\[
\omega_{\text{DIFF}}(t) = A_{\text{DIFF}}(t) \omega_0. \quad (3.20)
\]

The following relation exists under general conditions.

\[
A_{\text{CONV+DIFF}}(t) = \lim_{n \to \infty} (A_{\text{CONV}}(t/n) A_{\text{DIFF}}(t/n)) \quad (3.21)
\]

The evolution equation of the vorticity field, equation 3.11, is repeated here for convenience.

\[
\frac{D\omega}{Dt} = \omega(\nabla \cdot \vec{u}) + \nu \nabla^2 \omega.
\]

The evolution of the flow is considered in discrete time steps. Applying the fractional step methods, the algorithm of viscous splitting consists of substeps in which the convective and the diffusive effects are considered successively, and therefore, the equation is divided into two parts as follows.

- **convexion**
  \[
  \frac{\partial \omega}{\partial t} + \vec{u} \cdot \nabla \omega = 0 \quad (3.22a)
  \]

- **diffusion**
  \[
  \frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega \quad (3.22b)
  \]

In a Lagrangian frame, the splitting algorithm is expressed as

- **convection**
  \[
  \frac{dx_p}{dt} = \vec{u}(x_p) \quad (3.23a)
  \]
  \[
  \frac{d\omega_p}{dt} = 0 \quad (3.23b)
  \]
  \[
  \frac{dx_p}{dt} = 0 \quad (3.23c)
  \]

- **diffusion**
  \[
  \frac{d\omega_p}{dt} = \nu \nabla^2 \omega(x_p) \quad (3.24a)
  \]
  \[
  \frac{dx_p}{dt} = 0 \quad (3.24b)
  \]
where \( x_p \) and \( \omega_p \) represent the location and the vorticity, respectively. In the first substep, the fluid elements are advanced with the local flow velocity. Then, diffusion acts at these new locations to modify the vorticity field of the flow. At each time step, the displacements of vortices are updated as

\[
x_i^{k+1} = x_i^k + \vec{u}(x_i^k, t) \Delta t + \eta,
\]

where \( x_i^{k+1} \) and \( x_i^k \) are the positions of vortices at time \( t + \Delta t \) and \( t \) respectively, \( \vec{u}(x_i^k, t) \) is the velocity vector at time \( t \), \( \Delta t \) is the time step, and \( \eta \) is the Gaussian random number.

### 3.5 The solution of the convection

The fundamental solution to the velocity field is the ‘Biot-Savart law’ defined in equation 3.4. The flow is discretized in terms of vorticity particles as follows

\[
\omega(x) = \sum_{N} f_0(x - x_j) \Gamma_j
\]

where \( f_0 \) is the smoothing function. In the vortex method, these particles are followed as their positions and strengths evolve with the fluid flow at each time. The velocities associated with these particles are calculated using the ‘Biot-Savart’ law. However, as seen in equation 3.4, this calculation faces a singularity problem at \( x = 0 \). Also, the number of operations per velocity calculation is too large. Each time step, the velocity field needs to be calculated for each vorticity particle. The number of the vorticity particles increases with time, and thus, the computational cost of calculating the velocity field increases continuously.

The vortex in cell (VIC) method is another approach to computer the velocity field. For an incompressible flow, the velocity components can be expressed by
using the stream function as follows.

\[ u = -\frac{\partial \psi}{\partial y}, \quad (3.27a) \]

\[ v = \frac{\partial \psi}{\partial x}. \quad (3.27b) \]

The streamfunction is therefore defined as

\[ \vec{u} = \nabla \times \psi \quad (3.28) \]

Substituting equation 3.28 into equation 3.1, we obtain the Poisson equation of the vorticity.

\[ \nabla^2 \psi = -\omega \quad (3.29) \]

In VIC method, the velocity field is obtained by solving equation 3.29 in the grid system. At each time step, the vorticity is interpolated onto the grid. The stream function of each grid point is calculated by solving the Poisson equation. The velocity field is then obtained by using equations 3.27a and 3.27b.

We suppose to have a grid with \( M \) number of grid points and \( N \) number of vortices. The computational cost of a fast poisson solver is \( O(M \log M) \), and the cost of the vorticity interpolation onto a grid is \( O(N) \). Therefore, the total cost of computing the velocity field is \( O(N + M \log M) \) at each time step. Comparing with the Biot-Savart law which has a computational cost in the order of \( O(N^2) \), this method is computationally cheaper and easier to implement numerically. Hence, in the present study, the VIC method is used for the solution of the convection part.

### 3.6 The solution of the diffusion

The common way to solve the diffusion part in the vortex method is the random walk method. This method is based on the fact that the analytical solution of
equation 3.22b is identical to the probability density function of the Gaussian distribution. Solving equation 3.22b for \( t \geq 0 \) in an infinite domain, we have the solution equation of

\[
\omega(x, t) = \frac{1}{\sqrt{2\pi \nu \Delta t}} \exp\left(-\frac{x^2}{4\nu t}\right).
\]  
(3.30)

This is also known as the Greens function for the heat equation in one-dimensional.

Now suppose to have \( N \) number of vorticity particles in the flow. The vorticity particles are discretized as equation 3.26. By using the VIC method for the velocity field calculation, we can avoid the singularity problem associated with the discretization, and hence, the smoothing function can be replaced by the Dirac delta function. For a set of \( N \) discrete point vortices, the total vorticity field at time \( t \) is written as

\[
\omega(x, t) = \sum_{i=1}^{N} \Gamma_i \delta(x - x_i),
\]  
(3.31)

where \( \delta \) is the Dirac Delta function, and \( x_i \) is the position of \( i \)-th vortex. We suppose to give each of the vorticity particles a random displacement drawn from a Gaussian distribution, with mean zero, and variance \( 2\nu \Delta t \). The probability of finding a particle between \( x \) and \( x + \Delta x \) is

\[
p(x) = \frac{1}{\sqrt{4\pi \nu t}} \exp\left(-\frac{x^2}{4\nu t}\right)dx
\]  
(3.32)

for each particle. For \( N \) discrete vortices, it can be written as

\[
\lim_{N \to \infty} \frac{N}{\sqrt{4\pi \nu t}} \exp\left(-\frac{x^2}{4\nu t}\right)dx = \frac{1}{\sqrt{4\pi \nu t}} \exp\left(-\frac{x^2}{4\nu t}\right).
\]  
(3.33)

As already stated, this is the probability density function of a Gaussian distribution which is same as the analytical solution of the diffusion equation. Therefore, at each time step, the diffusion process is approximated by the random walk method.
3.7 Methods for the Pressure Calculation

In vortex methods, the pressure term is eliminated when the Navier-Stokes equation is expressed in terms of vorticity. In order to calculate the forces applied to the cylinder by the fluid flow, it is necessary to have the pressure. Since the pressure cannot be calculated directly from the numerical simulation, it has to be recovered from the existing values such as vorticity and velocity fields. Four different methods of the force calculation were introduced in this section.

3.7.1 Poisson equation of the pressure

A poisson equation of the pressure can be derived from the governing 2-dimensional Navier-Stokes equations as follows (Roache 1976).

\[
\nabla^2 p = 2 \rho \left( \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial v}{\partial x} \frac{\partial u}{\partial y} \right). \tag{3.34}
\]

In polar coordinates, this equation is expressed as

\[
\nabla^2 p = 2 \rho \left( u \frac{\partial u}{\partial r} + u_r \frac{\partial u}{\partial r} + \frac{u}{\partial \theta} \frac{\partial u_r}{\partial r} - \frac{\partial u_r}{\partial \theta} \right). \tag{3.35}
\]

In order to solve this equation, two boundary conditions are required; one at the cylinder surface and the other at the farfield. At the farfield, we use the dirichlet boundary condition setting \( p = 0 \). At the cylinder surface, the Neumann boundary condition, \( \frac{\partial p}{\partial r} = 0 \), is used. The poisson equation (3.35) is solved by SOR routine with these two boundary conditions. This method is a very robust way to calculate the pressure, if the boundary condition is properly imposed. Also, it gives the pressure values for the entire flow field. However, the computational cost increases greatly
due to additional poisson solver.

3.7.2 Surface vorticity calculation

\[
\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial \theta} + \nu \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_\theta}{\partial r} \right) \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2},
\]  

(3.36)

Equation (3.36) is the momentum equation in \( \theta \)-direction. Assuming the velocity field is zero at the cylinder surface, the equation is simplified as follows.

\[
\frac{1}{\rho} \frac{\partial p}{\partial \theta} = \nu \frac{\partial}{\partial r} \left( r \frac{\partial u_\theta}{\partial r} \right).
\]  

(3.37)

The right hand side can be written in terms of the vorticity (Smith and Stansby 1988; Meneghini and Bearman 1995),

\[
\frac{\partial p}{\partial \theta} = -\nu \frac{\partial \omega}{\partial r}.
\]  

(3.38)

The central finite difference scheme is used to solve equation (3.38).

This calculation method shows a strong dependence on the grid size and the time step in the present numerical investigations. It is due to the finite difference scheme that we used for the solution process. This method requires a finer grid size and a smaller time step than the other methods, and thus, the computational cost is higher.
3.7.3 Momentum flux

Let’s suppose there is a system which is filled by an aggregate of fluid particles. The rate of change of any gross property, B (mass, kinetic energy, momentum, etc.), of the system at that instant is calculated using the Reynolds transport theorem. The general relation is written as follows.

\[
\frac{dB}{dt} = \frac{d}{dt} \int_{cv} \frac{dB}{dt} \rho d(vol) + \int_{cs} \frac{dB}{dm} \rho \vec{v} \cdot dA \tag{3.39}
\]

where \(dA\) is the area vector having the direction of outward normal to the control surface, \(m\) is the mass, \(\rho\) is the density, and \(\vec{v}\) is the velocity vector. \(cv\) indicates the control volume, and \(cs\) indicates the control surface. This formulation can be applied to any property, \(B\). The total force, \(F\), both surface forces and body forces, acting on the system is equal to the rate of change of the liner momentum of the system. Let the property \((B)\) be the linear momentum \((mv)\). Then the Reynolds transport equation becomes

\[
F = \frac{d}{dt}(mv) = \frac{d}{dt} \int_{cv} \vec{v} \rho d(vol) + \int_{cs} \vec{v} (\rho \vec{v} \cdot dA) \tag{3.40}
\]

It should be noted that equation (3.40) is valid only for an inertial control volume. In the present study the force is calculated at an instant after the system reaches a steady state. Therefore, the time derivative term becomes zero, and equation (3.40) is simplified as

\[
F = \frac{d}{dt}(mv) = \int_{cs} \vec{v} (\rho \vec{v} \cdot dA) \tag{3.41}
\]

Here, the force is simply the sum of the momentum fluxes at the control surfaces of the system.
Chapter 3. Numerical Method

The force obtained using equation (3.41) is always same at any instant by conservation law of linear momentum. The calculated force coefficients are equivalent to the time averaged values. The time history of the force coefficients are not available. The sinusoidal oscillation of the lift coefficients can not be obtained.

3.7.4 Momentum equation in $r$-direction

Cheng et al. (1997) and Akbari (1999) also calculated the pressure field in their studies using a given velocity field. Instead of using the momentum equation in $\theta$-direction, they used the equation in the $r$-direction to obtain the pressure in the flow.

In polar coordinates, the Navier-Stokes equation in $r$-direction is written as

$$\frac{\partial u_r}{\partial t} + \frac{u_r}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\nu}{r} \frac{1}{r} \frac{\partial}{\partial r}(r \frac{\partial u_r}{\partial r}) + \frac{1}{r} \left( \frac{\partial^2 u_r}{\partial \theta^2} - u_r - 2 \frac{\partial u_\theta}{\partial \theta} \right).$$

(3.42)

The pressure term is moved to the left-side, and the rest of the terms are moved to the right-side. Using the log-polar transformation, equation 3.42 is rewritten as

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = -\frac{1}{r} \left[ u_r \frac{\partial u_r}{\partial \zeta} + u_\theta \frac{\partial u_r}{\partial \theta} - u_\theta^2 \right] + \frac{\nu}{r^2} \left[ \frac{\partial^2 u_r}{\partial \zeta^2} + \frac{\partial^2 u_r}{\partial \theta^2} - (u_r + 2 \frac{\partial u_\theta}{\partial \theta}) \right] - \frac{\partial u_r}{\partial t},$$

(3.43)

where $u_\theta$ and $u_r$ are the velocity components in the $z$-plane. Here, the only unknown is the pressure gradient. Integrating both sides from the outer boundary to the cylinder surface, we have

$$\int_{r=0}^{r=R} \frac{1}{\rho} \frac{\partial p}{\partial r} = \int_{r=0}^{r=R} \left[ -\frac{1}{r} \left[ u_r \frac{\partial u_r}{\partial \zeta} + u_\theta \frac{\partial u_r}{\partial \theta} - u_\theta^2 \right] + \frac{\nu}{r^2} \left[ \frac{\partial^2 u_r}{\partial \zeta^2} + \frac{\partial^2 u_r}{\partial \theta^2} - (u_r + 2 \frac{\partial u_\theta}{\partial \theta}) \right] - \frac{\partial u_r}{\partial t} \right] \, dr.$$

(3.44)

Here, $R$ is the outer boundary and $a$ is the cylinder radius. A second order finite difference scheme is used to obtain the partial derivatives in the right side. The left hand side, the pressure gradient, is approximated as

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{p_{i+1,j} - p_{i,j}}{\rho(r_{i+1,j} - r_{i,j})}.$$

(3.45)
This method is used in the present study because it is easy to implement during simulations. Akbari (1999) reported that this method does not give correct evaluations for $Re > 200$. However, it was proved in the present study and other numerical studies that the pressure filed can be properly evaluated at $Re > 200$ using this method. The only possible difference from the Akbari’s (1999) are the boundary conditions imposed at the grid boundaries.

### 3.8 Applications to the Present Study

The governing equations are the evolution equation of the vorticity and the continuity equation,

\[
\frac{D\omega}{Dt} = \omega(\nabla \cdot \vec{u}) + \nu \nabla^2 \omega,
\]

\[
\nabla \cdot \vec{u} = 0.
\]

with the boundary condition

\[
\vec{u} = 0 \quad \text{at the cylinder surface.}
\]

Applying the viscous splitting algorithm, the governing equation is divided into convective and diffusive parts, equations 3.22a and 3.22b. A polar coordinate system, $(r, \theta)$, is used in the present study in order to handle the cylinder boundary easier. $r$ is normal to the cylinder surface, and $\theta$ is tangential to the cylinder. Equations 3.22a and 3.22b are solved successively at each time step by using the VIC method and the random walk.
3.8.1 VIC method

The convection equation is solved by the VIC method. The vorticity field is interpolated onto the grid nodes using the weighting scheme

$$\omega_i = \frac{\Gamma_n a_i}{A^2},$$  \hspace{1cm} (3.46)

where $\Gamma_n$ and $A$ are the circulation of the $n$-th vortex and area of the cell, respectively, and $a_i$ is the area of each subsection of the cell (see figure 3.1). Then, the Poisson equation of the stream function, equation 3.29, is written in polar coordinates as

$$\frac{\partial \psi}{\partial r} + \frac{1}{r} \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} = -\omega.$$  \hspace{1cm} (3.47)

Figure 3.1: A polar cell containing a vorticity particle

Two boundary conditions are applied to solve the stream function on the given polar grid,

$$\psi = 0 \quad \text{at the cylinder surface} \hspace{1cm} (3.48a)$$

$$\psi = U_0 R \sin \theta \quad \text{at far field} \hspace{1cm} (3.48b)$$

where $U_0$ is the free stream velocity and $R$ is the radius of the outer boundary of the numerical domain. The no-slip boundary condition is automatically satisfied during the solution of the Poisson equation due to equation 3.48a.
Introducing a log-polar transformation,

\[ \zeta = \log r, \]

(3.49)

and using equation 3.49, equation 3.47 is rewritten as

\[ \frac{\partial^2 \psi}{\partial \zeta^2} + \frac{\partial^2 \theta}{\partial \theta^2} = -r^2 \omega. \]

(3.50)

Using a central difference scheme, equation 3.50 is expressed as

\[ \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta \zeta)^2} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta \theta)^2} = -\omega_{i,j} \exp(2(\Delta \zeta) i) \]

(3.51)

and is solved by a successive over relaxation (SOR) routine. The indices, \( i \) and \( j \), indicate the grid nodes in the \( \zeta \)(or \( r \)) and \( \theta \) directions, respectively. The radial and tangential velocity are obtained, again using a central finite difference scheme,

\[ u_{ri,j} = \frac{1}{r_i} \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta \theta} \]  

(3.52a)

\[ u_{\theta i,j} = -\frac{1}{r_i} \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta \zeta} \]  

(3.52b)

The nodal velocities are interpolated to each vorticity particle inside the cell by again using the area weighting scheme,

\[ \vec{u}_n = \sum_{i=1}^{4} \vec{u}_i a_i A \]  

(3.53)

Here, \( \vec{u}_n \) is the velocity of the \( n \)-th vortex. \( \vec{u}_i \) is the velocity at the node which corresponds to the each subsection.

The vortex’s position is updated after this step as follows.

\[ r_i^{n+1} = r_i^n + u(r_i^n, t) \Delta t \]

(3.54)

### 3.8.2 The random walk

The diffusion part is solved by the random walk method. The random walk method was introduced by Chorin (1973). The solution of diffusion is simulated
stochastically by a two-dimensional displacement of the vortex element in two orthogonal directions, using two independent sets of Gaussian random number. At the end of each time step, the displacements of the vortices are updated as

\[ r_{i}^{n+1} = r_{i}^{n} + u(r_{i}^{n}, t)\Delta t + \eta_{i} \]  

(3.55)

where \( r_{i}^{n+1} \) and \( r_{i}^{n} \) are the positions of vortices at time \( t + \Delta t \) and \( t \) respectively, \( u(r_{i}^{n}, t) \) is the velocity vector at time \( t \), \( \Delta t \) is the time step, and \( \eta_{i} \) is the Gaussian random variable.

### 3.8.3 Boundary condition

In an inviscid fluid, a particle of fluid with zero vorticity continues to have zero vorticity. This statement is no longer valid in a viscous fluid because diffusion of vorticity from nearby particles can occur. A solid boundary can be represented as a distributed source of vorticity. The no-penetration condition at the cylinder surface is automatically satisfied during the solution of the Poisson equation. However, the resulting tangential velocity at the surface may not satisfy the no-slip boundary condition. Therefore, new point vortices are created to satisfy the no-slip boundary condition at the cylinder surface at each time step. The cylinder surface is divided into a number of panels. Vortices are created on the panels to cancel out the tangential velocity at the surface. The strengths of vortices are determined as

\[ (\Gamma_{i})_{\text{total}} = -u_{\tau_{i}} \cdot s_{i} \]  

(3.56)

where \( u_{\tau_{i}} \) is the tangential velocity at panel \( i \), and \( s_{i} \) is the length of panel \( i \). As the number of vortices increases, the accuracy of the random walk method increases. Also the vorticity field gets smoother, when the circulation of each vorticity particle
is small. In the present study, the proper value of $\Gamma$ is estimated through the convergence test. The total circulation, $(\Gamma)_{\text{total}}$, is divided equally such that the strength of each vortex does not exceed the determined proper value of $\Gamma$.

