Finite Element Analysis of Reinforced Concrete Members - R.W. Spokowski
FINITE ELEMENT ANALYSIS OF REINFORCED

CONCRETE MEMBERS

by

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of Master of Engineering.

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TO MY PARENTS
FINITE ELEMENT ANALYSIS OF
REINFORCED CONCRETE MEMBERS

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ABSTRACT

Experimental results are reported for beam-column joints, isolated at the points of inflection in frames previously tested and loaded along the line joining the inflection points. A finite element analysis is conducted on this joint and on a simply supported beam.

The load-deflection and the moment-rotation curves, crack patterns and the failure loads obtained experimentally are compared with the results obtained from the analysis to examine the feasibility of using finite elements in structural concrete research.

The computer program developed can be applied to any plane problems in structural concrete. A stiffness matrix is assembled considering non-linear concrete characteristics, steel reinforcing, non-linear bond characteristics, and stirrups. Failure criteria are defined to describe the behaviour of the constituents.
Keywords: Analog, Beams, Bond, Concrete, Computers, Cracking, Deflection, Finite Element Method, Frames, Hinges, Joints, Matrix Method, Reinforced, Rotation, Steel-Reinforcement, Stiffness, Strength.
ANALYSE PAR ÉLÉMENTS FINIS DE
PIÈCES EN BÉTON ARMÉ

Robert W. Spokowski

Détartment de génie civil et mécanique appliquée

Thèse de Maîtrise

Mai 1972

RÉSUMÉ

Cette thèse présente les résultats d'une étude sur des pièces représentant l'assemblage monolithique d'une poutre à une colonne, comprenant la partie de la poutre et de la colonne située entre le noeud et les points d'inflexion établis dans une étude antérieure d'un portique simple, et mises en charge par une force dirigée selon la droite joignant les points d'inflexion. Une analyse par éléments finis est faite de cet assemblage, ainsi que d'une poutre sur appuis simples.

Une comparaison est faite entre les courbes charge-déflexion et moment-rotation, les charges à la rupture, et les fissurations obtenues expérimentalement, et celles calculées par éléments finis, afin d'établir la possibilité d'une analyse adéquate de charpentes en béton armé à l'aide d'éléments finis.
Le programme utilisé peut être appliqué à toute structure plane en béton armé, tenant compte de l'inélasticité du béton et des aciers longitudinal et transversal, des caractéristiques non-linéaires de l'adhérence, et de leurs critères de rupture respectifs.
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b = Width of compression face of member

d = Distance from extreme compression fiber to centroid of tension reinforcement

d_{bs} = Local bond slip

d_{b} = Diameter of bar

d_{m} = Relative nodal displacement

f = Stress in concrete

f_{c} = Maximum concrete stress at yielding of tensile steel

f_{f} = Concrete stress at maximum strain

f'_{c} = Maximum concrete stress

f_{r} = Modulus of rupture of concrete

f'_{s} = Stress in compression steel

f_{y} = Yield stress of steel

k = A constant

t = Thickness of the triangular element

u,\nu = Displacements in x and y directions

u_{bs} = Local bond stress

x,\nu = Global coordinates

x',\nu' = Local coordinates

\alpha_{1}^{6} = Generalized displacements

\varepsilon = Strain in concrete

\varepsilon_{f} = Maximum concrete strain at failure

\varepsilon_{o} = Concrete strain corresponding to maximum stress

\varepsilon_{s} = Steel yield strain

\varepsilon'_{s} = Strain in compression steel

\varepsilon_{sh} = Steel strain at onset of strain hardening
\( v_{xy} \) = Poisson's ratio for elongation in x-direction under load in y-direction

\( v_{yx} \) = Poisson's ratio for elongation in y-direction under load in x-direction

\( \delta \) = Nodal displacement vector

\( \gamma_s \) = A constant

\( \theta \) = Angle of inclination of local x-axis to global x-axis.
A. INTRODUCTION

The understanding of the complete behaviour of reinforced concrete members under static and dynamic loading is of prime importance in structural concrete research. Energy absorption and moment-rotation capacities, of both isolated and framed reinforced concrete members, are important factors that must be ascertained. One of the objectives of this continuing research program is to formulate an analytical model which simulates the complete behaviour of a given structural concrete member. However, the formulation of an analytical model that describes the complete behaviour of reinforced concrete, and produces the important prementioned factors, has yet to be achieved.

A number of analytical models have been proposed, but the problem of non-linearity has hindered their development and application. To form an analytical model, the non-linearities of the constitutive materials must be defined. The stress-strain curves of concrete and steel (into strain hardening), and the behaviour of the steel-concrete interface ('bond'), must be defined. The problem is further complicated by the introduction of cracks and the presence of the changing topology throughout the inelastic loading range.
During the past number of years researchers have attempted to formulate an analytical model by using finite elements. This entails the building of a stiffness matrix and the definition of failure criteria to describe the behaviour of the structure. Although many steps have been made towards the formulation of a more complete analytical model, a number of important problems have still to be solved. Previous models lacked some capabilities and some questions have still to be answered concerning the effect of loading history on the overall non-linear behaviour. A detailed computer examination of a structural concrete member under a given load system requires that the structure be subjected to monotonically increasing loads, the failure criteria be checked, and the structural stiffness be redefined as non-linearities appear. An analytical model possessing such capabilities could describe the behaviour of a reinforced concrete member throughout the entire loading range.

B. HISTORICAL BACKGROUND

One of the first applications of finite elements to reinforced concrete analysis was carried out by Ngo and Scordelis(1). They idealized a reinforced concrete beam as an assemblage of separate concrete and steel elements. The behaviour of the steel-concrete interface was idealized by spring linkages connecting corresponding nodes on the steel-concrete interface. Stress-strain curves of both steel and concrete were considered to be linear and
the bond linkage stiffnesses were assumed to be constant. A number of beams were analyzed with different predefined crack patterns, tension reinforcement, and a symmetrical two point loading. From these solutions, many interesting stress and bond force distributions were obtained.

Subsequent work by Nilson (2) introduced the non-linear material properties along with failure criteria for elements and bond spring elements. Nilson analyzed pullout specimens tested by Bresler and Bertero (20). When the principal stress at the centroid of an element reached the modulus of rupture, the computer program was stopped, the finite element grid was examined and the specimen topology was redefined to account for the crack formation. The structure was then reloaded from zero load until another element exhibited cracking after which the cycle was repeated.

Recently, Ngo, Franklin and Scordelis (3) formulated a more refined model of a reinforced concrete beam. Along with separate concrete elements, steel elements, and bond linkages, stirrups and aggregate interlock parameters were included. The crack patterns were predefined in this model, which was subjected to a single load, while the material properties were kept constant. They analyzed simply supported beams, with and without web reinforcement, and obtained a significant number of results depicting the overall behaviour of reinforced concrete members under static load. It must be noted that many of these results could not have been as easily obtained experimentally.
Finite element analysis has not only included simple reinforced concrete members, such as pullout specimens or simply supported beams, but is being used to solve more complicated problems in the field of structural concrete research. An analytical model of plain concrete has been proposed in a recent work by Buyukozturk (4) who idealized the mortar and aggregate constituents using finite elements. Three different failure criteria were established to describe the following types of possible crack formations:

1. Compression-shear bond cracks at the mortar-aggregate interface
2. Tension-shear bond cracks at the mortar-aggregate interface
3. Tensile mortar cracks.

This model was then subjected to uniform displacements at the boundaries to yield important information on the behaviour of plain concrete.

The difficult problem of analyzing prestressed concrete slabs using finite elements has been recently attempted by Youssef (5) while reinforced concrete slabs and shells were studied by Bell (6). The behaviour of frames, with and without shear walls, has been studied by Franklin (7), with the use of a finite element model.
Experimental study of moment redistribution and moment-rotation capacity of frames has presented several problems and has resulted in the following two schools of thought:

1. The analysis of beam-column joints, isolated at the points of inflection, to establish moment-rotation capacities, and

2. The analysis of complete frames, designed to fail in a predetermined failure mechanism, to establish the degree of moment redistribution.

Work by Sader (8) and Adenot (9) has resulted in complete load-deflection curves for one and two bay reinforced concrete frames. Both of these works considered frames which were designed to fail in a sidesway mechanism. Although the testing of frames would form a more rational basis for a Limit Design procedure (10), the testing of isolated joints yields a significant amount of detailed information on the behaviour of hinges. Viest and Morrow (11) tested tension reinforced concrete joints, as far back as 1957. More refined technique was incorporated by Burns (12) in the testing of beams with column stubs. Recently, Khan (13) tested three dimensional beam-column connections to the failure load. In all cases, the moment-curvature and the moment-rotation capacities were established for variation of different structural parameters.

C. INVESTIGATION OBJECTIVES

The object of this investigation is to construct an analytical model of reinforced concrete using finite elements. The behaviour
of this model will be examined using a piecewise linear idealization of material properties along with an incremental loading procedure, so that an analysis can be carried on throughout the entire loading range. This model will then be used to study the non-linear behaviour of reinforced concrete hinges, members and assemblies.
CHAPTER II

THEORETICAL CONSIDERATIONS

A. GENERAL

During the past decade the finite element method has received considerable attention in the field of continuum mechanics. This method formulates a structural stiffness matrix, by dividing the structure into a finite number of discrete elements connected at the nodes, and with compatibility assured along the element boundaries. The stiffness characteristics of the element can be determined using the minimum potential energy theorem. This has been done for several shapes of elements possessing varying numbers of degree of freedom. Nodal displacements are found by solving the system of equilibrium equations. The nodal displacements are then used to determine the strains and the stresses at the element centroids.

If a reinforced concrete structure is subjected to a monotonically increasing load, changes in stress and strain can be accumulated to give the overall strain and stress in any element. Non-linearities can be introduced by altering the stiffness matrix, with respect to constitutive relationships, or with respect to the failure criteria, at each load increment. This procedure would then give an accurate representation of the loading history. Therefore, two areas that must be defined to form an analytical model are:
B. ASSEMBLAGE OF THE GLOBAL STIFFNESS MATRIX

1. Concrete and Steel Elements

Inherent imperfections in all materials makes it difficult to define their properties on a global scale. Concrete, being a mixture of a number of basic materials, presents additional problems. The non-homogeneous nature of concrete makes it difficult to define the constitutive relationships at the microscopic level. The non-linearities of the constitutive relationships can be idealized, however, at the microscopic level, by the use of finite elements and a piecewise linear analysis. A constant stress triangular finite element is used in this analysis, to define the stiffness characteristics of the concrete and steel, under a biaxial state of stress. For a six degree-of-freedom plane stress triangular element (Fig. 1) the minimum potential energy theorem gives the following (14):

\[ P = K \Phi \]

where \[ K = t \int \int B^T D B \, dA = tA B^T D B \]

= element stiffness matrix (6 x 6 for the triangular element)

A = area of the element

t = thickness of the element

P = nodal force vector
\( \delta \) = nodal displacement vector

\( D \) = elasticity matrix for the orthotropic case

\[
D = \begin{bmatrix}
E_x & \frac{\nu_{yx} E_x}{1 - \nu_{xy} \nu_{yx}} & 0 \\
\frac{\nu_{xy} E_y}{1 - \nu_{xy} \nu_{yx}} & E_y & 0 \\
0 & 0 & G
\end{bmatrix}
\]

\( E \) = strain matrix

\( E_x \) and \( E_y \) = elastic tangent moduli in directions of principal strains

\( \nu_{xy} \) = Poisson's ratio for deformation in x-direction due to loads in y-direction

\( \nu_{yx} \) = Poisson's ratio for deformation in y-direction due to loads in x-direction

\( G \) = shear modulus of elasticity

The strain matrix can be derived using displacement functions in the form of two linear polynomials (14).

\[
\begin{align*}
u &= \alpha_1 + \alpha_2 x + \alpha_3 y \\
v &= \alpha_4 + \alpha_5 x + \alpha_6 y
\end{align*}
\]

For isotropic materials the moduli \( E_x \) and \( E_y \) are equal. Both steel and concrete have been assumed to be isotropic. The
lower bound solution of the finite element method, becomes notable with triangular elements, and negates any additional accuracy achieved by using the orthotropic elasticity matrix. In this analysis the elastic moduli are kept equal and dependent on the largest principal strain at the centroid of a particular element. To account for the non-linearity in the stress-strain curves the tangent moduli are changed at each loading increment depending on the state of strain at the centroid of the element.

It is thus possible to assemble a stiffness matrix for a reinforced concrete member with the concrete and steel constituents separate with respect to the degrees of freedom but occupying the same positions in space. The two dimensional aspect of this analysis requires the creation of a transformed equivalent section (2) which is facilitated by the conversion of the circular cross-section of the steel bars into a rectangular form (Fig. 2).

2. Bond

A mathematical representation of the force-slip relationship describing the behaviour of the steel-concrete interface ('bond') is difficult to produce. However, a coarse idealization can be achieved with the use of four degree-of-freedom spring linkages introduced by Ngo and Scordelis (1) (Fig. 3). These links would connect corresponding nodes on concrete and steel elements.
The stiffness matrix is obtained from the equation:

\[
K = \begin{bmatrix}
-c & s \\
-s & -c \\
c & -s \\
s & c \\
\end{bmatrix}
\begin{bmatrix}
K_h & 0 \\
0 & K_v \\
\end{bmatrix}
\begin{bmatrix}
-c & -s & c & s \\
-s & -c & -s & c \\
\end{bmatrix}
\]

(5)

where \( c = \cos \theta \)
\( s = \sin \theta \)
\( \theta = \) angle of inclination of local x-axis to global x-axis
\( K_h = \) 'bond' pullout spring stiffness(horizontal)
\( K_v = \) dowel spring stiffness(vertical)

The values of \( K_h \) and \( K_v \) are dependent on the relative displacement between the steel and steel concrete nodes and are non-linear and must, therefore, be altered at each loading increment. The determination of these non-linear relationships is a difficult experimental problem and will be discussed later.

