Switched Reluctance Motor Analysis and Design for a Lunar Roving Vehicle

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ABSTRACT

In the 21st century, many countries launched robotic space exploration programs, including the Discovery Program in United States, the Chandrayaan Program in India and the Chang’e Program in China. The lack of an atmosphere in outer space makes electric motors the only viable choice for the propulsion systems of a space exploration robot.

This thesis proposes the switched reluctance motor (SRM) as a potential candidate for a traction drive. The SRM has a simple and robust rotor structure, an inherent degree of fault tolerance, high efficiency over a wide speed range, and a low cost. This thesis also presents the complete design process of an SRM, starting with its user requirement. Numerical experiments are carried out to analyze both the motoring and the regenerative braking modes of the SRM, and to verify the proposed design. Additionally, evolutionary strategy (ES) single- and multi-objective optimizations are performed to refine the geometric parameters.
RÉSUMÉ

Au XXIème siècle, beaucoup de pays ont lancé les programmes de robotiques d'exploration spatiale, comprenant le programme Discovery des Etats Unis, Chandrayaan de l'Inde et le programme chinois Chang'e. Le manque d'une atmosphère dans l'espace fait que le moteur électronique est le seul choix viable pour les systèmes de propulsion des robots d'exploration spatiale.

Cette thèse propose le moteur à réluctance variable (SRM) comme une candidature potentielle de la traction. Le SRM a une structure simple et robuste de son rotor, une tolérance inhérente de panne, un rendement élevé sur une grande plage de vitesses et un prix bas. Le "design process" complet de SRM est aussi présenté commencé par le besoin de l'utilisateur. Les expériences numériques ont été réalisées afin d'analyser aussi bien le mode de moteur que le mode de freinage par régénération, et de vérifier la conception proposée. De plus, la stratégie évolutive (ES) des optimisations de l'objectif simple ou objectifs multiples a été réalisée pour affiner les paramètres géométriques.
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LIST OF SYMBOLS

$W_e$: Electric energy sent to the winding
$W_f$: Field energy
$W_m$: Mechanical energy
$W_{co}$: Coenergy
$P_{ele}$: Electric power
$P_{mech}$: Mechanical power
$\lambda$: Flux linkage
$\lambda_u$: Flux linkage at the unaligned position
$\lambda_a$: Flux linkage at the aligned position
$\theta$: Mechanical angular displacement
$T$: Mechanical torque
$v$: Transient voltage
$i$: Transient current
$\omega$: Rotational speed
$L$: Inductance
$\theta_{on}$: Switch turn-on angle
$\theta_{off}$: Switch turn-off angle
$v_{ph}$: Phase voltage
$i_{ph}$: Phase current
$L_u$: Inductance at unaligned position
$L_a$: Inductance at aligned position
rpm: Revolutions per minute
$\rho_A$: Absolute overlap ratio
$\varepsilon$: Stroke angle
$\theta_{rs}$: Phase shift
$\theta_0$: Rising slope angle
$m$: Phase number
$K$: Output coefficient
TRV: Torque per unit rotor volume
\( \sigma \): Air gap shear stress \\
\( \sigma \): Standard deviation \\
\( N_p \): Number of winding turns per pole \\
\( V_s \): DC supply voltage \\
\( B \): Magnetic flux-density \\
\( y_i \): Variable \\
\( \bar{y}_i \): Mutated variable \\
p(\bar{y}_i): Probability density function \\
\( M \): Mass of the motor \\
\( w_1, w_2 \): Weights in the objective function

The design parameters of the SRM are listed in Table 3-3.
Chapter 1 INTRODUCTION

1.1 Introduction to the LELR Project

In 2009, the Canadian Space Agency (CSA) launched a project to build a Lunar Exploration Light Rover (LELR) prototype. It would be an unmanned robotic rover for a human precursor mission for sample collection and scientific investigation [1]. McGill University’s contribution in this project was to design an integrated propulsion system (motor, controller, wheel, and ‘tire’). A portion of the work involves modeling lunar soil properties and the interactions between the ground and the tire. Other portions include the construction of a compliant wheel, a lunar dust mitigation system, and a propulsion motor, as well as the development of a power train model that ties everything together. The focus of this thesis is the propulsion motor analysis and design.

1.1.1 Mission Profile

In this project, the operation of the electric motor can be divided into three phases. These phases are the acceleration phase, the cruise phase and the deceleration phase. The electric motor converts electric energy to heat and kinetic energy in the acceleration phase, while the same motor
acts as an electric generator in the deceleration phase and converts kinetic energy back to electric energy. This process is called regenerative braking, and is an important issue when considering the propulsion drive of electric vehicles (EV) [2]. During the cruise phase, since the vehicle’s velocity does not change, the electric energy is only transformed into heat due to the friction. In particular, when the vehicle climbs up or goes down a hill, the control strategy can be identical to the strategy applied in the acceleration or deceleration phase.

1.1.2 Working Environment

The special working environment places a number of obstacles and constraints on the design of the propulsion system. Addressing these core issues is necessary in order to ensure a successful and robust design.

1. Tenuous Atmosphere

Due to the lack of an atmosphere on the Moon, the electric motor is preferred as a propulsion engine. However, the nature of the moon’s terrain makes lunar dust mitigation a very difficult task. Air convection (which is an effective and inexpensive means of cooling on Earth) is not possible in this environment.

2. Large Temperature Range
Because the lunar temperature ranges from nearly absolute zero to a few hundred degrees [3], regulating the motor’s temperature is a considerable challenge in this project. Since permanent magnets lose their magnetism under high temperature [4], they are not well adapted to the electric motor design. Besides, active heating should be taken into account when the ambient temperature is especially low.

3. Highly Abrasive Lunar Dust

The low mass of dust particles and the lack of moisture from the atmosphere offer a great opportunity for electric charge to bind the dust to the surface of the electric motor. Sealed enclosures may be an answer to this problem, but sealing a rotating part against the abrasive dust is a big challenge.

1.2 History of Electric Motors Used in Space Exploration

Space exploration vehicles such as the Lunar and Mars rovers play very important roles in exploration missions. According to Harrison Schmitt, one of two astronauts assigned to fly in the Apollo 17 program [5], “without it [the Lunar Roving Vehicle], the major scientific discoveries of Apollo 15, 16, and 17 would not have been possible; and our current understanding of lunar evolution would not have been possible.” The traction drive used
in the Lunar Roving Vehicles contained a harmonic drive reduction unit, a series-wound DC motor, and a brake assembly. To counteract the influence of abrasive dust and improve brush lubrication, the traction drive was hermetically sealed and the internal pressure was maintained at 7.5 PSIA [6]. Because the energy cannot be recovered through regenerative braking, and because the brushes require periodic replacement, this kind of motor is not a good choice for a long-term mission. In fact, for lunar missions, brush-type motors and brushless motors were developed simultaneously in order to assure that motors meeting the necessary criteria could be provided. The General Electric Company had already constructed brushless motors which were much lighter and more efficient than the conventional motors available at the time. However, to shorten the development time and testing, brush-type motors had to be chosen, even with the added complexity [7].

1.3 Classification of Electric Motors

As illustrated in Figure 1-1, electric motors usually can be divided into two classes: DC (direct current) motors and AC (alternating current) motors. This classification scheme is based on the ways that different motors are powered. Depending on the type of commutators, DC motors
can be further categorized as either brushed or brushless DC motors. Brushed DC motors are generally built with wound rotors and with either wound or permanent magnet stators [8]. When the wound stators are applied, the field coils and armature coils can be excited separately or connected in three formats: series-wound, shunt-wound and compound-wound. Brushless DC (BLDC) motors and switched reluctance motors (SRM) are two rapidly emerging types in the DC motor family. Strictly speaking, these motors cannot be classified as DC motors because they are, essentially, powered by rectangular alternating current. But thanks to the development of power electronics technology, BLDC motors and SRMs can be supplied by a DC source, and the motor combined with its controller can be regarded as a DC-powered system.

In contrast to DC motors, AC motors are powered by a sinusoidal AC source. The coils wound around the stator normally produce a rotational magnetic field within a narrow air gap. The net torque is given by the interaction between the stator and rotor magnetomotive forces (MMF). The induction motor and the synchronous motor are the two main types of AC motors, although these types do have different rotor structures. The armature current in induction motors is induced by the rotational magnetic field, while this current is independent of the stator magnetic field in
synchronous motors. Essentially, BLDC and AC synchronous motors have the same configuration. The only difference between them is the shape of the back-electromotive force (back-EMF). Indeed, these motors can work both in DC and AC modes regardless of the performance, in terms of efficiency and torque ripple [9]. They are also referred to as universal motors.

Figure 1-1: Family tree of electric motors.

Compared to industrial motors, EV motors often require frequent speed change, high-torque low-speed hill climbing, low-torque high-speed cruising, and a wide speed range [10]. In practice, all kinds of electric motors introduced above have been utilized for EV propulsion, even though the control strategies may differ [10]. Unlike BLDC and induction motors, SRMs remain less developed due to the difficulties encountered in
motor design and control. However, the features of the SRM make it a very good candidate for space exploration.

1.4 History of Switched Reluctance Motors

The utilization of SRMs can be traced back to 1838, when Robert Davidson built one as a traction drive for a locomotive on the Glasgow-Edinburgh railway, even though the top speed was especially low [11]. This took place only a few years after the invention of the commutator-type DC motor [12]. But in the decades ahead, more attention was given to brushed DC motors, which have been used commercially since the 1880s [13]. The interest in SRMs was renewed by the appearance of solid-state semiconductor devices such as transistors and thyristors in the 1960’s. Some important milestones during this time include the construction of an axial air-gap reluctance motor by Unnewehr and Koch of the Ford Motor Company [14], and the creation of a 4-phase, double-stack motor by Bausch and Rieke [15]. The main purpose of the latter research was to find an alternative traction motor for vehicle propulsion. Yet due to the complexity of the design, the subsequent development of these ideas never took place.

In the late 1970’s, the Universities of Leeds and Nottingham carried
out a research project aimed at the implementation of the SRM in a battery EV [16]. Even though the target of this endeavor was not reached, this research project laid the foundation for the practical design of SRMs of competitive size and performance [17]. A company named Switched Reluctance Drives Ltd. (SRDL), which is now part of Emerson Electric, was formed in 1980 by the researchers who had participated in the Leeds and Nottingham project. This company played a leading role in calling attention to the commercial potential of SRM technology. Many companies started to work on SRM and motor drive development. The best known manufacturer licensed by SRDL is Tasc Drives, which produces variable-speed drives for SRMs covering the range from 4kW to 80kW [11]. Since then, SRMs have been designed for a range of applications, from hard disk drives (4-phase miniature SRM with outer rotor structure) [18] to the technology used by the coal mining industry (1100V, 35kW, flameproof SRM) [19]. Although these applications have been successful, the once-expected widespread commercial use of SRMs has not occurred due to several factors: in particular, the development of permanent-magnet materials and detailed improvements in induction motors and frequency converters. Nonetheless, SRMs remain as “emerging and promising” as they used to seem in 1970s and 1980s [20].
1.5 Features of Switched Reluctance Motors

All the features of the SRM result from its simple structure and sophisticated power electronics controller. A SRM with eight stator poles and six rotor poles, and an asymmetric bridge converter are illustrated in Figure 1-2. Conventionally, this kind of motor is called an 8-6 SRM, and the SRMs with other pairs of stator and rotor poles are named in the same way. The 4 phase coils in the asymmetric bridge converter are corresponding to the phase coils of the SRM. For simplicity, the rest of the thesis just uses the word “phase” to represent the coil in each phase. Although many other controller topologies have been designed with the development of the SRM, the converter displayed in Figure 1-2 remains popular because of its control flexibility and high efficiency [11]. Since the thesis focuses on the SRM design, the function of the controller is only mentioned when it is necessary.