3.9 The Force Coefficients

The drag and lift coefficients are the important characteristics to distinguish the flow. They are obtained as follows.

\[ C_D = \frac{F_x}{\frac{1}{2}\rho U_o^2 D} \]  
\[ C_L = \frac{F_y}{\frac{1}{2}\rho U_o^2 D} \]

Here, $\rho$ is the fluid density, $U_o$ is the far field velocity, $D$ is the diameter of the cylinder. $F_x$, are the drag force and, $F_y$ is the lift force. They are calculated by taking the integrals of the pressure and the shear stress along the cylinder surface,

\[ F_x = a \int_0^{2\pi} (\tau \sin \theta - p \cos \theta)d\theta, \]  
\[ F_y = -a \int_0^{2\pi} (\tau \cos \theta + p \sin \theta)d\theta, \]

where $\tau$ and $p$ are the skin friction and pressure respectively. The calculation of the pressure field is explained in section (3.7). The skin friction is the tangential component of the force and is due to shear stress. It is obtained by

\[ \tau = -\mu \frac{\partial u_\theta}{\partial r} \]  
\[ \text{at the cylinder surface} \]

The equation 3.59 is approximated by using a first order different scheme in the present study.

\[ \tau_{i,j} = -\mu \frac{u_{\theta i+1,j} - u_{\theta i-1,j}}{\Delta \theta}. \]
3.10 Strouhal number

The Strouhal number is a non-dimensional variable describing oscillating flow mechanisms and defined as

\[ St = \frac{f D}{U_o} \]  

(3.61)

where \( f \) is the frequency of the vortex shedding, \( D \) is the diameter of a cylinder, and \( U_o \) is the far field velocity. Here, it is obtained by taking the power spectral density (PSD) of the lift. The Strouhal number can also be obtained by taking the PSD of the velocity profile near the wake position. The Strouhal numbers obtained from the lift coefficient and the velocity profile are the same for a single stationary cylinder in the present study.
Chapter 4

A Single Circular Cylinder

In this chapter, we present the results from a single stationary cylinder and a transversely oscillating cylinder. First, the results of a single stationary cylinder will be presented. The near wake structure, the drag and lift coefficients, and the Strouhal number will be thoroughly investigated at Re = 200. The Reynolds number effect on the mean drag coefficient will be studied and compared to previous experimental and numerical studies. Next, we will present results for a transversely oscillating cylinder. The lock-in diagram at Re = 200 will be presented for different frequency ratios, \( f_e / f_s \), where \( f_e \) is the frequency of the cylinder oscillation, and \( f_s \) is the Strouhal number of a single cylinder in this study. Finally, the simulation results for various \( f_e \) will be shown for a fixed amplitude of 0.44 at Re = 400 and Re = 800.

4.1 A stationary circular cylinder

In this section, the two-dimensional numerical study of a fixed single cylinder is presented. This particular study is done in order to validate the present numerical
Chapter 4. A Single Circular Cylinder

model. The simulations for a stationary cylinder is performed at Re = 200. The Reynolds number of 200 is the upper limit where the wake is still laminar. Vortex shedding at Re = 200 is still considered as two-dimensional, and thus, the numerical results obtained at Re = 200 should agree well with experimental results. The governing equations and the numerical method will be briefly reviewed in the following subsections. The simulation results of the vorticity structure, the force coefficients, and the Strouhal number are presented in this section, and a comparison with published data is provided. In the present study, the simulation of a single circular cylinder is only used for model validation.

4.1.1 Governing equations and the solution methods

The governing equations are the evolution equation of the vorticity and the continuity equation with two boundary conditions at the cylinder surfaces. The equations are repeated here for convenience.

\[
\frac{D\omega}{Dt} = \omega(\nabla \cdot \vec{u}) + \nu \nabla^2 \omega.
\]

\[
\nabla \cdot \vec{u} = 0.
\]

The boundary conditions are

\[
\vec{u} \cdot \hat{n} = 0,
\]

\[
\hat{n} \times \vec{u} = 0.
\]

The random vortex method is used to solve these governing equations. A polar grid, \((r, \theta)\), where \(r\) is normal to the surface and \(\theta\) is tangent to the surface, is used here. As described in Chapter 3, the vortex-in-cell method is used to solve the convection part, and the random walk method is used to solve the diffusion part.
4.1.2 The numerical procedures

In the previous chapter, the solution methods, the VIC method and the random walk, were described in detail. Here, we present the step-by-step numerical procedures for a single circular cylinder. Initially, there is no vorticity in the flow. The flow is impulsively started, and new vortices are generated from the cylinder surface at the beginning of each time step in order to satisfy the boundary conditions. The numerical procedures after the initial time step are given here in detail. The procedure is adopted from Akbari’s (1999) study.

1. Distribute all the vortices onto the grid and calculate the vorticity field using the weighting scheme of equation (3.46).

2. Solve the Poisson equation of the stream function, equation (3.50), for a given vorticity field.

3. Calculate the surface tangential velocity using equation (3.52b).

4. Create new vortices at the cylinder surface to satisfy the no-slip boundary condition. The vortex panel method is used to create the vorticity particles at the surface.

5. Distribute all the vortices onto the grid including the newly generated surface vortices.

6. Solve the Poisson equation of the stream function again for an updated vorticity field by equation (3.50).

7. Calculate the velocity field, equations (3.52a) and (3.52b), using a central fi-
nite difference scheme.

8. Interpolate the nodal velocities to vortices using equation (3.53).

9. Convect the vorticity particles with the given velocity.

10. Diffuse the point vortices with Gaussian random variables. At each time step, the displacements of vortices are updated as equation (3.25)

11. Repeat the steps from 1 to 10 for each time step.

### 4.1.3 Convergence test

The following parameters are fixed in the course of this study.

- Radius of the cylinder: \( a = 1 \)
- Radius of the flow boundary (outer grid boundary): \( R = 200a \)
- Free stream velocity: \( U_\infty = 1 \)
- Density of the fluid: \( \rho = 1 \)
- Reynolds number: \( Re = 200 \) for the convergence tests
  \( Re = 800 \) for two circular cylinders

The radius of the grid boundary was set to 200a. In order to prevent the parti-
cles from reaching the boundary, the vortices are removed from the computational domain once the particle position is greater than $160a$. Therefore, vortices beyond $r = 160a$ are not included when the vorticity field is calculated. The vorticity field on the grid nodes is calculated by interpolating the circulation of each vorticity particle (equation (3.46) in chapter 3). In this study, the mean drag coefficient and the Strouhal number were used to determine the convergence of the method. The test results are presented in table 4.1. $I$ is the grid number in the $r$-direction, $J$ is the grid number in the $\theta$-direction, $\Delta t$ is the time step, and $C_D$ is the time-averaged drag coefficient. The time-averaged drag coefficient, $\overline{C_D}$, was computed after $tU/D = 30$, where typically a periodic state is reached. $St.$ is the Strouhal frequency and is obtained by taking the power spectral density (PSD) of the lift coefficient. The results in test 17 are used as benchmarks for the convergence tests in the present study.

The results from table 4.1, are illustrated in figure 4.1, where it is clear that as the grid size is increased, the solutions converge to the benchmark mean drag coefficient of 1.291. The Strouhal number also depends on the grid size and the time step. In the first two test cases in table 4.1, a Strouhal number of 0.193 is obtained which is slightly smaller than the Strouhal number of 0.194 obtained in test 17. As the grid size increases (from tests 3 to 16), the Strouhal number converges to 0.194. We obtained the Strouhal number of 0.194 for all tests with $\Delta t \leq 0.025$. The dependence of the Strouhal number on the time step is clearly shown in figure 4.2.

In addition to the grid size and the time step, the effect of the maximum strength of vortices, $\Gamma_{max}$, is also checked and is shown in table 4.1. The cylinder surface is divided into a number of panels which the vortices are generated at each time step in order to satisfy the no-slip boundary condition. As the panel number increases, the circulation, $\Gamma$ of each particle decreases. $\Gamma_{max}$ is the maximum value of $\Gamma$ generated at the cylinder surface during one time step. The panel number can be same as the grid number. In that case, the panel coincides with the grid at
### Table 4.1: The convergence tests for the simulation of the flow around a single stationary cylinder at $Re = 200$ for different grid sizes and time steps

<table>
<thead>
<tr>
<th>No.</th>
<th>$Re$</th>
<th>$I$</th>
<th>$J$</th>
<th>$\Delta t$</th>
<th>$\Gamma_{\max}$</th>
<th>$C_D$</th>
<th>St.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>240</td>
<td>320</td>
<td>0.05</td>
<td>0.06</td>
<td>1.378</td>
<td>0.193</td>
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<tr>
<td>2</td>
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<td>240</td>
<td>320</td>
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<td>0.06</td>
<td>1.321</td>
<td>0.193</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>280</td>
<td>320</td>
<td>0.05</td>
<td>0.06</td>
<td>1.294</td>
<td>0.196</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>280</td>
<td>320</td>
<td>0.025</td>
<td>0.06</td>
<td>1.291</td>
<td>0.194</td>
</tr>
<tr>
<td>5</td>
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<td>320</td>
<td>0.025</td>
<td>0.015</td>
<td>1.287</td>
<td>0.194</td>
</tr>
<tr>
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<td>0.005</td>
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<tr>
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<td>320</td>
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<td>0.005</td>
<td>1.292</td>
<td>0.194</td>
</tr>
<tr>
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<td>0.06</td>
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<td>0.005</td>
<td>1.291</td>
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<td>0.06</td>
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<tr>
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<td>0.005</td>
<td>0.003</td>
<td>1.291</td>
<td>0.194</td>
</tr>
</tbody>
</table>

Table 4.1: The convergence tests for the simulation of the flow around a single stationary cylinder at $Re = 200$ for different grid sizes and time steps
4.1. A stationary circular cylinder

Figure 4.1: The convergence of the drag coefficient.

Figure 4.2: The convergence of the Strouhal number.
the cylinder surface. The tests are conducted for $\Delta t \leq 0.025$, and the results are shown in table 4.1. $\Gamma_{\text{max}}$ varied from 0.06 to 0.003. The Strouhal number remains unchanged as $\Gamma_{\text{max}}$ changes, and the change in $C_D$ is within 0.3% of 1.291. For the simulation of a flow around a single cylinder based on the convergence test as shown in table 4.1 and figures 4.1 and 4.2, we choose $I = 320$, $J = 320$, $PN = 960$, and $\Delta t = 0.025$ as it produced the drag coefficient and Strouhal number accurately up to three decimal places with the least number of grid points at the largest time step.

4.1.4 Results

The numerical results at $Re = 200$ are shown in this section. Figure 4.3 (a) shows the vorticity structure around a stationary cylinder obtained from the present study. This is a snapshot of the flow at $tU/D = 50$ where the flow has reached a periodic state. The periodic pattern of the vortex shedding, a single Karman vortex street, is observed in our simulation. Figure 4.3 (b) is the numerical results of Meneghini and Bearman (1995). They employed a discrete vortex method. In general the results show a good comparison, however, the simulation result obtained in the present study seems more diffusive than that of Meneghini and Bearman (1995). It may be due to the random walk method we used to solve the diffusion equation as it might be more diffusive than the finite difference method Meneghini and Bearman (1995) used in their study.

The time history of the drag and lift coefficients are shown in figure 4.4 (a). It can be seen that the flow reaches steady state after $tU/D = 30$. $\hat{C}_D$ and $\hat{C}_L$ converge to 0.082 and 0.902 for $tU/D \geq 30$, respectively. The periodic nature of the lift coefficient is clearly shown in the steady state. The PSD of the lift coefficient, $C_L$, is presented in figure 4.4 (b). Only the portion of $C_L$, after $tU/D = 30$ is used to obtain the PSD. The obtained frequency corresponds to the frequency of the vortex shedding of the
4.1. A stationary circular cylinder

cylinder, and this is called the Srouhal number of a single stationary cylinder. The
time averaged mean drag coefficient is 1.291 in the present study. Akbari and Price
(1996) reported $\overline{C_D}$ of 1.363 in their numerical study by the random vortex method.
This is slightly larger than that of the present study. Wanderley and Levi (2005) also
reported a similar value of $\overline{C_D}$, 1.361, in their numerical study where they calculated
the $\overline{C_D}$ value using a finite difference model. Persillon and Braza (1998) reported two
different $\overline{C_D}$ values, 1.321 with a two-dimensional simulation and 1.306 with a three-
dimensional simulation. The $\overline{C_D}$=1.291 obtained in the presented study is slightly
smaller than those obtained with other two-dimensional simulations, but is very
close to the experimental value of 1.30 from Relf and Simmons (1926) and the three-
dimensional result. The Strouhal number, $St$, is 0.194±0.0019 in our simulation.
The uncertainty is obtained by dividing the sampling frequency by the number
of Fast Fourier Transform(FFT) points. Williamson (1991) reported $St = 0.196$ in
his experimental study. Akbari and Price (1996) reported the Strouhal number
of 0.196 in their numerical study using the random vortex method. Herfjord
(1995) studied the flow using a finite element method and also obtained a Strouhal
number of 0.196. A recent study by Wanderley and Levi (2005) reported $St =
0.194$, which is the same as the present study. Persillon and Braza (1998) presented
Strouhal numbers of 0.198 (two-dimensional) and 0.181 (three-dimensional). The
two dimensional simulations seem to overestimate the Strouhal number and the
drag coefficient. Williamson (1996) observed the three-dimensional transitional
vortex shedding (mode A instability) around $Re = 200$ through the wind tunnel
experiment. Therefore, the differences in the drag coefficient and the Strouhal
number are probably due to the three-dimensional effects. The presented numerical
studies were conducted with two-dimensional models, and therefore the three-
dimensional effects were not accounted for in these results.

The mean pressure coefficients, $\overline{C_P}$, are calculated at $Re = 160$ and 200 using the
Figure 4.3: Near wake structure around a single circular cylinder at Re= 200 at $tU/D = 50$: (a) simulation result from the present study; (b) numerical simulation result from Meneghini and Bearman (1995).
4.1. A stationary circular cylinder

Figure 4.4: Simulation of the flow around a single circular cylinder at Re = 200: (a) time histories of the force coefficients, (b) PSD of $C_L$.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Re</th>
<th>$C_D$</th>
<th>St.</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relf and Simmons (1926)</td>
<td>200</td>
<td>1.30</td>
<td>-</td>
<td>Experiment</td>
</tr>
<tr>
<td>Williamson (1991)</td>
<td>200</td>
<td>-</td>
<td>0.196</td>
<td>Experiment</td>
</tr>
<tr>
<td>Herfjord (1995)</td>
<td>200</td>
<td>1.35</td>
<td>0.196</td>
<td>Finite elements $N^0$ of nodes</td>
</tr>
<tr>
<td>Akbari and Price (1996)</td>
<td>200</td>
<td>1.363</td>
<td>0.196</td>
<td>Vortex method $(200\times360)$</td>
</tr>
<tr>
<td>Persillon and Braza (1998)</td>
<td>200</td>
<td>1.321</td>
<td>0.198</td>
<td>Pressure-velocity formulation (2D)</td>
</tr>
<tr>
<td>Persillon and Braza (1998)</td>
<td>200</td>
<td>1.306</td>
<td>0.181</td>
<td>Pressure-velocity formulation (3D)</td>
</tr>
<tr>
<td>Wanderley and Levi (2005)</td>
<td>200</td>
<td>1.361</td>
<td>0.194</td>
<td>Finite difference $(120\times120)$</td>
</tr>
<tr>
<td>Present study (2011)</td>
<td>200</td>
<td>1.29</td>
<td>0.194</td>
<td>Vortex method $(280\times320)$</td>
</tr>
</tbody>
</table>

Table 4.2: The mean drag coefficient and the Strouhal number at Re = 200
equation (4.1).

\[ C_P = \frac{\overline{P} - P_\infty}{\frac{1}{2} U_o D} \]  

(4.1)

where \( U_o \) is the freestream velocity, and \( D \) is the diameter of the cylinder. \( \overline{P} \) is the time-averaged pressure in the present study, and \( P_\infty \) is the farfield pressure and is assumed to be zero. The \( C_P \) of the upper half, \( \theta = 0^\circ \) to \( \theta = 180^\circ \), is shown in figure 4.5. The separation angle was obtained around \( 100^\circ \sim 120^\circ \) here. Persillon and Braza (1998) also reported a similar range of the separation angle in their numerical study. The same pattern of \( C_P \) was shown in Henderson (1997) at \( \text{Re} = 1000 \). The \( C_P \) value at \( \theta = 180^\circ \), which is called the base pressure coefficient, \( C_{pb} \), are around -0.85 and -0.9 for \( \text{Re} = 160 \) and \( \text{Re} = 200 \), respectively, in this study. Figure 4.6 shows the numerical and experimental data of \( C_{pb} \). It is adapted from Posdziech and Grundmann (2001) who studied the wake of a circular cylinder numerically at Reynolds numbers of \( 190 \leq \text{Re} \leq 330 \) using a three-dimensional spectral element method. The \( C_{pb} \) values obtained in the present study are smaller than the published data presented in figure 4.6. Only Williamson and Roshko (1990) reported smaller \( C_{pb} \) than that of the present study at \( \text{Re} = 200 \). The data presented in figure 4.6 are widely scattered, and therefore, the discussion is inconclusive. Akbari and Price (1996) and Wanderley and Levi (2005) did not present \( C_{pb} \).

The mean drag coefficients, \( \overline{C_D} \), calculated for different Reynolds numbers are presented in figure 4.7. The published experimental and numerical data are also shown in figure 4.7. At \( \text{Re} = 100 \), the \( \overline{C_D} \) from the present study is larger than the numerical data from Braza et al. (1986) and Jordan and Fromm (1970). Braza et al. (1986) used a second order finite volume scheme, and Jordan and Fromm (1972) used a finite difference scheme for the flow simulation. For \( 200 \leq \text{Re} \leq 600 \), the results obtained in the present study agree well with the published data except Akbari’s (1995). Akbari (1995) reported \( \overline{C_D} \) with an increasing pattern for \( \text{Re} \geq 200 \), while the others presented \( \overline{C_D} \) with a decreasing pattern when \( \text{Re} \) is greater than 200. At \( \text{Re} > 600 \), the results from the present study agree well with
the numerical data. However, results are larger than the experimental data from Relf and Simmons (1926). As discussed in earlier (Persillon and Braza 1998), it might be due to the lack of the three-dimensional effects.