3. **Stirrups**

The idealization of stirrups and to accurately describe their effect on the behaviour of a structure is difficult to achieve. Conventional stirrups, either closed or U-shaped, confine a large portion of the concrete and increase the ductility of the section and its rotation capacity (15). Three dimensional finite elements may be required, to include this feature in the analysis.
For the sections tested in this program, a zig-zag shear reinforcement is used to prevent any confinement of the section (16). In this case, stirrups could be represented by a four degree-of-freedom bar element (Fig. 4) described by a stiffness matrix:

\[
K = \frac{E_s A_s}{L_s} \begin{bmatrix}
c^2 & cs & -c^2 & -cs \\
-\frac{cs}{s^2} & s^2 & cs & -s^2 \\
-\frac{c^2}{s^2} & cs & c^2 & cs \\
-\frac{cs}{s^2} & -s^2 & cs & s^2 \\
\end{bmatrix}
\]

where

\[c = \cos \theta\]
\[s = \sin \theta\]
\[\theta = \text{angle of inclination of local x-axis to global x-axis}\]
\[E_s = \text{elastic tangent modulus (steel) of stirrup}\]
\[A_s = \text{area of cross-section of the stirrup}\]
\[L_s = \text{length of the stirrup}\]

These elements can be connected between any concrete nodes and the spring linkages, used on the longitudinal steel, would not be required to describe the behaviour of the stirrup steel-concrete interface if the stirrup is sufficiently anchored. Large steel forces and large relative displacements between the steel and concrete are present for the longitudinal steel but not for the stirrups, and thus the spring linkages will not be required for the latter.
C. FAILURE CRITERIA

1. **Steel**

Failure criteria for steel can be established by using the stress-strain curve as a basis. Principal strains in the elements are compared with the stress-strain curve to determine an appropriate value for the tangent modulus. When the principal strain in any steel element exceeds the yield strain, a nominal value (10 psi.) for the tangent modulus is used in the elasticity matrix. The presence of a large number of yielded elements could lead to an ill-conditioned stiffness matrix. However, since only one or two elements yield in a given loading step, this ill-conditioning of the matrix is eliminated. Similarly, when the principal strain exceeds the strain hardening limit, the initial strain hardening modulus is used in the stress matrix. Further steel deformations are not accounted for in this analysis.

A similar failure criterion is used for stirrups, and their contribution is neglected after they yield. The stress-strain curve of the steel obtained experimentally, and used in the analyses, is shown in Fig. 5.

2. **Concrete**

The procedure used to describe the structural behaviour of the concrete elements is similar to that used for the steel elements. The concrete stress-strain curve will be used to determine a value for the tangent modulus for use in the elasticity matrix. In addition,
appropriate failure criteria will define the structural capacity of an element. The concrete stress-strain curve can be described by Saenz's equation (17), plotted in Fig. 6 along with a curve obtained experimentally. Saenz's equation is:

\[ f = \frac{E_c \varepsilon}{1 + (R + R_2) \left( \frac{\varepsilon}{\varepsilon_o} - (2R - 1) \left( \frac{\varepsilon}{\varepsilon_o} \right)^2 + R \left( \frac{\varepsilon}{\varepsilon_o} \right)^3 \right)} \quad (7) \]

where

\[ R = \frac{R_E (R_E - 1)}{(R_E - 1)^2} - \frac{1}{R_E} \]

\[ R_E = \frac{E_c}{E_o} \]

\[ R_f = \frac{f'_c}{f_f} \]

\[ R_e = \frac{\varepsilon_f}{\varepsilon_o} \]

\( f \) = concrete stress

\( \varepsilon \) = concrete strain

\( \varepsilon_o \) = concrete strain corresponding to maximum stress

\( f'_c \) = maximum concrete strain

\( \varepsilon_f \) = maximum concrete strain at failure

\( f_f \) = concrete stress at maximum strain

\( E_c \) = initial tangent modulus

\( E_o \) = secant modulus \( = \frac{2f'_c}{\varepsilon_o} \)

Substitution for \( R \) in equation (7) yields:
Using Saenz's equation it is then possible to determine a tangent modulus, for use in the \((m+1)\)th loading step, from strains obtained in the \(m\)th loading step. For elements in which the largest principal strain is positive, the initial tangent modulus is used, and is kept constant throughout the loading sequence.

Termination of the constitutive relationship requires the formulation of failure criteria for concrete under a biaxial stress condition. The establishment of a general failure criterion for concrete has presented researchers with a number of problems, but notable work has been carried out by Bresler and Pister (18) and McHenry and Karni (19). However, for this analysis, the lack of sophistication of the finite element formulation nullifies the advantages of using a more accurate failure criterion. A concrete element is considered to have failed in tension if the principal stress at its centroid reaches the modulus of rupture (Fig. 7).
The thickness of this element is set equal to zero, for subsequent loading increments. Concrete is considered to have failed in compression if the principal compressive strain exceeds the maximum permissible concrete strain $\varepsilon_f$ (Fig. 6). The nodal forces on the element which has failed in tension or compression are applied to the nodes of the adjacent elements and redistributed into the remaining structure in the following load increment (Fig. 8).

3. **Bond**

Criteria for failure at the steel-concrete interface consist of the determination of the load-deformation characteristics in pullout and dowel actions and the subsequent evaluation of values for spring constants $K_h$ and $K_v$. The pullout stiffness $K_h$ may be approximated from pullout tests similar to those conducted by Bresler and Bertero (20). Although realistic values will depend on the type of bar, bar spacing, the type of concrete, and the amount of concrete cover, the following non-linear relationship, derived by Nilson (2), is used in this investigation.

$$u_{bs} = 3606 \times 10^3 d_{bs} - 5356 \times 10^6 d_{bs}^2 + 1986 \times 10^9 d_{bs}^3$$

(9)

where $u_{bs} =$ local 'bond' stress (psi)

$d_{bs} =$ local 'bond' slip ($x \times 10^{-6}$ inches)

from which

$$K_h = \frac{d_{u_{bs}}}{dd_{bs}} M^b L^b n^b d_b$$
where $M_b$ = the number of bars in the cross section

$L_b$ = the length over which each spring acts

$d_b$ = diameter of the bar

The 'bond'-slip equation (9) was derived from experimental results obtained by Bresler and Bertero (20). These particular tests used 16 inch long by 6 inch diameter specimens containing one No. 9 bar. Relative displacement between the concrete and reinforcement, at the face of the concrete, was measured along with steel strains at locations inside the bar. From these results, and an assumption of variation of slip between the concrete face and the center line of the specimen, an equation for the concrete displacement was derived. Using this equation, and the steel displacements found from the strain readings, values for the local 'bond'-slip were obtained. Nilson then used the method of least squares to fit a third degree polynomial to the data, and obtained equation 9.

The measurement of the relative displacement between the steel and concrete is extremely complicated. The validity of using relationships derived from large specimen lengths and applying them to spring linkages over very small lengths, is open to question. However, Nilson's equation will be used for this analysis, but could be replaced when a more improved relationship becomes available in the near future (21).

Failure criteria to be established from Nilson's 'bond'-slip curve (Fig. 9) fall into two categories. When the local 'bond'-slip
exceeds $449 \times 10^{-6}$ inches, the 'bond'-slip curve begins to descend and $K_h$ becomes negative. As the significance of this portion of the curve is open to question, a slip limit of $449 \times 10^{-6}$ inches will be used as the termination point of the constitutive relationship. Thus, for spring elements at or near an open face of concrete, it is likely that the 'bond' stress will drop to zero when the relative displacements exceed the slip limit. This would be attributed to the presence of high tensile splitting forces in the area, finally leading to the pulling out of a wedge of concrete at the concrete face. However, for spring elements far removed from the open face, confinement prevents such cracking and the shearing action of the lugs will prevent the 'bond' stress from diminishing.

The effect of the failure of both exterior and interior springs is similar, with respect to the overall behaviour of the structure and the mechanics of the force redistribution are identical. If the relative displacement, between concrete and steel nodes connected by a spring, exceeds the slip limit, the spring force is calculated, and reapplied to the structure in the following loading step (Fig. 10). For further loading increments the stiffness of the pullout spring is set equal to zero.

Although it is possible to obtain a $K_h$ value for any displacement, a problem arises if displacements from the (m)th loading step are used to determine a stiffness value for the (m+1)th loading step. This direct procedure will lead to a diverging 'bond'-slip curve (Fig. 11) and a stiffer 'bond'-slip relationship. To bring the
computed 'bond'-slip relationship close to the theoretical curve a predictor formula must be employed. This involves the addition of a fraction of the (m)th load displacement increment to the total displacement to obtain a possible slip value for the (m+1)th loading step.

$$d_{m+1} = d_m + C(d_m - d_{m-1})$$ (10)

where

- $d_m$ = the relative nodal displacement at the end of the (m)th loading step
- $d_{m+1}$ = the possible relative nodal displacement at the end of the (m+1)th loading step
- $C$ = a constant
- $d_{m-1}$ = the relative nodal displacement at the end of the (m-1)th loading step.

This predicted slip value is then used to obtain a stiffness value to be used in the (m+1)th loading step. The value of C will be dependent on the loading procedure employed, but a value of 0.7 is used in this analysis. This value was used by both Nilson (2) and Mufti et al (22) in the analysis of pullout tests.

A failure criteria for relative displacements perpendicular to a reinforcing bar is even more difficult to determine. The problem is similar to that involved in the determination of a pullout stiffness, in that an experimental procedure must be developed to measure the relative displacement along the concrete-steel interface. At present there is insufficient data available, however, experimental research is underway (21,23) to derive suitable failure criteria and a force-displacement relationship which will yield the values of $K_v$. 
necessary for computational purposes. For this analysis no failure criteria are used and the dowel stiffness is kept constant. This constant value is proportional to the initial tangent modulus of concrete and dependent on the number of bars, the diameter of the bars and the width of the member.

\[ K_v = \frac{E_c M_b d_b L_b}{b} \]  

(11)

where

- \( E_c \) = the initial tangent modulus of concrete
- \( M_b \) = the number of bars in the cross section
- \( d_b \) = the diameter of the bars
- \( b \) = the width of the member
- \( L_b \) = the length over which the spring acts.

D. CRACK STABILITY

The modification of the structure stiffness matrix before the application of each loading increment leads to a problem which could be classified as one of crack stability. If a structure, whose stiffness matrix has been altered, either by removal of elements or changes in \( E_c \), is reanalyzed at the same load, the deflection of the revised structure will differ from the value obtained from the first analysis. If new elements break, the initial crack configuration would have been classified as an unstable crack pattern. This concept was not included in this analysis. The difference between the load-deflection curve obtained from the two analyses is insignificant if small loading steps are used. It may be noted, that
using a small loading increment would cause only two or three elements to fail and would have an insignificant effect on the structural stiffness. The consequent effect on the redistribution of forces would also be minimal.

E. COMPUTER PROGRAM

The computer program used in this analysis is coded in Fortran IV, G. Level, and is compatible with the O/S system of the I.B.M. 360/75 computer. A listing appears in the Appendix along with the requirements for the input data. In the form presented the program is capable of analyzing a structure with 325 nodes, 450 elements, 20 boundary conditions, 120 spring elements, and 40 stirrups, requiring a core storage of over 200,000 bytes.

Equations of equilibrium are solved using the method of tridiagonalization where inversion of the individual sub-matrices is performed using Cholesky's method.

The computer input consists of the following:

1. Structure parameters
2. Nodal coordinates
3. Element parameters
4. Boundary conditions
5. Initial material properties
6. Initial loads
7. Loading steps and increments
8. Spring parameters
9. Stirrup parameters
10. Concrete stress-strain curve constants
11. Steel stress-strain curve constants.

The computer output consists of the following:
1. Reprint of input data
2. Spring stiffness values
3. Nodal displacements
4. Residual forces
5. Stress, strain and elastic modulus of each triangular element
6. Pullout stresses, pullout forces and local displacements of each spring element.

The program is an extended version of the one developed by Mufti et al. (22), and a flow chart appears on the following page.
checking of failure criteria

All sustain failure criteria

To Global Stiffness Matrix

Stirrups have not failed in this step or previously

Add triangular element stiffness matrices to global stiffness matrix

Add stirrup stiffness matrix to global stiffness matrix

Bond linkages have not failed in this step or previously

Solve for incremental displacements

Add incremental stresses and strains to total stress and strain matrices

Zero load matrix

Zero global stiffness matrix

Redefine element thickness or tangent modulus make contributions to load matrix, if any

Triangular elements have failed in this loading step

Zero stiffness of broken springs make contributions to load matrix

Bond springs have failed in this loading step

Printout

N < Nsteps

N = N + 1

Add load increment to load matrix

STOP

Computer program flow chart
CHAPTER III

EXPERIMENTAL PROCEDURE

A. GENERAL

The question of whether to test beam-column joints isolated at points of inflection, or complete frames is difficult to answer. In the testing of frames the failure mechanism can be a beam mechanism, a sidesway mechanism, or a combination of these mechanisms. Adenot (9) tested two bay portal frames which were subjected to a constant vertical load and an increasing horizontal load. Using a 1/6-scale model (Fig. 12), he obtained load-deflection curves, joint rotation curves, and the ultimate load for different column steel percentages. The frames tested were designed in such a way that the failure mechanism was one of sidesway.