Figure 1-2: An 8-6 SRM and an asymmetric bridge converter.
1.5.1 Simple and Robust Rotor Structure

Unlike its companions such as the BLDC motor or induction motor, the SRM does not have any windings, commutators, or brushes on the rotor. The bulk of the power losses occur in the stator, which is relatively easy to cool [21]. Therefore, the SRM is best suited to a high-temperature environment and high-speed operation. Since the rotor does not use a permanent magnet, the issue of a loss of magnetism due to the high temperature of the motor can be neglected.

1.5.2 Inherent Degree of Fault Tolerance

Since SRMs are operated based on the sequential excitation of diametrically opposite stator coils, a motor having three or more phases could work even under the condition that one phase is out of action [11].

1.5.3 Extended Constant Power Operation

A 6-4 or 8-6 SRM is typically capable of extended constant power operation, often up to 3-7 times the base speed. This is the speed at which the back-EMF equals the bus voltage and the motor reaches its rated power [22]. The operation at constant power is achieved through advancing the excitation current until an overlap between successive
phases occurs [11]. The high-speed performance of an SRM mainly depends on the shape of the rotor and requires a compromise between the constant torque and the constant power capacity. For example, the constant power range of 6-4 and 8-6 SRMs could be extended to more than 6 times the base speed when the rotor pole arc is narrower than the stator pole arc. But the torque density of these machines is relatively low [22]. The number of rotor and stator poles also has a heavy influence on the constant power capacity. Machines with large pole number usually have poor capacity. One example of this is the 24-16 SRM, which only has a constant power range of 2 times the base speed.

1.5.4 Torque Ripple and Acoustic Noise

When an SRM works at low rotational speeds, the acoustic noise is predominantly due to resonances induced by the high torque ripple. In order to reduce the torque ripple, complex profiling of the phase current waveforms and accurate measurement of the rotor position are required [9]. At high rotational speeds, the acoustic noise is dominated by the radial magnetic force-induced resonance with the circumferential mode-shapes of the stator [21]. The acoustic noise significantly increases with the rotational speed and load. However, it can be reduced effectively by
employing a relatively thick stator yoke, and by lowering the supply voltage [9]. Because of the particularity of the project, the acoustic noise reduction is not taken into consideration.

1.5.5 High Saturation

Practical SRMs are operated with a high magnetic flux density in order to obtain the maximum benefit from the magnetic material which is in saturation [23]. However, the saturation limits the amount of torque available from the SRM and introduces difficulties in machine analysis and design. The nonlinear method or finite element method (FEM) has to be used to achieve an accurate analysis of the SRM [24].

1.5.6 Inverter Capacity and Stator Ohmic Loss

Due to the lack of permanent magnets, the excitation burden falls on the stator winding and the converter. Therefore, compared with a BLDC motor, the inverter volt-ampere rating for a given SRM output power is relatively high. Besides, due to the high current going through the stator coils, the stator ohmic loss is significant.

1.6 Thesis Objective and Organization

Currently, SRMs are utilized in a range of applications such as
laboratory centrifuges, washing machines and aircraft engine fuel pumps, while switched reluctance generators (SRG) are often combined with wind turbines to convert kinetic energy to electric energy. Although certain aspects of the SRMs have been discussed in the literature, no universal design methodologies are constructed until now. The objective of the thesis is to develop a process for determining the design parameters and performance of an SRM, and to explore the possibility of using it in lunar applications.

In Chapter 2, the electromechanical energy conversion principles will be described. The natural and demanded torque-speed characteristics of SRMs will also be compared to find an appropriate control strategy. In Chapter 3, the engineering design process will be illustrated and the preliminary design of the SRM based on the user requirements will be investigated directly. The simulation results for both the motoring and the generating modes of the SRM will be presented and discussed in Chapter 4. Then an evolutionary strategy will be introduced and applied to SRM design optimization in Chapter 5. Finally, the possible future work will be addressed.
2.1 Magnetization Curve

The magnetization curve depicts the relation between the flux linkage and the excitation current. It can be used to calculate the instantaneous torque and the total energy converted in each period, which will be discussed later in this chapter.

![Magnetization Curves](image)

Figure 2-1: Nonlinear energy conversion.

Figure 2-1 illustrates a typical nonlinear energy conversion process of an SRM. Line OU and curve OSA represent the magnetization curves in the unaligned and aligned position, respectively. The straight line AU shows that the excitation current is constant during the energy conversion.
It is readily proven that the mechanical energy output is equal to the area enclosed by OSAU.

Since the current does not vary during energy conversion, the electric energy $W_e$ sent to the winding is given by

$$ W_e = \int iv \, dt = \int i \frac{d\lambda}{dt} \, dt = \int i \, d\lambda = \text{CBAU} \tag{2.1} $$

where $i$ and $v$ are the transient current and voltage, $\lambda$ is the flux linkage.

The areas enclosed by OCU and OBAS represent the stored magnetic energy in the aligned and unaligned positions, so that the change in the stored energy $W_f$ is

$$ \Delta W_f = \text{OBAS} - \text{OCU} \tag{2.2} $$

According to the law of conservation of energy, the mechanical energy $W_m$ is

$$ W_m = W_e - \Delta W_f = \text{CBAU} - (\text{OBAS} - \text{OCU}) $$
$$ = \text{CBAU} + \text{OCU} - \text{OBAS} = \text{OBAU} - \text{OBAS} \tag{2.3} $$
$$ = \text{OSAU} $$

### 2.1.1 Aligned Position

When any pair of rotor poles is exactly aligned with a stator phase, the phase is said to be in the aligned position. For instance, Figure 2-2 illustrates phase A in the aligned position. The inductance of the motor is
at its maximum because the air gap between the rotor and the stator is minimized in this position. Due to the small air gap, a large portion of MMF is absorbed by the stator yoke so that the optimization of the stator shape can increase the magnetic flux and, further, the average torque.

2.1.2 Unaligned Position

When the interpolar of the rotor is exactly aligned with phase A, as shown in Figure 2-3, the phase is said to be in the unaligned position. When either phase B or phase C is excited, the rotor will leave this position and not return. Therefore, the situation is one of unstable equilibrium [11]. In normal circumstances, the unaligned magnetization curve is a straight line, since the air gap reluctance is very large.

Figure 2-2: 6/4 SRM - aligned position on phase A.
2.2 Energy Conversion Principles

2.2.1 Singly-Excited Magnetic System

To analyze an SRM quantitatively, the motor should be regarded as a lossless magnetic energy storage system. The electrical losses, such as the ohmic losses in the windings, and mechanical losses, such as friction losses, can all be separated from this system [23]. The electric and mechanical terminals are coupled through the magnetic field, and the time rate of change of the magnetic stored energy is equal to the difference between the input electric power and the output mechanical power [23]

$$\frac{dW_f}{dt} = P_{\text{elec}} - P_{\text{mech}}$$  \hspace{1cm} (2.4)

The electric power is given by the product of the terminal voltage and
the current, and the terminal voltage is equal to the time rate of change of
the flux linkage according to Faraday's law of induction

\[ P_{\text{elec}} = i v = i \frac{d\lambda}{dt} \quad (2.5) \]

The output mechanical power of a motor is given by the product of the
mechanical torque and the rotational speed

\[ P_{\text{mech}} = T \frac{d\theta}{dt} \quad (2.6) \]

By substituting Eq. (2.5) and Eq. (2.6) into Eq. (2.4) and multiplying
Eq. (2.4) by \( dt \), we get [23]

\[ dW_f = id\lambda - Td\theta \quad (2.7) \]

From Eq. (2.7), the storage energy \( W_f \) in the magnetic field can be
seen as a function of two independent variables \( \lambda \) and \( \theta \), so that the total
differential of \( W_f \) is [23]

\[ dW_f = \frac{\partial W_f}{\partial \lambda} \bigg|_\theta d\lambda + \frac{\partial W_f}{\partial \theta} \bigg|_\lambda d\theta \quad (2.8) \]

Eq. (2.7) and Eq. (2.8) must be equal for all values of \( d\lambda \) and \( d\theta \), so
that [23]

\[ T = - \frac{\partial W_f}{\partial \theta} \bigg|_\lambda d\theta \quad (2.9) \]

A non-physical quantity coenergy is defined as a function of \( i \) and \( \theta \):

\[ W_{\text{co}}(i, \theta) = i\lambda - W_f(\lambda, \theta) \quad (2.10) \]
and the differential of the coenergy is

$$dW_{co}(i, \theta) = d(i\lambda) - dW_f(\lambda, \theta) \quad (2.11)$$

The differential of $i\lambda$ can be expressed as:

$$d(i\lambda) = id\lambda + \lambda di \quad (2.12)$$

Substituting Eq. (2.7) and Eq. (2.12) into Eq. (2.11) results in

$$dW_{co}(i, \theta) = \lambda di + Td\theta \quad (2.13)$$

The coenergy $W_{co}$ can be seen as the function of variables $i$ and $\theta$. Therefore, its differential is

$$dW_{co} = \frac{\partial W_{co}}{\partial i} \bigg|_\theta di + \frac{\partial W_{co}}{\partial \theta} \bigg|_i d\theta \quad (2.14)$$

Comparing Eq. (2.13) and Eq. (2.14), the instantaneous torque can be formulated as

$$T = \frac{\partial W_{co}}{\partial \theta} \bigg|_i d\theta \quad (2.15)$$

Here the torque is equal to the partial derivative of $W_{co}$ while the current is held constant. But this does not mean the excitation current should be constant during the energy conversion.
The magnetization curves in Figure 2-4 and Figure 2-5 depict how the instantaneous torque is calculated by Eq. (2.9) and Eq. (2.15). The zones
\( \Delta W_f \) in the figures differ by a triangle \( \text{abc} \) whose area is proportional to the product of two infinitesimals. Therefore, the shaded zones resulting from \( \Delta \theta \) at constant \( \lambda \) or at constant \( i \) should be equal, and Eq. (2.9) and Eq. (2.15) should give approximately identical results. Physically, the torque depends on the rotor position and the magnetic field. The field, whose energy and coenergy are \( W_f \) and \( W_{co} \) in the equations, can be specified in terms of the flux linkage \( \lambda \) or the current \( i \). Therefore, the selection of Eq. (2.9) or Eq. (2.15) is determined by the variables used to depict the magnetic field. The algebraic signs in the equations show how the positive force works to decrease the stored energy \( W_f \) and increase the coenergy \( W_{co} \).

### 2.2.2 Multiply-Excited Magnetic System

In practice, the SRM is often considered a singly-excited magnetic system since only one phase is excited a time. However, because demagnetization in each phase cannot be achieved instantaneously, the SRM would become a doubly-excited magnetic system during this period. This is also the situation when a two-phase excitation strategy is applied [25]. Therefore, it is useful to discuss the multiply-excited magnetic system which is one consequence of the analysis in the previous section. Since a three-phase excitation would never happen in the SRM, the discussion
here focuses on two-phase excitation.