Though Akbari’s (1996) calculation was chosen to calculate the pressure in this study, we have tried to calculate the pressure with all four methods which were introduced in Chapter 3. The results are presented in figure 4.7. The obtained results are in a close range with each other and agree well with the published numerical data except the results of the surface vorticity method. The $C_D$ values obtained from the surface vorticity method are smaller than the published data for $Re > 300$. As discussed in Chapter 3, this might be due to the strong dependence on the grid sizes of the surface vorticity calculation. As $Re$ increases, the gradients at the cylinder surface become sharp, and therefore finer grid sizes are required. In the present study, the grid sizes are fixed, and therefore, the $C_D$ values obtained from the surface vorticity might differ from the results obtained from the other calculations for $Re > 300$.

![Figure 4.5: The pressure coefficients on the cylinder surface at Re =160 and 200.](image-url)
Figure 4.6: Mean base pressure coefficients versus Reynolds number adopted from Posdziech and Grundmann (2001); −, Henderson (1995); □, Thompson et al. (1996); ○, Persillon and Braza (1998); △, Kravchenko et al. (1999); ●, Posdziech and Grundmann (2001); +, Williamson and Roshko (1990); ◊, present study.
4.2 A transversely oscillating cylinder

Flow past a stationary circular cylinder results in an alternating vortex shedding from the body. Vortices of opposite rotational direction are shed from each side of the cylinder at a certain frequency. This alternate shedding process generates an unsteady force on the cylinder and may induce the cylinder to vibrate. This phenomena is termed vortex-induced vibration. When the structural natural frequency is close to the vortex shedding frequency of the cylinder, the wake structure

![Figure 4.7: The drag coefficients for different Reynolds numbers obtained in the present study compared with the experimental data; ○: mean drag coefficients obtained from the momentum flux method; ◦: mean drag coefficients obtained from the momentum equation in r-direction; □: mean drag coefficients obtained from the surface vorticity; ▽: mean drag coefficients obtained from the Poisson equation; ∗: Relf and Simon (1926); △: Braza et al. (1986); +: Jordan and Fromm (1972); ◄: Akbari and Price (1997).]
and the forces on the cylinder are significantly altered. A convenient way to look into this fluidelastic problem is to study the forced oscillation case. The simplified case where the cylinder oscillations are forced shows many features similar to those in vortex induced vibration.

In this section, we present the numerical results of the flow over a transversely oscillating cylinder. First, the "lock-in" phenomena was investigated at Re = 200 in order to validate the present numerical model for an oscillating cylinder. The lock-in diagram will be presented. Then, we studied the vortex patterns and the shedding frequency for different oscillation frequencies at a fixed amplitude at Re = 400 and Re = 800. The Reynolds number of 800 was chosen in order to get a better insight of the later study - a flow around two circular cylinders. The cylinder oscillation frequency ranged between 0.07 and 0.4, and the oscillating amplitude is fixed at $A/D = 0.22$. The vortex pattern and the vortex shedding frequency of each case will be presented. In the present study, the $C_D = 1.2$ and $St = 0.202$ are obtained for a stationary cylinder at Re = 800.

4.2.1 The Governing Equations

The Governing equations are the evolution equation of the vorticity and the continuity equation. The equations are repeated here for the convenience.

\[
\frac{D\omega}{Dt} = \omega(\nabla \cdot \vec{u}) + \nu \nabla^2 \omega,
\]

\[
\nabla \cdot \vec{u} = 0.
\]

The boundary conditions on the cylinder surface are

\[
u = V_b(t) \quad \text{at the cylinder surface}\quad (4.2)
\]
$V_b$ is the transverse velocity at the cylinder surface. The cylinder is oscillating transversely to the flow, and the position of the cylinder is represented as

$$y = A \cos(2\pi f_e t)$$  \hspace{1cm} (4.3)

where $A$ is the oscillating amplitude and $f_e$ is the oscillating frequency of the cylinder. Therefore, the velocity at the cylinder surface is

$$V_b(t) = -A2\pi f_e \sin(2\pi f_e t).$$  \hspace{1cm} (4.4)

The numerical procedure for an oscillating cylinder is the same as that for a stationary cylinder. The only difference is the boundary condition at the cylinder surface. Unlike the case of a stationary cylinder, equation 4.4 is used as a boundary condition at the cylinder surface. However, when we calculate the surface vortices in order to satisfy the no-slip boundary condition at the beginning of each time step, we should take into account the relative velocity of the flow on the cylinder surface. Therefore, at this stage of the calculations, we use the same boundary condition, $\psi = 0$, as that in the stationary cylinder case. The procedures creating the surface vortices are the same as those in a stationary cylinder.

For a stationary cylinder, the streamfunction is zero at the cylinder surface. When the cylinder is oscillating transversely, the streamfunction at the farfield is

$$\psi = U_o [1 - (\frac{a}{r})^2] y + \text{constant}.$$  \hspace{1cm} (4.5)

Here, $U_o$ is the free stream velocity, and $a$ is the cylinder radius. The constant can be any number because it has no effect on the flow. This is the boundary condition at the cylinder surface. The far field velocity is not affected by the cylinder oscillation if the outer boundary is large enough. Assuming that, we can use the same boundary condition as that in the case of a stationary cylinder.

The polar grid, $(r, \theta)$, which is moving with the cylinder, is used to simulate the flow. The grid size and the time step was chosen based on the convergence tests in
a stationary single cylinder. In section 4.1, the effects of the grid size and the time step were thoroughly investigated, and the grid numbers, \( I = 320 \) and \( J = 320 \), \( \Gamma_{\text{max}} \) and the time step of 0.025 were chosen (test 2 in table 4.3). Here, we conducted several tests for the first case, \( A/D = 0.18 \) and \( f_e D / U \approx 0.15 \), to check if these grid numbers and the time step generate a reasonable \( C_D \) and Strouhal number. The case of \( A/D = 0.18 \) and \( f_e D / U = 0.15 \) was randomly chosen. The results are shown in table 4.3. The changes in \( C_D \) is within 1.2% of that in test 2. The Strouhal number remains unchanged for all test cases. Therefore, the grid numbers of \( I = 320 \) and \( J = 320 \), \( \Gamma_{\text{max}} \) and the time step of 0.025 employed are sufficient in modeling this flow.

<table>
<thead>
<tr>
<th>No.</th>
<th>I</th>
<th>J</th>
<th>( \Delta t )</th>
<th>( \Gamma_{\text{max}} )</th>
<th>( C_D )</th>
<th>St.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>320</td>
<td>320</td>
<td>0.025</td>
<td>0.015</td>
<td>0.838</td>
<td>0.155</td>
</tr>
<tr>
<td>2</td>
<td>320</td>
<td>320</td>
<td>0.025</td>
<td>0.005</td>
<td>0.837</td>
<td>0.155</td>
</tr>
<tr>
<td>3</td>
<td>320</td>
<td>360</td>
<td>0.025</td>
<td>0.015</td>
<td>0.828</td>
<td>0.155</td>
</tr>
<tr>
<td>4</td>
<td>320</td>
<td>360</td>
<td>0.025</td>
<td>0.005</td>
<td>0.827</td>
<td>0.155</td>
</tr>
<tr>
<td>5</td>
<td>360</td>
<td>360</td>
<td>0.025</td>
<td>0.005</td>
<td>0.836</td>
<td>0.155</td>
</tr>
</tbody>
</table>

(a) \( \Delta t=0.025 \)

<table>
<thead>
<tr>
<th>NO.</th>
<th>I</th>
<th>J</th>
<th>( \Delta t )</th>
<th>( \Gamma_{\text{max}} )</th>
<th>( C_D )</th>
<th>St.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>320</td>
<td>320</td>
<td>0.02</td>
<td>0.005</td>
<td>0.837</td>
<td>0.155</td>
</tr>
<tr>
<td>7</td>
<td>360</td>
<td>360</td>
<td>0.02</td>
<td>0.005</td>
<td>0.838</td>
<td>0.155</td>
</tr>
</tbody>
</table>

(b) \( \Delta t=0.02 \)

Table 4.3: The mean drag coefficients and the Strouhal numbers for different grid numbers and the time step.
4.2.2 \( \text{Re} = 200 : \) Lock-in

The lock-in phenomena were investigated for a Reynolds number of 200. In general, lock-in is defined as when the vortex shedding frequency is coincident with the frequency of the cylinder oscillation. This definition is used to determine the lock-in in this study. The non-dimensional vortex shedding frequency, \( f_w D/U \), is obtained by taking the PSD of the lift coefficient. \( f_s \) is the Strouhal number of a stationary cylinder which is 0.194 in the present study. For \( f_e/f_s > 1 \), Anagnostopolous (2000) reported that the PSD of the lift coefficient is unable to detect the "beating effect" which is caused by the interference between two oscillations of different frequencies. He suggested to use the PSD of the velocity trace when \( f_e \) is larger than \( f_s \). For \( f_e/f_s > 1 \), we define lock-in where both PSDs of the lift coefficient and the velocity profile are coincident with the frequency of the cylinder oscillation.

The results in figures 4.8 show the case at \( f_e \approx 0.184 (f_e/f_s = 0.95) \) where lock-in did not occur. The time histories of the force coefficients are shown in (a), and the PSD of the lift coefficient is shown in (b). The force coefficients and pressure are calculated as in a single cylinder case. The dominant frequency is obtained at 0.193±0.0019 which is larger than the frequency of the cylinder oscillation. The mean drag coefficient, \( \overline{C_D} \) of 1.11 is obtained, and this is smaller than that of a stationary cylinder. Anagnostopoulos (2000) reported \( \overline{C_D} \) of 1.28 at \( f_e/f_s = 0.95 \) and \( A/D = 0.04 \). It was a bit smaller than 1.30 of a stationary cylinder he obtained at \( \text{Re} = 106 \) in his numerical study (1997). Both numerical studies reported smaller values but the results presented here show a larger reduction. We believe this may be due to the additional dissipation from the random walk approach. Figure 4.9 shows the results of lock-in at \( f_e = 0.165 (f_e/f_s = 0.85) \). Only one peak of shedding frequency 0.165 is shown in figure 4.9 (b). It is the same as that of the frequency of the oscillating cylinder. This indicates that vortices are shed at the same frequency as that of the cylinder oscillation. Time histories of the force coefficient is shown in
figure 4.9 (a). The force coefficients become periodic after some initial time. The $C_D$ is 1.04, and, again this value is smaller than that of a stationary cylinder. Meneghini (1993) obtained the $C_D$ value of 1.125 for $A/D = 0.15$ and $f_c D/U \sim 0.167$ at $Re = 200$. They obtained $C_D = 1.23$ for a stationary cylinder. Anagnostopoulos (2000) reported $C_D = 1.22$ for the case of $f_c/f_s = 0.8$ and $A/D = 0.22$ at $Re = 106$. These results confirm that the observed reduction in the drag coefficient as the ratio of frequencies is reduced is in line with the numerical studies of Meneghini (1993) and Anagnostopoulos (2000).

Figure 4.8: Simulation of the flow around a transversely oscillating cylinder at $f_c D/U = 0.184$ and $A/D = 0.04$: (a) time histories of the force coefficients, (b) PSD of $C_L$.

Figures 4.10 and 4.11 are the results at $f_c = 0.209$ ($f_c/f_s = 1.075$). Figure 4.10 shows the time histories of the force coefficients and the PSD of the lift coefficient. Figure 4.11 presents the velocity profile taken at $X/D = 2.5$ and $Y/D = -0.75$ and PSD, where $X$ is the streamwise direction, and $Y$ is the spanwise direction. The "beating effect" is clearly shown in the time history of the lift coefficient. Two frequencies with the dominant frequency of 0.209 are obtained from the PSD of the lift co-
4.2. A transversely oscillating cylinder

Figure 4.9: Simulation of the flow around a transversely oscillating cylinder at \( f_e D / U = 0.165 \) and \( A/D = 0.16 \): (a) time histories of the force coefficients, (b) PSD of \( C_L \).

The beating frequency, the difference of two frequencies, is 0.012. The dominant frequency is coincident with the cylinder oscillation frequency. However, the PSD obtained from the velocity profile shows a different result. In this case, the dominant frequency is 0.197 which is the second peak in the PSD of the lift coefficient. As mentioned by Anagnostopolous (2000), the different results obtained from the lift coefficient and the velocity profile are due to the "beating effect". He observed the non-periodic lift coefficient with beats and found that the modulation in the lift coefficient does not affect the shedding frequency. Due to this reason, he used the PSD of the streamwise velocity profile to determine the lock-in for \( f_e / f_s > 1 \). As seen in figures 4.10 and 4.11, the obtained dominant frequencies are different. Therefore, \( A/D = 0.08 \) is not in the lock-in region.

The lock-in diagram is shown in figure 4.12. The presented results with the rectangular symbol represent the lowest value of \( A/D \) for which lock-in was observed.
Figure 4.10: Simulation of the flow around a transversely oscillating cylinder at \( f_e D/U = 0.209 \) and \( A/D = 0.08 \): (a) time histories of the force coefficients, (b) PSD of \( C_L \).

In this study, \( A/D \) is varied by 0.01. The experimental results from Koopmann (1967) and the numerical results from Menighini and Bearman (1995) and Anagnostopolous (2000) are also shown in figure 4.12. The results from Koopmann (1967) and Meneghini and Bearman (1993) are obtained at \( \text{Re} = 200 \) which is the Reynolds number in the present study. The results from Anagnostopolous (2000) are calculated at \( \text{Re} = 106 \). As can be seen in figure 4.12, at \( f_e/f_s < 1 \), the results agree well with Koopman’s experimental data (1967). Meneghini and Bearman (1993) reported slightly higher \( A/D \), but the results are still in close agreement with those in the present study. For \( f_e/f_s < 1 \), the lock-in amplitude increases as \( f_e/f_s \) decreases. There is considerable difference between the results from the numerical studies and the experiments at \( f_e/f_s > 1 \). Meneghini and Bearman (1993) and Anagnostopolous (2000) obtained much higher lock-in amplitudes for this range of frequencies compared to Koopmann’s data (1967). Only one case in the present study, \( f_e/f_s = 1.075 \), can be directly compared to Koopman’s results. Also it is shown that the lock-in
amplitude obtained in the present study is much higher than that of Koopmann (1967). This discrepancy can be explained with the three-dimensional nature of the flow (Griffin and Hall 1995; Anagnostopoulous 2000). As \( f_e \) increases, in particular, \( f_e > f_s \), the wake pattern becomes very irregular and three-dimensional. The simulated results in the present study are two-dimensional, and therefore, the three-dimensional features were not captured. This could be a primary cause of disagreement between the present study and Koopmann (1967). For \( f_e/f_s > 1 \), the lock-in amplitude increases as \( f_e/f_s \) increases. The magnitudes of the PSD peaks at the lock-in frequencies are shown in figure 4.13 for each frequency ratio. It is observed that the magnitude of the peak increases as the amplitude of the cylinder oscillation increases inside the lock-in region. Outside lock-in, the peak at the lock-in frequency is not dominant, and hence, the magnitude of the peak is taken as zero in the present study.
4.2.3 Vortex modes

The lock-in phenomena was investigated at \( \text{Re} = 200 \). In this subsection, we will look at the vortex patterns for different oscillating frequencies at a fixed amplitude at a Reynolds numbers of 400 and 800. The simulations were conducted for six different frequencies, \( f_D/U \), at a fixed amplitude of \( A/D=0.22 \). The results are discussed based on the map of Williamson and Roshko (1988) (figure 4.14). They used the notations of ‘P’ for a pair of vortices and ‘S’ for a single vortex. In figure 4.14, 2S indicates two single vortices shed per cycle, 2P indicates two vortex pairs shed per cycle, P+S is a pattern in which one pair and a single vortex are shed per cycle, and 2P+2S is a pattern in which a vortex pair is shed at the top and bottom of the body trajectory with single vortices in between. Williamson and Roshko (1988) conducted experiments at \( \text{Re} = 392 \).
4.2. A transversely oscillating cylinder

4.2.3.1 Re = 400

Figure 4.13: The magnitudes of the PSD peaks for different frequency ratios, $f_c/f_s$.

Figure 4.15 shows the near wake structures at $f_c D/U = 0.07$. It corresponds to $\lambda/D = 14.3$ where $\lambda$ is the wavelength of the cylinder oscillation (figure 4.14). The overall pattern approximately represents a single Kármán vortex. At each half cycle of the cylinder oscillation, three vortices are shed from the upper and lower sides of the cylinder alternatively. The first vortex is shed from the lower side during (c)~(d), and the second is shed from the upper side at (f). Then the third vortex is shed from the lower side, (g)~(i). This process is repeated in the next half cycle. Williamson and Roshko (1988) observed 2P+2S pattern for this frequency which also sheds 6 vortices during one cycle of the cylinder oscillation. However, the vortex pattern observed in the present study is not 2P+2S as no paired vortices are observed. This is due to the very low amplitude used in the present study. At a larger amplitude, the vortex pattern of 2P+2S, is observed in the present study.
The wake structure at $f_e D / U = 0.15$ is shown in figure 4.17. The vortex pattern shown in figure 4.17 (q) is virtually similar to that shown in figure 4.17, but the wake periodicities are not synchronized with the frequency of oscillation. This result agrees with Willamson and Roshko (1988) as they reported either 2P vortex pattern or an unorganized vortex pattern for the case of $f_e D / U = 0.15$ and $A / D = 0.22$ (figure 4.14). Figure 4.18 shows the vortex pattern at $A / D = 0.5$. One pair of vortices and one single vortex (P+S mode) is shown in figure 4.18. Blackburn and Henderson (1995) also did not observe the 2P mode in their two-dimensional numerical study at $Re = 500$.

At $f_e D / U = 0.24$, two vortices are shed during one cycle of the cylinder oscillation, as shown in figure 4.19. The vortices with opposite rotational directions are shed alternatively from each side of the cylinder, and this process is the same as that in the single cylinder case. A single Kármán vortex street is formed behind the cylinder. Figure 4.20 shows the wake structure at $f_e D / U = 0.3$. One vortex is shed during each half cycle. Williamson and Roshko (1988) observed two single vortex shedding for this case that a coalescence of the scattered small vortices are combined into two large scale single vortices. However, in this study, no scattered vortices are observed.

### 4.2.3.2 $Re = 800$

It is assumed that the vortex modes reported in figure 4.14 remain invariant for $300 < Re < 1000$. Here, the vortex modes at $Re = 800$ are studied for different oscillation frequencies.

The wake structure at $f_e D / U = 0.07$ is shown in figure 4.21. The vortex pattern is similar to that observed at $Re = 400$. The first vortex is shed from the upper side
4.2. A transversely oscillating cylinder

during (d)~(e), and the second is shed from the lower side at (f). The third vortex is shed from the upper side, (g)~(i). This process is repeated in next half cycle. Six vortices are shed during one complete cycle of the cylinder oscillation, but the pattern does not correspond to $2P+2S$ mode owing to the low amplitude. The time histories of the force coefficients and the PSD of the lift coefficients are shown in figure 4.22. From each side of the cylinder, three vortices are shed, and therefore, a $1/3$-subharmonic frequency, $f_w/f_e \approx 3$ is observed. Williamson and Roshko (1988) also found $1/3$-subharmonic wake frequency when $2P+2S$ pattern is observed at low amplitudes.