Under any set of loads, joint E2 (Fig. 12) may be idealized as a pin connected L-shaped member if it is isolated at inflection points (Fig. 13). The increasing horizontal load, however, has the effect of moving the position of the inflection points away from the joint. Assuming a joint moment-rotation curve as shown in Figure 14 it can be seen that the relative difference between $M_u$ and $M_y$ will not make an appreciable difference in the position of the inflection points. The position of the inflection
points at the yield moment was therefore used to establish the
dimensions of the isolated joint.

Hognestad (24) has developed equations for the yield moment
of a section, containing tension and compression reinforcing, sub-
jected to combined bending and axial loading. Using Ritter's
parabola:

\[ f_c = \varepsilon E_c \left( 1 - \frac{\varepsilon}{E_c} \right) \]

(12)

and assuming a tension failure (Fig. 15) an equation for the
yield moment may be derived.

\[ M_y = \frac{1}{2} f_c b k d \left( d - \frac{kd}{3} \right) + A_s f'_s \nu d \]

(13)

The equilibrium of forces yields the equation:

\[ P + A_s f_y = A'_s f'_s + \frac{1}{2} f_c b k d \]

(14)

and the strain distributions (Fig. 15) leads to the following
equations:

\[ f'_s = \varepsilon'_s E_s = \varepsilon'_s E_s \left( \frac{\gamma d - (d - kd)}{d - kd} \right) = f_y \left( \frac{\gamma + k - 1}{1 - k} \right) \]

(15)

\[ \frac{\varepsilon_c}{\varepsilon_s} = \frac{kd}{d - kd} = \frac{k}{1 - k} \]

(16)
Substitution of equations 15 and 16 into equation 14 results in:

\[
\left( f_c \frac{\varepsilon_s}{\varepsilon_o} b d \right) k^2 + \left( 2A_s f_y + P \right) k + A_s f_y (y_s - 1) - P - A_s f_y = 0 \quad (17)
\]

Solving for \( k \) and substitution into the moment equilibrium equation (13) gives a value for the yield moment for a given applied load.

A computer analysis, employing the stiffness method, was performed on a similar frame but at 1/3 scale. For a monotonically increasing horizontal load, axial loads in column E1-E2 (Fig. 12) were calculated, thus giving a corresponding yield moment. The horizontal load was increased until the moment at joint E2 exceeded the calculated yield moment, signifying the formation of a hinge. The inflection points were then noted for various column steel percentages and steel yield strengths (Fig. 16). The ultimate concrete strength \( f'_c \) for these calculations was maintained constant at 4000 psi. Using a 38.5 ksi steel (Fig. 5) and a 4000 psi concrete (Fig. 6) a relationship was found for the 'X' dimension of the knee for varying percentages of column steel (Fig. 17). Letting the 'Y' dimension remain constant at twenty two inches, the midpoint of the column, it was thus possible to establish dimensions for an isolated joint for any percentage of column steel reinforcing. Using this criteria, joints were isolated to be tested under a load, applied along the line joining the inflection points (Fig. 13).
B. EXPERIMENTAL SETUP

1. Specimens

The analysis of the 1/3 scale portal frame resulted in the standardization of the following structural parameters:

1. A beam and column width of four inches
2. A column depth of five inches
3. A beam depth of eight inches
4. Steel percentage in the beam stub = 2.05%
5. Concrete cover in the column stub = 1/2 inch
6. Concrete cover in the beam stub = 1 inch
7. Length of the column stub measured along the centerline = 22 inches.

The reinforcing bars were welded to a 1/4 inch thick steel plate at the ends of the column and beam stubs. Closed stirrups were provided at extreme ends of the stubs to prevent any local failure. However, the zig-zag shear reinforcing (Fig. 18), consisting of two D2.5 bars, was used throughout the entire column stub to prevent the formation of a confined concrete section in the region of the hinge.

The stress-strain curve of the steel used, is given in Fig. 5. A comparison of the experimental stress-strain curve obtained for concrete to the curve of Saenz's equation is shown in Fig. 6. The experimental curve was obtained by testing 3" by 6" concrete cylinders in a constant strain Instron machine. The machine
speed was .05 inches/min. and strains were averaged from the readings of four strain gauges placed symmetrically around the cylinder. Saenz's equation gave a maximum concrete stress of 4000 psi. at a strain of $1800 \times 10^{-6}$ in./in. while the experimental value of the maximum concrete stress was 3930 psi. at a strain of $1959 \times 10^{-6}$ in./in.

The question of whether to base the concrete tensile failure criterion on the tensile strength ($f_{ct}$) or the modulus of rupture ($f_r$) is difficult to answer. The modulus of rupture is used for this analysis since the members analyzed, were subjected to loadings which produced linearly varying stress fields. For the analysis of pullout test specimens, which have a constant stress field across the section, the direct concrete tensile strength would be a more appropriate failure criterion. The modulus of rupture of concrete was obtained by testing simply supported plain concrete beams with a cross section identical to that of the column stub. These beams were twenty inches long and were loaded at 1/3 points to produce a region of constant moment. The modulus of rupture was noted to be 640 psi. and was the average of tests on ten beams.

A joint containing 1.42% of steel in the column stub (Fig. 18) was tested to form a basis for comparison with the results obtained from the finite element analysis. Further tests included joints containing 1.79%, 2.14%, and 2.84% of steel in the column stub.
2. Loading and Measurement

The specimens were tested in an Instron constant strain machine. The load was applied through pillow blocks to prevent any shearing force across the section. The pillow blocks were bolted to the end plates by cast-in-place bolts (Figs. 18, 33).

Loads and machine head deflections were recorded automatically on the Instron recorder, while the relative displacements across the pillow blocks and the end plates were measured by dial gauges, with a least count of .0001 inches (Fig. 18). Hereafter, the joint deflection will be referred to as the relative displacement between the center-lines at the ends of the beam and column stubs.

Strain gauges were placed on the tension and compression steel in the column section, and on the column compression face, and along one side of the column stub (Fig. 20). All gauges were located so that their strain readings could be compared directly to the discrete point results obtained from the finite element analysis. Gauges were of 5mm. gauge length and were either the PL-5 or the PS-5 type. Strain readings were obtained using an automatic electronic strain recorder giving instantaneous results before any substantial load decrease occurred.
3. Experimental Results

The experimental load-deflection curve of the joint, with 1.42% of steel in the column stub, is given in Figure 31. Deflections of the other three joints are presented in Table I. The moment-rotation curve for the joint (1.42% steel), is shown in Fig. 30. The failure of the joint was characterized by the formation of three distinct cracks (Figs. 32a, 34). The initial crack, labelled 'A', formed at a load of 2400 lbs. and propagated at a relatively constant rate until a load of 3500 lbs. was reached, where it ceased to propagate. The second crack, labelled 'B', began to propagate at a load of 3150 lbs. and continued to propagate past the load at which the tension steel yielded. The third crack, labelled 'C', began to propagate when the tension steel first yielded and did not stop until a displacement of .058 inches was attained. The first visible signs of compression failure in concrete appeared at a displacement of .089 inches (Fig. 31). The formation of the compression hinge, is defined by the formation of the two primary cracks 'A' and 'B'. It can be seen that the failure occurs within a very localized region in the column stub, extending approximately three inches from the face of the beam stub.

The results of the experimentally measured concrete and steel strains, at various load levels, are presented in Table II and indicate:
1. That the tension reinforcement in the column stub yielded at a load of 3500 lbs.

2. The strains on the concrete compression face of the column stub never approached a failure value at the yield load.

3. A stress concentration is present in the region of the joint corner. The great differences between gauges No. 8, 13 and 9, indicate the presence of large strain gradients in this area, even in the elastic range (load = 2400 lbs).
CHAPTER IV

ANALYSIS OF RESULTS

A. GENERAL

Two finite element analyses were conducted to determine the feasibility of formulating an analytical model of reinforced concrete by the finite element method. The first analysis was conducted on a simply supported beam, as it is a more general problem, and to make qualitative comparisons with the work by Ngo and Scordelis (1) and Ngo, Franklin, and Scordelis (3). The second analysis was performed on a joint identical to the one described in the experimental procedure (with 1.42% steel in the column stub). The purpose of this analysis was to make quantitative comparisons between theory and experiment, throughout the entire loading sequence.

B. FINITE ELEMENT IDEALIZATION

The beam analyzed was simply supported, contained only tension reinforcement, and was subjected to two, symmetrically placed, loads. For the analysis of the beam, symmetry was used to obtain a finite element grid of 176 elements and 120 nodes (Fig. 19). This finite element idealization gave an error in the elastic deflection of 12.8%, when compared to a transformed section elastic
analysis. Material properties for this analysis were identical to those obtained experimentally for the joint tests.

The finite element grid for the joint consisted of 312 nodes, 432 elements, 118 spring elements, 10 stirrups and 2 boundary conditions (Fig. 20). Accuracy of such a grid was checked by the use of a plain concrete idealization, consisting of 186 nodes and 314 elements using both a single precision and a double precision analysis. A plain concrete grid of 364 nodes and 636 elements was also used as a check for accuracy. The results of these analyses appear in Table III. It can be seen that the analysis by a double precision program did not improve the accuracy significantly above the single precision results. The results from the fine grid gave only 6.5% increase in deflection. It was noted that the added cost of either the fine grid or the double precision analysis did not warrant the increased accuracy.

Simulation of the anchorage of the reinforcing bars to the steel end plates was accomplished by giving the exterior bond links an increased relative stiffness. For this analysis the effective lengths of each bond linkage was quadrupled. The starting loads for both analyses were established by finding the initial crack load. This load is defined as the load which would cause a single concrete element to fail in tension. Loading increments were kept as small as was economically possible, with a maximum of two or three elements breaking in a particular loading step.
C. ANALYSIS OF A SINGLY REINFORCED BEAM

According to the A.C.I. Building Code 318-63 the simple beam analyzed (Fig. 19) had a yield load of 2170 lbs. and an ultimate shear capacity of 2140 lbs. An elastic analysis indicated that the initial cracking load was 1125 lbs. Loading increments were set at 100 lbs. The computed load deflection curve obtained (Fig. 21) exhibits a yield load of 2025 lbs. The first steel element to exceed the yield stress was on section 2 (Fig. 19). The final crack pattern indicates that the beam failed in a flexural mode.

The stress and bond force distributions (Figs. 22-26) indicate the following:

1. Two primary cracks formed within the loading sequence. The first crack formed within the region of constant moment (section 2) at a load of 1125 lbs., while the second appeared at section 4 at a load of 1425 lbs. The rate of progression of both cracks was identical after the initial breakage. The formation of the first crack at section 2 could be attributed to the inaccuracy of the finite element formulation. Although sections 1, 2 and 3 are in a region of constant moment where the compressive and tensile extreme fiber stresses should be equal, the nearness of the centerline boundary conditions and the applied load would produce slight discrepancies.
2. A third crack appeared, between the two primary cracks at section 3 at a load of 1525 lbs. This crack was internal throughout the loading sequence, and propagated further until a load of 1625 lbs. The cracks formation near the longitudinal steel and the fact that extreme bond failure occurred at this load level indicates that its presence can be attributed to a form of longitudinal splitting.

3. The principal stresses at the tip of the two primary cracks are shown in Fig. 27. There is a general indication that the principal stress at the crack tip decreases as the crack propagates upwards. This would indicate that crack stability increases as the crack propagates towards the compression zone.

4. Steel longitudinal stresses increase significantly around crack locations, but the effect is localized. This increase could amount to over 200% when compared with an elastic stress distribution (Fig. 23).

5. 'Bond' forces also increase significantly around the cracks, but reverse in direction across the crack. The percentage increase is greater than that of the steel stresses as shown in Fig. 24. Here, a spring next to the crack possessed a 'bond' force of 2434 lbs. while its immediate neighbour possessed a 'bond' force of only 200 pounds.
6. The position of the neutral axis rises significantly as the applied moment increases. At the initial cracking load the neutral axis was at an average of ten inches from the compression face of the beam. At a load of 2025 lbs., the yield load (Fig. 25), the neutral axis was within 25% of the depth from the compressive face at some sections.

7. At or near the yield load, 'bond' failure is complete in regions of maximum moment. However, in regions of minimum applied moment, 'bond' forces remain small.

8. Increase in steel transverse shear forces (dowel forces) is also localized around cracks. This force amounted to approximately ten percent of the total shear force in the beam tested. For the elastic region, at a load of 1125 lbs., the force amounted to less than two percent of the total shear force.

9. Results are given for the concrete shear stresses on section 5. The general form of the distribution is parabolic as predicted by the elastic theory. However, at a load of 1725 lbs. (Fig. 24), it can be seen that the shear forces have reversed direction near the tension face.

10. A concrete element failed in compression at a load level of 2325 lbs., the final loading step. The occurrence of this failure is questionable since the primary cracks had progressed to such an extent that only two concrete elements
remained in section 2 at a load of 2225 lbs. All forces required to keep the beam in equilibrium had to be transmitted through these remaining elements and failure was inevitable.

11. The steel element that yielded at a load of 2025 lbs. reached the strain hardening stage in the following loading step. Stresses in this element had reached 40 ksi in the final loading step.

Two methods were used to obtain moment-rotation characteristics of the beam. The first method used, defined beam rotations, as the rotation of the elements along the compression face. In this method, therefore, rotations were calculated directly from the nodal displacements. The second method derived rotations by finding the area under a calculated beam curvature curve. Curvatures of individual sections were established by dividing the compressive strains of the extreme concrete element, by the depth of the neutral axis. Since a constant stress finite element was used in this analysis, the strain at the element centroid would be equal to the strain near the element boundary. Results of the two methods are presented in Tables IV and V and in graphical form in Figs. 28 and 29, for sections 3 and 10 of the beam.