![Diagram of Lossless Switched Reluctance Machine](image)

**Figure 2-6**: Energy conversion of a doubly-excited SRM.

As shown in Figure 2-6, the system has two electric terminals and one mechanical terminal. Consequently, the system must be described in terms of three variables, which are the rotor position $\theta$ along with two flux linkages $\lambda_1, \lambda_2$, or two currents $i_1, i_2$. When the flux linkages are used, the differential of the stored energy corresponding to Eq. (2.7) is

$$dW_f = i_1 d\lambda_1 + i_2 d\lambda_2 - T d\theta$$  \hspace{1cm} (2.16)

and the torque is

$$T = - \frac{\partial W_f}{\partial \theta} \bigg|_{\lambda_1, \lambda_2} d\theta$$  \hspace{1cm} (2.17)

Using an arrangement that recalls Eq. (2.15), the torque can also be expressed in terms of the coenergy and the rotor position

$$T = \frac{\partial W_{co}}{\partial \theta} \bigg|_{i_1, i_2} d\theta$$  \hspace{1cm} (2.18)
2.2.3 Energy-Conversion Loop

Instantaneous torque has been discussed in the last two sections. But from a user's point of view, the average torque may be more important if the torque ripple is set aside. One natural step in calculating the average torque is to integrate Eq. (2.9) or Eq. (2.15). A more convenient way is to derive it from the magnetization curve described in the first section. This model is presented under the condition that both the rotational speed and the supply voltage are time-invariant.

Figure 2-7: Energy-conversion loop during the period of transistor conduction.

Figure 2-7 depicts an energy conversion process during a typical motoring stroke. Line OA and OU represent the magnetization curves in the aligned and unaligned positions, respectively. In the unaligned position
the flux linkage increases linearly with the current as the inductance remains small and constant. But once the poles overlap at the point B, the inductance increases and the back EMF builds up. Therefore, the rate of rise of the current slows down and finally stops at a steady value. The phase is commutated at the point C to avoid generating any negative torque. According to Eq. (2.3), the mechanical work done between O and C is $W_{m1}$, which is also the output energy from the motor during the period of transistor conduction illustrated by the image on the left in Figure 2-8.

During the period of diode freewheeling represented by the right-hand image in Figure 2-8, the voltage across the coil is reversed and the energy is returned to the supply. Figure 2-9 shows the energy conversion process
during this period. Because the rotor does not reach the aligned position at point C, the inductance continues to rise although the current begins to decrease. Then, the rotor arrives at the aligned position and follows the magnetization curve OA until the point D. After that, the motor does the negative work and the mechanical work done in this period is $W_{m2} - W_{m3}$.

![Magnetization Curves](image)

Figure 2-9: Energy-conversion loop during the period of diode freewheeling.

2.3 Torque-Speed Characteristics of Switched Reluctance Motors

2.3.1 Natural Torque/Power-Speed Characteristics

It is unwise to discuss the torque-speed characteristics of an electric motor without considering the control strategy. But before undertaking a detailed discussion of the controlled motor, reviewing the “natural” or “inherent” characteristics of an SRM will be very helpful.
According to Eq. (2.15), torque can be expressed as the change in the energy stored in the motor. Assuming that the motor is a linear magnetic system, the flux linkage and the current are linearly related by the inductance, which does not depend on the current. The stored energy is found to be

\[ W_t(i, \theta) = \frac{1}{2} L(\theta) i^2 \tag{2.19} \]

and the magnitude of the torque is

\[ T = \frac{i^2}{2} \frac{dL(\theta)}{d\theta} \tag{2.20} \]

where \( \frac{dL(\theta)}{d\theta} \) is a constant determined only by the geometry of the motor.

We can write Eq. (2.20) as

\[ T = k_1 i^2 \tag{2.21} \]

From Eq. (2.21), the output power is found to be

\[ P_{\text{mech}} = T\omega = k_1 i^2 \omega \tag{2.22} \]

If neither the change in the energy stored in the magnetic field nor the energy losses are taken into account, the input power should be equal to the output power.

\[ P_{\text{ele}} = vi = k_1 i^2 \omega \tag{2.23} \]

Assuming that a voltage source is applied and the terminal voltage is constant,
\[ i = \frac{v}{k_1 \omega} = \frac{k_2}{\omega}, \quad k_2 = \frac{v}{k_1} \quad (2.24) \]

According to Eq. (2.21), the torque is proportional to the square of the current, which also makes the torque inversely proportional to the square of the rotational speed from Eq. (2.24).

\[ T = \frac{k_1 k_2^2}{\omega^2} \quad (2.25) \]

The output power can be calculated as

\[ P_{\text{mech}} = \frac{k_1 k_2^2}{\omega} \quad (2.26) \]

Figure 2-10 illustrates the natural characteristics of SRMs based on Eq. (2.25) and Eq. (2.26). In the figure, the speed is normalized by the base speed while the torque and power are normalized by their corresponding values at the base speed.

Figure 2-10: The natural torque/power-speed characteristics of SRMs.
2.3.2 Idealized Torque/Power-Speed Characteristics

Figure 2-11 shows that the torque/power requirements for a traction machine used in an EV are obviously different from the natural characteristics of SRMs. The torque and power at low speeds are very high in Figure 2-10. This never happens in practice because the excitation current is restricted by the current rating of the power electronics converter [9]. At a speed higher than the base speed, the idealized power should be constant and the torque should decrease with the rotational speed. When the electric motor operates in the reduced power region, a constant power is impossible to maintain and the torque/power characteristics are exactly the same as that shown in Figure 2-10.

![Figure 2-11: Idealized torque/power-speed characteristics [9].](image-url)
2.3.2.1 Constant torque region

In the constant torque region, the rotational speed is lower than the base speed so that the back EMF is always smaller than the supply voltage. The maximum torque capability in this region is determined by the current rating of the motor drive [9], and a control strategy is often employed to limit the excitation current. The chopping illustrated in Figure 2-12 is a widely used method for regulating the current level. There are two strategies for carrying out the chopping. The first is called soft chopping, which leaves the transistor $T_1$ on during the conduction period and switches $T_2$ on and off. When $T_2$ is turned off, the voltage $v_{ph}$ across the coil will simultaneously jump to zero, and the phase current $i_{ph}$ will start to decrease gradually. The second strategy is called hard chopping which switches these two transistors on and off at same time. It is more effective in current regulation because the voltage across the coil $v_{ph}$ becomes negative when the transistors are turned off. But this process also involves a higher torque ripple and greater power losses.
The EV usually accelerates in the constant torque region since the vehicle rated speed is higher than the base speed. The motor is required to offer a maximum torque that is four to five times the rated torque for temporary acceleration and hill-climbing [10]. The constant torque in this region can be determined based on this constraint.
2.3.2.2 Constant power region

Most well-designed electric motors have a wide constant power operating range that is around 3-4 times the base speed [9]. If designed and controlled appropriately, the SRM is capable of extended constant power operation, typically 3-7 times the base speed [22]. In this region, the back-EMF is higher than the DC bus voltage and the current starts to decrease whenever the rotor and stator poles overlap. Therefore, the inverter current may never reach the rated value and using a chopping control to lower the current value is no longer a possibility. The control strategy applied in this region is a single-pulse mode, which is sketched out in Figure 2-13. Through an adjustment of the conduction angle, the controller can deliver more effective current to the phase and maintain the torque inversely proportional to the speed [21]. Since the power is equal to the torque multiplied by the rotational speed, the power remains constant in this region.
2.3.2.3 Reduced power region

The power and torque are further reduced in this region due to the increment of the back-EMF. As shown in Figure 2-14, the advancement of the turn-on position of phase A is limited to the position at which the phase inductance has a negative slope. If the speed continues to increase, the single-pulse mode control could not maintain the constant power. This mode is also referred as the natural mode of operation because the torque in this region is governed by the natural characteristics of the motor [21]. The protracted operation of electric motors in this region should be avoided.
Figure 2-14: Advanced turn-on position of phase A.
3.1 Design Process

The first step in the engineering design process illustrated in Figure 3-1 is to define a problem to be solved — or, in other words, defining the general objective. The goal of the present project is to design an electric motor satisfying certain specifications for a lunar roving vehicle. After a brief and abstract response to the problem is generated conceptually, designers need to discuss and establish the design requirements. Although the requirements should remain constant during the design process, they are continuously reviewed and revalidated in order to ensure they reflect the objectives of the project [27]. After that, the designers need to combine the elements of an objective into a whole through a process called “synthesis”. Finally, a prototype device is built, and a design space is defined.

The three procedures described above are referred to as preliminary design stages because only the overall system configuration is defined and only a schematic drawing is developed. Nonetheless, the design space established during this phase will be deeply explored and exploited later. In the detailed design phase, the processes of modification and
analysis are repeated until the performance of the device meets the requirements. Instead of using "Brute Force" (that is, trying every possibility in the design space), we can use engineering knowledge to determine the direction of our inquiry.

Figure 3-1: The engineering design process [28].

Nowadays, computer-aided design (CAD) is one of the most significant and widely-used technologies employed in product design; it shortens the product design cycle and improves the quality of the products. Among the many advantages of using CAD, possibly the greatest one is the analytical capability made available by various software packages [27]. As depicted in Figure 3-2, CAD technology can be effectively applied in SRM design. Nevertheless, more traditional design procedures are still followed. For example, based on the requirements specified by users, a rough configuration for the motor can be constructed using MotorSolve (an
electric machine design software developed by Infolytica Corporation [29]). Then, a Finite Element Method (FEM) based simulation in MagNet (a 2D/3D electromagnetic field simulation software, also developed by Infolytica Corporation [30]) is carried out to test whether the performance of the prototype motor matches the user requirements. It is worth noting that the geometry of the SRM can also be modified in MagNet, which means that MotorSolve will not be included in the optimization phase. The optimization of an SRM corresponds to the detailed design described above, in which MagNet and OptiNet (an automated design optimization software developed by Infolytica Corporation [31]) are running in parallel. The former program analyzes and modifies the electric motor while the latter provides the search direction within the design space.

This chapter will focus on how to design a prototype motor roughly matching particular user requirements. The optimization of the SRM is going to be discussed primarily in Chapter 5.
3.2 Design Requirements

As illustrated in Figure 3-1 [28], the design process starts from the requirements that must be established prior to initiating the design. These requirements drive and control the design throughout the process, but
they are especially important during the preliminary phase because the
design team uses them as the principal guidelines at this point [27].
Furthermore, the requirements should be determined by the project
participants together in order to avoid misunderstanding. Although the
electric motor is designed by the author alone in our project, cooperation
with the researchers responsible for designing the wheels and the power
train system was necessary.

To increase the robustness of the propulsion system, each wheel is
equipped with a driving motor. Some important parameters leading to the
motor design requirements are listed below:

- The number of motors: 4
- Weight of the rover with payloads: 800 kg
- Maximum cruise velocity: 15 km/h
- Minimum cruise velocity: 5 km/h
- Diameter of the wheel: 0.69 m
- Width of the wheel: 0.48 m
- Battery voltage: 46.8 V
- The number of battery packs: 2

Since the lunar terrain and the friction coefficient remain unknown, it
is difficult to decide the rated power of the electric motor. However, the
propulsion motors used in Apollo program are good samples to consider because they propelled a vehicle having similar parameters to the one designed in this project. (In both cases, the number of motors was 4.) The motor was capable of outputting 190 W at 10,000 rpm and was attached to the wheel through an 80:1 harmonic drive [6]. Considering that the lunar roving vehicle in the Apollo program has a slightly lower weight and higher maximum velocity, a 200 W output torque would be a rational requirement for the motor designed in this project. In practice, the vehicle would be tested mostly at speeds of 5 km/h, 10 km/h, and 15 km/h. Hence, the rated velocity of the vehicle is 10 km/h. It is worth noting that the rated velocity of a vehicle is different from that of an electric motor. The former always falls into the constant power region beyond the base speed while the latter is exactly the base speed [32]. Therefore, the rated velocity of the electric motor is set to 5 km/h. The radius of the wheel is 0.35 m (as determined by the wheel design team), so that the rated rotational speed is about 38 RPM and the torque to be produced by the machine is 50.4 N.m.