At $f_e D/U = 0.15$, three vortices are shed during one cycle of the cylinder oscillation (figure 4.23). The first vortex is shed from the lower side of the cylinder ((a)~(f)), the second is shed from the upper side ((g)~(l)), and the last is shed from the lower side ((m)~(q)). The number of vortices corresponds to $P+S$ modes, but no paired vortices are observed in the present study. Figure 4.24 shows the force coefficients and the PSD of the lift coefficient. The obtained mean drag coefficient, $\bar{C}_D$, is 1.08 which is smaller than that of a stationary cylinder. Two frequencies are obtained here. The dominant frequency is coincident with the cylinder oscillation indicating that this case is in lock-in. The other frequency is approximately $\frac{3}{2}f_e D/U$.

Figure 4.26 shows the wake structure at $f_e D/U=0.3$. Virtually, two vortices are shed from the cylinder during one cycle. As shown at $Re = 400$, no coalescence is observed here. Figure 4.27 shows the velocity profile taken near the wake and the PSD of the velocity trace. The velocity profile is an alternate source for calculating the vortex shedding frequency and is better able to detect the "beating effect" than the lift coefficient for $f_e > f_s$. The velocity is taken at $X/D=2.5$ and $Y/D=-0.75$. The time history of the velocity is shown in figure 4.27 (a). The pattern is similar to that of the lift coefficient. A single peak is obtained at 0.188 which is a bit smaller than that of a single cylinder. The dominant frequency of 0.188 is also obtained in the PSD of the lift coefficient.
Figure 4.28 shows the vortex shedding frequencies for different excitation frequencies in the range of $0.07 \leq f_e D/U \leq 0.8$. Both frequencies obtained from the lift coefficient and the velocity profile are shown in figure 4.28. We take the PSD of the lift coefficient after some initial time where the lift coefficient reaches a periodic state ($t U/D > 40$). The velocity profiles are taken at $X/D = 2.5$ and $Y/D = -0.75$, and the PSD is also taken after some initial time. Higher shedding frequencies including a 3-superharmonic frequency are obtained for $f_e D/U < 0.1$. For larger frequencies, $f_e D/U > 0.4$, the wake frequency shows the convergence to 0.2 which is the Strouhal number of a single stationary cylinder at $Re = 800$. Two different results are shown for $0.1 < f_e D/U < 0.4$. In this range of the excitation frequencies, the dominant vortex shedding frequencies obtained from the lift coefficients are coincident with the frequencies of the cylinder oscillation, while the results obtained from the velocity profiles are around 0.2. However, the other frequencies obtained from the lift coefficients are the same as the dominant frequencies obtained from the velocity profile. It is because the "beating effect" is not properly detected in the lift coefficient.
4.2. A transversely oscillating cylinder

Figure 4.14: Maps of vortex synchronization regions on the wavelength-amplitude ($\lambda/D$, $A/D$) plane at $Re = 392$ (adopted from Williamson and Roshko (1988)).
Figure 4.15: Near wake structure at $fD/U = 0.07$ during one cycle of a cylinder oscillation: (a) nominal position, (e) minimum, (i) nominal, (m) maximum, (q) nominal position.
Figure 4.16: Vortex pattern at $A/D = 0.5$ for $f_{c}D/U = 0.07$. 
Figure 4.17: Near wake structure at $f_{c}D/U = 0.15$ during one cycle of a cylinder oscillation: (a) nominal position, (e) minimum, (i) nominal, (m) maximum, (q) nominal position.
Figure 4.18: Vortex pattern at $A/D = 0.5$ for $f_cD/U = 0.15$. 
Figure 4.19: Near wake structure at $f_cD/U = 0.24$ during one cycle of a cylinder oscillation: (a) nominal position, (e) minimum, (i) nominal, (m) maximum, (q) nominal position.
Figure 4.20: Near wake structure at $f_rD/U = 0.3$ during one cycle of a cylinder oscillation: (a) nominal position, (c) minimum, (g) maximum, (i) nominal.
Figure 4.21: Near wake structure at $f_c D/U = 0.07$ during one cycle of a cylinder oscillation: (a) nominal position, (e) minimum, (i) nominal, (m) maximum, (q) nominal position.
Figure 4.22: Simulation of the flow around an oscillating cylinder at $f_D U = 0.07$ and $A/D = 0.22$: (a) time histories of the force coefficients, (b) PSD of $C_L$. 
Figure 4.23: Near wake structure at $f_c D/U = 0.15$ during one cycle of a cylinder oscillation: (a) nominal position, (e) minimum, (i) nominal, (m) maximum, (q) nominal position.
Figure 4.24: Simulation of the flow around an oscillating cylinder at $f_D/U = 0.15$ and $A/D = 0.22$: (a) time histories of the force coefficients, (b) PSD of $C_L$. 
Figure 4.25: Near wake structure at $f_cD/U = 0.24$ during one cycle of a cylinder oscillation: (a) nominal position, (e) minimum, (i) nominal, (m) maximum, (q) nominal position.
4.2. A transversely oscillating cylinder

Figure 4.26: Near wake structure at $f_c D/U = 0.3$ during one cycle of a cylinder oscillation: (a) nominal position, (c) minimum, (g) maximum, (i) nominal.
Figure 4.27: Simulation of the flow around an oscillating cylinder at $f_c D/U = 0.3$ and $A/D = 0.22$: (a) velocity profile at $X/D = 2.5$ and $Y/D = -0.75$, (b) PSD of the velocity profile.
Figure 4.28: Wake frequencies obtained from the lift coefficients and the velocity profiles for different excitation frequencies
4.3 Discussion

The flow around a single circular cylinder was investigated using a random vortex method. This numerical model was validated at \( Re = 200 \) for both stationary and oscillating cases. The results are summarized as follows.

For a stationary cylinder, the simulated wake structure showed a single Kármán vortex street at \( Re = 200 \). The simulated result agreed well with the published numerical result. Although not shown here, the near wake structures at \( Re \leq 1000 \) agreed well with previous studies. The mean drag coefficient, \( \overline{C_D} \), the Strouhal number, \( St \), and the pressure coefficient, \( C_p \), were also calculated. The \( \overline{C_D} \) at \( Re = 200 \) was a bit smaller than other numerical results (Herfjord 1995; Akbari and Price 1996; Wanderley and Levi 2005) but agreed well with the experimental data (Relf and Simmons 1926). The \( \overline{C_D} \) was calculated for different Reynolds numbers. The results were in good agreement with the numerical results except at \( Re = 100 \). The results also agreed well with the experimental results at \( Re \leq 600 \). At higher Reynolds numbers, the results obtained from the present study and the other two-dimensional numerical studies slightly overestimated the experimental data. The overall pattern of the drag coefficients for different Reynolds numbers were fairly good compared to the published data. The \( St \) and \( \overline{C_D} \) at \( Re = 200 \) were in close agreement with the published data.

For an oscillating cylinder, the lock-in region was calculated. The results obtained in this study agreed well with the numerical and experimental results at \( f_e/f_s < 1 \) where the flow is two-dimensional. At \( f_e/f_s > 1 \), the lock-in diagram did not agree with the published data due to the beating effect (Anagnostopolous 2000). The vortex patterns were predicted for different excitation frequencies for a fixed amplitude \( A/D = 0.22 \) at \( Re = 400 \) and 800. The results were discussed based on Williamson and Roshko’s study. The vortex pattern did not exactly agree with
Williamson and Roshko (1988) at a small amplitude ratio, $A/D = 0.22$. It might be because the amplitude of $A/D = 0.22$ is near the onset of each vortex mode. At higher amplitudes, the results agreed well. At $Re = 800$, the vortex shedding frequencies were also obtained by taking PSDs of the lift coefficients and the velocity profiles near the wake. $1/3$-subharmonic frequencies were obtained at low $f_cD/U$. The vortex shedding frequency converged to 0.2 when $f_cD/U > 0.3$.

In this chapter, the obtained results were in good agreement with the published data. However, due to the lack of three-dimensionality, as reported in other two-dimensional numerical studies, overestimations of the force coefficients and the Strouhal number were found when $Re > 600$. Despite these weaknesses, the results obtained in this study were in comparable range with the experimental and higher order numerical studies. The vortex method is easy to implement and computationally less expensive. Therefore, it is concluded in this chapter that the random vortex method produces meaningful results to understand the flow physics at a low cost and can be used for further extensive studies. We now extend this study for two circular cylinders.
Chapter 4. A Single Circular Cylinder
Chapter 5

Two Stationary Circular Cylinders

Flow around two circular cylinders is investigated for different configurations. The arrangement of the cylinders with respect to the free stream flow can be classified as either tandem, side-by-side, or staggered (schematically shown in figure. 5.1). In the tandem configuration, the cylinders are arranged in-line and parallel to the flow. The tandem configuration is defined with the centre-to-centre longitudinal pitch ratio, $L/D$. The side-by-side configuration where the cylinders are arranged parallel to one another in a row and transverse to the flow is defined with the centre-to-centre transverse pitch ratio, $T/D$. The staggered configuration is a combination of the above two configurations and defined with the centre-to-centre pitch ratio, $P/D$, and the incidence angle, $\alpha$. Flow around these configurations was studied numerically at $Re = 800$ using a random vortex method in this chapter. The simulation results of the near wake structures, the force coefficients, and the vortex shedding frequencies are presented in this chapter.
Figure 5.1: Cylinder configuration for two circular cylinders
5.1 Governing equations and numerical method

The 2-dimensional flow around circular cylinders is described by the evolution equation of the vorticity field and the Poisson equation of the stream function. These are solved by using the random vortex method. More details on the random vortex method is found in Chapter 3. The velocity field is obtained by solving the Poisson equation of the stream function, equation 3.29. Equation 3.22b is solved by the random walk method.

Figure 5.2 shows the grid system in the present study. Two polar grids associated with each circular cylinder were used to solve the Poisson equation. The primary cylinder in grid 1 is cylinder 1, with the secondary cylinder termed as cylinder 2. The vice versa is found in grid 2. All the vortices generated from both cylinders are interpolated onto each grid. The Poisson equation is solved for the primary cylinder on each grid with the same boundary conditions as those of a single cylinder. For the secondary cylinder, the no-slip boundary condition is satisfied by distributing the newly generated surface vortices. The no-flow boundary condition, however, can not be satisfied by the solution of the Poisson equation since the grid does not coincide with the surface of the second cylinder. Therefore, it needs to be forced.

In the present study, a source panel method is used to force the no-flow boundary condition at the surface of the second cylinder. The surface of the second cylinder is divided into \( n \) number of panels. At each time step, the normal velocities at \( n \)-collocation points on the second cylinder surface are calculated, and the sources to cancel out these normal velocities are obtained by the panel method. The velocities induced by these source panels cancel out the normal velocity components at the second cylinder surface induced by the other cylinder.

The simulations were done at a Reynolds number of 800. The outer boundary is set at 200 times the cylinder radius. The grid sizes and the time step were chosen
Figure 5.2: Grid system for two stationary circular cylinders
5.2. The Numerical procedures

Based on the convergence tests in Chapter 4. A grid with $320 \times 320$ points together with a time step of 0.025 is used to simulate the flow.

5.2 The Numerical procedures

The fundamental numerical procedures are the same with that of the single cylinder, as described in Chapter 4; however the following modifications were introduced to accommodate the second grid.

1. The flow is started impulsively and vortices are generated at each cylinder surface to satisfy the no-slip boundary condition.

2. Next we obtain the vorticity field on each grid by combining all the vortices from the two grid system. When we calculate the vorticity field, we interpolate all the vortices onto the grid.

3. Solve the Poisson equation on each grid, respectively.

4. Calculate the tangential velocity at each cylinder surface using the stream function for the corresponding grid. In order to eliminate the tangential velocity, new vortices are generated at each cylinder surface using the vortex panel method.

5. The surface of the second cylinder is divided into a number of panels, and the
normal velocity is calculated on the collocation point of each panel using the stream function for the primary cylinder. The source of each panel is obtained using the source panel method.

6. Calculate the vorticity field again on each grid including the newly generated vorticity particles. All the vortices are again included.

7. Solve the Poisson equation of the stream function on each grid, respectively.

8. Calculate the velocity field on each grid for the primary cylinder, respectively. Correct the normal velocity at the surface of the second cylinder by distributing the previously obtained sources.

9. Convect the vorticity particles by using the velocity field for the primary cylinder on the corresponding grid.

10. Diffuse the vortices using the random walk method.

11. Repeat the steps from 2 to 10 for each time step.
5.3 Tandem configuration

When the gap is small, there is no vortex shed from the upstream cylinder. The spacing where the upstream cylinder starts to shed vortices is termed as the critical spacing. Vortices are observed from the upstream cylinder when the gap is larger than a certain spacing. (Igarashi 1981; Kuo et al. 2008; Sumner 1999; Slouti and Stansby 1992; Kitagawa and Ohta 2008). Six different flow regimes were classified for the tandem arrangement by Igarashi (1981): (A) The shear layers separated from the upstream cylinder do not reattach onto the downstream cylinder; (B) The shear layer separated from the upstream cylinder reattaches to the downstream cylinder and this process is synchronized with the vortex formation of the shear layer and vortex shedding in the near wake of the downstream cylinder; (C) Quasi-stationary vortices are formed between the cylinders; (D) Intermittent vortex shedding is detected; (E) The shear layers separated from the upstream cylinder intermittently roll up very near the downstream cylinder. This pattern is a bistable flow in the transition region between patterns D and F; (F) Vortices are shed in the gap region (shown in figure 5.3).

In this section, we present the numerical results of flow around two tandem cylinders. The simulations were conducted at Re = 800, and L/D was varied from 1.5 to 4.5. This range of L/D is chosen to have both flow patterns with and without the vortex shedding from the upstream cylinder. The force coefficients and the Strouhal numbers are also calculated and compared with the published data.
When $L/D < 3.5$, there is no distinct vortex shedding behind the upstream cylinder. A single Kármán vortex street forms behind the downstream cylinder. The same observations have been made in most published data (Igarashi (1981, 1984); Sumner 1999; Meneghini et al. 2001; Xu and Zhou 2004; Deng et al. 2006; and Kitagawa and Ohta 2008). Figure 5.4 shows the simulation result obtained in the present study, the numerical result in Kitagata and Ohta (2008), and the experimental result in Xu and Zhou (2004). Although small scale structures are better observed in Kitagawa and Ohta (2008) than in the present study as three-dimensional structures were included during the simulation, the result obtained in the present study is compatible with the other results. It clearly shows a single vortex street behind the downstream cylinder. A shed vortex is observed at the upper side, and a vortex
starts to form at the lower side of the downstream cylinder. A similar pattern is also shown in Kitagawa and Ohta (2008) and Xu and Zhou (2004). Small vortices which are not developed into a complete vortex are observed at the downstream cylinder surface in these studies. The time histories of the near wake structures at $L/D = 1.5$ and $L/D = 3$ are shown in figures 5.5 and 5.6, respectively. The flow pattern at $L/D = 1.5$ shows the alternating reattachment behaviour. In this flow pattern, a shear layer shed from either side of the upstream cylinder reattaches to the downstream one, and the other shear layer wraps around the downstream cylinder rolling-up into Kármán vortices in the near wake of the downstream cylinder. Vortices shed from the downstream cylinder synchronized with this alternating reattachment of the shear layers and form a single Kármán vortex street. As $L/D$ increases up to 3, a shear layer shed from the upstream cylinder rolls up right behind the downstream cylinder and produces small vortices near the downstream cylinder. These small vortices are not fully developed as a large scale vortex and dissipate forward of the downstream cylinder. At $L/D = 3.0$, there is still no sign of the vortex shedding from the upstream cylinder. Sumner (1999) reported that these shed gap vortices are entrained into the near wake region of the downstream cylinder, and participate in the formation of "gathers". The "gathers" are known as Z-shaped notches developed at the downstream edges of the recirculation zone. The "gathers" are also observed in the present study (figure 5.6). Kitagawa and Ohta (2008) and Xu and Zhou (2004) did not observe "gathers". Kitagawa and Ohta (2008) observed two different flow patterns at $L/D = 3.0$ in their numerical study. The first flow pattern was observed in the present study. In the second flow pattern, both shear layers are shed from the upstream cylinder and simultaneously reattach to the downstream cylinder. No vortex street is formed behind the downstream cylinder in this flow pattern. Kitagawa and Ohta (2008) reported that this symmetrical shear layer reattachment behaviour is a weak flow pattern. This flow pattern was not observed in the wake structure in the present study, however, the existence of this weak flow pattern was implied in the
Strouhal number results. Sumner (1999) did not observe the simultaneous shear layer reattachment to the downstream cylinder in his experiments. For $L/D < 3.5$, the vortex shedding process behind the downstream cylinder behaves in a nearly identical fashion to a single cylinder.

Figure 5.7 shows the streamlines at several instants in time. Similar size eddies in the gap between the cylinders are clearly shown. The eddies are constrained by the other cylinder thus the size of the eddy remains unchanged. Igarashi (1981) confirmed the existence of the eddy pair at this range of $L/D$; However, he did not provide the flow visualization. Sharman et al (2005) presented a pair of standing eddies in the gap between two cylinders at $L/D = 2$. Their numerical study was conducted at $Re = 100$. The result in Sharman et al. (2005) agrees well with the result in figure 5.7. Slaouti and Stansby (1992) and Sumner (1999) did not observe the stationary pair of vortices in the gap. Slaouti and Stansby (1992) observed a large standing eddy at $L/D = 2$, and is shown in figure 5.8 (a).