It can be seen that both methods give comparable results until loading step 7. After this loading step discrepancies arise with the direct method giving larger rotations than the curvature method. At
the yield load this increase amounted to 24% for section 10 and 27% for section 3. The discrepancy could be explained by the fact that the strain at the centroid of the extreme element is an underestimate of the actual extreme fiber strain. This underestimate would become more pronounced as the neutral axis moved upward and increased the strain gradient.

D. ANALYSIS OF A BEAM-COLUMN JOINT

The reinforced concrete joint, described previously, was also analyzed to check the capabilities of the finite element program. In this case, however, comparisons between the experimental and the analytical results were of prime importance. The load-deflection curve of both the experimental and analytical specimens appear in Fig. 31. The load at which tension steel first yielded was 3500 lbs. for the experimental specimen and 3415 lbs. in the computer analysis. The initial crack load for the analysis was 2415 lbs., after which, loading increments of 100 lbs. were added until a maximum load of 3715 lbs. was reached. The initial elastic deflection of the finite element analysis was approximately 18 percent smaller than the deflection for the experimental specimen, however, initial cracking occurred in both cases at approximately the same load level. It may be noted that a finite element idealization, using conforming elements, results in a lower bound solution and this would lead to the discrepancy noted.
A comparison of the experimental and analytical crack patterns appears in Fig. 32. Three primary cracks appeared in the experimental specimen, however, the crack labelled 'C' did not appear until after the tension steel had yielded. The crack appearing first, labelled 'A', appeared in both specimens at a load level of approximately 2400 lbs. The location of the crack and its propagation towards the inner corner of the joint was similar in both specimens. In both the analysis and the experimental specimen this crack had stopped propagating before the steel had yielded. The second crack, labelled 'B', appeared in both specimens, at similar load levels, with initial cracking occurring in the experimental specimen at 3150 lbs. and in the analytical specimen at 3315 lbs. However, the crack in the analytical specimen never propagated to the same depth as the crack of the experimental specimen. This could be attributed to the fact that at a load level of 3215 lbs., concrete compression failure had occurred in section 1-1. This resulted in a complete absence of any concrete elements at this section, leading to a discontinuity in the structure. As was shown in the analysis of the beam, the principal stress at the tip of a crack, tended to decrease as the crack propagated towards the compression zone. Principal stresses at the tip of crack 'A' are shown in Fig. 35.

Element stresses and bond forces are shown in Fig. 36 through 39 and show similar trends as were seen in the beam results and are as follows:
1. Localized increase in steel longitudinal stress and bond forces along with the reversal of bond forces across a crack (Fig. 36).

2. A significant rise in the position of the neutral axis as the cracking progressed (Fig. 38).

3. The presence of small bond forces at the ends of the bars, even when total bond failure had occurred in the regions with severe cracking of concrete.

A concrete compression failure occurred in the analytical specimen in the corner elements (Elements 227-228, Fig. 32). This failure appeared at a load of 3015 lbs. and a deflection of .0254 inches while the experimental compression failure did not occur until the deflection value reached .089 inches. The complete separation of the concrete elements along this section undoubtedly affected the behaviour of the analytical specimen beyond this load level. The cause of the failure at such a low load could be attributed to the presence of a stress concentration at this corner. This would cause large strain gradients which could not be defined by a plane stress finite element unless a very fine grid was used. The effect of the presence of the compression steel on the transverse steel stresses is indicated in Fig. 40. It can be seen that when the crack tip was in the region of the tension steel, the tension steel received a greater portion of the shear force. However, as the crack propagated towards the compression face, transverse stresses in the compression steel increased, while those in the tension steel decreased.
realized that most of these results can be accepted only qualitatively, since crack width and aggregate interlock relationships will also influence the final results.

Moment-rotation curves of the joint, analytical and experimental, appear in Fig. 30 (Table VI) while rotations of the column compression face are presented in Fig. 41. In both cases analytical results were obtained directly from nodal point displacements. Rotation of the column compression face elements (Fig. 41) indicate the localized nature of the hinge. It can be seen that the rotations increase almost linearly from section 7-7 to section 2-2. At this point the rotations increase drastically with rotations at section 1-1 becoming counter-clockwise.

The load-deflection curve obtained from the computer analysis indicates that the introduction of cracks (and the resulting deletion of elements) decreased the stiffness of the structure significantly. The large discrepancies in deflections is reflected in the poor correlation between the experimentally recorded strains and the analytical element strains (Table II). Therefore, if accurate strain readings are to be obtained, a more refined analytical model must be formulated to produce more accurate load-deflection curves. In the case of this analysis the errors in deflections increased with the introduction of non-linearities. Determination of those variables which contribute to the greater portion of this error is a problem for future research. The final two points on the load-deflection curve were plotted for completeness,
but their validity is questionable since the analytical structure had reached a badly disintegrated state by this time.

E. HINGE ROTATION

The analytical model used in this analysis was not capable of fully developing a reinforced concrete hinge as the model generally broke down after a member reached its yield load. In the loading sequence prior to the yield load the model was able to produce the following results characteristic of the impending formation of a hinge:

1. An increase in member curvature in the area of the impending hinge (Table V).

2. The localized increase in longitudinal steel stresses, crack patterns, and movement of the neutral axis towards the compression face.

An analytical model capable of dealing with the formation of a hinge will require more refined technique in dealing with concrete compression failures and steel elongation into the strain hardening range.

F. CONVERGENCE OF 'BOND'-SLIP AND CONCRETE STRESS-STRAIN CURVES

A number of graphs are given (Fig. 42) to compare the convergence of the concrete stress-strain curves and 'bond'-slip curves obtained experimentally and also plotted from the computer output. Two concrete elements and two spring elements were taken from both
the beam and the joint analyses to illustrate the problem. In Fig. 42a and 42c a satisfactory correlation, for both the beam and joint, can be seen. Inaccuracies do arise as the stresses approach $f'_c$, but general compatibility of strains (within $\pm 10\%$) can be noted. Fig. 42b shows that significant error has accumulated, up to 30% in strains, even before the stress in the element reaches $0.5 f'_c$. Results for this element are recorded for the complete loading sequence (13 steps). In Fig. 42d, element strains never reached failure, but stresses had exceeded $f'_c$. In other words, a diverging stress-strain curve had been produced. The preceding results indicate that a predictor formula should be used for the determination of tangent moduli, but its form will probably depend on the boundary conditions of the particular element. Use of a multi-linear stress-strain curve is another alternative.

'Bond'-slip correlation was more satisfactory in both analyses, with inaccuracies being apparent only near the slip limit (Fig. 42 e,f,g,h) and the maximum difference in the experimental and the computed slips being 28.5% at the slip limit.
A. CONCLUSIONS

The finite element method has been shown to be a useful tool in analyzing the complicated behaviour of reinforced concrete. Individual behavioural patterns can be defined explicitly to form an integrated analytical model. Loading history can be accounted for if an incremental loading procedure is adopted. Load-deflection curves, moment-rotation relationships, stress distributions, and crack patterns can be obtained from the computer analysis results. A number of results are obtainable from a finite element analysis which would not be as easily derived experimentally. However, more research is needed to establish confidence in these results.

Quantitative comparisons between the analytical results and the experimental results indicate that:

1. The computed yield loads were slightly lower than the experimental values.

2. In the non-linear range, the analytical model gave an overestimate of deflections, leading to corresponding overestimates of strain readings.

3. Steel longitudinal strains increased at a greater rate in the experimental specimen. This would indicate that the 'bond'-slip relationships were not compatible and more refined relationships are required.
Although the accuracy of the present model was questionable, with respect to deflections in the non-linear range, a number of important qualitative results could be obtained.

1. An indication of the relative magnitude of the dowel force.
3. The discrepancies between the moment rotation capacities, found from the moment curvature method, and the direct method.
4. A general description of the rate of bond failure and its subsequent effect on the steel stresses.
5. The model could indicate the localized nature of the impending formation of a reinforced concrete hinge.

An analytical model should be able to establish accurate moment-rotation characteristics of members, well into their non-linear range of behaviour, and even to the descending portion of the load deflection curve. The present model did not have these capabilities and requires refinement if the behaviour of reinforced concrete hinges is to be studied more thoroughly.

B. FUTURE CONSIDERATIONS

In this type of analysis it is clear that a rational failure criteria for concrete, under biaxial stress (tensile and compressive) is required. This will also entail the establishment of a more refined method of forming cracks and dealing with compression failure. As far
as crack formation is concerned, a number of methods are possible. One alternative is to define strains at element nodes and then define failure criteria with respect to these strains. If the strains at a particular node exceed the failure criteria a bifurcation of the node could be performed (Fig. 43). This would give a value for the crack width from which the aggregate interlock stiffness can be defined. With the additional use of quadrilateral elements the entire method would become more accurate (30). One drawback to this procedure is that cracks must follow a predefined mesh pattern.

An alternative method is to define a crack within the element and then split the element in two if element strains exceed the tensile failure criteria (Fig. 44). The basic problem with this method is that an adjacent element will have to assume some temporary configuration. Cracks, however, would then be able to propagate at any angle.

The two ideas just presented are easily visualized but present some formidable programming problems. The primary problem concerns the increase in the number of degrees of freedom in the stiffness matrix. This necessitates the renumbering of both the nodes and the elements. In addition, spring linkages must be dynamically introduced to describe the aggregate interlock relationships.

With more refined models, errors in the non-linear deflections would decrease. With the improved accuracy and the use of strain controlled loading, moment-rotation characteristics of hinges could be obtained throughout the entire loading sequence.
BIBLIOGRAPHY


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### Table I

**DEFLECTIONS OF JOINT - EXPERIMENTAL**

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**ALL DEFLECTIONS IN INCHES**

1. Initial Appearance of First Crack
2. Initial Appearance of Second Crack
3. Initial Appearance of Third Crack

\(^{(w)}\) Yield Load of Joint

### Table III

**ELASTIC DEFLECTIONS OF JOINT FOR CHECKING ACCURACY OF FINITE ELEMENT ANALYSIS**

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<td>3-3</td>
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<td>809(26.2)</td>
<td>842(24.4)</td>
<td>860(22.4)</td>
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Fig. 1 TRIANGULAR FINITE ELEMENT

Fig. 2 TRANSFORMATION OF REINFORCED CONCRETE SECTION

Fig. 3 SPRING LINKAGE ELEMENT

Fig. 4 STIRRUP ELEMENT
Fig. 5 STRESS-STRAIN CURVE OF STEEL

Fig. 6 STRESS-STRAIN CURVE OF CONCRETE
Fig. 7 FAILURE CRITERIA FOR CONCRETE UNDER BIAXIAL STRESSES

Fig. 8 REDISTRIBUTION OF ELEMENT NODAL FORCES AT FAILURE

Fig. 9 NILSON'S BOND-SLIP CURVE
Fig. 10 REDISTRIBUTION OF SPRING NODAL FORCES AT FAILURE

Fig. 11 DIVERGING BOND-SLIP CURVE

Fig. 12 ADENOT'S TWO BAY PORTAL FRAME
Fig. 13 ISOLATED BEAM-COLUMN JOINT

Fig. 14 ASSUMED MOMENT-ROTATION CURVE OF JOINT

Fig. 15 HOGNESTAD'S STRESS AND STRAIN DISTRIBUTIONS FOR COLUMN SECTIONS
Fig. 16 LOCATION OF INFLECTION POINTS VS. STEEL PERCENTAGE AND YIELD STRESS

Fig. 17. CENTER LINE LENGTH OF COLUMN STUB FOR STEEL YIELD STRESS OF 38.5 KSI.
Fig. 18 JOINT DETAIL
(COLUMN STEEL PERCENTAGE = 1.42%)
Fig. 20 FINITE ELEMENT IDEALIZATION OF JOINT WITH LOCATION OF STRAIN GAUGES
Fig. 21 LOAD-DEFLECTION CURVE FOR THE SINGLY REINFORCED BEAM

Concrete compression failure

Steel element reaches strain hardening

$R_y = 2170$ lbs

$R_y = 2025$ lbs

First steel element yields

Initial formation of first crack

Initial formation of second crack

Interior cracking
Fig. 22 STRESS AND BOND FORCE DISTRIBUTIONS OF BEAM AT LOAD OF 11,25 LBS.
Fig. 23 STRESS AND BOND FORCE DISTRIBUTIONS OF BEAM AT LOAD OF 1425 LBS.
Fig. 24 STRESS AND BOND FORCE DISTRIBUTIONS OF BEAM AT LOAD OF 1725 LBS.
Fig. 25 STRESS AND BOND FORCE DISTRIBUTIONS OF BEAM AT LOAD OF 2025 LBS.
Fig. 26 STRESS AND BOND FORCE DISTRIBUTIONS OF BEAM AT LOAD OF 2325 LBS.
Fig. 27 PRINCIPAL STRESS AT TIP OF CRACKS FOR SIMPLE BEAM

Fig. 28 MOMENT-ROTATION CHARACTERISTIC FOR SECTION 10 (BEAM)
Fig. 29 MOMENT-ROTATION CHARACTERISTIC FOR SECTION 3 (BEAM)

Fig. 30 MOMENT-ROTATION CURVE OF JOINT (COLUMN STEEL PERCENTAGE = 1.42%)
Fig. 31 LOAD-DEFLECTION CURVE OF JOINT (COLUMN STEEL PERCENTAGE = 1.42%)
Fig. 32 EXPERIMENTAL AND ANALYTICAL CRACK PATTERNS FOR THE BEAM-COLUMN JOINT
Fig. 33 Experimental Setup for Tests on Joints

Fig. 34 Typical Crack Patterns for the Joints Tested
Fig. 33 Experimental Setup for Tests on Joints

Fig. 34 Typical Crack Patterns for the Joints Tested
Fig. 35 PRINCIPAL STRESS AT TIP OF CRACK 'A' IN THE BEAM-COLUMN JOINT
FIG. 36 STRESS AND BOND FORCE DISTRIBUTIONS OF THE BEAM-COLUMN JOINT AT A LOAD OF 2415 LBS.
Fig. 37 Stress and bond force distributions of the beam-column joint at a load of 2815 lbs.
FIG. 38: TENSION AND BOND FORCE DISTRIBUTIONS OF THE BEAM-COLUMN JOINT AT A LOAD OF 3415 LBS.
Fig. 39 Stress and Bond Force Distributions of the Beam-Column Joint at a Load of 3715 lbs.
Fig. 40 TRANSVERSE SHEAR STRESS IN COLUMN
STUB STEEL BARS (SECT. 1-1)
Fig. 41 ROTATION OF COLUMN Stub COMPRESSION FACE
Fig. 42: EXPERIMENTAL AND ANALYTICAL BOND-SLIP AND CONCRETE STRESS-STRAIN CURVES
Fig. 42 (cont.)