The specifications of the SRM are concluded as follows.

- Rated power output $P_{\text{rate}} = 200$ W
- Rated rotational speed $\omega_{\text{rate}} = 38$ RPM
- Rated torque $T_{\text{rate}} = 50.4 \text{ N.m}$
- Input voltage $V = 46.8 \text{ V}$ (the voltage is not fixed, a DC-DC converter can be applied if necessary)
- Peak current $i_p = 12 \text{ A}$

### 3.3 Selecting the Numbers of Poles

Unlike the components of many other DC machines, both the stator and rotor of the SRM have salient poles. The rotor and stator poles in a regular SRM are symmetric and equally distributed around the rotor and stator respectively. Irregular SRMs are also designed for some special purposes. In this project, a constant mechanical torque is required in all positions so that our discussion remains restricted to the regular SRM.

![Figure 3-3: 2-phase 4/2 regular SRM.](image)
To reduce the switching frequency and the minimum inductance, the rotor pole number is usually set at less than the stator pole number [2]. A regular 2-phase 4/2 SRM illustrated in Figure 3-3 has two “singular points” [11] at which the torque is zero. This is because the aligned position on phase A is exactly the unaligned position on phase B. In practice, the torque is quite small for several degrees on either side of these two positions. If the initial position of the rotor coincidentally falls in one of the “dead zones,” it will be impossible to start the motor without help from an external force. A conclusion can be generalized to state that the ratio of the stator pole number to the rotor pole number should not be an integer; otherwise, there would be zero-torque positions.

Avoiding the zero-torque position is a basic requirement in motor design, which can be measured by the absolute overlap ratio $\rho_A$ [11]. It is the ratio of the absolute torque zone to the stroke angle. The absolute torque zone is the angle through which one phase generates non-zero torque and the maximum in a regular motor is $\pi/N_r$, where $N_r$ is the number of rotor poles. This is because the maximum rotor pole angle is $2\pi/N_r$, and only half of this can be applied to produce the positive torque. The stroke angle $\varepsilon$ is given by

$$\varepsilon = \frac{2\pi}{mN_r} \quad (3.1)$$
where \( m \) is the phase number. A value of at least 1 is necessary for the absolute overlap ratio, which is equal to \( \frac{m}{2} \), to eliminate the zero-torque position. Thereby, the phase number of most SRMs is larger than 2.

The most widely used SRM structures are four-phase 8/6 and three-phase 6/4 configurations [2]. In addition, the three-phase 12/8 configuration is also involved in many industrial applications. (It is technically a 6/4 configuration with double stator and rotor poles.) A motor with higher phase numbers tends to produce a smaller torque ripple, but the price paid for this advantage is a lower inductance ratio and less output torque. For that reason, motors with five or more phases are not discussed in this thesis.

Compared to the 8/6 configuration, the 6/4 configuration has more room for turn-on angle advancing in high-speed operations and weaker mutual coupling between adjacent phases. Therefore, the torque ripple is generally high and the starting torque can be a problem. Although the 8/6 configuration can alleviate these disadvantages, it puts a greater strain on the power electronics controller. In the design set forth in this thesis, a 6/4 configuration is used.
3.4 Sizing the Motor

The specifications consist not only of requirements but also of constraints. The constraints on motor volume are specified as follows: the diameter should not be greater than 20 cm, and the stack length should be limited to 30 cm. These constraints should not be violated when the overall dimensions of the motor are determined.

The information we have includes the rated torque and rated power, which are possible starting points for the electric motor design. A simple form of the output torque equation is [11]

\[ T = KD_r^2L_{stk} \]  

(3.2)

where \( K \) is the output coefficient, and \( D_r \) and \( L_{stk} \) are rotor diameter and the stack length respectively. \( K \) is a new coefficient which is proportional to the product of the electric and magnetic loadings. Two other coefficients related to \( K \) are the torque per unit rotor volume, \( TRV \), and the air gap shear stress, \( \sigma \). The meaning of the \( TRV \) is suggested by its name, while \( \sigma \) represents the pressure on rotor’s surface. The coefficients are expressed by the following equations [11]:

\[ TRV = \frac{T}{\frac{\pi}{4} D_r^2 L} \]  

(3.3)
\[ \sigma = \frac{T}{\pi \frac{D_t^2}{L}} \]  
(3.4)

\[ K = \frac{\pi}{2} \sigma = \frac{\pi}{4} \text{TRV} \]  
(3.5)

The three parameters are tightly related so that any one of them can be selected to determine the size of the electric motor. Some typical values of \( \sigma, K, \) and TRV are given in Table 3-1.

<table>
<thead>
<tr>
<th>Type of Machine</th>
<th>( \Sigma )</th>
<th>( K )</th>
<th>TRV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small totally-enclosed motors</td>
<td>0.1 ~ 0.5</td>
<td>0.15 ~ 0.8</td>
<td>2.5 ~ 7</td>
</tr>
<tr>
<td></td>
<td>(0.7 ~ 3.5)</td>
<td>(1.0 ~ 5.5)</td>
<td></td>
</tr>
<tr>
<td>Integral-hp industrial motors</td>
<td>0.5 ~ 2</td>
<td>0.8 ~ 3</td>
<td>7 ~ 30</td>
</tr>
<tr>
<td></td>
<td>(3.5 ~ 13.8)</td>
<td>(5.5 ~ 20.7)</td>
<td></td>
</tr>
<tr>
<td>High-performance servomotors</td>
<td>1 ~ 3</td>
<td>1.5 ~ 5</td>
<td>15 ~ 50</td>
</tr>
<tr>
<td></td>
<td>(6.9 ~ 20.7)</td>
<td>(10.3 ~ 34.5)</td>
<td></td>
</tr>
<tr>
<td>Aerospace machines</td>
<td>2 ~ 5</td>
<td>3 ~ 7.5</td>
<td>30 ~ 75</td>
</tr>
<tr>
<td></td>
<td>(13.8 ~ 34.5)</td>
<td>(20.7 ~ 51.7)</td>
<td></td>
</tr>
<tr>
<td>Large liquid-cooled machines</td>
<td>10 ~ 15</td>
<td>15 ~ 200</td>
<td>100 ~ 250</td>
</tr>
<tr>
<td></td>
<td>(68.9 ~ 103.4)</td>
<td>(103.4 ~ 1378.8)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-1: Typical values of three important coefficients [11].

After a rough calculation, we find that the machine capable of
producing the required torque should have a rotor whose volume is near 1 m$^3$. This violates the constraints specified earlier. As a result, a 50:1 gear box is employed to reduce the rated torque to 1 N.m and increase the rated rotational speed to 1900 RPM. It is worth noting that the characteristics of the SRM are quite suitable for high speed operation [9].

As listed in Table 3-1, different types of motors usually have different TRVs, which depend entirely on the cooling method applied. The goal of this project is to design a high-performance traction motor. Therefore, TRV = 20 kNm/m$^3$ is chosen and the required rotor volume $D_r^2 L_{stk}$ is obtained from Eq. (3.2) and Eq. (3.5) as $\frac{4T}{\pi} \cdot \frac{1}{\text{TRV}} = 6.36 \times 10^{-5}$ m$^3$. In order to determine the stack length and the rotor diameter respectively, it is necessary to find their ratio. According to [11], a typical value for the ratio $L_{stk}/D_r$ is 1, and thus the rotor diameter and the stack length are determined to be $D_r = 40.0$ mm, $L_{stk} = 40.0$ mm.

The simplest way to figure out the stator diameter $D$ is from the ratio $D_r/D$, which can vary over a wide range between 0.4 and 0.7, and is around 0.5 to 0.55 for most designs [11]. The selection of an appropriate ratio depends on the number of stator and rotor poles, and on the operation requirements. Some widely used values are suggested in Table 3-2, where $\beta_r$ and $\beta_s$ are rotor and stator pole arcs, respectively. From the
table we notice that the ratio tends to increase with rotor and stator pole number. In other words, the SRM with a larger pole number usually has a smaller volume.

<table>
<thead>
<tr>
<th>Phase</th>
<th>$N_s$</th>
<th>$N_r$</th>
<th>$D_{r}/D$</th>
<th>$\beta_r$</th>
<th>$\beta_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>4</td>
<td>0.5</td>
<td>32°</td>
<td>30°</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>8</td>
<td>0.57</td>
<td>16°</td>
<td>15°</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>6</td>
<td>0.53</td>
<td>23°</td>
<td>21°</td>
</tr>
</tbody>
</table>

Table 3-2: Suggested parameters for the most common pole numbers [11].

Since an SRM with a 6/4 configuration is going to be designed, the ratio $D_{r}/D = 0.5$ is chosen, so $D = D_{r}/0.5 = 80.0$ mm.

### 3.5 Sizing the Internal Dimensions

After determining the configuration and the frame dimensions of the SRM, we need to pay attention to its internal dimensions. The principal design parameters are illustrated in Figure 3-4, and the rules for selecting the appropriate values are presented in this section.
3.5.1 Air Gap Length

The SRM requires a uniform air gap and good concentricity [11] to minimize torque ripple and acoustic noise. Since the magnetic reluctance of the yoke is small compared to that of the air gap, the air gap length determines the output torque and the volt-ampere requirement in the motor drive. However, the air gap cannot be made as small as possible due to manufacturing constraints, and it is roughly proportional to the motor size. For instance, given that the stack length/rotor diameter ratio
\( L_{\text{stk}}/D_r \) is 1, the air gap length is 0.5\% of the rotor diameter \([11]\). For a motor of the frame size mentioned in Section 3.4, it is reasonable to assume that the air gap length is 0.3 mm.

### 3.5.2 Stator and Rotor Pole Arcs

Although a set of stator and rotor pole arcs, \( \beta_s \) and \( \beta_r \), was proposed in Table 3-2, the ranges of the pole arcs, which will be used to constrain the design space during the optimization phase, still need to be discussed. There are two primary criteria guiding pole arc selection. The first involves ensuring that the motor meets the self-starting requirement, and the second is a matter of achieving a torque versus position waveform with minimum ripple \([33]\). In fact, these requirements place upper and lower limits on the stator and rotor pole arcs, which will be derived analytically in the following paragraphs.

The phase shift \( \theta_{rs} \) between two successive stator phases is the stroke angle \( \varepsilon \) mentioned in Eq. (3.1)

\[
\theta_{rs} = \varepsilon = \frac{2\pi}{N_r} \cdot \frac{2}{N_s}
\]  

(3.6)

where \( N_r \) and \( N_s \) are the numbers of rotor and stator poles. \( \theta_{rs} \) is equal to 30 degrees in a 6/4 SRM.
A necessary condition for the continuous starting torque is that the rising slope angle $\theta_0$ in each phase should be equal to or larger than the stroke angle. The rising slope angle $\theta_0$ in Figure 3-5 can be expressed as

$$\theta_0 = \theta_2 - \theta_1$$  \hspace{1cm} (3.7)

Therefore

$$\theta_2 - \theta_1 \geq \theta_{rs}$$  \hspace{1cm} (3.8)

Based on the geometry of the SRM, we can conclude that

$$\theta_2 - \theta_1 = \min [\beta_r, \beta_s]$$  \hspace{1cm} (3.9)

where $\beta_r$ and $\beta_s$ are the rotor and stator arcs. Considering the design, manufacturing, and performance constraints, practical designs always

Figure 3-5: Inductance vs. rotor position.
have $\beta_r > \beta_s$ [34]. Thereby,

$$ \beta_s \geq \theta_{rs}, \text{and} \theta_{rs} = 30^\circ \quad (3.10) $$

To ensure no overlap between the stator and rotor poles in the unaligned position, the angular separation between the adjacent rotor poles must be wider than the stator pole arc; otherwise, the starting torque will be very small. Based on this principle, we get

$$ \frac{2\pi}{N_r} - \beta_r \geq \beta_s \quad (3.11) $$

The rotor and stator pole arcs determine the aligned/unaligned inductance ratio and further the torque generation. The optimal combination will be found during the optimization phase.