**5.3.1.2 $L/D \geq 3.5$**

A distinct change in the flow characteristics occurs for $L/D \geq 3.5$. The upstream cylinder starts to shed vortices. This indicates that the critical spacing for the appearance of the vortex shedding from the upstream cylinder is within the range of $3.0 \leq L/D \leq 3.5$ in the present study. Table 5.1 shows the critical spacings presented in previous experimental and numerical studies. Sumner (1999) and Kuo et al. (2008) conducted the water tunnel experiments at $Re = 900 \sim 1000$. Sumner (1999) found the critical spacing around $L/D = 5$, and Kuo et al. (2008) observed the critical spacing in the range of $5.5 < L/D < 6$. Their results are very similar because their water tunnel experiments were conducted at similar Reynolds numbers. Mittal et al. (1997) and Jester and Kallinderis (2003) numerically studied the flow around two tandem cylinders at $Re = 1000$. They observed the critical
Figure 5.4: Vortex shedding; (a) the simulation result at $L/D = 2.0$ in the present study; (b) Kitagawa and Ohta (2008), $L/D = 2.0$; (c) Xu and Zhou (2004), $L/D = 2.5$
Chapter 5. Two Stationary Circular Cylinders

Figure 5.5: Time history of the vortex shedding process at L/D=1.5
5.3. Tandem configuration

Figure 5.6: Time history of the vortex shedding process at L/D=3
Figure 5.7: Streamlines plotted at several instants of time at $L/D = 2$
Figure 5.8: Streamlines at $L/D = 2.0$; (a) Slaouti and Stansby (1992); (b) Sharman et al. (2005)
spacing within the range of $2 < L/D < 2.5$. Their results are much smaller than the experimental data in Sumner (1999) and Kuo et al. (2008). The result obtained in the present study is larger than the numerical results in Mittal et al. (1997) and Jester and Kallineris (2003), but smaller than the experimental data in Sumner (1999) and Kuo et al. (2008). At a large Re, Igarashi (1981) found the critical spacing for $3.09 < L/D < 3.53$ in his wind tunnel experiment. Kitagawa and Ohta (2008) reported the critical spacing around 3.25 using a LES method at $Re = 2.2 \times 10^4$. Meneghini et al. (2001) and Deng et al. (2006) conducted the numerical studies at the low Reynolds numbers. Meneghini et al. (2001) observed the critical spacing in the range of $3.0 < L/D < 4$ at $Re = 200$, and Deng et al. (2006) found the critical spacing in a similar range of $3.5 < L/D < 4$ at $220 < Re < 270$. The critical spacing obtained in the present study is in a close range with Igarashi (1981), Meneghini et al. (2001), Deng et al. (2006), and Kitagawa and Ohta (2008). However, these results are from largely different Reynolds numbers. Sumner (1999) and Kuo et al. (2008) conducted the experimental studies at a similar Reynolds number with that in the present study. These studies, however, reported large differences in the critical spacing. These differences could be attributed to the three-dimensional effect. The appearance of three-dimensional structures increases the formation length behind the cylinder, thus causing a larger critical spacing. However, a further investigation is needed to clarify the three-dimensional effect. Mittal et al. (1997) and Jester and Kallinderis (2003) also simulated the flow at $Re = 1000$ using two-dimensional models which is close to $Re = 800$ in the present study. The smaller formation regions are observed behind the cylinders in these numerical studies compared to Sumner (1999) and Kuo et al. (2008) and eventually result in smaller critical spacings. Figure 5.9 shows the experimental flow visualization in Xu and Zhou (2004) and the simulation result in the present study. The shed vortices from the upstream cylinder are clearly shown in both figures. However, the formation region observed in the present study is smaller than that in Xu and Zhou (2004), and therefore vortex shedding occurs slightly earlier in the present
5.3. Tandem configuration

Figure 5.10 shows the time history of the near wake structure at $L/D = 4$. As presented in figure 5.10, a vortex shed from the upstream cylinder impinges on the downstream cylinder and deforms. The flow behind the downstream cylinder becomes highly unsteady compared to the previous simulation results for $L/D < 3.5$. Sumner (1999) observed that the Kármán vortex street is absent in the flow behind the downstream cylinder due to the turbulence and the three dimensional flow generated by the Kármán vortices from the upstream cylinder. The impingement of the vortex on the downstream cylinder induced turbulence in the wake of the downstream cylinder. In the present study, a highly disturbed, irregular vortex street was observed behind the downstream cylinder, but the absence of the vortex street was not observed. Slaouti and Stansby (1992) and Kitagawa and Ohta (2008) both reported that the impinged vortex is synchronized with a vortex shed from the downstream cylinder. This synchronization behaviour was not observed in the present study. It might be because the synchronization behaviour is sensitive to the angle of the vortex impingement and the strengths of the vortices (Kitagawa and Ohta (2008)). As shown in figure 5.11 (a), in the present study, a shed vortex from the upstream cylinder impinged on the upper side of the downstream cylinder, while the downstream cylinder sheds a vortex from the lower side. However, Kitagawa and Ohta (2008) reported the synchronization behaviour of the vortex shedding from the upstream and the downstream cylinders, and it is shown in Figure 5.11 (b). The streamlines plotted at several instances of time are presented in figure 5.12. It shows the early stages of the vortex shedding behind the upstream cylinder. In figure 5.12 (b), two standing eddies are observed in the gap between the cylinders. As time increases, the eddies become asymmetric and finally separate from the cylinder. Sharman et al (2005) presented the streamline plot at $L/D = 4.0$ (figure 5.13). Their results shows the vortex shedding from the upstream cylinder and agrees well with figure 5.12 (a).
<table>
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<tr>
<th>Reference/method</th>
<th>Re</th>
<th>Critical spacing</th>
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<tr>
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<td>$3.53 \leq L/D \leq 3.90$</td>
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<td>A stabilized finite element formulation</td>
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<tr>
<td>Sumner (1999)</td>
<td>900 ~ 950</td>
<td>$\approx 5$</td>
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<td>Water tunnel experiment</td>
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<tr>
<td>Meneghini et al. (2001)</td>
<td>100, 200</td>
<td>$3 &lt; L/D &lt; 4$</td>
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<td>A fraction step method</td>
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<td></td>
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<tr>
<td>Jester and Kallinderis (2003)</td>
<td>1000</td>
<td>$2 &lt; L/D &lt; 2.5$</td>
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<tr>
<td>Petrov-Galerkin projection scheme</td>
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<tr>
<td>Deng et al. (2006)</td>
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<td>$3.5 &lt; L/D &lt; 4$</td>
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<tr>
<td>A virtual boundary method</td>
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<td></td>
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<tr>
<td>Kitagawa and Ohta (2008)</td>
<td>22000</td>
<td>$L/D \sim 3.25$</td>
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<td>Large eddy simulation</td>
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<td>Kuo et al. (2008)</td>
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<td>Water channel experiment</td>
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Table 5.1: The critical spacing for the tandem arrangement
Figure 5.9: The near wake structure at L/D = 4.0; (a) the simulation result in the present study; (b) Xu and Zhou (2004)
Figure 5.10: Time history of the vortex shedding process at L/D=4
Figure 5.11: Vortex shedding processes; (a) the simulation result in the present study at L/D = 4.0; (b) Kitagawa and Ohta (2008) at L/D = 3.25.
Figure 5.12: Streamlines at L/D = 4
5.3. Tandem configuration

Figure 5.13: Streamlines at $L/D = 4$ in Sharman et al. (2005).

5.3.2 Force coefficients

The force coefficients, $C_D$ and $C_L$, were calculated for each cylinder using the same equations, (3.57a) and (3.57b) in Chapter 3, as the case of a single stationary cylinder. The pressure was calculated using equation (3.44). The pressure of cylinder 1 was calculated on grid 1 using the corresponding velocity field and vice versa for the cylinder 2. Figures 5.14 and 5.15 show the time histories of $C_D$ and $C_L$ at $L/D = 2$ and $L/D = 3.5$, respectively. A negative $C_D$ is observed for the downstream cylinder where $L/D < 3.5$ (see figure 5.14). In this range of tandem arrangement, there is no vortex shed from the upstream cylinder, and the downstream cylinder is in the wake of the upstream cylinder. Thus, $C_D$ of the downstream cylinder has the opposite sign of that of the upstream cylinder. The fluctuating amplitude of $C_L$ of the upstream cylinder is much smaller than that of the downstream cylinder for $L/D < 3.5$. As shown in figure 5.14, there is very small amplitude fluctuations in the lift coefficient. This is due to the fact that there is no vortex formation from the upstream cylinder. For $L/D \geq 3.5$, where the upstream cylinder also shed vortices, the positive $C_D$ values for both cylinders were observed, as shown in figure 5.15. The fluctuating amplitude of the lift coefficient of the upstream cylinder became noticeably larger within this range of $L/D$ in the present study.
Table 5.2 and figure 5.16 present the time averaged mean drag coefficients for different L/D. The data from the previous studies are also plotted in figure 5.16 for a comparison. A two-dimensional numerical study was conducted by Meneghini et al. (2001) at Re = 200. Kitagawa and Ohta (2008) investigated the flow around two tandem cylinders at Re = 65000 using a three-dimensional numerical model. Ljungkrona et al. (1991) conducted an experimental study at Re = 20000, and the results are plotted in figure 5.16. All of these results demonstrated the sudden increases of the $C_D$ values for both cylinders at the critical spacing. Igarashi (1981), Sumner (1999), Deng et al. (2006), and Kuo et al. (2008) did not present $C_D$.

The negative $C_D$ values are observed for the downstream cylinder for L/D < 3.5 in the present study. For L/D ≥ 3.5, the $C_D$ values of the downstream cylinder are large positive values, and this indicates that the upstream cylinder has started to shed vortices. This sudden increase of the $C_D$ value is observed for both cylinders at the critical spacing in the present study. Additionally, it should be noted that, for L/D < 3.5, the combined $C_D$ value of the two cylinders is less than $C_D$ of the
Figure 5.15: Time histories of the $C_D$ and $C_L$ on (a) first and (b) second cylinder at $L/D = 3.5$.

<table>
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<th>Configuration (L/D)</th>
<th>$\overline{C_D}$ upstream</th>
<th>$\overline{C_D}$ downstream</th>
<th>$\overline{C_L}$ upstream</th>
<th>$\overline{C_L}$ downstream</th>
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Table 5.2: $\overline{C_D}$ and $\overline{C_L}$ for the upstream and downstream cylinders in the tandem configurations
Figure 5.16: Mean drag coefficient, $\overline{C_D}$. ○ and ●: present study at Re = 800; ○ and ◆: Kitagawa and Ohta (2009), numerical study at Re = 65000; □ and ■: Meneghini et al. (2001), numerical study at Re = 200; △ and ▲: Ljungkrona et al. (1991), experimental study at Re = 20000, open symbol: upstream cylinder; closed symbol: downstream cylinder.
flow past a single cylinder. The results obtained in the present study agree with the published data in Ljungkrona et al. (1991), Meneghini et al. (2001), and Kitagawa and Ohta (2008). These results showed the same pattern that $C_D$ increases abruptly at the critical spacing. It is therefore concluded that the behaviour of $C_D$ strongly depends on the critical spacing, but is less sensitive with the Reynolds numbers. The mean lift coefficients are near zero for all L/D considered in the present study, and thus they are not discussed in the present study.

<table>
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Table 5.3: $C_D'$ and $C_L'$ for the upstream and downstream cylinders in the tandem configurations

The $C_D'$ and $C_L'$ values are also measured and shown in table 5.3. $C_D'$ and $C_L'$ are the root-mean-square of the drag and lift coefficients, respectively. As shown in table 5.3, the values are larger for L/D $\geq$ 3.5 than for L/D $<$ 3.5. $C_D'$ and $C_L'$, are presented as a function of L/D in figures 5.17 and 5.19, respectively. The numerical data in Kitagawa and Ohta (2008) and the experimental data in Alam et al. (2003) are also presented in these figures. Ljungkrona et al. (1993), Sumner (1999), Meneghini et al. (2001), and Kuo et al. (2008) did not present $C_D'$ and $C_L'$. It was observed in the present study that the $C_D'$ value of the upstream cylinder increases right after the critical spacing. The same pattern is shown in Kitagawa and Ohta (2008) and Alam
et al. (2003). For the downstream cylinder, the gradual increase of $C'_D$ was observed in the present study, while Kitagawa and Ohta (2008) and Alam et al. (2003) observed a sudden change in $C'_D$ around the critical spacing which was caused by the vortex shedding form the upstream cylinder. The gradual increase in $C'_D$ of the downstream cylinder seems to be related to the flow pattern. Kitagawa and Ohta (2008) reported a synchronized vortex shedding from both cylinder where $L/D$ is greater than the critical spacing. However, in the present study, the synchronized shedding pattern was not observed at $L/D \geq 3.5$. As shown in figure 5.18, vortices are shed from different sides of the cylinders. When two vortices are synchronized, the change in the force acting on the downstream cylinder is greater. Therefore, the sudden change in $C'_D$ was not observed for the downstream cylinder in the present study. For $C'_L$, Alam et al. (2003) and Kitagawa and Ohta (2008) also presented the results with a similar pattern. In their results, through out the $L/D$ values, the $C'_L$ of the downstream cylinder is larger than that of the upstream cylinder. However, in the present study, the $C'_L$ of the downstream cylinder is smaller than that of the upstream cylinder for $L/D \geq 3.5$. The synchronized flow behaviour contributes towards the fluctuating amplitude on the lift coefficient of the downstream cylinder, and therefore, the $C'_L$ of the downstream cylinder is larger than that of the upstream cylinder. In the present study, a vortex shed from the opposite side of the upstream cylinder reduces the fluctuating amplitude of the lift coefficient of the downstream cylinder.

### 5.3.3 Strouhal data

The Strouhal numbers for the different $L/D$ values are presented in figure 5.20. The Strouhal number is obtained by taking the PSD of the velocity profile. For $L/D < 3.5$, the velocity profile is taken at $x/D = 1.5$ and $y/D = -1$ from the downstream cylinder. For $L/D \geq 3.5$, the velocity profile is taken at $x/D = 1.5$ and $y/D = -1$ from
5.3. Tandem configuration

Figure 5.17: Fluctuating drag coefficients, $C_D'$. ○ and ●: present study at $Re = 800$; □ and ■: Kitagawa and Ohta (2009), numerical study at $Re = 65000$; ◊ and ◆: Alam et al. (2003), experimental study at $Re = 65000$. Open symbol: upstream cylinder, closed symbol: downstream cylinder.
Figure 5.18: The near wake structures at (a) L/D = 4, (b) L/D = 5, (c) L/D = 5.5, and (d) L/D = 6.
5.3. Tandem configuration

Figure 5.19: Fluctuating lift coefficients, $C_{D}'$. ○ and ●: present study at Re = 800; □ and ■: Kitagawa and Ohta (2009), numerical study at Re = 65000; ◇ and ◦: Alam et al. (2003), experimental study at Re = 65000. Open symbol: upstream cylinder, closed symbol: downstream cylinder.
each cylinder. The data from the previous experimental (Xu and Zhou 2004; Kuo et al. 2008) and numerical studies (Deng et al. 2006; Kitagawa and Ohta 2008) are also plotted, as well as the data obtained in the present study. When L/D is smaller than the critical spacing, only one vortex street is formed behind the downstream cylinder and one Strouhal number is obtained. The Strouhal number decreases in this range of L/D and has the smallest value right before the critical spacing. The Strouhal number starts to increase when L/D is greater than the critical spacing. As L/D increases further, the Strouhal number converges to that of a single stationary cylinder. In the present study, the smallest Strouhal number is shown at L/D = 5. The Strouhal number decreases for L/D < 5, and it increases when L/D is greater than 5. Two Strouhal numbers are obtained for 3 < L/D < 5 in the present study. Although the simulation results only showed one flow pattern, the vortex shedding from the upstream, however, the second Strouhal number implies the existence of the second flow pattern as mentioned earlier. The results obtained in the present study agree well with the experimental data obtained at Re = 1000-1200. Xu and Zhou (w004) obtained the Strouhal numbers for different Reynolds numbers, Re = 1200 and Re = 42000. They showed that the critical spacing depends on the Reynolds number, however the behaviour of the Strouhal number for different L/D is insensitive to the Reynolds number.
Figure 5.20: Strouhal number. ◇: present study at Re = 800, ◐: Xu and Zhou (2004), experimental study at Re = 1200; +: Xu and Zhou (2004), experimental study at Re = 42000; □: Deng et al. (2006), numerical study at Re = 220; ▽: Kitagawa and Ohta (2008), numerical study at Re = 65000; ◊: Kuo et al. (2008).
5.4 Side-by-side configuration

For the side-by-side configuration, we see largely three different flow regimes. First, a regime in which the wake behind the cylinder pair is asymmetric and the gap flow between the two cylinders is biased either upwards or downwards. In this regime, the flow is flip-flopping between two quasi-stable, asymmetric states, and it occurs for $T/D < 2$ (Kim and Durbin (1988)). When $T/D > 6$, no flow interaction is generated between two vortex streets. For $2 < T/D < 6$, in-phase or anti-phase vortex streets are observed. These flow patterns are reported in Bearman and Wadcock (1973), Chang and Song (1990), and Sumner et al. (1997).

Slaouti and Stansby (1992) described four different flow regimes adapted from Zdravkovich (1977) and Ohya et al. (1988). It is shown in figure 5.21. For $T/D < 1.2$, the two cylinders behave in a similar way to a single bluff body. The unstable biased flow pattern is shown in the range of $1.2 < T/D < 2$. The anti-phase vortex shedding is observed in the range of $2 < T/D < 3.5$, and the independent vortex streets are shown for $T/D > 3.5$.

5.4.1 Flow structures

5.4.1.1 $T/D \leq 1.2$

When the two cylinders are in contact, the cylinders act like a single cylinder. A slight modification is found in the single bluff body behaviour, where the pitch ratio, $T/D$, is larger than 1. Figure 5.22 shows the time sequences of the near wake structure. The parallel gap flow and the weak biased gap flow are observed intermittently. At further downstream, a single vortex street is found. Sumner (1999) described three different flow patterns for $1.0 < T/D \leq 1.2$ for a steady mean
5.4. Side-by-side configuration

Figure 5.21: Sketches of flow patterns in the flow around pairs of circular cylinders in tandem configurations. Extracted and adapted from Slaouti and Stansby (1992)
flow. The first was a symmetrical near wake structure. He found that a single vortex street parallel to the flow axis is formed behind the cylinders. The second is an asymmetrical near wake. This pattern shows the biased gap flow with a single vortex street downstream. The third has no distinct gap flow. Sumner (1999) found that the second flow pattern were dominant in his experiments. The flow visualizations were not provided in his study. Xu et al. (2003) investigated flow around two side-by-side cylinders at $Re = 150 \sim 14300$. They observed two different gap flows depending on the Reynolds number as shown in figure 5.23. The gap flow immediately swerved to the upper cylinder at $Re = 450$, while the gap flow is deflected to the upper cylinder at $Re = 1500$. Alam et al. (2003) also reported the gap flow swerving around the upper cylinder at $Re = 5.5 \times 10^4$ (figure 5.23).

Figure 5.24 shows the snap shots of the streamlines. The weak biased gap flow is observed in figure 5.24 (a) ~ (d), and the parallel gap flow is shown in figure 5.24 (e) ~ (h). A single vortex street is formed further downstream. Liang et al. (2008)’s numerical results is shown in figure 5.25, and they observed the swerving gap flow around the lower cylinder at $T/D = 1.1$. 