Fig. 43 NODE SEPARATION METHOD

Fig. 44 ELEMENT PARTITION METHOD
APPENDIX

COMPUTER PROGRAM

THE LISTING OF THE COMPUTER PROGRAM, USED IN THIS ANALYSIS IS GIVEN; ALONG WITH THE REQUIREMENTS FOR THE INPUT DATA. THE VERSION PRESENTED HAS A MAXIMUM CAPACITY OF:

(1) 325 NODES
(2) 450 TRIANGULAR ELEMENTS
(3) 120 SPRING ELEMENTS
(4) 20 NODAL POINT BOUNDARY CONDITIONS
(5) 15 PARTITIONS
(6) 40 STIRRUPS
(7) 5 NODES AT WHICH LOADS MAY BE APPLIED
(8) 35 NODES IN A PARTICULAR PARTITION

COMPUTER CORE STORAGE REQUIRED IS OVER 200,000 BYTES AND FOUR DISCS ARE REQUIRED FOR AUXILIARY STORAGE.
A finite element program which analyzes reinforced concrete structures subjected to monotonically increasing loads.

INPUT DATA

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<th>NOTATION</th>
<th>DESCRIPTION</th>
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<td>NPART</td>
<td>total number of partitions</td>
</tr>
<tr>
<td>5-8</td>
<td>NPOIN</td>
<td>total number of nodal points</td>
</tr>
<tr>
<td>9-12</td>
<td>NELEM</td>
<td>total number of elements</td>
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<tr>
<td>13-16</td>
<td>NBOUND</td>
<td>total no. of nodes with prescribed displacements</td>
</tr>
<tr>
<td>17-20</td>
<td>NCOLN</td>
<td>total no. of different loadings</td>
</tr>
<tr>
<td>21-24</td>
<td>NYM</td>
<td>total no. of different elastic properties</td>
</tr>
<tr>
<td>25-28</td>
<td>NP</td>
<td>type of analysis; 1 = plane stress</td>
</tr>
<tr>
<td>29-32</td>
<td>NFREE</td>
<td>no. of degrees of freedom per element node</td>
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<tr>
<td>33-36</td>
<td>NCONC</td>
<td>total no. of concentrated loads</td>
</tr>
<tr>
<td>37-40</td>
<td>NS</td>
<td>total no. of stirrups</td>
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A. STRUCTURE PROPERTIES 10I4, ONE CARD

B. NODAL POINT ARRAY 14,2F14.5, ONE CARD PER NODAL POINT

C. ELEMENT ARRAY 5I4,2F16.9, ONE CARD PER ELEMENT

D. BOUNDARY ARRAY 3I4,3F16.8, ONE CARD PER BOUNDARY POINT
E. PARTITION ARRAY
4I4, ONE CARD PER PARTITION

1-4 NSTART(I) NO. PRETAINING TO FIRST ELEMENT IN A PARTITION
5-8 NEND(I) NO. PRETAINING TO LAST ELEMENT IN A PARTITION
9-12 NFIRST(I) NO. PRETAINING TO FIRST NODAL POINT IN A PARTITION
13-16 NLAST(I) NO. PRETAINING TO LAST NODAL POINT IN A PARTITION

F. ELASTIC PROPERTIES
5F15.5, ONE CARD PER ELASTIC PROPERTY

1-15 EL(I) MODULUS OF ELASTICITY IN X-DIRECTION
16-30 EL(I) MODULUS OF ELASTICITY IN Y-DIRECTION
31-45 P1(I) POISSON’S RATIO IN X-DIRECTION
46-60 P2(I) POISSON’S RATIO IN Y-DIRECTION
61-75 G(I) SHEAR MODULUS G

G. INITIAL LOAD ARRAY
I4, 2F14.5, ONE CARD PER CONCENTRATED LOAD

1-4 K NODAL POINT NUMBER
5-18 U(2*K-1, 1) INITIAL LOAD IN X-DIRECTION
19-32 U(2*K, 1) INITIAL LOAD IN Y-DIRECTION

H. STRUCTURE PROPERTIES
2I4, 3F10.4, ONE CARD

1-4 NELEMS NUMBER OF SPRING ELEMENTS
5-8 NSTEPS NUMBER OF LOADING STEPS
9-18 DELTAX LOAD INCREMENT IN X-DIRECTION *
19-28 DELTAY LOAD INCREMENT IN Y-DIRECTION *
29-38 BTH WIDTH OF MEMBER

I. SPRING ELEMENT ARRAY
4I4, 4F10.4, ONE CARD PER SPRING ELEMENT

1-4 I SPRING NUMBER
5-8 NODS(I, 1) CONCRETE NODE CONNECTED TO SPRING
9-12 NODS(I, 2) STEEL NODE CONNECTED TO SPRING
13-16 NSSS(I) SPRING POSITION INDICATOR; 1 = INTERNAL, 0 = EXTERNAL
17-20 WDI(I) EFFECTIVE LENGTH OF SPRING
27-36 BNI(I) ANGLE OF INCLINATION TO GLOBAL X-AXIS OF SPRING
37-40 DIA(I) AVERAGE DIA. OF BARS CONNECTED TO SPRING
47-50 BNUM(I) NUMBER OF BARS CONNECTED TO SPRING

J. SPRING ELEMENT COUNTER ARRAY
2I4

1-4 NCHK(I) ONE CARD PER PARTITION, NO. OF SPRING ELEMENTS IN A PARTICULAR PARTITION

IF NCHK(I) .NE. 0 READ ONE CARD

1-4 NSPGST(I) NO. OF FIRST SPRING ELEMENT IN PARTITION
5-8 NSPGED(I) NO. OF LAST SPRING ELEMENT IN PARTITION
K. STIRRUP COUNTER ARRAY

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******

1-4 NSTR(I) ONE CARD PER PARTITION, NO. OF STIRRUPS TERMINATING IN A PARTICULAR PARTITION

IF NSTR(I).NE.0 READ ONE CARD

1-4 NENDST(I,1) NO. OF FIRST STIRRUP IN PARTITION

5-8 NENDST(I,2) NO. OF LAST STIRRUP IN PARTITION

***************************************************

L. STIRRUP ARRAY

314,3F16.8, ONE CARD PER STIRRUP

******

1-4 K STIRRUP NUMBER

5-8 NODSTR(I,1) NODE I OF STIRRUP

9-12 NODSTR(I,2) NODE J OF STIRRUP

13-28 STIR(I,1) AREA OF STIRRUP

29-44 STIR(I,2) LENGTH OF STIRRUP

45-60 STIR(I,3) ANGLE OF INCLINATION TO GLOBAL X-AXIS

***************************************************

M. CONCRETE PARAMETERS

4F15.5, ONE CARD

******

1-15 FO MAXIMUM CONCRETE STRESS

16-30 FF CONCRETE STRESS AT MAXIMUM STRAIN

31-45 SEF MAXIMUM CONCRETE STRAIN

46-60 RUPTR MODULUS OF RUPTURE OF CONCRETE

***************************************************

N. STEEL PARAMETERS

4F15.5, ONE CARD

******

1-15 EY YIELD STRAIN OF STEEL

16-30 ESTRH STRAIN AT ONSET OF STRAIN HARDENING

31-45 E INITIAL STRAIN HARDENING MODULUS

46-60 P POISSON'S RATIO IN STRAIN HARDENING RANGE

***********************************************************

* VECTOR IN POSITIVE SENSE OF GLOBAL AXES IS CONSIDERED AS POSITIVE

***********************************************************
-90-

CALL ERRSET(207,256,-1,2,1,209)

DIMENSION X(325,2),NOD(450,3),NEP(450),THICK(450),STRAIN(450,2)

1 NNUM(120),NODS(120,2),TBREAK(120),ICKLS(120),WDT(120),ASTRS(120)
2 NSSS(120),DIA(120),BN(120),E1(2),E2(2),P1(2),P2(2),GE(2),ITAG(1)
3 NBRK(1),SKS(4,1),XXE(3,2),XE(3,2),NF(2),NP(20),(20,2)
4 NSTART(15),NEND(15),NFIRST(15),NLAST(15),NCHK(15),NSPGST(15)
5 NSPGED(15),NRST(15),NSTR(15),NODS(40,2),STIR(40,3),ILD(10)

COMMON C(6,6),DRA(3,6),B(3,6),A(6,6),ST(7,140),U(650,4),U(650)

18(3,3),TSTRES(450,3,1),TSRN(450,3,1),U1(120),U2(120)

PEADING AND PRINTING OF INPUT DATA

READ (5,10) INPART, NPOIN, NELEM, NBOUN, NCOL, NVAL, NP1, NFRE, NCONC, NS
WRITE(6,10) INPART, NPOIN, NELEM, NBOUN, NCOL, NVAL, NP1, NFRE, NCONC, NS
DO 30 II=1,NPOIN
READ (5,20) I,X(I,1),X(I,2)
30 WRITE(6,20) I,X(I,1),X(I,2)
READ(5,10) NCARD
IF(NCARD-NPOIN) 7000, 7000
DO 10 II=1,NELEM
READ (5,230) I,(NOD(I,J),J=1,3),NEP(I),THICK(I),AN
WRITE(6,230) I,(NOD(I,J),J=1,3),NEP(I),THICK(I)
READ(5,10) NCARD
IF(NCARD-NELEM) 7000, 80, 7000
DO 50 II=1,NPOIN
READ (5,240) IF(I),NB(I,1),NB(I,2),BV(I,1),BV(I,2)
50 WRITE(6,240) IF(I),NB(I,1),NB(I,2),BV(I,1),BV(I,2)
DO 260 II=1,NPOIN
READ (5,10) NSTART(I),NEND(I),NFIRST(I),NLAST(I)
WRITE(6,10) NSTART(I),NEND(I),NFIRST(I),NLAST(I)
DO 100 II=1,NYM
READ (5,270) EL(I),E2(I),P1(I),P2(I),GE(I)
GE(I)=E1(I)/(2.*(1.+P1(I)))
100 WRITE(6,270) EL(I),E2(I),P1(I),P2(I),GE(I)
NP0IN2=NP01N+2
DO 110 II=1,NPOIN2
U1(I,1)=0.
U1(I)=0.
110 U4(I)=0.
ICOUNT=0
DO 140 II=1,NCONC
READ (5,20) K,U(2*K-1,1),U(2*K,1)
IF(U(2*K-1,1),NF1,0,AND, U(2*K,1),NE1) GO TO 130
GO TO 120
130 ICDNT=ICOUNT+2
ILD(I CDNT-1)=2*K-1
ILD(I CDNT)=2*K
GO TO 140
120 IF(U(2*K-1,1),NE1,0,AND, U(2*K,1),NE1) GO TO 90
GO TO 150
90 ICOUNT=ICOUNT+1
ILD(ICOUNT)=2*K-1
GO TO 140
150 IF(U(2*K-1,1).EQ.0.AND.U(2*K,1).NE.0) GO TO 160
GO TO 140
160 ICOUNT=ICOUNT+1
ILD(ICOUNT)=2*K
140 WRITE(6,20) K,U(2*K-1,1),U(2*K,1)
READ (5,250) NELEMS,NSTEPS,DELTAX,DELTAY,BTH
WRITE(6,250) NELEMS,NSTEPS,DELTAX,DELTAY,BTH
DO 170 II=1,NELEMS
ILCKS(II)=0
U2(II)=0.0
IBREAK(II)=0
READ (5,60)II,(NODS(I,J),J=1,2),NSSS(I),WDT(I),BN(I),DIA(I),BNUM(I)
170 WRITE(6,60)II,(NODS(I,J),J=1,2),NSSS(I),WDT(I),BN(I),DIA(I),BNUM(I)
DO 180 II=1,NELEMS
READ (5,10) NCHK(I)
WRITE(6,10) NCHK(I)
IF(NCHK(I).EQ.0) GO TO 180
190 READ (5,10) NSPGST(I),NSPGED(I)
WRITE(6,10) NSPGST(I),NSPGED(I)
180 CONTINUE
DO 200 I=1,NPART
READ(5,10) NSTR(I)
WRITE(6,10) NSTR(I)
IF(NSTR(I).EQ.0) GO TO 200
210 READ (5,10) NENDST(I,1),NENDST(I,2)
WRITE(6,10) NENDST(I,1),NENDST(I,2)
200 CONTINUE
DO 220 I=1,NS
READ (5,240) K,(NODSTR(I,J),J=1,2),STIR(I,J),J=1,3
220 WRITE(6,240) K,(NODSTR(I,J),J=1,2),(STIR(I,J),J=1,3)
READ (5,270) FF,FSE,RPTR
WRITE(6,270) FF,FSE,RPTR
READ (5,270) EY,ESTHR,E
WRITE(6,270) EY,ESTHR,E
270 FORMAT(5F15.5)