Once the pole arcs have been fixed, the stator and rotor widths $w_s$ and $w_r$ can be calculated using the following equations

$$ w_s = 2 \left( \frac{D_r}{2} + g \right) \sin \frac{\beta_s}{2} \quad (3.12) $$

$$ w_r = D_r \sin \frac{\beta_r}{2} \quad (3.13) $$

Assuming that $\beta_s = 30^\circ$ and $\beta_r = 32^\circ$ are the values of the starting point for the optimization, the stator and rotor widths will be $w_s = 10.5$ mm and $w_r = 11.0$ mm.
3.5.3 Rotor Yoke Thickness

The requirement for the rotor yoke is that it should be able to carry the peak flux without any saturation. In other words, saturation is only expected to happen at the rotor poles. In a singly excited SRM, the flux divides into two equal parts after leaving the rotor pole. Therefore, the yoke thickness \( y_r \) should be at least half of the rotor width \( \frac{w_r}{2} \). Because practical SRMs do operate with their material in saturation, \( y_r \) is usually 20-40\% more than \( \frac{w_c}{2} \) [11]. Here \( y_r \) is chosen as \( y_r = 0.65w_r = 7.2 \text{ mm} \).

3.5.4 Rotor Slot Depth and Shaft Diameter

The rotor slot depth \( d_r \) is the length of the rotor pole, which is at least 20-30 times the air gap required to gain a low unaligned inductance [11]. But since the flux from the stator pole tends to fringe into the rotor pole, deepening the rotor slot further has no effect in reducing unaligned inductance. Besides, the rotor slot depth is limited by the need to maximize the shaft diameter \( D_{sh} \). The purpose of choosing a large shaft diameter is to improve the lateral stiffness of the rotor [11]. Since the rotor diameter and rotor yoke thickness have already been decided, the value of \( D_{sh} \) is entirely dependent on that of \( d_r \). Here the rotor depth is chosen as \( d_r = 20g = 6 \text{ mm} \) and the shaft diameter is chosen as \( D_{sh} = D_r - \).
2(d_r + y_r) = 40.0 − 2 \times (6 + 7.2) = 13.6 \text{ mm}.

3.5.5 Stator Yoke Thickness and Stator Slot Depth

The stator yoke thickness, \( y_s \), can be determined based on the principle stated above. In addition, the stator yoke needs to be made thicker to maximize the stiffness of the stator against the radial force. Unlike a synchronous AC or a BLDC motor, the SRM has a much smaller pole area, which makes it easier for the motor to be deflected when the load is heavy. The deflection may cause vibration and acoustic noise and can be reduced by a stiff stator yoke. On account of these factors, setting the stator yoke thickness as \( y_s = w_s \) is not inappropriate. However, this increases the weight of the machine and compresses the slot area. The selection of the stator yoke thickness is an optimization problem with two conflicting objectives. However, it remains unsolved in this thesis because the electromagnetic field simulation software MagNet cannot analyze the stiffness of magnetic material. The stator yoke thickness is set at \( y_s = 0.75w_s = 7.9 \text{ mm} \). Because the stator diameter, the rotor diameter, the air gap, and the stator yoke thickness have already been determined, the stator slot depth can be calculated as \( d_s = \frac{1}{2}(D − D_r − 2(g + y_s)) = \frac{1}{2} \times (80.0 − 40.0 − 2 \times (0.3 + 7.9)) = 11.8 \text{ mm} \).
At this point, the principal design parameters of the specified SRM have been decided and are listed in Table 3-3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of stator poles</td>
<td>(N_s)</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>No. of rotor poles</td>
<td>(N_r)</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Stator diameter</td>
<td>(D)</td>
<td>80.0</td>
<td>mm</td>
</tr>
<tr>
<td>Rotor diameter</td>
<td>(D_r)</td>
<td>40.0</td>
<td>mm</td>
</tr>
<tr>
<td>Stack length</td>
<td>(L_{stk})</td>
<td>40.0</td>
<td>mm</td>
</tr>
<tr>
<td>Air gap length</td>
<td>(g)</td>
<td>0.3</td>
<td>mm</td>
</tr>
<tr>
<td>Stator pole arc</td>
<td>(\beta_s)</td>
<td>30</td>
<td>deg</td>
</tr>
<tr>
<td>Rotor pole arc</td>
<td>(\beta_r)</td>
<td>32</td>
<td>deg</td>
</tr>
<tr>
<td>Stator pole width</td>
<td>(w_s)</td>
<td>10.5</td>
<td>mm</td>
</tr>
<tr>
<td>Rotor pole width</td>
<td>(w_r)</td>
<td>11.0</td>
<td>mm</td>
</tr>
<tr>
<td>Rotor yoke thickness</td>
<td>(y_r)</td>
<td>7.2</td>
<td>mm</td>
</tr>
<tr>
<td>Rotor slot depth</td>
<td>(d_r)</td>
<td>6.0</td>
<td>mm</td>
</tr>
<tr>
<td>Shaft diameter</td>
<td>(D_{sh})</td>
<td>13.6</td>
<td>mm</td>
</tr>
<tr>
<td>Stator yoke thickness</td>
<td>(y_s)</td>
<td>7.9</td>
<td>mm</td>
</tr>
<tr>
<td>Stator slot depth</td>
<td>(d_s)</td>
<td>11.8</td>
<td>mm</td>
</tr>
</tbody>
</table>

Table 3-3: Principal design parameters of the specified SRM.
3.6 Evaluating the Number of Turns per Pole and Strand Area

The frame of the SRM has been designed, but the motor cannot work without excitation coils. Thus, the number of winding turns per pole $N_p$ should be estimated in advance of the FEM-based simulation. Assuming that the SRM operates at the rated speed and that the conduction angle of each power transistor is exactly the stroke angle specified in Eq. (3.1), the peak flux linkage $\lambda_{\text{peak}}$ per phase is

$$\lambda_{\text{peak}} = \frac{V_s \varepsilon}{\omega} \quad (3.14)$$

where $V_s$ is the DC supply voltage, $\varepsilon$ is the transistor conduction angle and also the stroke angle, $\omega$ is the rated rotational speed, and $\omega = \text{rpm} \times \frac{2\pi}{60}$.

The peak flux linkage occurs at the aligned position. According to the definition of flux linkage, we generate another equation

$$\lambda_{\text{peak}} = Bw_s L_{\text{stk}} \times 2N_p \quad (3.15)$$

where $B$ is the magnetic flux-density on the pole surface, $w_s \cdot L_{\text{stk}}$ represents the pole surface area, and $2N_p$ is the number of turns on a stator pole pair.

Combining Eq. (3.14) and Eq. (3.15),

$$N_p = \frac{15V_s \varepsilon}{\text{rpm} \times \pi Bw_s L_{\text{stk}}} \quad (3.16)$$
All the values of parameters in Eq. (3.16) have been given except $B$ which can be found with the help of Figure 3-6. For the steel used to construct the machine, the saturation flux-density $B$ is about 1.4 T.

![Magnetization curve of M19-29 Ga.](image)

Therefore, the number of turns will be

$$N_p = \frac{15 \times 46.8 \pi}{1900 \times \pi \times 1.4 \times 10^{-3} \times 40.0 \times 10^{-3}} = 104.7 \approx 105 \text{ turns/pole.}$$

This is only an estimated value used in the preliminary design and may be adjusted according to the simulation results.

The slot area of the SRM can be calculated with the data provided in Table 3-3. However, due to the presence of insulation material and liner, the coils cannot fill up the slot area. A constant called “the fill factor” is introduced to measure the effective conducting area, and this constant is the ratio of the area of the conductor without insulation to the area of the
slot without liner [29]. The fill factor ranges from 0.4 to 0.5 for most SRMs [35] and it can be as high as 0.625 for a well-designed SRM [36]. Here, the fill factor is 0.5. The area of the slot without liner is about 66 mm² and there are two half windings in a slot, so that the strand area is \( \frac{66}{2} \times \frac{1}{105} = 0.31 \text{ mm}^2 \).
Chapter 4 DESIGN VERIFICATION

4.1 Prototype Construction

An SRM prototype can be developed in MotorSolve [29] with the given parameters. The start-up position depicted in Figure 4-1 is 40 degrees ahead of the aligned position. The rotor will rotate counter clockwise when the windings at phase B, A, C are excited sequentially.

![Figure 4-1: SRM prototype with specified parameters.](image)

A conventional three-phase controller is constructed in MagNet to drive the motor. The switches PCS1 to PCS6 in Figure 4-2 are ideal components without any loss and are turned on and off according to the rotor position. In practice, these switches are solid-state semiconductor devices such as GTO thyristors and IGBTs which do bring extra losses to
the system. The diodes used in the controller are not ideal and the voltage drop across each one is 5 V during conduction. The power loss incurred by the diodes should be taken into account when the efficiency is calculated.

![Diagram of SRM control circuit](image)

Figure 4-2: SRM control circuit.

### 4.2 Preliminary Simulation Results

First, the performance of the SRM operating at the rated speed is examined. The transient 2D with motion analysis is carried out by MagNet and the results of the first 90-degree rotation are illustrated in Figure 4-3, Figure 4-4, and Figure 4-5.
Figure 4-3: Phase A, B and C flux linkage waveforms.

Figure 4-4: Torque versus time waveform.
The peak flux linkage in Figure 4-3 is about 0.11 Wb, which occurs well before the aligned position because of the advanced excitation. This typically happens when the overlap between the stator and the rotor pole is about 2/3 of the stator pole arc [11]. Based on Eq. (3.15), the maximum flux density on the stator pole surface is about \( B_{ \text{max} } = \frac{\lambda_{ \text{peak} } }{ \frac{2}{3} W_4 L_{ \text{atk} } \times 2 N_p } = \frac{0.11}{\frac{2}{3} \times 10.5 \times 40 \times 10^{-6} \times 2 \times 85} = 2.31 \text{ T} \). The number of turns \( N_p \) applied in the simulation is 85 instead of the earlier estimate of 105, yet other parameters are kept the same. This is because the estimated value is simply calculated through the empirical equation, and the corresponding simulation result does not match the user requirement exactly. The maximum flux density only exists in the air gap, and the flux density in the
stator pole is approximately $B = \frac{2}{3} B_{\text{max}} = 1.54 \, \text{T}$. These values are verified by the simulation results depicted in Figure 4-6. Because the average flux density is higher than the saturation value specified in Figure 3-6, the SRM operates with its magnetic material in saturation, as expected.

Figure 4-6: Magnetic flux density in the fully excited SRM.