5.4. Side-by-side configuration

Figure 5.22: Time history of the vortex shedding in one cycle for T/D=1.2
5.4.1.2 $1.2 < T/D \leq 2$

For $1.2 < T/D \leq 2$, the biased flow pattern is reported in many literatures (Bearman and Wadcock 1973; Kim and Durbin 1988; Sumner 1999; Alam et al. 2003; Kang 2003; Chen et al. 2003). The biased flow pattern consists of a narrower wake behind one of the cylinders and a wider wake region behind the other cylinder. Bearman and Wadcock (1973) reported that the $C_{pb}$ values are different due to the deflection of the flow. Kim and Durbin (1988) also reported the higher base pressure for the wider wake and the lower base pressure for the narrower wake. We also observed this behaviour at $1.2 < T/D < 2$ (figure 5.4). The deflection of the gap flow changes from one cylinder to the other cylinder, and thus the biased flow pattern is known to be bistable. Kim and Durbin (1988) investigated this flip-flopping phenomenon and found that the transition from one cylinder to the other cylinder occurs randomly. In the present study, it was also observed that the flip-flopping phenomenon occurs randomly. Figure 5.26 shows the near wake structure at $T/D = 1.5$. In this simulation, the biased flow pattern is more apparent in the relative lengths of the near wake of the two cylinders rather than in the deflection of the gap flow. The noticeable deflected gap flow is found in figure 5.26 (r) ~ (u). Williamson (1985) studied the flow behind a pair of bluff bodies and identified two different vortex interactions for $Re \leq 200$ within this range of $T/D$. In the first flow interaction, vortices which form on inner sides are squeezed and enveloped into the stronger vortices shed from the outer shear layer on the side of the narrower wake. The second mode is marked by pairs of vortices behind the cylinder with the narrow wake and a single vortex behind the neighboring cylinder. The first interaction was observed throughout time (figure 5.26) in the present study, but the second interaction was not found. Sumner (1999) conducted an experimental study at $Re = 1000 \sim 3000$, but also did not observe the second interaction in his experiment. Therefore, it is suggested that the absence of the second interaction is largely due to the different Reynolds number. Sumner (1999) observed the biased flow pattern
Figure 5.23: Flow visualizations and the corresponding sketch published in experimental studies, (a) $\text{Re} = 450$ Xu et al. (2003); (b) $\text{Re} = 1500$ Xu et al. (2003); (c) $\text{Re} = 5.5 \times 10^4$ Alam et al. (2003), (d) sketch of (c) Alam et al. (2003).
Figure 5.24: Streamlines at T/D=1.2
Having validated the order of spatial accuracy using test problems with steady flow solutions, we will look at the simulation of the unsteady flow past an isolated cylinder at Re = 100 with comparison between our results and other published results.

Figure 6 shows the computational grid for the unsteady flow past a single cylinder. There are 32 cells around the circumference of the cylinder. The first cell next to the cylinder wall has a spacing around 11% cylinder radius in the normal direction. The level of grid resolution is much coarser than the one used in Maeneghini et al. [18] who employed 128 points around the cylinder wall and the first node had a distance about 1% of cylinder radius for an isolated cylinder case. The computation for this case is performed using the fifth-order SD method and a cubic curved wall boundary condition is employed for the cylinder surface. Dirichlet boundary condition is used for the inlet and fixed-pressure is adopted for the outlet boundary condition. Inviscid symmetry boundary conditions are applied on the two lateral sides. The initial condition is provided according to the free-stream condition as $u = u_1$, $v = 0$, $p = p_1$ and $q = q_1$.

The SD method offers a flexibility in adjusting the number of degrees of freedom for different grid resolution. We only need to vary one parameter in our solver for the polynomial degree $N$. The difference of the fluctuating lift coefficient $C_{l0}$ predicted by the 4th-order (total DOFs 21,376) and the fifth-order SD methods (total DOFs 33,400) and the difference of coefficient $C_{d0}$ are all less than 2%. In the following for an isolated cylinder case, we only present the results obtained by the fifth-order SD method.

Table 3 reports the comparison between present computation of compressible viscous flow at Mach number 0.2 to other numerical and experimental studies for incompressible viscous flow at the same Reynolds number 100. The Strouhal number predicted by the SD method on a mesh with degree-of-freedom 33,400 is identical to the one predicted by Sharman et al. [25] and the measured value by Williamson [32]. There is a separate compressible flow simulation which is not included in the table. Mittal and Tezduyar [20] also predicted 0.164 using a finite-element compressible flow solver at Re = 100 and Mach number 0.2. The SD method predicted $C_{l0}$ is identical to the one predicted by the

at Re = 1000 ~ 3000 in his experiments, and the flow visualizations are shown in figure 5.27. Meneghini (2000) also reported the biased flow pattern at Re = 200. His simulation result is shown in figure 5.28. The streamlines are shown in figure 5.29. The weak deflection of the gap flow is observed throughout time. The deflection changes from the upper cylinder to the lower cylinder time to time. Sumner’s (1999) also shows the biased flow pattern (figure 5.30).

<table>
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Table 5.4: $C_{pb}$ of the cylinders in biased flow regimes, $1.2 < T/D < 2$
Figure 5.26: Time history of the vortex shedding process at $T/D=1.5$
Figure 5.27: Flow patterns of two side-by-side cylinders in cross flow (a) T/D = 1.5 (b) T/D = 2 from Sumner et al. (1999)

Figure 5.28: Near wake structure at T/D = 1.5 in Menegini (2000), This numerical study was conducted at Re = 200.
Figure 5.29: Streamlines plotted at several instants of time at $T/D = 1.5$
5.4. Side-by-side configuration

5.4.1.3 $T/D > 2$

At large pitch ratios, the flow around two cylinders behaves independently. The biased flow pattern disappears, and the anti-phase, or sometimes in-phase, vortex shedding is observed. (Williamson 1985; Sumner 1999; Chen et al. 2003; Kang 2003). Figures 5.31 and 5.32 show the near wake structures at $T/D = 3.0$ and $T/D = 4.5$, respectively. It was noticed that the streamwise gap between the vortices increases as $T/D$ increases. Therefore, the flow pattern is held at further downstream as the $T/D$ value increases. The anti-phase vortex shedding is observed for $T/D = 3.0$, while the in-phase vortex shedding is found for $T/D = 4.5$. Figure 5.33 shows the flow visualizations from Sumner (1999). Unlike the result in the present study, Sumner (1999) observed the anti-phase vortex shedding at $T/D = 4.5$. Meneghini (2000) simulated the flow around the two side-by-side cylinders and obtained the anti-phase vortex shedding at $T/D = 3.0$ and $T/D = 4.0$. The simulation results are shown in figure 5.34. Williamson (1985) discussed the in-phase shedding in his
Figure 5.31: Time history of the vortex shedding process at T/D=3

experimental study. Figure 5.35 is the smoke visualization at Re = 100 which shows the in-phase vortex shedding at T/D = 3.0. Liang et al. (2003) also reported the in-phase vortex shedding in their numerical study. Williamson (1985) further showed that the vortices are paired and developed into a large scale combined wake in the downstream region. In the present study, the in-phase vortex shedding was observed at T/D = 4.5, but the pairing process was not observed. It might be due to the larger T/D value compared to Williamson’s (1985). Figure 5.36 shows the streamlines at T/D = 3.0. The anti-phase synchronization is clearly shown. Liang et al. (2009) also reported the streamline at T/D = 3.0, which is shown in figure 5.37. Their simulation result captured in-phase synchronization.
5.4. Side-by-side configuration

Figure 5.32: Time history of the vortex shedding process at T/D=4.5
Figure 5.33: Anti-phase flow patterns of two side-by-side cylinders in steady cross flow (a) $T/D = 2.5$ (b) $T/D = 4.5$ from Sumner et al. (1999).

Figure 5.34: Near wake structures at (a) $T/D = 3$ and (b) $T/D = 4$ in Meneghini (2000). The numerical study was conducted at Re = 200.
Figure 5.35: In-phase shedding at $T/D = 3.0$ observed in Williamson (1985). The flow visualization is obtained from a wind tunnel experiments at $Re = 100$.

5.4.2 The force coefficients

The force coefficients, $C_D$ and $C_L$, are calculated for each cylinder. The time histories of the force coefficients are shown in figures 5.38 and 5.39. At $T/D = 1.5$, the biased flow pattern is shown, and hence, the $C_D$ and $C_L$ patterns of the lower cylinder are different from those of the upper cylinder. Two Kármán vortex streets are formed behind the two cylinders at $T/D = 3.0$, and thus, the patterns of $C_D$ and $C_L$ are close to those in a single cylinder. The time averaged mean drag and lift coefficients are shown in Table 5.5. The $\overline{C_D}$ value of the lower cylinder increases as $T/D$ increases for $T/D \leq 3.5$ and decreases for $T/D > 3.5$. For the upper cylinder, the $\overline{C_D}$ value increases as $T/D$ increases except when $T/D = 3.5$. At $T/D = 3.5$, the smallest $\overline{C_D}$ is observed. This indicates that there exists a repulsive force in the gap between the cylinders. On both cylinders, the lift forces act in an outward direction. As $T/D$ increases, the repulsive force in the gap of the cylinders decreases. Therefore, it is suggested that there is less interaction of the wake structures when the gap between the cylinders is larger.

Figures 5.40 and 5.41 show the $\overline{C_D}$ and $\overline{C_L}$ values for different $T/D$, respectively.
Figure 5.36: Streamlines plotted at several instants of time at T/D=3
Figure 5.37: Streamlines at T/D = 3 in Liang et al. (2009). The simulation result is obtained at Re = 100 using a spectral difference method.

Figure 5.38: The force coefficient at T/D = 1.5
Figure 5.39: The force coefficients at T/D = 3.0

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<th>$\bar{C}_L$</th>
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Table 5.5: $\bar{C}_D$ and $\bar{C}_L$ for the upstream and downstream cylinders in the side-by-side configurations
The overall pattern shown in figure 5.40 is that the $C_D$ increases for $T/D < 3.5$ and then appears approximately at 1.8 for $T/D > 3.5$. The previous experimental and numerical results are also presented in figure 5.40 for a comparison. In these results, it was observed that $C_D$ increases in the deflected flow pattern (a bistable flow) and then decreases as $T/D$ increases. Slaouti and Stansby (1992) calculated $C_D$ at $Re = 200$, and the results are lower than that in the present study. In their results, $C_D$ increases for $T/D \leq 2$ and then decreases as $T/D$ increases further. Meneghini et al. (2001) also reported $C_D$ with similar results to Slaouti and Stansby (1992) at $Re = 200$. The data (Slaouti and Stansby 1992; Meneghini et al. 2001) presented in figure 5.40 are larger than those in the present study, and it may be due to the lower Reynolds they employed. Bearman and Wadcock (1973) reported $C_D$ for only $T/D = 1.5$ and 2, and the increasing pattern with $T/D$ was observed for $T/D \leq 2$. Kang (2003) conducted a numerical study at $Re = 160$ using an immersed boundary method. Their results increased up to $T/D = 3.0$. The $C_L$ values are shown in figure 5.41. The results in the previous experimental and numerical studies are also presented in figure 5.41. The negative sign of $C_L$ is removed here because all these results reported the repulsive force on both cylinders. $C_L$ decreases as $T/D$ increases in all the results presented in figure 5.41 indicating that the repulsive force on the cylinders decreases as $T/D$ increases. The $C_D$ values obtained in the present study are close to those reported in the literatures (Alam et al. 2003; Kang 2003; Meneghini et al. 2001).

The r.m.s. of the force coefficients, $C'_D$ and $C'_L$, are shown in table 5.6. Figure 5.42 shows $C'_D$ for different $T/D$ values. In the present study, the $C'_D$ values obtained from the upper and lower cylinders are almost the same throughout the range of $T/D$ values. The $C'_D$ increases for $T/D < 2.5$ and then decreases for $T/D \geq 2.5$ in the present study. Alam et al. (2003) reported $C'_D$ values for the wide and narrow wakes at $Re = 5.5 \times 10^4$. In their results, two different values of $C'_D$ are observed for $T/D < 2.5$ due to the deflected flow pattern. There exists very little difference
Figure 5.40: Mean drag coefficient, $C_D$. ○ and •: present study at Re = 800; +: Bearman and Wadcock (1973), numerical study at $2.5 \times 10^4$; ◆ and ◇: Slaouti and Stansby (1992), numerical study at Re = 200; □: Meneghini et al. (2001) (same for both cylinders), numerical study at Re = 200; ◄: Kang (2003), experimental study at Re = 160, open symbol: upstream cylinder; closed symbol: downstream cylinder.
Figure 5.41: Mean lift coefficient, $\overline{C_L}$. ○ and ●: present study at Re = 800; ◊: Meneghini et al. (2001), numerical study at Re = 200; ◊: Kang (2003), experimental study at Re = 160; □: Alam et al. (2003), open symbol: upstream cylinder (wide wake in Alam et al. (2003)); closed symbol: downstream cylinder (narrow wake in Alam et al. (2003)).
between the $C'_D$ values obtained from the upper and lower cylinders in the present study. The deflected flow pattern is less clear here than in Alam et al.’s (2003) and no difference is found in $C'_D$. Their results also decrease for $T/D \geq 2.5$. The $C'_L$ values are shown in figure 5.43. There is no distinguishable difference between the $C'_L$ values obtained from the upper and lower cylinders in the present study. It seems that $C'_L$ is insensitive to the $T/D$ value in the present study. Alam et al. (2003) and Kang (2003) reported the smaller $C'_L$ values for the biased flow pattern, $T/D \leq 2.0$. In particular, at $T/D = 1.5$, Alam et al. (2003) obtained $C'_L$ of near 0.1. Again, the difference between the wide and narrow wakes is larger in Alam et al. (2003) and Kang (2003) than that in the present study, and thus, two different $C'_L$ values exist for $T/D \leq 2.0$ in these experimental studies. Sumner (2000) and Meneghini et al. (2001) did not present the $C'_D$ and $C'_L$.

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Table 5.6: $C'_D$ and $C'_L$ for the upstream and downstream cylinders in the side-by-side configurations
5.4. Side-by-side configuration

Figure 5.42: Fluctuating drag coefficients, $C_D'$. ◦ and ●: present study at Re = 800; ◇ and ◆: Alam et al. (2003), experimental study at Re = 55000. Open symbol: upstream cylinder (narrow wake in Alam et al. (2003)), closed symbol: downstream cylinder (wide wake in Alam et al. (2003)).
5.4.3 Strouhal data

The Strouhal number is obtained by taking the PSD of the velocity profile. The velocity profile is taken at $x/D = 2$ and $y/D = -1$ for the lower cylinder. For the upper cylinder, $x/D = 2$ and $y/D = 1$. The Strouhal number behaves in three different ways. For $T/D \leq 1.2$, there exists only one Strouhal number. It corresponds to the fact that a single vortex street is formed behind two side-by-side cylinders. Multiple Strouhal numbers are found for $1.2 < T/D \leq 2.0$ where the deflected flow pattern is observed. In the region of the biased flow pattern, the wide near wake region is formed behind one of the cylinders and shed Kármán vortices at the lower frequency. The higher frequency is observed from the narrower near wake. As $T/D$ increases, the difference of the two frequencies decreases, and only one shedding frequency is obtained for $T/D > 2$. Within this region of $T/D$, the shedding frequency converges to that in the single cylinder case as $T/D$ further increases. Figure 5.44 presents the Strouhal numbers for different $T/D$ values. Three shedding frequencies were obtained for the biased flow pattern in the present study. A higher shedding frequency is obtained from the narrow wake, and the lower shedding frequency is obtained from the wide wake region. The intermediate shedding frequency is associated with the parallel gap flow (Alam et al. 2003). Sumner (1999) also reported the intermediate frequency associated with the parallel gap flow in three side-by-side cylinder cases but not in two side-by-side cylinder cases. For two side-by-side cylinders, they reported two shedding frequencies for $T/D \leq 2.5$ suggesting that the biased flow pattern exist for $T/D \leq 2.5$. The highest frequency he reported is 0.4 at $T/D = 1.5$ which is larger than that in the present study. It might be due to the different sizes of the wakes formed behind the cylinders. In the present study, the difference between the wide and narrow wakes is small. Sumner’s (1999) results indicate that the biased flow pattern exist for $T/D < 3$, however, in the present study, the sign of the biased flow appears for $T/D < 2.5$. This could be due to the three-dimensional effect. Slouti and Stansby (1992) obtained the shedding frequencies
Figure 5.43: Fluctuating lift coefficients, $C_D'$. ○ and ●: present study at Re = 800; □: Kang (2003), numerical study at Re = 160; ◊ and ◆: Alam et al. (2003), experimental study at Re = 65000. Open symbol: upstream cylinder (narrow wake in Alam et al. (2003)), closed symbol: downstream cylinder (wide wake in Alam et al. (2003)).
Figure 5.44: Strouhal number. ○: present study at Re = 800, *: Sumner (1999), experimental study at Re = 500 - 1300; □: Slaouti and Stansby (1992), numerical study at Re = 200, ◇: Bearman and Wadcock (1973), experimental study at 2.5×10^4. at Re = 200 using a vortex method. The shedding frequencies obtained at the bistable region are smaller than the data reported in other studies (Sumner1999; Bearman and Wadcock 1973; the present study). The low Reynolds number they used could be a source for this discrepancy. Bearman and Wadcock (1973) obtained the shedding frequencies in their experiment at Re = 2.5×10^4. The presented data in figure 5.44 have a similar variation with those of the present study.

5.5 Staggered Configuration

The staggered arrangement of a pair of circular cylinders is the configuration most commonly found in engineering applications. Due to its complexity, less investi-
Staggered Configuration

gation has been conducted compared to the tandem or side-by-side configurations. Sumner et al. (2000) classified nine flow regimes in detail (figure 5.45). They categorized the flow patterns into three groups with the proximity and the incidence angle, $\alpha$. First, three flow patterns are observed when the cylinders are in close proximity. Sumner et al. (2000) identified three single bluff body behaviours, and these are shown in figure 5.45 (a), (b), and (c). The first flow pattern (figure 5.45 (a)) shows the instabilities in the shear layer from the upstream cylinder. It is termed SBB1. The second single bluff body (SBB2) behaviour is shown in figure 5.45 (b). A single, low frequency Kármán vortex street is formed behind the two cylinders; the same as that in a single stationary cylinder. The last one (figure 5.45 (c)) shows the weak gap flow of variable deflection angle. Again, a single vortex street is formed downstream. Sumner et al. (2000) named it as base-bleed, BB.

The next three patterns are observed when the incidence angle, $\alpha$, is small. Figure 5.45 (d) shows that the shear layer shed from the upstream cylinder reattaches to the downstream cylinder preventing the mean flow through the gap between the cylinder. It is the shear layer reattachment (SLR) flow pattern. In figure 5.45 (e), it is shown that a separation occurs on the inner side of the downstream cylinder due to the gap flow between the cylinders. This flow pattern is termed induced separation, IS. The last pattern in this group (figure 5.45 (f)) shows the vortex shed from the upstream cylinder is impinged to the downstream cylinder. This is the vortex impingement (VI). The flow patterns in the last group are observed when the incidence angle is large.

The key feature of the last three flow patterns in the last group (figure 5.45 (g)~(i)) is that vortices shed from the inner sides of the cylinders are paired and enveloped by the outer shear layer shed from the upstream cylinder. Sumner et al. (2000) named this flow pattern as vortex pairing and enveloping (VPE). In figure 5.45 (h), the complete enveloping is no longer observed anymore. The paired inner vortices are enveloped incompletely by the outer shear layer shed from the upstream cylinder.
and split later. This flow pattern is called vortex pairing, splitting, and enveloping (VPSE). The last flow pattern is synchronized vortex shedding (SVS). In this flow pattern, the shedding of anti-phase gap vortices is synchronized.