C FORMAT STATEMENTS FOR INPUT DATA
C
10 FORMAT(11I4,2F16.8)
20 FORMAT(14,5F14.5)
230 FORMAT(5I4,3F16.8)
240 FORMAT(5I4,3F16.8)
250 FORMAT(2I4,3F10.4)
60 FORMAT(4I4,4F10.5)
270 FORMAT(5F15.5)
C CALCULATION OF CONSTANTS FOR CONCRETE STRESS-STRAIN CURVE
C
STF=SQRT(SQRT(FO))
SED=0.00001*STF*(31.5-STF)
RSE=SEF/SED
RSF=FO/FF

C FORMAT STATEMENTS FOR CONCRETE STRESS-STRAIN CURVE
C
10 FORMAT(11I4,2F16.8)
20 FORMAT(14,5F14.5)
230 FORMAT(5I4,3F16.8)
240 FORMAT(5I4,3F16.8)
250 FORMAT(2I4,3F10.4)
60 FORMAT(4I4,4F10.5)
270 FORMAT(5F15.5)
**RE=E1(1)*SE0/FO**  
RR=RE*(RSF-1)/((RSE-1)**2)  
RR=RR-1./RSE  
C1=2.*RR-1.  
C2=RR  
C3=RR+RE-2.  
DO 280 K=1,NELEM  
DO 280 I=1,3  
TSR(I,K+1)=0.  
T1(K)=0.  
STRAIN(K,1)=0.0  
STRAIN(K,2)=0.0  
280 TSTRES(I,K+1)=0.0  
**C ADDITION OF LOAD INCREMENTS TO LOAD MATRIX**  
DO 300 LD=1,NSTEPS  
IMAX=1  
WRITE(6,290) LD,IMAX  
ITAG(I)=0  
REWIND 1  
REWIND 4  
310 IF(LD-I) 310,310,320  
310 DO 330 I=1,NPOIN2  
310 DO 330 II=1,ICOUNT  
320 IF(LD(I)-I) 340,350,340  
320 CONTINUE  
330 GO TO 330  
330 IF(MOD(I,2)) 360,370,360  
340 U(I,1)=U(I,1)+DELTAX  
340 CONTINUE  
350 IF(MOD(I,2)) 360,370,360  
350 U(I,1)=U(I,1)+DELTAY  
330 CONTINUE  
360 CONTINUE  
370 INTER=0  
**C FORMATION OF GLOBAL STIFFNESS MATRIX BY PARTITIONS**  
DO 5000 II=1,NPART  
DO 1010 J=1,140  
DO 1010 I=1,70  
1010 ST(I,J)=0.0  
NST=NSART1(I)  
NEN=ENED(I)  
K=NFIRST(I)  
L=NLAST(I)  
MINUS=K-1  
IF(NCHK(I)) 1020,1020,1030  
1020 NSST=NSPGST(I)  
NSEN=NSPGED(I)  
1030 NSST=NSPGST(I)  
NSEN=NSPGED(I)  
DO 1040 LK=NST,NEN  
MH=LK-INTER  
DO 1050 I=1,3  
JJ=MOD(LK-1)  
XE(I,JJ)=X(JJ,1)
CALCULATION OF ELASTIC PROPERTIES FOR FINITE ELEMENTS

IF(J-1) 1060, 1060, 1090
1060 AX=ABS(STRAIN(LK,1)):
AY=ABS(STRAIN(LK,2))
DIFS=AX-AY
IF(DIFS) 1070, 1080, 1080
1080 IF(STRAIN(LK,1)) 1100, 1090, 1090
1070 IF(STRAIN(LK,2)) 1110, 1090, 1090
1110 COFF=(STRAIN(LK,2))/SEO
1100 COFF=(STRAIN(LK,1))/SEO
1120 TNUM=El(1)*(1.+Cl*COFF**2,-2.*C2*(COFF**3))
DENUM=1.+C3*COFF-Cl*COFF**2)+C2*(COFF**3)
VM1=TNUM/(DENUM**2)
VM2=VM1
G=(0.5*VM1)/(1.+P1(1))
PR1=P1(J)
PR2=P2(J)
GO TO 1130
1090 IF(THICK(LK),NE.0.,AND.T1(LK),EQ.0.) GO TO 1140
1100 IF(THICK(LK),EQ.0.,AND.T1(LK),NE.0.) GO TO 1140
1110 IF(THICK(LK),NE.0.,AND.T1(LK),NE.0.) GO TO 1150
1140 YH1=E1(J)
1150 YH2=E2(J)
PR1=P1(J)
PR2=P2(J)
G=GE(J)
GO TO 1130
1160 YH1=E
1170 YH2=E
PR1=P
PR2=P
G=YH1/(2.*(1.+PR1))
1130 CONTINUE

ADDITION OF ELEMENT STIFFNESS MATRIX TO GLOBAL STIFFNESS MATRIX

CALL FEM(XE,YH1,YH2,PR1,PR2,G,ANG,NP,TH,MM,J,LK)
1040 DO 1190 LL=1,3
1050 DO 1140 KK=1,3
1060 IF(ND(LK,LL)=K) 1040, 1160, 1160
1160 IF(ND(LK,LL)=L) 1170, 1170, 1140
1170 N=NFREE*(ND(LK,LL)-K)
1180 I=NFREE*(KK-1)
1190 J=NFREE*(LL-1)
1040 N=1040 N180, 1180, 1180
1180 DD 1190 NJ=1,NFREE
DO 1190 MI=1,NFREE
 MHI=M+MI
 NNJ=N+NJ
 IMI=I+MI
 JNJ=J+NJ
1190 ST(MHI,NNJ)=ST(MHI,NNJ)+C(IMI,JNJ)
1040 CONTINUE
 IF(NSTR(II)>2010,Z010,Z010,NN1=ENDST(II,1))
 NN2=ENDST(II,2)

C C ADDITION OF STIRRUP STIFFNESS MATRIX TO GLOBAL STIFFNESS MATRIX AND C CHECKING OF FAILURE CRITERIA.

C DO 2040 NN=NN1,NN2
 IF(STIR(NN,1)) 2040,2040,2030
2030 JJS1=NODSTR(NN,1)
 JJS2=NODSTR(NN,2)
 BNR=STIR(NN,3)*.017453
 XXE(1,1)=ABS(U1(2*JJS2-1)-U1(2*JJS1-1))
 XXE(1,2)=ABS(U1(2*JJS2)-U1(2*JJS1))
 RATIO=XXE(1,2)/XXE(1,1)
 ANGLE=ATAN(RATIO)
 XXE(1,2)=SQRDXXE(1,1)**2+XXE(1,2)**2
 XXE(1,1)=XXE(1,2)*COS(ANGLE-BNR)
 ANGLE=XXE(1,1)/STIR(NN,2)
 IF(ANGLE-EV) 2060,2050,2050
2050 STIR(NN,1)=0.0
 WRITE(6,2070) NN
 GO TO 2040
2060 STIFF=E1(2)*STIR(NN,1)/STIR(NN,2)
 CO=COS(BNR)
 S=SIN(BNR)

C SKS(1,1)=STIFF*CO*CO
 SKS(1,2)=STIFF*CO*S
 SKS(1,3)=-STIFF*CO*CO
 SKS(1,4)=-SKS(1,3)
 SKS(2,1)=SKS(1,2)
 SKS(2,2)=STIFF*S*S
 SKS(2,3)=-SKS(1,2)
 SKS(2,4)=-SKS(2,3)
 SKS(3,1)=SKS(1,3)
 SKS(3,2)=-SKS(1,2)
 SKS(3,3)=SKS(1,1)
 SKS(3,4)=-SKS(1,3)
 SKS(4,1)=-SKS(1,2)
 SKS(4,2)=-SKS(2,2)
 SKS(4,3)=SKS(1,2)
 SKS(4,4)=SKS(2,2)

C DO 2040 LL=1,2
 DO 2040 KK=1,2
 IF(NODSTR(NN,KK)) 2040,2080,2080
2080 IF(NODSTR(NN,KK)==L) 2090,2090,2040
2090 M=2*(NODSTR(NN,KK)-K)
N=2*(NODSTR(NN,LL)-K)
I=2*(KK-1)
J=2*(LL-1)
IF(I==1) 2040,2040,2100
2100 DO 2110 N=1,2
DO 2110 M=1,2
MM=M+M
NN=N+N
IM=I+I
JN=J+N
2110 ST(MM,NN)=ST(MM,NN)+SKS(IM,JN)
2040 CONTINUE
2010 IF(NCHK(II)) 3010,3010,3020
C ADDITION OF SPRING STIFFNESS MATRIX TO GLOBAL STIFFNESS MATRIX AND C CHECKING OF FAILURE CRITERIA.
C
3020 DO 3030 LKS=NSST,NSEN
ANGE=BN(LKS)*.017453
JJS1=NODS(LKS,1)
JJS2=NODS(LKS,2)
IF(IBREAK(LKS),EQ,1) GO TO 3040
XE(1,1)=ABS(U1(2*JJS1-1)-U1(2*JJS1-1))
XE(1,2)=ABS(U1(2*JJS2-1)-U1(2*JJS1))
RATIO=XE(1,2)/XE(1,1)
ANGLE=ATAN(RATIO)
XE(1,2)=SQRT(XE(1,1)**2+XE(1,2)**2)
XE(1,1)=XE(1,2)*COS(ANGLE-ANGE)
AAREA=1.5708*BN(LKS)*WDT(LKS)*DIA(LKS)
SLIP=.000749
CHECK=XE(1,1)-SLIP
IF(CHECK>3050,3040,3040)
3040 XE(1,1)=SLIP+XE(1,1)
IF(REM(LKS)==2
3070 SK=0.3666+0.07*XXE(1,1)*0.0712E+11+XXE(1,1)**2+0.5898E+13
SKB=SK*AAREA
3060 SKD=EL(1)*BHN(LKS)*DIA(LKS)*WDT(LKS)*BTH
WRITE(6,3090) LKS,SK,SKB,XXE(1,1),XXE(1,1),SKD
C =COS(ANG)
S=SIN(ANG)
C

SKS(1,1) = SKB*(CO**2) + SKD*(S**2)
SKS(1,2) = SKB*CO*S - SKD*CO*S
SKS(1,3) = -SKB*(CO**2) - SKD*(S**2)
SKS(1,4) = -SKS(1,2)
SKS(2,1) = SKS(1,2)
SKS(2,2) = SKB*(S**2) + SKD*(CO**2)
SKS(2,3) = -SKS(1,2)
SKS(2,4) = -SKB*(S**2) - SKD*(CO**2)
SKS(3,1) = SKS(1,3)
SKS(3,2) = SKS(2,3)
SKS(3,3) = SKS(1,1)
SKS(3,4) = SKS(1,2)
SKS(4,1) = -SKS(1,2)
SKS(4,2) = SKS(2,4)
SKS(4,3) = SKS(1,2)
SKS(4,4) = SKS(2,2)

JJS3 = 2*(JJS1*K)+1
IF (ST(JJS3, JJS3)) 3100, 3030, 3100

3100 DO 3030 LL = 1, 2
3030 DO 3030 KK = 1, 2
IF (NODS(LKS, KK) - K) 3030, 3030

3110 IF (NODS(LKS, KK) - L) 3120, 3120, 3030

3120 M = 2*(NODS(LKS, KK) - K)
N = 2*(NODS(LKS, LL) - K)
I = 2*(KK - 1)
J = 2*(LL - 1)
IF (N) 3030, 3130, 3130

3130 DO 3140 NJ = 1, 2
3140 DO 3140 MI = 1, 2

MM = MN + MI
NNJ = NJ + NN
IMI = I + MI
JNJ = J + NJ

3140 ST(MMI, NNJ) = ST(MMI, NNJ) + SKS(IMI, JNJ)
3030 CONTINUE

C ADDITION OF PRESCRIBED BOUNDARY CONDITIONS TO GLOBAL STIFFNESS MATRIX

3010 DO 4010 I = 1, NBOUND

M = NF(I) - K
MM = NF(I) - 1
IF (M) 4010, 4020, 4020

4020 IF (M = 34) 4030, 4030, 4010
4030 DO 4040 J = 1, NFREE
4040 IF (NBI(J)) 4040, 4040, 4040

4060 NM = NFREE - J - 1
ST(NMI, NMI) = ST(NMI, NMI) + 1E+12

DO 4040 JJ = 1, MCOLN
4040 CONTINUE
4050 IF (NJN(JJ)) = ST(NMI, NMI) + BV(I, J)
4040 CONTINUE
4010 INTER = NEN

C
MI=NFREE*MINUS + 1
NJ=NFREE*L
M=NJ-MI+1
IF(II-NPART) 4070,4080,4070
4070 NA=NFREE*(NLAST(II+1)-MINUS)
GO TO 4090
4080 NA=M+1
4090 NA=NA-M
MM=M+1

ADDITION OF ARBITRARY STIFFNESS TO ACCOUNT FOR ISOLATED JOINTS.