Upon analyzing the torque waveform in Figure 4-4, we find that the average output torque is about 1 N.m, a value that succeeds in fulfilling the requirement. However, the torque ripple is very high in the waveform. Although there is no explicit constraint on the torque ripple value, it is always preferable for this value to be minimized for application to the vehicle itself. The reason for the high torque ripple is shown in Figure 4-5. During the phase switching, the previous phase current drops so fast that
the following phase current cannot increase to a sufficient value to maintain the torque level. One natural course for reducing the torque ripple is to retard the previous phase current or to advance the following phase current. It seems that the two methods have the same function. In practice, advancing the following phase current does not decrease the torque ripple effectively since it increases the peak torque simultaneously. After retarding the previous phase current by 4 degrees, we get the new results displayed in Figure 4-7 and Figure 4-8.

![Figure 4-7: Torque waveform with retarded commutation.](image)

0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
Torque Waveform
Time (ms)
Torque (N.m)
Figure 4-8: Phase A, B and C current waveforms with retarded commutation.

Compared with the situation depicted in Figure 4-5, the commutation interval pictured in Figure 4-8 is much smaller, which leads to a relatively smooth torque waveform. After we have tuned the advance excitation angle to 39 degrees and the number of turns to 80, a better torque waveform is attained in Figure 4-9. Further discussion is also based on this result. It can be concluded that elimination of the torque ripple is achievable at the expense of very precise shaft position feedback.
Figure 4-9: Torque waveform with adjusted parameters.

Figure 4-10: Phase A, B and C current waveforms with adjusted parameters.

4.3 Energy Conversion Loop

4.3.1 Magnetization Curve

The measurement of the magnetization curve is of importance in
validating the performance of the SRM and determining the optimal control strategy. Different rotor positions and energizing currents are specified, and the flux linkages under these conditions can be calculated by the multi-problem static 2D solver in MagNet.

![Magnetization Curves](image)

Figure 4-11: Flux linkage versus current at different positions

The magnetization curve in Figure 4-11 displays the static characteristic of the SRM, and the flux linkage-current trajectory in Figure 4-12 exhibits the dynamic performance of the SRM in a complete stroke. According to Eq. (2.3), the mechanical work done by the SRM is equal to the area surrounded by the black dotted line in Figure 4-12.
4.3.2 Optimal Excitation Current

It is obvious in Figure 4-12 that the area surrounded by the flux linkage-current trajectory increases with the stator current. However, the increment in mechanical energy is not uniform, especially in the saturated region [37]. The incremental mechanical energy per current is defined as \(\frac{dW_m}{di}\) and it is maximized at the point where the tangent of the aligned position curve is equal to the tangent of the unaligned position curve.

The tangent is easy to determine at the unaligned position because the flux linkage is proportional to the stator current. But due to the high magnetic saturation in the aligned position, a high order polynomial should be employed to fit the flux linkage curve as shown in Figure 4-13.
At the unaligned position, the flux linkage is

\[ \lambda_u = 0.0027i \]  \hspace{1cm} (4.1)

And at the aligned position, the flux linkage is

\[ \lambda_a = -8.7377 \times 10^{-5}i^4 + 0.0025i^3 - 0.0276i^2 + 0.1402i - 0.1631, \quad 3.5 \leq i \leq 9 \]  \hspace{1cm} (4.2)

Once the calculation is completed, we find that the tangent of \( \lambda_a \) is equal to 0.0027 at the point \( i = 8.7338 \) A. This value is close to the average current shown in Figure 4-10, which means that the SRM is operating near the optimal point.
4.4 Output Efficiency and Losses

Output efficiency is a very important parameter for measuring the quality of an electric motor. It is defined as the ratio of the output power to the input power. Because the position-controlled switches used in the simulation are ideal components, calculating the efficiency of the motor drive is meaningless and only the SRM efficiency is considered.

![Phase B Voltage Waveform](image)

Figure 4-14: Phase B voltage waveform.

The power input to phase B can be calculated by multiplying the voltage waveform in Figure 4-14 and the current waveform in Figure 4-10. The power input to the other two phases can be obtained in the same way. The total input energy is equal to the integral of the input power over time.

The program provided in Appendix I calculates the input and output energy when the rotor rotates from 30 degrees to 60 degrees. During this
period the input and output energy are 0.666 J and 0.504 J, respectively. So the average input and output power are 253.2 W and 191.7 W, and the efficiency is 75.7%.

The power losses in the SRM can be divided into two classes: ohmic loss and iron loss. The ohmic loss is also called the copper loss, which only occurs in copper coils and is equal to $i^2 R$. The iron loss can be further classified as either hysteresis loss or eddy current loss, and both types are determined by the characteristics of the magnetic material used in the motor. The simulation results demonstrate that the ohmic loss represents 64.7% of the total losses, which can be reduced by expanding the effective conducting area or by shortening the stack length. For example, if the effective conducting area is doubled, the ohmic loss will be only half of the existing value and the efficiency will increase to 82.2%. This analysis assumes the excitation current is constant and therefore that the ohmic loss is proportional to the resistance. In practice, the motor is driven by a constant voltage source instead, which means that the simulation result may be different. As shown in Figure 4-15, the SRM with the doubled conducting area is able to produce more torque than the previous model. This is because the resistance is lower and more excitation current is injected into the motor. However, the increment in the current is
relatively small, so that the ohmic loss illustrated in Figure 4-16 is almost solely determined by the resistance. This characteristic ensures that analysis based on a current-driven SRM can also be applied to a voltage-driven SRM. The actual efficiency of the SRM with the doubled conducting area is 79.4%, which is a little lower than the expected value. Nevertheless, the simulation results confirm that the motor efficiency can be effectively increased by the larger conducting area.

Figure 4-15: Output torques with different conducting areas.
The iron loss can be reduced by using low-loss silicon steel [35]. Additionally, the efficiency is not constant and varies with respect to the rotational speed. The losses discussed above are all at the rated speed.

4.5 Constant Power Range

Because the SRM operates mostly in the constant power region, the output torque over this region needs to be investigated. When the rotational speed rises from 1900 RPM to 3800 RPM, the required torque will decrease to 0.5 N.m. If the turn on/off angle of switches is kept the same as in the former simulation, the torque will follow the “natural characteristic” illustrated in Figure 2-10. To counteract the high back EMF and inject more current into the stator coils, the turn on angle should be
further advanced and the dwell angle should be prolonged at the same time. The simulation results are depicted in Figure 4-17 and Figure 4-18 where the torque ripple is high. Here, the phase current is no longer close to a square waveform.

Figure 4-17: Torque waveform at the rotational speed of 3800 RPM.
If the rotational speed is tripled, the demand torque will decline to one third of the previously mentioned value. The simulation result is shown in Figure 4-19 in which the average torque is about 0.33 N.m.
Based on the discussion above, we can conclude that the SRM that we have designed is able to produce considerable torque in the constant power region. But because the current waveform can no longer be regulated, the torque ripple is very high compared to that of the current waveform at the base speed.

4.6 Regenerative Brake of Switched Reluctance Motors

In conventional braking systems, the excess kinetic energy is converted to heat and therefore wasted. When the electric traction motors of a vehicle are used as generators, the kinetic energy can be partially transformed into electric energy that returns to the supply line. In the SRM, positive torque is produced when the rotor approaches the position of minimum reluctance. If the stator phase is excited when the rotor moves away from its aligned position, the torque created will be in the opposite direction to the rotor movement and the SRM will work in the generating mode [2].

Since the switched reluctance generator (SRG) does not have either a field winding or a permanent magnet on the rotor, a magnetic field needs to be produced initially by the external power supply. From this point of view, the SRG is very similar to an induction generator. The power supply
stops energizing the phase windings as soon as the magnetic field is created. After that, the energy stored in the magnetic field returns to the power source. The energy conversion process is illustrated in Figure 4-20, which shows how the prime mover keeps doing the positive work on the SRG.

Figure 4-20: Energy conversion during the regenerative brake.

4.6.1 Simulation Results

The rotor is set to the position shown in Figure 4-21 prior to the simulation and the turn on/off angles of the position controlled switches are adjusted accordingly. The controller circuit is the same as the one used to drive the SRM.
The simulation is carried out under the condition that both the supply voltage and the rotational speed are constant. The mechanical torque and the current going through the power source are presented in Figure 4-22 and Figure 4-23. The negative current in Figure 4-23 indicates that the current is drawn out from the power source, and vice versa.
As the voltage across the power source is constant, the shape of power waveform is exactly the same as that of the current waveform.

Based on the simulation results, it is not difficult to ascertain that the
average input mechanical power to the SRG is 37.2 W and that the net electric power is 24.2 W. Here the electric power is the power going back to the voltage source, and this quantity is a bit smaller than the power from the SRG due to the 0.5 V voltage drop on each diode, as illustrated in Figure 4-2. The average power loss in the diodes is 1.68 W, so that the efficiency of the SRG is 69.6%. The simulation is based on a fixed rotational speed and the single-pulse control strategy. Hence, it is very difficult to maintain a constant output torque. Actually, the braking torque required by the lunar rover (or any EV) is much larger than the torque that the SRM can produce, so a mechanical friction brake should be applied in conjunction with the electrical brake [2]. Consequently, the torque ripple introduced by the regenerative brake does not have a great impact on the overall braking performance of the rover. The final goal of the design is to ensure both safe braking and the ability to recover as much energy as possible [2].

4.6.2 Energy Conversion Loop

The energy conversion loop is clearly presented in Figure 4-24 where the area surrounded by the flux linkage-current trajectory is much smaller than that in Figure 4-12. This is because the phase is only excited for 15
degrees in the SRG while the phase in the SRM is energized for at least 30 degrees. Additionally, the direction of this trajectory is different from the direction employed in the SRM. The rotor of the SRG moves away from the aligned position when the phase is excited, so that the direction of the trajectory is clockwise. But in the SRM, it is counter-clockwise. If the net energy is calculated by taking the integral around the trajectory, the different directions will be responsible for the different signs of the results.

![Magnetization Curves](image)

Figure 4-24: Flux linkage-current trajectory for the SRG.

### 4.6.3 Separately Excited Switched Reluctance Generator

It is obvious in Figure 4-23 that only a half cycle is used to generate the power, and another half cycle is applied to create a magnetic field that the rotor can interact with. One natural course for overcoming this difficulty is to convert the SRG into a doubly-excited machine similar to a
synchronous AC machine. However, attaching coils to the rotor is not viable because almost all the advantages of the SRG come from the simple structure of its rotor. Therefore, the coil on each stator pole is divided in two, as illustrated in Figure 4-25. Taking phase A as an example, coil A is responsible for setting up a magnetic field and coil A1 delivers the electric power to the electrical loads.

![Figure 4-25: A separately excited switched reluctance generator.](image)

Since the number of coils has been doubled, the power electronics controller should be modified. The controller driving coil A, B, C is the same as the one used for a normal SRG, but the controller connected to coil A1, B1, C1 is different. Figure 4-26 depicts the basic configuration of this controller, which only contains diodes.
With a 50 Ω resistor connected in the circuit (to represent the source), the simulation results are demonstrated in Figure 4-27, Figure 4-28, and Figure 4-29. The mechanical torque on the SRG is close to the previously resulting value, but the excitation current is higher and the energy returning to the power source is smaller. By integrating these waveforms over time, we can calculate the average input mechanical power as 34.7 W, the power on the resistor as 37.8 W, and the power drawn out from the source as 21.5 W. The efficiency of this system is 62.1% and this is close to the efficiency of the previous design.
Figure 4-27: Mechanical torque waveform in the generating mode.

Figure 4-28: The power source current waveform.
Figure 4-29: The electrical load current waveform.