In the present study, four different cylinder configurations, classified by the pitch ratio between centres, $P/D$, and the incidence of the line between the centres to the free stream flow, $\alpha$, are discussed (see figure 5.1). For two staggered cylinders, two different vortex shedding frequencies are often obtained. Sumner et al. (2000) showed that vortex shedding from the inner shear layer of the downstream cylinder is often synchronized with vortex shedding from the upstream cylinder, and the outer shear layer of the downstream cylinder shed vortices at a different frequency. It is investigated in the present study for four different cylinder configurations. In addition to the near wake structures and Strouhal numbers, the force coefficients are also presented here.

5.5.1 $P/D = 2, \alpha=16^\circ$

For $P/D = 2.0$ and a small incidence angle, $\alpha = 16^\circ$, the inner shear layer of the upstream cylinder roll-up into small diameter Kármán vortices immediately behind the downstream cylinder, and induces a separation of the flow from the downstream cylinder. The inner shear layer of the upstream cylinder is deflected due to the penetrating mean flow. This shear layer rolls up at the rear surface of the downstream cylinder and causes a flow separation. Figure 5.46 is the experimental flow visualization demonstrated by Hayder (2008), and the result compares favourably to that of the present study in the immediate near wake. Sumner et al. (2000) also reported the IS flow pattern in this range of the cylinder configuration.

It is noticed that the wake structure observed in the present study is significantly different from that in Hayder’s (2008) after approximately 5 ∼ 6 cylinder diameters
Figure 5.45: Classification of the flow patterns in the staggered configurations in Sumner et al. (2000).
downstream of the downstream cylinder. In Hayder’s (2008) results, small scale three-dimensional structures are observed downstream. Williamson (1996) and Prasad and Williamson (1997) observed three-dimensional instabilities in their experimental studies conducted at $190 \leq \text{Re} \leq 260$. They found two three-dimensional instabilities; mode A and mode B. Mode A instability is a large scale instability observed for $\text{Re} \approx 180 \sim 190$, and mode B instability is a small scale instability observed for $\text{Re} \geq 250$. Mode B instability is a dominant three-dimensional phenomena up to $\text{Re} = 1000$ or even for $\text{Re} \geq 1000$. The difference at approximately 5-6 cylinder diameters downstream could be explained with this three-dimensional instability (mode B) which the present two-dimensional model cannot detect during the simulation. Therefore, the discussion and comparison with Hayder’s (2008) is focused on the immediate near wake, approximately within 5 cylinder diameters downstream through this section.

The sequence of plots of the wake structure are shown in figure 5.47. The flow separation from the downstream cylinder occurs due to the penetrating mean flow (Figure 5.47 (c)). This induced shear layer shed from the inner side of the downstream cylinder is enveloped by the outer shear layer of the upstream cylinder and forms a combined vortex (figure 5.47 (d)). As this combined vortex moves downstream, another shear layer is induced from the inner side of the downstream cylinder (figure 5.47 (h)). This second induced vortex is enveloped by the deflected inner shear layer shed from the upstream cylinder and also forms a combined vortex (figure 5.47 (i) and (j)). These two combined vortices move downstream as one large vortex and shed together, while one vortex is shed from the outer shear layer of the downstream cylinder. The streamlines are shown for $P/D = 2$ and $\alpha = 16^\circ$ in figure 5.48. The separation of the inner shear layer shed from the downstream cylinder and the deflection of the inner shear layer shed from the upstream cylinder are observed throughout the sequence of plots. The first induced separation is shown in figure 5.48 (b), and the second is shown in figure
5.5. Staggered Configuration

Figure 5.46: Near wake structures at P/D = 2 and $\alpha = 16^\circ$; (a) Flow visualization in Hayder (2008); (b) Vorticity structure simulated in the present study

5.48 (e). These two vortices are combined and shown in figure 5.48 (f) and (g).

The time history of $C_D$ and $C_L$ for both cylinders are shown in figure 5.49. The fluctuating amplitude of the $C_D$ and $C_L$ for the upstream cylinder is very small due to absences of vortex formation. The $\overline{C_D}$ and $\overline{C_L}$ values are presented in table 5.7. There is no available data for low Reynolds number, Re = 800 for the staggered configurations, thus we compare the results with the data in Sumner et al. (2005). Sumner (1999) conducted the experimental study at Re = 1000 $\sim$ 3000 for the staggered configuration, however, no force coefficients were provided at that time. The different Reynolds number could explain the discrepancy of the obtained values in the present study from the experimental data. A large negative $\overline{C_L}$, which is known as the "inner" lift peak (Zdravkovich and Pridden 1977), is observed for the downstream cylinder. This is due to the high speed flow being forced through the gap between the cylinders. A significantly lower pressure is caused by this strong gap flow around the downstream cylinder, and the downstream cylinder experiences the high "inner" lift. The $\overline{C_D}$ values for both cylinders are smaller than that of a single cylinder. Especially, $\overline{C_D}$ of the downstream cylinder is very small, and this corresponds to the fact that the minimum drag coefficient occurs in the
Figure 5.47: Time history of the vortex shedding process at $P/D=2$ and $\alpha = 16^\circ$
Figure 5.48: Streamlines plotted at several instants of time at \( P/D = 2 \) and \( \alpha = 16^\circ \)
vicinity of the "inner" lift peak. Table 5.8 shows $C_D'$ and $C_L'$ and comparison with data in Alam et al. (2003). In both studies, the $C_D'$ and $C_L'$ values of the downstream cylinder are larger than that of the upstream cylinder. This is because vortices are only shed from the downstream cylinder. The higher Reynolds number, Re = 55000, and the small incidence angle is considered in Alam et al. (2003), and this could explain the difference of the data from the present study.

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Table 5.7: $\overline{C_D}$ and $\overline{C_L}$ for the upstream and downstream cylinders at $P/D = 2$ and $\alpha = 16^\circ$

Figure (5.50) shows the frequency peaks obtained in Hayder (2008) and the present
5.5. Staggered Configuration

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<td>Kiya et al. (1980)</td>
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Table 5.9: Strouhal frequency comparisons for P/D = 2, α = 16°

study. One dominant frequency is obtained in the present study. This indicates that vortices are shed at the same frequency from the combined wake and the outer shear layer of the downstream cylinder. Hayder (2008) observed three different frequencies: higher frequency from the upstream near wake, and lower frequencies from the downstream cylinder. The published data are summarized in Table 5.9. Price et al. (2007) and Kiya et al. (1980) reported two frequencies, and the data in Price et al. (2007) are in similar range with those in the present study. Sumner and Richards (2003) reported only one frequency of 0.15 at Re = 32000. Despite a similar Reynolds number, the Strouhal frequency obtained in the present numerical study is different from those in Hayder’s (2008).

Table 5.8: \( C_D' \) and \( C_L' \) for the upstream and downstream cylinders at P/D = 2 and \( \alpha = 16° \)
Figure 5.50: PSD peaks at $P/D = 2$ and $\alpha = 16^\circ$ in Hayder (2008) (a) upstream, (b) downstream; in the present study (c) upstream, (d) downstream. In the present study, the velocity profiles are taken at $x/D = 5$, $y/D = -0.75$ from the upstream and at $x/D = 3$, $y/D = 1.25$ from the downstream cylinder.
5.5.2 $P/D = 3.25, \alpha = 16^\circ$

At a large pitch ratio and small angle of incidence, the flow pattern resembles the flow obtained from the tandem configuration after the critical spacing. Vortices are shed from the upstream cylinder at a frequency close to that of a single circular cylinder and then impinge upon the downstream cylinder. Hayder (2008) presented the vortex impingement (VI) pattern for $P/D = 3.25$ and $\alpha = 16^\circ$ in his experimental study (figure 5.51 (a)). The near wake region of the downstream cylinder is highly disturbed by the impingement of a vortex street from the upstream cylinder. Figure 5.51 (b) is the simulation result in the present study. It also shows that a vortex shed from the upstream cylinder is impinged to the downstream cylinder. A large combined vortex behind the downstream cylinder is observed instead of a highly disturbed flow in the present study. Compared with Hayder’s (2008) results, the formation length is significantly smaller in the present study. This is likely due to the appearance of three dimensional structures. Figure 5.52 shows the time series of the near wake structure at $P/D = 3.25$ and $\alpha = 16^\circ$. A vortex shed from the outer shear layer of the upstream cylinder slows down when it approaches the downstream cylinder (figure 5.52 (c) ~ (f)). A vortex shed from the inner shear layer of the upstream cylinder is impinged to the downstream cylinder and is combined with a vortex shed from the inner side of the downstream cylinder as shown in figure 5.52 (g) ~ (j). This combined vortex moves downstream faster than the previous vortex shed from the outer shear layer of the upstream cylinder. These vortices are combined and formed a large combined vortex. It is shown in figure 5.52 (l).

The streamline patterns are shown in figure 5.53. In figure 5.53 (a) ~ (d), the shed vortex from the inner shear layer of the upstream cylinder is impinged to the downstream cylinder. Another vortex shed from the outer shear layer of the upstream cylinder passes the inner side of the downstream cylinder (figure 5.53...
Figure 5.51: Near wake structures at P/D = 3.25 and $\alpha = 16^\circ$; (a) Flow visualization in Hayer (2008); (b) Vorticity structure simulated in the present study.

Table 5.10: $\overline{C_D}$ and $\overline{C_L}$ for the upstream and downstream cylinders at P/D = 3.25 and $\alpha = 16^\circ$.

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Figure 5.54 shows the time histories of $C_D$ and $C_L$. The times series of $C_D$ and $C_L$ of the upstream cylinder are approximately identical to those of a single cylinder. For the downstream cylinder, due to the vortex impingement, it was observed that the fluctuating amplitudes of $C_D$ and $C_L$ were different from those of the upstream cylinder. The fluctuating amplitude of $C_D$ is larger than that of the upstream cylinder, while the fluctuating amplitude of $C_L$ is smaller than that of the upstream cylinder. Table 5.10 shows the time averaged $C_D$ and $C_L$ values. The $\overline{C_D}$ value of the upstream cylinder is close to that of a single cylinder. However, $\overline{C_D}$ of the
5.5. Staggered Configuration

Figure 5.52: Time history of the vortex shedding process at P/D=3.25 and $\alpha = 16^\circ$
Figure 5.53: Streamlines plotted at several instants of time at $P/D = 3.25$ and $\alpha = 16^\circ$
Figure 5.54: Time series of the $C_D$ and $C_L$ at $P/D = 3.25$ and $\alpha = 16^\circ$

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Table 5.11: $C_D'$ and $C_L'$ for the upstream and downstream cylinders at $P/D = 3.25$ and $\alpha = 16^\circ$
downstream cylinder is much smaller than that of the upstream cylinder due to the vortex-body interaction. A negative value, which is known as “outer lift peak” (Zdravkovich and Pridden 1977), is observed in the $C_L$ for the downstream cylinder. Sumner (1999) explained the mechanism of causing the outer lift peak in VI flow pattern. He described that the vortex impingement and the circulation induced thereby create the higher lift on the downstream cylinder. Sumner et al. (2005) reported maximum outer lift peak for $P/D = 3$ (and 4) and $\alpha \simeq 18^\circ$ at $Re = 72000$. $C_D'$ and $C_L'$ are also shown in table 5.11. Compared with the data in Alam et al. (2005), the $C_L'$ data obtained in the present study is very high, especially, $C_L'$ of the upstream cylinder being twice of that of the downstream cylinder. The different Reynolds number and the different configuration considered in Alam et al (2005) could be a source of the discrepancy. The other possible reason is the appearance of three-dimensional structures. Williamson (1996) explained that the shorter formation length in the two-dimensional simulation causes a higher Reynolds stress on the body and eventually results in the higher drag and lift. In the present study, a two-dimensional numerical model was used to simulate the flow, and thus, it could be a source for the higher $C_D$ and $C_L$ observed in the present study compared with those in Sumner et al. (2005).

The single Strouhal number of 0.2 is obtained in the present study. Hayder (2008) presented the Strouhal number of 0.17 at $Re = 550 \sim 750$. The power spectra obtained from Hayder’s (2008) experiment and the present study are shown in figure 5.55, respectively. The results are compared with those of previous experimental studies in Table 5.12. Sumner et al. (2005), Alam and Sakamoto (2005), and Kiya et al. (1980) also observed the single Strouhal number of 0.17 or 0.18 which is smaller than that of a single stationary cylinder. The higher frequency obtained in the present study could be due to the shorter formation length.
Figure 5.55: (a) PSD peaks in Hayder (2008); (b) PSD peaks at $P/D = 3.25$ and $\alpha = 16^\circ$ in the present study. The velocity profiles are taken at $x/D = 5$, $y/D = -1.5$ for the upstream and at $x/D = 4.5$, $y/D = 1.75$. 
Chapter 5. Two Stationary Circular Cylinders

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<td>0.17</td>
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Table 5.12: Strouhal frequency comparisons for P/D = 3.25, ρ = 16°

5.5.3 P/D = 2.5, ρ=21°

At a moderate incidence angle, ρ = 21° and pitch ratio, P/D = 2.5, the gap is appreciably wider, and the flow pattern transforms from the induced separation to the vortex pairing and enveloping. The induced separation on the inner side of the downstream cylinder is developed to a separated, free shear layer and forms a vortex. Then, Kármán vortices formed by the inner shear layer of the downstream cylinder are shed at the same frequency as those from the upstream cylinder. This flow pattern is seemed vortex pairing and enveloping (VPE), and it is shown in figure 5.56. The flow visualization in Hayder (2008) (5.56 (a)) and the simulation result (5.56 (b)) in the present study show a good agreement. In these figures, vortices shed from the inner shear layers from the upstream and the downstream cylinders are paired and completely enveloped by the outer shear layer shed from the upstream cylinder. Sumner et al. (2000) also reported observing VPE pattern in their experimental study. Figure 5.57 shows the time history of the flow pattern. The outer shear layer shed from the upstream cylinder reaches to the inner side of the downstream cylinder in figure 5.57 (c) ~ (e), and the inner shear layers of the upstream and downstream cylinders are paired in figure 5.57 (f) ~ (h). These paired vortices are completely enveloped by the outer shear layer shed from the upstream
5.5. Staggered Configuration

Figure 5.56: Near wake structures at P/D = 2.5 and $\alpha = 21^\circ$; (a) Flow visualization in Hayder (2008); (b) Vorticity structure simulated in the present study

cylinder (figure 5.57 (i) ~ (k)), and a large combined vortex is formed (figure 5.57 (l) and (m)). This composite vortex shed from the downstream cylinder as the outer shear layer shed from the upstream cylinder reaches the inner shear layer of the downstream cylinder. The process of the vortex pairing, enveloping and shedding of a composite vortex is repeated with time. During the period shown in figure 5.57, there are two vortices shed from the cylinders, a composite vortex and a single vortex. Figure 5.58 shows the streamline patterns. The vortex pairing and enveloping processes are hardly seen in figure 5.58. Only the enveloping process is observed behind the downstream cylinder in figure 5.58 (e) and (f).

The $C_D$ and $C_L$ values were measured for each cylinder. The time series of $C_D$ and $C_L$ are shown in figure 5.59. The fluctuating amplitude of $C_D$ is larger in the wake of the downstream cylinder than in the wake of the upstream cylinder because of a large combined vortex produced by the enveloping process. For $C_L$, the fluctuating amplitudes are similar. The $\overline{C_D}$ and $\overline{C_L}$ values are presented in table 5.13. The higher $\overline{C_D}$ and $\overline{C_L}$ are obtained in the present study compared with those in Sumner et al. (2005). The large difference in Reynolds numbers could be a reason for this discrepancy. Sumner et al. (2005) conducted an experiment
Figure 5.57: Time history of the vortex shedding process at P/D=2.5 and $\alpha = 21^\circ$
5.5. Staggered Configuration

Figure 5.58: Streamlines plotted at several instants of time at $P/D = 2.5$ and $\alpha \approx 21^\circ$
at the same configuration for higher Reynolds number of Re = 72000, and thus, the values might be different due to Reynolds number effects. As discussed in the previous subsection, the shorter formation length results in the higher $C_D$ and $C_L$. The "inner" lift force is observed for the downstream cylinder. Sumner (1999) explained this inner lift peak with a mechanism associated with the VPE and VPSE flow patterns. The static pressure distribution is created on the inner surface of the downstream cylinder by the gap flow and causes the high, inward directed lift force. The gap flow speed is lower than that of the IS flow pattern, and thus, the peak is smaller than the "inner" lift peak found in IS flow pattern. Table 5.14 shows $C_D'$ and $C_L'$. The $C_D'$ and $C_L'$ values of the downstream cylinder are higher than those of the upstream cylinder in both studies. However, the values obtained from the present study are much larger than the experimental data. Again, the difference could be due to three-dimensional effect or the Reynolds number.