C DO 4100 I=1,M
IF(ST(I,I) .GE. 0.) 4100,4110,4100
4110 ST(I,I)=ST(I,I)+0.1E+18
4100 CONTINUE
5000 WRITE(4) HI,M,((ST(I,J),I=1,M),J=1,M),((ST(I,J),I=1,M),J=MM,NA),
1 ((U(I,J),I=MI,NJ),J=1,NCOLN)
REWIND 1
REWIND 2
REWIND 3
REWIND 4

SOLUTION OF EQUATIONS OF EQUILIBRIUM

C CALL SOLVE(NPART,NCOLN)
REWIND 3

CALCULATION OF ELEMENT STRESSES AND STRAINS, CHECKING OF FAILURE CRITERIA, AND ADDING ELEMENT NODAL FORCES TO LOAD MATRIX IF REQUIRED.

C CALL STRESS (NPART,NFIRST,NLAST,NCOLN,NFLM,NOD,NFREE,NPOIN,
1 THICK,EY,ESTHR,SEF,NBRK,RUPTR,STRAIN)
WRITE(6,6440)

CALCULATION OF BOND SPRING STRESSES AND FORCES, AND ADDING OF SPRING NODAL FORCES TO LOAD MATRIX IF FAILURE HAS OCCURRED.

C DO 6000 I3=1,NPART
IF(CHK(I3)) 6000,6000,6010
6010 NSST=NSPGST(I3)
NSEN=NSPGED(I3)
DO 6000 LKS=NSST,NSEN
ANCH=B(LKS)*0.017453
JJS1=NNDS(LKS,1)
JJS2=NNDS(LKS,2)
XE(1,1)=U1(2*JJS2-1)-U1(2*JJS1-1)
XE(1,2)=U1(2*JJS2)-U1(2*JJS1)
IF(IBREAK(LKS),EQ,1) GO TO 6140
XE(1,1)=ABS(U1(2*JJS2-1)-U1(2*JJS1-1))
XE(1,2)=ABS(U1(2*JJS2)-U1(2*JJS1))
RATIO=XE(1,2)/XE(1,1)
ANGLE=ATAN(RATIO)
XE(1,2)=SQRT(XE(1,1)**2+XE(1,2)**2)
XE(1,1)*XE(1,2)*COS(ANGLE-ANG)
BSTRS(LKS)=XE(1,1)*0.3606E+07-(XE(1,1)**2)*0.5356E+10+(XE(1,1)**3)*0.1986E+13
FRICTN=0.
BFORC=BSTRS(LKS)*WOT(LKS)*1.5708*DIA(LKS)*BNUM(LKS)
BFORPC= BFORCE*COS(ANG)
BFORPS= BFORCE*SIN(ANG)
CHECK=XE(1,1)-SLIP
IF (BREAK(LKS),FQ.2) GO TO 6130
IF (CHECK) 6020,6130,6130
6020 IF (NSSS(LKS)) 6040,6030,6030
6030 IF (ICLKS(LKS)-4) 6050,6050,6050
6040 DO 6060 LI=1,NELEH
6050 DO 6060 LI=1,NELEH
6060 CONTINUE
6120 IF (NSSS(LKS)) 6110,6120,6110
6110 IF (ICLKS(LKS)-8) 6460,6460,6460
6100 IBREAK(LKS)=1
6130 ITAG(1)=ITAG(1)+1
6140 XE(1,1)+SLIP
IF (NSSS(LKS)) 6150,6150,6150
6150 IF (U2(LKS)) 6180,6170,6180
6170 IF (XXE(1,1)) 6200,6190,6190
6190 U(2*JJS1-1,1)=U(2*JJS1-1,1)-BFORPC
U(2*JJS2-1,1)=U(2*JJS2-1,1)-BFORPC
GO TO 6210
6200 U(2*JJS1-1,1)=U(2*JJS1-1,1)+BFORPC
U(2*JJS2-1,1)=U(2*JJS2-1,1)+BFORPC
6210 IF (XXE(1,2)) 6230,6220,6220
6220 U(2*JJS1,1)=U(2*JJS1,1)-BFORPS
U(2*JJS2,1)=U(2*JJS2,1)+BFORPS
GO TO 6180
6230 U(2*JJS1,1)=U(2*JJS1,1)+BFORPS
U(2*JJS2,1)=U(2*JJS2,1)+BFORPS
6240 U(2*LKS)=1.0
6250 BSTRS(LKS)=0.0
GO TO 6100
6160 IF (U2(LKS)) 6240,6240,6240
6240 BSTRS(LKS)=720.
6250 IF (XXE(1,1)) 6260,6250,6260
6260 U(2*JJS1-1,1)=U(2*JJS1-1,1)+BFORPC+FRICTN*COS(ANG)
U(2*JJS2-1,1)=U(2*JJS2-1,1)+BFORPC+FRICTN*COS(ANG)
GO TO 6270
6270 U(2*JJS1-1,1)=U(2*JJS1-1,1)+BFORPC+FRICTN*COS(ANG)
U(2*JJS2-1,1)=U(2*JJS2-1,1)+BFORPC+FRICTN*COS(ANG)
BSTRS(LKS)=720.
6280 IF (XXE(1,2)) 6290,6280,6280
6280 U(2*JJS1,1)=U(2*JJS1,1)-BFORPS+FRICTN*STN(ANG)
U(2*JJS2,1)=U(2*JJS1,1)+8FORPS-FRICTN*SIN(ANG)
GO TO 6300

6290 U(2*JJS1,1)=U(2*JJS1,1)+8FORPS-FRICTN*SIN(ANG)
U(2*JJS2,1)=U(2*JJS2,1)-8FORPS+FRICTN*SIN(ANG)
IF(BFORPC-BFORPS) 6310,6300,6300

6310 BSTRS(LKS)=720.
GO TO 6300

6320 IF(XXE(1,1)) 6350,6340,6340

6340 U(2*JJS1-1,1)=U(2*JJS1-1,1)+FRICTN*COS(ANG)
U(2*JJS2-1,1)=U(2*JJS2-1,1)-FRICTN*COS(ANG)
GO TO 6360

6350 U(2*JJS1-1,1)=U(2*JJS1-1,1)-FRICTN*COS(ANG)
U(2*JJS2-1,1)=U(2*JJS2-1,1)+FRICTN*COS(ANG)

6360 IF(XXE(1,2)) 6380,6370,6370

6370 U(2*JJS1,1)=U(2*JJS1,1)+FRICTN*SIN(ANG)
U(2*JJS2,1)=U(2*JJS2,1)-FRICTN*SIN(ANG)
GO TO 6300

6380 U(2*JJS1,1)=U(2*JJS1,1)-FRICTN*SIN(ANG)
U(2*JJS2,1)=U(2*JJS2,1)+FRICTN*SIN(ANG)

6300 U2(LKS)=1.0
6100 BFORCE=BSTRS(LKS)*WDT(LKS)*1.5708*DIAM(LKS)*BNUM(LKS)
GO TO 6430

6460 IF(BFORPC-BFORPS) 6410,6390,6390
6390 IF(XXE(1,1)) 6400,6430,6430
6400 BSTRS(LKS)=BSTRS(LKS)
BFORCE=-BFORCE
GO TO 6430

6410 IF(XXE(1,2)) 6420,6430,6430
6420 BSTRS(LKS)=BSTRS(LKS)
BFORCE=BFORCE
6430 BSTRS1=BSTRS(LKS)/WDT(LKS)
BFOR=BFORCE/WDT(LKS)
WRITE(6,6450) LKS,BSTRS(LKS),BSTRS1,BFORCE,BXE(1,1)

C
C POSITIVE BOND FORCE = POSITIVE FORCE ON CONCRETE.
C
C 6000 CONTINUE
WRITE(6,6470) ITAG(1),LD
290 FORMAT('1',1X,'LOADING STEP',1X,'I4',1X,'ITERATION NO',1X,I4,'/',LKS',RX',BNUM,LKS)
1'SK',12X,'SKB',10X,'XXE(1,1)',6X,'XXE(1,1)',11X,'SKD',11/)
6440 FORMAT('1',1X,'ELEMENT BOND STRESS STRESS/INCH BOND FORCE',BNUM)
2 'FORCE/INCH SLIP')
6450 FORMAT(18,4F15.2,F15.6)
3090 FORMAT('1',1X,'SPRING ELEMENTS BROKEN DURING LOADING STEP',1X,I4,'/')
6470 FORMAT('1',1X,'STIRRUP NUMBER',1X,I4,' HAS REACHED YIELD')
2070 FORMAT(10X,'ITAG')

C
C USE ARITHMETIC 'IF' STATEMENT HERE TO LIMIT DEFLECTIONS OF STRUCTURE
C
C 300 CONTINUE
.7000 STOP
END
C
SUBROUTINE STRESS(NPART,NFIRST,NLAST,NCOLN,NELEM,NOD,NFREE,NPOIN)
1 THICK,EY,ESTHR,SEF,NBRK,RPTR,STRAIN)

SUBROUTINE TO FIND TOTAL STRESSES AND STRAINS IN ELEMENTS AND TO
CHECK FAILURE CRITERIA.

DIMENSION NOD(450,3),THICK(450),STRAIN(450,2),NFIRST(15),NLAST(15)
1,0(650),NBRK(1),Z(6,1),T(6,1),D(3,3),BB(3,6),XE(3,2)
COMMON C(6,6),DBA(3,6),DB(3,6),A(6,6),ST(70,140),U(650,4),U1(650),U3(650)
1(3,6),TSTRES(450,3,1),TSRN(450,3,1),U2(120),U3(650),T1(450)

NBRK(1)=0
DO 10 I=1,NPART
10 READ(3) ((U(I,J),I=1,6,N),J=1,NCOLN)
NPOIN2=2*NPOIN
DO 20 I=1,NPOIN2
20 U(I,1)=0.0
WRITE(6,600)
WRITE(6,610) (U(I,2*I-1),U(I,2*I),I=1,NPOIN)
WRITE(6,620)

C ADDITION OF INCREMENTAL STRESSES AND STRAINS TO TOTAL STRESS AND
C STRAIN MATRICES.

DO 1000 LL=1,NELEM
READ (1) ((DBA(I,J),I=1,3),J=1,6),ORX,RAY
1,YM1,YM2,PR1,PR2,GRJEP,C
DO 30 I=1,3
J=NOD(LL,I)
T(2*I-1,1)=U(2*I-1,1)
T(2*I,1)=U(2*I,1)
Z(2*I-1,1)=D(2*I-1,1)
30 Z(2*I,1)=D(2*I,1)
IF((JEP-1).EQ.1.AND.THICK(LL).EQ.0) GO TO 60
DO 40 J=1,NCOLN
40 D(I,J)=0.
DO 50 K=1,6
U3(K)=0.
50 U3(K)=0.
DO 40 J=1,NCOLN
DB(I,J)=DB(I,J)+DBA(I,K)*Z(K,J)
DO 50 J=1,NCOLN
STRES(LL,I,J)=STRES(LL,I,J)+DB(I,J)
DO 60 J=1,3
D(I,1)=1.0/YM1
D(I,2)=PR2/YM2
D(I,3)=U3(K)
60 D(I,2)=1.0/YM1
D(I,2)=PR2/YM2
D(I,3)=U3(K)
DO 70 I=1,3
DO 70 J=1,6
BB(I,J)=0.
DO 80 I=1,3
DO 80 J=1,6
TSRN(LL,I,J)=TSRN(LL,I,J)+BB(I,J)*Z(J,1)
DO 90 K=1,3
10 BR(I,J)=BR(I,J)+OBA(K,J)*OBA(K,J)
DO 80 1=1,3
DO 80 J=1,6
80 TSRN(LL,I,J)=TSRN(LL,I,J)+BB(I,J)*Z(J,1)
DO 90 1=1,3
DO 90 J=1,6
90 DB(I,J)=TSRES(LL,I,J)
CALL PRIN(DB) NCOLN)
STRAIN(LL,1)=A(I,1)/YM1-PR1*A(I,1)/YM1
STRAIN(LL,2)=A(I,1)/YM2-PR2*A(I,2)/YM2
TA=STRAIN(LL,1)
TB=STRAIN(LL,2)
WRITE(b,630)LL,ORX,ORY,(BOB(I,J),I=1,3),(AC(I,J),I=1,3),J=1,NCOLN)
WRITE(b,630)LL,ORX,ORY,(TSRN(LL,I,J),I=1,3),STRAIN(LL,1),STRAIN(LL,2),YM1
150 IF(JEP=1) 160,160,420
C CHECK FAILURE CRITERIA FOR CONCRETE ELEMENTS, AND ADD NODAL FORCES TO LOAD MATRIX IF NECESSARY.
C
160 IF(A(I,1)) 170,170,190
170 IF(TA) 180,180,210
180 IF(ABS(TA)>SEF) 210,340,340
190 CHECK=A(I,1)-PUPTR
IF(CHECK) 210,200,200
200 IF(THICK(LL)>EQ.0.0) GO TO 330
WRITE(b,640)
GO TO 260
210 IF(A(I,2)) 220,220,240
220 IF(TB) 230,230,1000
230 IF(ABS(TB)>SEF) 1000,340,340
240 CHECK=A(I,2)-PUPTR
IF(CHECK) 1000,250,250
250 IF(THICK(LL)>EQ.0.0) GO TO 330
WRITE(b,720)
260 DD 270 I=1,6
DD 270 J=1,6
270 U3(I)=U3(I)+C(I,1)*TZ(J,1)
DD 280 I=1,3
JJ=1ND(LL,1)
U2(JJ-1,1)=U2(JJ-1,1)+U3(2*I-1)
280 U2(JJ+1,1)=U2(JJ,1)+U3(2*I)
THICK(LL)=0.0
NBRK(1)=NBRK(1)+1
GO TO 1000
330 WRITE(b,650)
GO TO 1000
340 IF(THICK(LL)>EQ.0.0) GO TO 410
WRITE(6,660)
DO 350 I=1,6
DO 360 J=1,6
350 U3(I)=U3(I)*C(I,J)*TL(J,1)
DO 360 J=1,3
JJ=NODE(LL,I)
U(J,2*JJ-1)=U(J,2*JJ-1)+1.0*U3(2*I-1)
THICK(LL)=0.0
GO TO 1000
360 UC=JJ,1)=U(2*JJll)+1.0*U3(2*I)
THICK(LL)=0.0
WRITE(6,680)
GO TO 1000
410 WRITE(6,670)
GO TO 1000
C CHECK FAILURE CRITERIA FOR STEEL ELEMENTS.
C
420 IF.THICK(LL)) 490,490,430
430 IF(T1(LL)) 440,440,1000
440 IF(ABS(TA)=EY) 450,450,460
450 IF(ABS(TB)=EY) 1000,460,460
460 T1(LL)=THICK(LL)
THICK(LL)=0.0
WRITE(6,680)
GO TO 1000
490 WRITE(6,690)
IF(ABS(TA)=ETHR) 500,510,510
500 IF(ABS(TB)=ETHR) 1000,510,510
510 THICK(LL)=T1(LL)
WRITE(6,700)
1000 CONTINUE
WRITE(6,710) RUPTR,NEBRK(1)
C 600 FORMAT(101,8H NODE,17H X-DISPLACEMENTS,17H Y-DISPLACEMENTS) FEMC0603
610 FORMAT(1H,3(4X,13,1,1X,2E16.8)) FEMC0604
620 FORMAT(101,4H CENTROID CARTESIAN STRESSES PRINCIPAL STRESSES,) FEMC0605
31, NO. XCORD YCORD X Y XY STRESS1 STRESS2 ANG
4 EX,10X,10X,10X,10X,10X,10X,10X,10X,10X,10X,10X,10X,10X
630 FORMAT(1,13,2F6.2,2F5.0,3F12.3,2F12.3,3F12.0) FEMC0609
640 FORMAT(1 NEW TENSILE CRACK NORMAL TO 1-1 DIRECTION) FEMC0610
650 FORMAT(1 ELEMENT CRACKED PREVIOUSLY) FEMC0611
660 FORMAT(1 CONCRETE ELEMENT REACHES MAX COMPRESSIVE STRAIN) FEMC0612
670 FORMAT(1 CONCRETE ELEMENT CRUSHED PREVIOUSLY) FEMC0613
680 FORMAT(1 STEEL ELEMENT HAS REACHED YIELD) FEMC0614
690 FORMAT(1 STEEL ELEMENT IN YIELD PLATEAU) FEMC0615
700 FORMAT(1 STEEL ELEMENT HAS REACHED STRAIN HARDENING) FEMC0616
710 FORMAT(1 MOD. OF RUPTR=9.2,8X,NO. OF ELEMENTS BROKEN=1,13) FEMC0617
720 FORMAT(1 NEW TENSILE CRACK NORMAL TO 2-2 DIRECTION) FEMC0618
RETURN
END
SUBROUTINE PRIN(DB,A,NCOLN)