The advantage of this system is that it can uninterruptedly power the electrical load, as is illustrated by Figure 4-29. This feature is vital for the power system application because it reduces the power conversion and regulation work. However, uninterrupted charging is not required in the lunar rover and additional semiconductor devices are required to construct the system. Therefore, this configuration of the SRG is only conceptually proposed in this thesis, and the previous design is a better choice for the regenerative brake.
5.1 Introduction

After the prototype SRM has been constructed, the performance of the machine may prove to be close to the specifications. However, we are interested in an optimal design which minimizes or maximizes some particular aspect such as weight, overall dimensions, average torque or torque ripple. As with many other electric motors, the performance of the SRM can be optimized through the choice of appropriate geometric parameters. Once the objective function (also called the cost function) is defined, both deterministic and stochastic optimization methods can be implemented to determine the optimal geometric parameters. The former method usually offers faster convergence while the latter needs a long computational time to arrive at the optimum solution. However, the deterministic methods are all gradient-based, which means that they may be unable to move beyond a local minimum. In contrast, a random walk of the design variables allows the stochastic optimization process to easily overcome the local minimum, so that the probability of finding the global minimum is relatively high. Additionally, deterministic optimization requires the gradient information of the design space, which is not necessary for
stochastic optimization. Another advantage of stochastic optimization is that it can be applied in a design space with a large number of dimensions where the gradient information is very complicated.

This chapter is going to briefly describe the evolutionary strategy optimization technique, which is a widely used stochastic algorithm. Then, the single- and multi-objective optimization results will be presented and compared with the results from the initial design. The goal of the multi-objective optimization process is to maximize the average torque and minimize the mass of the motor at the same time.

5.2 Introduction to the Evolutionary Strategy

The evolutionary strategy (ES) is a stochastic optimization technique inspired by two biological concepts - adaptation and evolution. The names of three important operators in ES algorithm, selection, mutation and recombination, are also borrowed from biological terminology. These operators are applied in a loop and each iteration of the loop is called a generation. The loop will stop once one of the termination conditions is met. Since a (1+1)-ES (one parent and one child) is applied in this project, there is always one member in the parent pool during the iterations and the recombination is simply a copy of the parent [38]. Therefore, more attention will be paid to the other two operators, mutation and selection.
5.2.1 Mutation

Mutation is a basic operator in the ES algorithm, and is the source of the genetic variation. In fact, the performance of the ES is largely dependent on the adjustment of the mutation strength, which is also called "the step length" [38]. In contrast to the selection operator which exploits fitness information to guide the search into a promising direction, the mutation operator explores the design space using only the information from the parental state. This means that no fitness information should be used, and the mutation process should not introduce any bias. This requirement leads us directly to the application of a normal distribution in an unconstrained real-value search space [38]. The simplest case of the normal distribution mutation is to use the same standard deviation \( \sigma \) on all the variables.

\[
\tilde{y} := y + z \quad (5.1)
\]

with

\[
z := \sigma(N_1(0,1) \ldots N_N(0,1)) \quad (5.2)
\]

where the number of design variables is \( N \) and each mutated variable \( \tilde{y}_i \) obeys the Gaussian function

\[
p(\tilde{y}_i) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{1}{2} \frac{(\tilde{y}_i - y_i)^2}{\sigma^2} \right) \quad (5.3)
\]
The standard deviation $\sigma$ presents the mutation strength which controls the width of the Gaussian bell.

### 5.2.2 Selection

As happens during natural selection, the weakest “species” (objects) in each generation are eliminated. The selection operator guides the search into the most promising regions of the design space. The (1+1)-ES applies the plus selection that keeps both the parental and the offspring population in the selection pool. It has already been proved that the convergence velocity of the (1+1)-ES is inversely proportional to the number of design space dimensions and the objective function [38].

### 5.2.3 Termination Conditions

Since the ES is applied in a loop, termination conditions should be defined to stop the iterations. In general, there are two types of standard stopping rules, resource criteria and convergence criteria [38]. Both of them should be defined in order to find an improved solution within a specified time-span. During the optimization, the maximum CPU time and maximum number of generations are set as the resource criterion while a minimum search area determined by the mutation strength is set as the
convergence criterion.

The reason why the mutation strength can be used as a termination condition is made evident by the search process. An obvious difference between stochastic and deterministic optimization algorithms is the way the step length changes. In a deterministic algorithm, the search region is always narrowed down around the improved solution to converge to the nearest minimum quickly. As mentioned before, the minimum found during the process is very likely to be a local minimum. Contrarily, if a better solution is found, the stochastic algorithm will become ‘braver’ and cover a larger region to retrieve another good solution in the neighborhood. Assuming that no improved solution is found, the size of the search region will keep decreasing and finally be smaller than the specified value.

It is worth noting that the larger the minimum search area, the shorter time it will take to reach the global optimum [31]. However, when the minimum search area increases to a certain value, the search may become too ‘brave’ to converge.

5.3 Single-Objective Optimization for Switched Reluctance Motor Design

This next discussion starts with single-objective optimization problems and later extends to multi-objective optimization. A general single-
A single objective optimization problem is defined as minimizing or maximizing $f(x)$ subject to $g_i(x) \leq 0, i = \{1, \cdots, m\}$, and $h_j(x) = 0, j = \{1, \cdots, p\}, x \in \Omega$, where $f(x)$ is the objective function, $g_i(x) \leq 0$ and $h_j(x) = 0$ represent constraints that must be fulfilled while optimizing $f(x)$, and $\Omega$ is the feasible region of $x$ [39]. The rigorous definition of global optimum is demonstrated as follow:

Given the function $f : \Omega \subseteq \mathbb{R}^n \rightarrow \mathbb{R}, \Omega \neq \emptyset$, for $x \in \Omega$ the value $f^* \triangleq f(x^*) > -\infty$ is called a global minimum if and only if $\forall x \in \Omega: f(x^*) \leq f(x)$ [39].

Based on this definition, we notice that a single-objective optimization problem may have a unique solution.

### 5.3.1 Objective, Design Variables, and Constraints

The objective, design space, and constraints should be explicitly defined before solving the optimization problem. At this point, the optimization target is to maximize the average mechanical torque of the SRM. As the performance of the SRM is influenced not only by the geometry of the machine but also by the control strategy, a current source instead of a voltage source should be employed to produce an ideal square-wave excitation current. Consequently, the optimization will focus on the motor configuration refinement. The magnitude of the excitation current is set to 8 A, which is close to the optimal value decided in Section
4.3.2.

The design variables in this problem are the geometric parameters that have a profound effect on the output torque. According to Eq. (3.2), the torque is proportional to the stack length $L_{stk}$ and also to the square of the rotor diameter $D_r$. However, enlarging these two parameters will increase both the volume of the machine and the overall weight. Therefore, $L_{stk}$ and $D_r$ are kept constant while optimizing the output torque. Three parameters, including the stator pole arc $\beta_s$, rotor pole arc $\beta_r$, and rotor radius $r_{in}$, as illustrated in Figure 5-1, are selected to construct the design space.

![Figure 5-1: Three design variables.](image)

The feasible intervals for these variables are summarized in Table 5-1.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_s$</td>
<td>30</td>
<td>30</td>
<td>33</td>
<td>deg</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>32</td>
<td>30.5</td>
<td>35</td>
<td>deg</td>
</tr>
<tr>
<td>$r_{in}$</td>
<td>14</td>
<td>12</td>
<td>15</td>
<td>mm</td>
</tr>
</tbody>
</table>

Table 5-1: Design variables for the single-objective optimization.

Constraints should be defined before the optimization to ensure that the optimal configuration will only be located among devices satisfying the specified conditions [31]. The only constraint assigned to this problem is that the minimum torque is 0.75 N.m. It is solution-based because a field solution is required for evaluation. The initial solution, also known as the starting point of the optimization, should always fulfill this constraint.

### 5.3.2 Optimization Results

The instantaneous torques at 30 rotor positions, which are equally distributed in a period, should be calculated. Then, the average value can be determined. Figure 5-2 illustrates how the output torque varies over 94 different SRM designs and that the objective finally converges to a certain value. To ensure that the solution is a global optimum instead of a local one, another optimization is performed with a different initial point and the convergent performance of the objective is presented in Figure 5-3. Although the search routes are different in two optimizations, they finally
arrive at the same point, so that the solution can be regarded as the global optimum. Here the search tolerance is set as 0.01 in the first optimization and 0.02 in the second one. That is why the number of iterations to reach convergence is larger in Figure 5-2.

**Figure 5-2:** The convergent performance of the objective.

**Figure 5-3:** The convergent performance of the objective with different start point.
Table 5-2 evaluates the variations of design variables from their initial values to their optimized values and the actual influence of each variable on the objective shown in Figure 5-2. It is obvious that the stator and rotor pole arcs have a heavier impact on the objective than the rotor radius. This conclusion can be verified by adding another parameter to the design space and analyzing the variations again.

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Start</th>
<th>End</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_s$</td>
<td>30</td>
<td>32.85</td>
<td>+9.5%</td>
</tr>
<tr>
<td>$\beta_r$</td>
<td>32</td>
<td>30.52</td>
<td>-4.6%</td>
</tr>
<tr>
<td>$r_{in}$</td>
<td>14</td>
<td>14.20</td>
<td>+1.4%</td>
</tr>
</tbody>
</table>

Table 5-2: The comparison between the initial values and optimized values of design variables.

The sensitivity analysis gives the designer a sense of which design variable should be prioritized to optimize the objective. Besides, the variable that has a tiny influence on the result can be eliminated from the design space, so that a predominantly similar design result can be achieved with a lower computational cost.

As mentioned in Section 5.3.1, the optimized result is obtained from the SRM driven by the constant current source. However, if the optimal design does not produce more torque than the initial design when driven by a voltage source, then the optimization process will have no meaning in
the real world. Hence, another simulation of the optimal design driven by a voltage source must be carried out, and the result is compared with the result from the initial design.

When the voltage source is applied, the control strategy should be taken into account since it determines how much input current will be sent to the SRM. The goal of the control is to make sure that the excitation currents in the initial and optimized design are identical.

![Current Waveforms](image)

Figure 5-4: Phase C current of the initial and optimized designs.
The current injected in phase C and the output torques are depicted in Figure 5-4 and Figure 5-5, respectively. With approximately identical input currents, the optimal design is able to produce more torque than the initial one. Because the source voltage and the rotational speed of the designs are also the same, the efficiency of the optimal design is much higher.

5.4 Multi-Objective Optimization for Switched Reluctance Motor Design

As proposed in Section 5.1, we want to design an SRM with the maximum output torque and the minimum weight, subject to certain constraints. Since these two objectives are conflicting, solutions attempting to further improve one objective may work to the detriment of
the other as a result. The concept of “Pareto improvement” is introduced to describe the increase in both objectives. This term is defined as a modification to a solution that improves one objective without worsening the other. When no further Pareto improvements can be made, a solution is defined as “Pareto efficient” [39]. A set of these solutions is known as a Pareto frontier, and is very difficult to obtain.

5.4.1 Aggregate Objective Function, Design Variables and Constraints

The aggregate objective function (AOF) is defined to integrate two objectives into one. The way to construct the AOF is called the weighted sum method [40], in which an essentially subjective weight is assigned to each objective. Because the objectives will be optimized in opposite directions, different signs are utilized before the torque $T$ and the mass $M$. At this point, the overall objective of the optimization is to minimize the value of the AOF. Before assigning a weight to each objective, we need to make sure that the absolute values of the torque and the mass are of comparable magnitude. The initial values of the output torque and the mass are 0.92 and 1.00, respectively. Since they are very close to each other, no reference value needs to be specified for the normalization.