The power spectra of the near wake are shown in figure 5.60. The peaks represent the non-dimensional vortex shedding frequencies. Three distinct Strouhal frequen-
5.5. Staggered Configuration

Table 5.13: $C_D$ and $C_L$ for the upstream and downstream cylinders at $P/D = 2.5$ and $\alpha = 21^\circ$

<table>
<thead>
<tr>
<th>Reference</th>
<th>$C_D$</th>
<th>$C_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study, Re = 800</td>
<td>0.944</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>0.739</td>
<td>-0.198</td>
</tr>
<tr>
<td>Sumner et al. (2005), Re = 72000</td>
<td>0.65</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

Table 5.14: $C_D'$ and $C_L'$ for the upstream and downstream cylinders at $P/D = 2.5$ and $\alpha = 21^\circ$

<table>
<thead>
<tr>
<th>Reference</th>
<th>$C_D'$</th>
<th>$C_L'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study, Re = 800</td>
<td>0.126</td>
<td>0.383</td>
</tr>
<tr>
<td></td>
<td>0.229</td>
<td>0.446</td>
</tr>
<tr>
<td>Alam et al. (2005), Re = 5500, P/D = 2.5, $\alpha = 25^\circ$</td>
<td>0.05</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>0.133</td>
<td>0.235</td>
</tr>
</tbody>
</table>
cies of 0.18, 0.36, and 0.54 were obtained in the present study (table 5.15). All of them are in harmonic. The harmonic frequencies are obtained from the upstream cylinder and show an integral relationship between the largest and the smallest. Hayder (2008) reported the frequencies of 0.14 and 0.46 at \( \text{Re} = 550 \sim 750 \) with the integral relationship of \( 1/3 \). The integral relationship of \( 1/3 \) is also found in this study. The slightly higher Strouhal number obtained in the present study could be due to the shorter formation length in the present study compared to Hayder’s (2008). This shorter formation region leads to an early shedding and results in the higher vortex Strouhal number. Sumner and Richards (2003) and Sumner et al. (2005) also reported two Strouhal numbers. The different integral relationship is observed between Hayder (2008) and the other experimental results (Sumner and Richards 2005; Sumner et al. 2005; Kiya et al. 1980). The source of this discrepancy might be the Reynolds number employed in their experiments.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Configuration</th>
<th>Re</th>
<th>( \text{St up} )</th>
<th>( \text{St down} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>( \frac{P}{D} = 2.5, \alpha = 21^\circ )</td>
<td>800</td>
<td>0.54, 0.36</td>
<td>0.18</td>
</tr>
<tr>
<td>Hayder (2008)</td>
<td>( \frac{P}{D} = 2.5, \alpha = 21^\circ )</td>
<td>500 \sim 750</td>
<td>0.46</td>
<td>0.14</td>
</tr>
<tr>
<td>Sumner &amp; Richards (2005)</td>
<td>( \frac{P}{D} = 2.5, \alpha = 20^\circ )</td>
<td>32000</td>
<td>0.15, 0.27</td>
<td></td>
</tr>
<tr>
<td>Sumner et al. (2005)</td>
<td>( \frac{P}{D} = 2.5, \alpha = 25^\circ )</td>
<td>72000</td>
<td>0.14, 0.28</td>
<td></td>
</tr>
<tr>
<td>Kiya et al. (1980)</td>
<td>( \frac{P}{D} = 2.5, \alpha = 15^\circ )</td>
<td>15800</td>
<td>0.14, 0.23</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.15: Strouhal frequency comparisons for \( \frac{P}{D} = 2.5, \alpha = 21^\circ \)

5.5.4 \( \frac{P}{D} = 2, \alpha = 45^\circ \)

At a large incidence angle, the complete enveloping of the gap vortices by the shear layer shed from the outer side of the upstream cylinder does not occur. The gap flow
5.5. Staggered Configuration

Figure 5.60: (a), (b): PSD peaks in Hayder (2008); (c), (d): PSD peaks at P/D = 2.5 and $\alpha = 21^\circ$ in the present study. The velocity profiles are taken at $x/D = 5$, $y/D = -1.5$ for the upstream and at $x/D = 3.5$, $y/D = 1.5$. 
becomes weaker and each inner gap vortex split into two concentrations of vorticity during the enveloping process. Hayder (2008) reported two flow patterns for the same configuration in his experimental study. One is the vortex pairing, splitting, and enveloping (VPSE) (5.61 (a)), and the other is the synchronized vortex shedding (SVS) (5.61 (c)). In this study, we observed two flow patterns of VPE (figure 5.61 (b)) and SVS (figure 5.61 (d)). Hayder’s (2008) result shows an incomplete enveloping, while the result in the present study shows a complete enveloping like the case of $P/D = 2.5$ and $\alpha = 21^\circ$. Hayder (2008) observed that the gap vortices are paired and enveloped by the outer shear layer shed from the upstream cylinder incompletely, and this incomplete enveloping causes splitting of a small vortex from the inner shear layer of the downstream cylinder. The other flow pattern, SVS, shows that the vortices shed from the inner shear layers of the upstream and downstream cylinders are synchronized. At a slightly smaller incidence angle of $\alpha = 40^\circ$, Sumner (1990) reported only VPSE flow pattern. The snap shots of the near wake structure are shown in figure 5.62. Anti-phase vortices shed from the inner shear layers of the upstream and downstream cylinders are shown in figure 5.62 (d) ~ (g). The synchronization is repeated in figure 5.62 (k) ~ (m). The vortex paring and the enveloping process is followed in figure 5.62 (p) ~ (t). The streamline patterns are shown in figure 5.63. The SVS flow pattern is shown in figure 5.63 (e) and (f).
Two different flow patterns were observed here. The vortex pairing, splitting, and enveloping pattern (Fig. 40 (a) and (b)), VPSE, and the synchronized vortex shedding (Fig. 40 (c) and (d)), SVS, pattern appears one after the other. Sumner et al. (2000) report VPSE pattern for \( P/D = 2 \) and \( \alpha = 40^\circ \). Recently, Hayder (2008) observed two flow patterns, VPSE and SVS, in this study for a slightly larger incidence angle of \( 45^\circ \) at \( Re = 550 \sim 750 \). The flow visualization and the numerical simulation are shown in Fig. 40. The vortices are paired and enveloped by the outer shear layer shed from the upstream cylinder incompletely. The incomplete enveloping causes splitting of the small vortex from the inner shear layer of the downstream cylinder (VPSE). In SVS pattern, anti-phase vortices are synchronized. The vortex shedding process is shown in Fig. 41. Two complete shedding cycles are shown. The vortex shedding starts with SVS pattern (Fig. 41 (a) - (c)), then VPSE pattern follows (Fig. 41 (e) - (g)). This process is repeated with SVS (Fig. 41 (h) - (j)) and VPSE (Fig. 41 (k) - (n)).

Figure 5.61: Near wake structures at \( P/D=2 \) and \( \alpha = 45^\circ \); (a), (c) Flow visualization in Hayer (2008); (b), (d) Vorticity structure simulated in the present study
Figure 5.62: Time history of the vortex shedding process at P/D=2 and $\alpha = 45^\circ$
Figure 5.64 present the time histories of $C_D$ and $C_L$. The patterns of $C_D$ and $C_L$ are more complicated due to the interaction of vortices. $C_D$ and $C_L$ are presented in Table 5.16. Unlike the other cases considered in this study, $C_D$ of the downstream cylinder is larger than that of the upstream cylinder. Sumner et al. (2005) also reported the larger $C_D$ for the downstream cylinder. The negative $C_L$ is observed for the downstream cylinder. As seen at $P/D = 2$ and $\alpha = 16^\circ$, this large inward directed force is an "inner" lift peak which is followed by the large $C'_D$. The fluctuating amplitudes of $C_D$ and $C_L$ are larger for the downstream cylinder, and it is confirmed with $C'_D$ and $C'_L$ values presented in table 5.17. The obtained values are higher than the experimental data. This might be due to the appearance of three-dimensional structures in the wake or the different Reynolds number.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\bar{C}_D$</th>
<th>$\bar{C}_L$ at $P/D = 2$ and $\alpha = 45^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>upstream</td>
<td>downstream</td>
</tr>
<tr>
<td>Present study, Re= 800</td>
<td>0.930</td>
<td>1.245</td>
</tr>
<tr>
<td>Sumner et al. (2005) Fe = 72000</td>
<td>1</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Table 5.16: $\bar{C}_D$ and $\bar{C}_L$ for the upstream and downstream cylinders at $P/D = 2$ and $\alpha = 45^\circ$

<table>
<thead>
<tr>
<th>Reference</th>
<th>$C'_D$</th>
<th>$C'_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>upstream</td>
<td>downstream</td>
</tr>
<tr>
<td>Present study, Re= 800</td>
<td>0.090</td>
<td>0.312</td>
</tr>
<tr>
<td>Alam et al. (2005), Re = 5500</td>
<td>0.06</td>
<td>0.14</td>
</tr>
<tr>
<td>$P/D = 2.1$, $\alpha = 45^\circ$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.17: $C'_D$ and $C'_L$ for the upstream and downstream cylinders at $P/D = 2$ and $\alpha = 45^\circ$

One dominant peak is found for each cylinder (figure 5.65 (c) and (d)). Two
Figure 5.63: Streamlines plotted at several instants of time at $P/D = 2$ and $\alpha = 45^\circ$
Strouhal numbers of 0.185 and 0.325 were observed in the present study. Hayder (2008) reported two Strouhal numbers of integral relationship with $1/3$ (of the smallest and the largest). This discrepancy seems due to the different dominant flow patterns between this study and Hayder (2008). The SVS flow pattern is the dominant flow in the present study while VPSE is the dominant flow pattern in Hayder (2008). Alam and Sakamoto (2005), Sumner and Richards (2003), and Kiya et al. (1980) also reported two Strouhal numbers without any integral relation (table 5.18).
Figure 5.65: (a), (b) PSD peaks in Hayder (2008); (c), (d) PSD peaks in the present study. The velocity profiles are taken at $x/D = 2.75$, $y/D = -0.75$ for the upstream and at $x/D = 2.5$, $y/D = 1.5$. 
Table 5.18: Strouhal frequency comparisons for $P/D = 2, \alpha = 45^\circ$

<table>
<thead>
<tr>
<th>Reference</th>
<th>Configuration</th>
<th>Re</th>
<th>St up</th>
<th>St down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>$P/D = 2, \alpha = 45^\circ$</td>
<td>800</td>
<td>0.325</td>
<td>0.185</td>
</tr>
<tr>
<td>Hayder (2008)</td>
<td>$P/D = 2 \alpha = 45^\circ$</td>
<td>500 – 750</td>
<td>0.46</td>
<td>0.14</td>
</tr>
<tr>
<td>Alam &amp; Sakamoto (2005)</td>
<td>$P/D = 2 \alpha = 45^\circ$</td>
<td>55000</td>
<td>0.14, 0.34</td>
<td></td>
</tr>
<tr>
<td>Sumner &amp; Richards (2005)</td>
<td>$P/D = 2, \alpha = 45^\circ$</td>
<td>32000</td>
<td>0.15, 0.34</td>
<td></td>
</tr>
<tr>
<td>Kiya et al. (1980)</td>
<td>$P/D = 2, \alpha = 45^\circ$</td>
<td>15800</td>
<td>0.12, 0.34</td>
<td></td>
</tr>
</tbody>
</table>

5.6 Discussion

Flow around two circular cylinders was investigated at $Re = 800$ using the random vortex method. Three different arrangements (tandem, side-by-side, and staggered) were studied in this chapter. For the tandem arrangement, no vortex shedding from the upstream cylinder was observed within the $L/D$ range, $L/D < 3.5$. The vortex shedding from the upstream cylinder appeared for $L/D \geq 3.5$ resulting in the critical spacing of $3.0 < L/D \leq 3.5$. The critical spacing obtained in the present study is smaller than those reported in the experimental studies (Sumner 1999; Kuo et al. 2008) despite similar Reynolds numbers. The $C_D$ value increases abruptly at the critical spacing in the present study, and this behaviour agrees well with other previous studies. Although it is difficult to make a direct comparison of these values due to the difference in the Reynolds number, the general pattern and the characteristics of $C_D$ agree well with the results in previous studies (Xu and Zhou 2004; Deng et al. 2006; Kitagawa and Ohta 2008; Kuo et al. 2008). A good agreement is also found between the Strouhal numbers obtained in the present study and those in the experimental studies (Kuo et al. 2008; Xu and Zhou 2004). In the present study, the smaller formation region is formed behind the cylinder due to the three-dimensional effect, and therefore the vortex shedding from the
upstream cylinder arises at smaller a L/D compared with Sumner (1999) and Kuo et al. (2008).

For the side-by-side arrangement, we observed the biased flow pattern for T/D < 2, the anti-phase vortex shedding for 2 ≤ T/D ≤ 4.5, and the in-phase vortex shedding at T/D = 4.5 in this study. Due to the shorter formation length, the deflected flow patterns seems to be weak compared to the published data, however, overall flow patterns agree favourably with the previous experimental and numerical results (Sumner 1999; Meneghini 2000). The force coefficients and the Strouhal numbers also show a fairly good agreement.

For the staggered arrangement, four different cylinder configurations were discussed in this chapter. For a small pitch ratio, P/D = 2 and α = 16°, the upstream cylinder’s inner shear layer is deflected between the two cylinders. Only one Strouhal number of 0.159 is observed in the combined wake. For a larger pitch ratio, P/D = 3.25 and α = 16°, vortices shed from the upstream cylinder impinges on the downstream one inducing the vortex formation from the inner shear layer of the downstream cylinder. In this case, Kármán vortex shedding is observed downstream of the cylinder pair with a single Strouhal number of 0.2. At a moderate incidence angle, α = 21°, a pair of vortices is shed via the inner shear layers of the two cylinders, and then enveloped by the outer shear layer from the upstream cylinder. This process yields a large combined vortex, which is followed by a vortex shed from the downstream cylinder’s outer shear layer. This gives three Strouhal numbers where the largest Strouhal number is approximately three times the smallest. At a moderate incidence angle, α = 45°, the dominant flow pattern of SVS, in which the opposite signed gap vortices are synchronized, is observed, and two Strouhal numbers of 0.325 and 0.185 are obtained in the present study. The immediate near wake structure observed in the present study is in good agreement with Hayder’s (2008). The Strouhal numbers also show a reasonable agreement.
The present study was performed with a two-dimensional numerical model, and therefore, no three-dimensional features was included in the simulation. As discussed earlier in this chapter, the appearance of three-dimensional structures tends to increase the formation length of the recirculation zone. Therefore, in the present study, a relatively shorter formation length was observed due to the lack of three-dimensional effects. The drag and lift forces were overestimated likely due to the shorter formation length, and the early vortex shedding arose in the wake resulting in a higher shedding frequency. Despite these differences, the obtained results were still in comparable range with the published data. The observations of the critical spacing in the tandem arrangements and the bi-stable flow in the side-by-side arrangements were in fairly good agreement with those in the experimental and numerical studies. For the staggered configurations, the observed flow patterns were in good agreement with those classified in Sumner’s (1999) experiments. The mean and fluctuating force coefficients also showed reasonable agreement with the published data.
Chapter 6

Conclusion and Recommendation

6.1 Conclusion

In this study, the flow around two circular cylinders arranged in the staggered configuration was numerically investigated. The two-dimensional Navier-Stokes equations were solved by the random vortex method, which has been well established in solving low Reynolds number flows over cylinders. It is easy to implement and computationally less expensive compared to other methods such as FEM, LES, and DNS. Therefore, a large volume of cases can be simulated by the random vortex method. During this study, we found some weaknesses such as a short formation length and slight overestimations of the Strouhal number and force coefficients compared to experimental and higher order numerical studies. In a single stationary cylinder case, we found that the drag coefficient was overvalued when Re > 600 due to the lack of three-dimensional effects. The short formation length which was observed in the two circular cylinder cases caused a slight overestimation of the force coefficients and the Strouhal number by increasing the Reynolds stress on the body. However, this model still gave us reasonable results to understand the
vortical structures and aerodynamic characteristics. The results obtained by the random vortex method were consistent with published data and therefore can be used to understand various flow phenomena in many academic and engineering applications.

The emphasis in this study was to validate the vortex method and to understand the flow around two circular cylinders by numerically reproducing Sumner’s (1999) experiments. First, the model validation was conducted at Re = 200 for a single stationary cylinder. The vorticity structure, drag coefficient and Strouhal number were used to validate this numerical model. Secondly, in order to achieve a better understanding of the flow, we performed the simulations for the cases of a transversely oscillating cylinder and two stationary circular cylinders (tandem, side-by-side, and staggered configurations). The near wake structures, mean and fluctuating force coefficients, and Strouhal numbers were thoroughly investigated and compared with published data. An in-depth investigation was carried out for the dynamics of the vortical structure and shedding frequencies. The flow evolutions and the shear layer separation and interaction were observed in detail. The present study is summarized in the following subsections.

6.1.1 Model Validation and Background Study

The random vortex method, which was used to simulate the flow in this study, was validated at a Reynolds number of 200. The near wake structure, force coefficients, and Strouhal number compared favourably with those in previous experimental and numerical studies. In addition, simulations of a single oscillation cylinder and two cylinder pairs with both tandem and side-by-side configurations were also carried out. For the single oscillating cylinder, the vortex patterns were predicted for different excitation frequencies at a fixed amplitude, $A/D = 0.22$ at Re = 400. The flow patterns compared favourably with the results in Williamson and Roshko
(1988). Investigations of the tandem and the side-by-side arrangements were also carried out at a Reynolds number of 800. For the tandem arrangement, comparable flow structures with previous studies were found. No vortex shedding from the upstream cylinder was observed within the range, $L/D < 3.5$. The upstream cylinder started to shed vortices for $L/D \geq 3.5$, and the critical spacing was found in $3.0 < L/D < 3.5$ in the present study. The force coefficients and Strouhal numbers compared favourably with experimental results (Alam et al. 2003; Xu and Zhou 2004; Kuo et al. 2008). For the side-by-side arrangement, three flow regimes were observed in this study: (i) the biased flow pattern for $T/D < 2$, (ii) the anti-phase vortex shedding for $2 \leq T/D < 4.5$, and (iii) the in-phase vortex shedding at $T/D = 4.5$. The flow patterns agreed with previous experimental and numerical results (Sumner 1999; Meneghini 2000). The force coefficients and the Strouhal numbers also showed good agreement with previously obtained values.

6.1.2 Two Staggered Cylinders: Stationary Case

Four different cylinder configurations were thoroughly investigated for the staggered arrangement. The near wake structure, measurement of vortex shedding frequencies, and force coefficients were examined. For a small pitch ratio, $P/D = 2.0$, and at $\alpha = 16^\circ$, the IS flow pattern was observed in the present study. The inner shear layer from the upstream cylinder was deflected behind the downstream cylinder owing to the penetrating mean flow and induced a flow separation from the downstream cylinder. Three vortices are shed periodically, yielding a Strouhal number of 0.159. For a large pitch ratio, $P/D = 3.25$, and $\alpha = 16^\circ$, the VI flow pattern was observed. Vortices shed from the upstream cylinder impinge on the downstream cylinder and cause a highly disturbed flow in the wake of the downstream cylinder. A single vortex street was observed with a Strouhal number of 0.2. At moderate incidence angles, $\alpha = 21^\circ$, at $P/D = 2.5$ the VPE flow pattern was observed.
in this study. A pair of the gap vortices was enveloped by the outer shear layer of the upstream cylinder. This process yields a large composite vortex, which is followed by a vortex shed from the downstream cylinder’s outer shear layer. Three Strouhal numbers, 0.18, 0.36, and 0.54, were obtained in the present study, and a 1/3 integral relationship was found between the smallest and the largest Strouhal numbers. At a large incidence angle, $\alpha = 45^\circ$ and $P/D = 2.0$, the VPE flow pattern was again observed along with the SVS flow pattern. In the present study, the SVS flow pattern seemed to be dominant. Two Strouhal numbers of 0.185 and 0.325 were observed.

### 6.2 Future work

Recently, Hayder (2008) experimentally investigated the flow around two circular cylinders, where one of them was oscillating. His study revealed the leading mechanisms of the vortex shedding in the combined wake and the effect of forced oscillation on the flow patterns and shedding frequencies. Price et al. (2006, 2007) also carried out experimental studies for different staggered configurations. However, no numerical or additional experimental investigations have been carried out for the configurations reported in the work of Price et al. (2006, 2007) and Hayder (2008), and therefore, it would be interesting to re-examine the experiments from the aspect of numerical simulations.

While flow structures around two cylinders with one oscillating have been examined in some configurations, the flow structures around two oscillating circular cylinders still remain to be explored. Particularly, the flow structure around cylinders oscillating streamwise has hardly been reported on until now. Therefore, further investigation on this subject is required.
6.2. Future work

The present numerical study was performed with a two-dimensional random vortex method. A two-dimensional model was chosen in order to enable a large number of simulations. In the near wake structure, the absence of three-dimensional structures leads to shorter formation lengths as compared to Sumner’s (1999) experimental results. The shorter formation length induces early vortex shedding, as compared to experimental results, and thus, the Strouhal number is overestimated in the two-dimensional simulations. The Reynolds stress is increased due to the shorter formation length behind the cylinder. Therefore, the drag and lift forces are also overestimated. Although the obtained results are within an acceptable accuracy, a three-dimensional investigation is strongly recommended in the future for more accurate results.
Bibliography