There is no rule guiding how to define the weights, so three typical
sets of weights ([0.7, 0.3] [0.5, 0.5] and [0.3, 0.7]) are applied. The simulation results are presented in the next section.

\[
\text{Objective}(T, M) = -w_1T + w_2M
\]  \hspace{1cm} (5.4)

The design variables are listed in Table 5-3. Beyond the variables involved in the single-objective optimization, two more variables are added to the design space, which are the stack length \(L_{stk}\) and the rotor outer radius \(r_{out}\). Because the mass of the machine is taken into account now, the frame of the motor can be slightly adjusted.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_s)</td>
<td>30</td>
<td>30</td>
<td>33</td>
<td>deg</td>
</tr>
<tr>
<td>(\beta_r)</td>
<td>32</td>
<td>30.5</td>
<td>35</td>
<td>deg</td>
</tr>
<tr>
<td>(r_{in})</td>
<td>14</td>
<td>12</td>
<td>16</td>
<td>mm</td>
</tr>
<tr>
<td>(r_{out})</td>
<td>20</td>
<td>19</td>
<td>22</td>
<td>mm</td>
</tr>
<tr>
<td>(L_{stk})</td>
<td>40</td>
<td>38</td>
<td>42</td>
<td>mm</td>
</tr>
</tbody>
</table>

Table 5-3: Design variables for the multi-objective optimization.

The constraint defined in this problem is also the minimum torque of 0.85 N.m. In the single-objective optimization problem, the optimal design should always satisfy the constraint. However, the situation is different in the multi-objective optimization problem, as some designs may be
promising when seen from the perspective of the AOF, even though the constraint is not fulfilled. Consequently, these designs have to be eliminated from consideration.

5.4.2 Simulation Results

When \( w_1 = 0.7 \) and \( w_2 = 0.3 \) in Eq. (5.4), the variation of the AOF is the variation shown in Figure 5-6. After 89 iterations, the minimum value has finally been reached and this solution should be Pareto efficient. The optimization processes for the objectives are individually illustrated in Figure 5-7 and Figure 5-8, where the mass of the SRM increases a little from the initial value. This is because the multi-objective optimization only ensures that the AOF arrives at the optimum and the objective with a lighter weight is compromised.
Figure 5-6: Minimize the aggregate objective function.

Figure 5-7: Minimize the mass of the SRM.
After implementing the optimizations with different AOFs, we have the three sets of results presented in Table 5-4. All three optimizations start from the same design, whose output torque is 0.922 N.m and whose mass is 1.004 kg.

<table>
<thead>
<tr>
<th>Number</th>
<th>Weights $[w_1, w_2]$</th>
<th>Torque (N.m)</th>
<th>Mass (kg)</th>
<th>Torque to Mass Ratio (N.m/kg)</th>
<th>AOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[0.7, 0.3]</td>
<td>1.029</td>
<td>1.118</td>
<td>0.92</td>
<td>-0.385</td>
</tr>
<tr>
<td>2</td>
<td>[0.5, 0.5]</td>
<td>0.935</td>
<td>0.929</td>
<td>1.01</td>
<td>-0.003</td>
</tr>
<tr>
<td>3</td>
<td>[0.3, 0.7]</td>
<td>0.875</td>
<td>0.867</td>
<td>1.01</td>
<td>0.344</td>
</tr>
</tbody>
</table>

Table 5-4: Optimization results.

When comparing these results, we find that the second and third solutions in Table 5-4 have a better ratio of torque to mass. The second
design is much closer to the user requirement of 1.0 N.m, which explains why it is chosen as the final solution. All the optimized design variables of the second solution are listed in Table 5-5 and the parameters of another two solutions are given in Appendix II.

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Start</th>
<th>End</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_s$</td>
<td>30</td>
<td>31.28</td>
<td>+4.3%</td>
</tr>
<tr>
<td>$\beta_r$</td>
<td>32</td>
<td>30.96</td>
<td>-3.3%</td>
</tr>
<tr>
<td>$r_{in}$</td>
<td>14</td>
<td>12.82</td>
<td>-8.4%</td>
</tr>
<tr>
<td>$r_{out}$</td>
<td>20</td>
<td>19.01</td>
<td>-5.0%</td>
</tr>
<tr>
<td>$L_{stk}$</td>
<td>40</td>
<td>41.81</td>
<td>+4.5%</td>
</tr>
</tbody>
</table>

Table 5-5: The optimized design variables when $w_1 = 0.5$ and $w_2 = 0.5$. 

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Chapter 6 SUMMARY AND FUTURE WORK

6.1 Thesis Summary

In this thesis, we reviewed different types of electric motors and proposed the SRM as a very good candidate for the traction drive on the lunar roving vehicle due to its robust rotor structure, inherent degree of fault tolerance, and high efficiency over a considerable speed range. In addition, we explained the principle of electromechanical energy conversion in SRMs and analyzed the performance of SRMs over three speed ranges. The user requirement was discussed, and this requirement enabled us to determine all the geometric parameters of the SRM. FEM-based simulations were carried out to confirm that the proposed design met the requirement, and also to assess the motoring and the regenerative braking performance of the SRM. At last, the Evolutionary Strategy optimization technique was been implemented on the roughly designed motor, and the optimal point in the design space was consequently located. Single- and multi-objective optimizations were performed separately and the resulting designs were compared with the initial ones. The project requirements and the motor performance are listed in Table 6-1, from which we can tell that the designed SRM is able
to fulfill all the basic requirements, but its performance under extreme temperatures and cooperation with the real world power electronics converter may need to be further investigated.

<table>
<thead>
<tr>
<th>Requirements</th>
<th>Motor Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Produce required mechanical torque</td>
<td>Fully met</td>
</tr>
<tr>
<td>Work with the battery voltage</td>
<td>Fully met</td>
</tr>
<tr>
<td>Avoid the violation of the space constraints</td>
<td>Fully met</td>
</tr>
<tr>
<td>Regenerative brake ability</td>
<td>Fully met</td>
</tr>
<tr>
<td>Very high efficiency</td>
<td>Partially met</td>
</tr>
<tr>
<td>Optimal design</td>
<td>Partially met</td>
</tr>
<tr>
<td>Work under extreme temperatures</td>
<td>Not tested</td>
</tr>
<tr>
<td>Controlled by the real world power electronics devices</td>
<td>Not tested</td>
</tr>
</tbody>
</table>

Table 6-1: The list of the project requirements and the final motor performance.

In conclusion, we have succeeded both in constructing an SRM for the lunar roving vehicle and in presenting a general design process which can be utilized to build SRMs with different specifications.

6.2 Future Work

Some interesting research topics that serve as extensions of the work in this thesis are suggested in this section. First, the thesis does not
explore the temperature distribution of the SRM, which determines the ratio of the torque to the volume. Besides, the working temperature for the lunar rover is extremely high during the daytime and low at night, making thermal analysis is of great importance in the motor design. Second, the control strategy is briefly mentioned in Chapter 2 and Chapter 4 since it has a profound effect on the output torque. In the real world, a closed-loop instead of an open-loop controller is usually chosen to drive the SRM and the control strategy can be very complex. Third, the optimization technique applied in the project is (1+1)-ES which will always converge to a global minimum value. However, the number of the steps that this process will take remains unknown [28]. A better optimization algorithm can be created to save computation time and improve computation accuracy. Finally, Chapter 3 lists all the rules required to construct an SRM. One worthwhile extension of this last inquiry would be to build a rule-based expert system capable of designing the SRM automatically.
LIST OF REFERENCES


[22] K. M. Rahman, B. Fahimi, G. Suresh, A. V. Rajarathnam, and M. Ehsani, "Advantages of switched reluctance motor applications to EV and


APPENDIX I

Curve fitting program

clear

I = [0,1,3,3.5,4,4.5,5,5.5,6,6.5,7,7.5,8,9,10,11,13];
I1=I(4:14);

M=dlmread('curvefitting.txt');

M1=zeros(2,17);
for i =1:2
    M1(i,:) = M(17*(i-1)+1:17*i,3)';
end

plot(I, M1(1,:), '-k','LineWidth',2);
hold on
plot(I, M1(2,:), '-','LineWidth',2);

grid on
set(gca,'XTick',0:1:15);
set(gca,'YTick',-0.01:0.01:0.16);
axis([0 15 -0.01 0.16])
title('Magnetization Curves');
xlabel('Current (A)');
ylabel('Flux Linkage (Wb)');
hold on

unalign_flux=M1(1,:); %define a matrix to store values of unaligned flux-linkage
align_flux=M1(2,4:14); %define a matrix to store values of aligned flux-linkage

A=polyfit(I,unalign_flux,1);
B=polyfit(I1,align_flux,4);

X=3.5:0.1:9;
f=polyval(B,X);
plot(X,f,'-r','LineWidth',2);
grid on

K = polyder(B);
K(4)=K(4)-A(1);
x=roots(K);
Efficiency calculation program

clear
M01=dlmread('phaseA.txt');
time=(10^-3)*M01(40:80,2);
M02=dlmread('phaseB.txt');
M03=dlmread('phaseC.txt');

M11=dlmread('VphaseA.txt');
M12=dlmread('VphaseB.txt');
M13=dlmread('VphaseC.txt');

M2=dlmread('torque vs time.txt');

Current_A=M01(40:80,3);
Current_B=M02(40:80,3);
Current_C=M03(40:80,3);

Voltage_A=M11(40:80,3);
Voltage_B=M12(40:80,3);
Voltage_C=M13(40:80,3);

Torque=M2(40:80,3);
Speed=(1900/60)*2*pi;

Power_A=Current_A.*Voltage_A;
Power_B=Current_B.*Voltage_B;
Power_C=Current_C.*Voltage_C;
in_power=Power_A+Power_B+Power_C;

in_energy=trapz(time,in_power);
out_power=Torque*Speed;
out_energy=trapz(time,out_power);

eff=out_energy/in_energy;
## APPENDIX II

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Start</th>
<th>End</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_s$</td>
<td>30</td>
<td>32.75</td>
<td>+9.2%</td>
</tr>
<tr>
<td>$\beta_r$</td>
<td>32</td>
<td>30.52</td>
<td>-4.6%</td>
</tr>
<tr>
<td>$r_{\text{in}}$</td>
<td>14</td>
<td>14.34</td>
<td>+2.4%</td>
</tr>
<tr>
<td>$r_{\text{out}}$</td>
<td>20</td>
<td>20.48</td>
<td>+2.4%</td>
</tr>
<tr>
<td>$L_{\text{stk}}$</td>
<td>40</td>
<td>41.83</td>
<td>+4.6%</td>
</tr>
</tbody>
</table>

*Appendix Table 1: The optimized design variables when $w_1 = 0.7$ and $w_2 = 0.3$."

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Start</th>
<th>End</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_s$</td>
<td>30</td>
<td>32.19</td>
<td>+7.3%</td>
</tr>
<tr>
<td>$\beta_r$</td>
<td>32</td>
<td>30.75</td>
<td>-3.9%</td>
</tr>
<tr>
<td>$r_{\text{in}}$</td>
<td>14</td>
<td>14.05</td>
<td>+0.4%</td>
</tr>
<tr>
<td>$r_{\text{out}}$</td>
<td>20</td>
<td>19.00</td>
<td>-5.0%</td>
</tr>
<tr>
<td>$L_{\text{stk}}$</td>
<td>40</td>
<td>38.73</td>
<td>-3.2%</td>
</tr>
</tbody>
</table>

*Appendix Table 2: The optimized design variables when $w_1 = 0.3$ and $w_2 = 0.7."