SUBROUTINE TO CALCULATE PRINCIPAL STRESSES OF TRIANGULAR ELEMENTS.

DIMENSION DB(3,6),A(6,6)
DO 3 J=1,NCOLN
  T1=DB(1,J)+DB(2,J)/2.0
  T2=((DB(1,J)-DB(2,J))/2.0)**2
  T3=(DB(3,J))**2
  T4=2.0*DB(3,J)/(DB(1,J)-DB(2,J))
  A(1,J)=T1+SORT(T2+T3)
  A(2,J)=T1-SORT(T2+T3)
  THA=0.5*ATAN(T4)
  THB=THA+0.5*3.14159
  SIG = T1+SORT(T2)*COS(2.*THA) + SORT(T3)*SIN(2.*THA)
  IF(ABS(A(I,J)-SIG)-0.0001) 2,2,1
  A(3,J)=THA*180./3.14159
  GO TO 3
1  A(3,J)=THB*180./3.14159
3 CONTINUE
RETURN
END

SUBROUTINE FEN(XE,YM1,YM2,PR1,PR2,G,ANG,NP,TH,MH,JEP,LK)

SUBROUTINE TO FORMULATE ELEMENT STIFFNESS MATRIX.

DIMENSION D(3,3),ATDBA(3,3),XE(3,2),R(6,6),Z(3,3),R(3,3)
COMMON C(6,6),A(3,6),ST(70,140),U(650,4),U(1,650)
1B(3,6),TSTRES(450,3,1),TSRN(450,3,1),U(120),U(3,6),U(650),T1(450)
DO 10 J=1,6
  DO 10 I=1,3
    D(I,J)=0.
10  DBA(I,J)=0.0
DO 20 I=1,6
  R(I,J)=0.
20  A(I,J)=0.
  BTDBA(I,J)=0.
DO 30 J=1,3
  C(I,J)=0.
30  DO 30 I=1,3
    D(I,J)=0.
30  DBA(I,J)=0.0
DO 40 I=1,3
  ORX=XE(I,1)+XE(I,2)+XE(3,1)*333333
  ORY=(XE(I,1)+XE(I,2)+XE(3,2))*333333
  IF(TH) 40,40,50
40  IF(JEP-L) 60,60,50
  GO TO 50
50  DO 50 I=1,3
    XE(I,1)=XE(I,1)-ORX
50  XE(I,2)=XE(I,2)-ORY
    IF(TH,E0,0,AND,JEP,EQ,2) GO TO 80
    GO TO 90
ELASTIC PROPERTIES OF STEEL ELEMENTS ARE ALTERED IF ELEMENT IS IN YIELD PLATEAU.

80 TH=T1(LK)
VM1=10.
VM2=10.
PR1=.30
PR2=.30
G=VM1/(2.*((1.+PR1))
90 Z(1)=XE(2,2)-XE(3,2)
Z(2)=XE(3,2)-XE(1,2)
Z(3)=XE(1,2)-XE(2,2)
Z(1)=XE(3,1)-XE(2,1)
Z(2)=XE(1,1)-XE(3,1)
Z(3)=XE(2,1)-XE(1,1)
ZK=XE(2,1)*XE(3,2)-XE(3,1)*XE(2,2)
Z=3.*ZK

A(1,1)=ZK/Z
A(2,1)=Z(1)/Z
A(3,1)=Z(1)/Z
A(4,2)=A(1,1)
A(5,2)=A(2,1)
A(6,2)=A(3,1)
A(1,3)=ZK/Z
A(2,3)=Z(2)/Z
A(3,3)=Z(2)/Z
A(4,4)=A(1,3)
A(5,4)=A(2,3)
A(6,4)=A(3,3)
A(1,5)=ZK/Z
A(2,5)=Z(3)/Z
A(3,5)=Z(3)/Z
A(4,6)=A(1,5)
A(5,6)=A(2,5)
A(6,6)=A(3,5)
B(1,2)=1
B(3,3)=1
B(3,5)=1
B(2,6)=1
DEN=(1.-PR1*PR2)

ELASTICITY MATRIX FOR PLANE STRESS CASE.

D(1,1)=VM1/DEN
D(2,1)=PR1*VM2/DEN
D(1,2)=PR2*VM1/DEN
D(2,2)=VM2/DEN
D(3,3)=G
IF(ANG) 100, 110, 100
CS=COS(ANG*.017453)
SS=SIN(ANG*.017453)
R(1,1) = CS * SS
R(2,1) = SS * SS
R(3,1) = SS * CS
R(1,2) = R(2,1)
R(2,2) = R(1,1)
R(3,2) = R(3,1)
R(1,3) = 2 * R(3,2)
R(2,3) = 2 * R(3,1)
R(3,3) = R(1,1) - R(2,1)

C
DO 120 J = 1, 3
DO 120 I = 1, 3
RL(I, J) = 0.
DO 120 K = 1, 3
120 RL(I, J) = RL(I, J) + D(I, K) * R(J, K)
DO 130 J = 1, 3
DO 130 I = 1, 3
D(I, J) = 0.
DO 130 K = 1, 3
130 D(I, J) = D(I, J) + R(I, K) * RL(K, J)
110 DO 140 J = 1, 6
DO 140 I = 1, 3
DO 140 K = 1, 3
140 DB(I, J) = DB(I, J) + D(I, K) * B(K, J)
DO 150 J = 1, 6
DO 150 I = 1, 3
DO 150 K = 1, 6
150 DBA(I, J) = DBA(I, J) + DB(I, K) * A(K, J)
VOL = 0.5 * TH * Z
C
DO 160 J = 1, 6
DO 160 I = 1, 3
DO 160 K = 1, 3
160 BTDBA(I, J) = BTDBA(I, J) + B(K, I) * DBA(K, J) * VOL
C
C STIFFNESS MATRIX C IS FORMED.
C
DO 170 J = 1, 6
DO 170 I = 1, 3
DO 170 K = 1, 3
170 C(I, J) = C(I, J) + A(K, I) * BTDBA(K, J)
69 IF (MM) 190, 190, 180
C
180 WRITE(1) ((DBA(I, J), I = 1, 3), J = 1, 6), ORX, ORY
12YM1, YM2, PR1, PR2, G, JEP, C
190 RETURN
END
C
C
C
C
C
SUBROUTINE SOLVE (NPART, NCOLN)

DIMENSION AM(70,70), BM(70,70), YM(70,70), TF(70,4), RS(70,4), F(70,4)

1 DIS(70,4)

COMMON C(6,6), DA(3,6), DB(3,6), A(6,6), ST(70,140), U(650,4), U1(650)

1B(3,6), TSTRES(450,3,1), TSRN(450,3,1), U2(120), U3(6), U4(650), T1(450)

EQUIVALENCE (AM(I,J), ST(I,J)), (BM(I,J), ST(I,J)), (TF(I,J), U(I,J))

1 (DIS(I,J), U(I,J)) (RS(I,J), U(I,J)) (F(I,J), U(I,J))

DO 20 I=1,70
DO 10 J=1,NCOLN
TF(I,J)=0.

10 RS(I,J)=0.0
DO 20 J=1,70

20 YM(I,J)=0.0
DO 160 LL=1,NPART
READ(4) M, N, ((AM(I,J), I=1,M), J=1,N), ((BM(I,J), I=1,M), J=1,N),

1 ((FC(I,J), I=1,M), J=1,NCOLN)

DO 110 I=1,M
DO 110 J=1,NCOLN
F(I,J)=F(I,J)-TF(I,J)

110 DIS(I,J)=F(I,J)
DO 110 J=1,M

120 CALL DCDMP(M,AM,LL)
CALL INVERT(M,AM)

CALL MATH(M,F,DIS,M, M, NCOLN)
WRITE(2) M, N, ((AM(I,J), I=1,M), J=1,N), ((BM(I,J), I=1,M), J=1,N),

1 ((FC(I,J), I=1,M), J=1,NCOLN)
IF(NPART-LL) 170, 170, 120

120 CALL MATH(M,F,DIS,M, M, NCOLN)
CALL MATHM(BM,DIS,TF,N,M, NCOLN)
DO 130 J=1,N
DO 130 I=1,M
YM(I,J)=0.

130 YM(I,J)=YM(I,J)+AM(I,K)*BM(K,J)
DO 140 J=1,N
DO 140 I=1,N
AM(I,J)=0.
DO 140 K=1,M

140 AM(I,J)=AM(I,J)+BM(K,I)*YM(K,J)
DO 150 I=1,N
DO 150 J=1,N

150 YM(I,J)=AM(I,J)
160 CONTINUE

170 REWIND 4
WRITE(3) ((DIS(I,J), I=1,M), J=1,NCOLN)
IF(NPART-1) 4000, 4000, 180

180 NA=NPART-1
DO 1010 LL=1, NA
BACKSPACE 2
BACKSPACE 2
1 CONTINUE
9 FORMAT('O', 'SUM IS LESS THAN ZERO AND SUBROUTINE FAILS!', '314', 'F12.2')
RETURN
END

C

SUBROUTINE INVERT(N, U)
C SUBROUTINE TO INVERT THE SYMMETRIC MATRIX U(NxN).
C
DIMENSION U(70,70)
N = N-1
DO I = 1, N
J = I+1
DO J = J, N
SUM = 0.0
K = J-1
DO 2 K = 1, K
2 SUM = SUM - U(K; I)*U(K; J)
U(J; I) = SUM*U(J; J)
DO 3 K = J, N
3 SUM = 0.0
DO 4 K = J, N
4 SUM = SUM + U(K; I)*U(K; J)
U(I; J) = SUM
RETURN
END

C

SUBROUTINE MATM(A, B, C, L, M, N)
C SUBROUTINE USED IN CONJUNCTION WITH SUBROUTINE SOLVE.
C
DIMENSION A(70,70), B(70,4), C(70,4)
DO I = 1, L
DO K = 1, N
C(I; K) = 0.0
DO 100 J = 1, M
100 C(I; K) = C(I; K) + A(I; J)*B(J; K)
RETURN
END

C

SUBROUTINE MATM(A, B, C, L, M, N)
C SUBROUTINE USED IN CONJUNCTION WITH SUBROUTINE SOLVE.
C
DIMENSION A(70,70), B(70,4), C(70,4)
DO I = 1, L
DO K = 1, N
C(I; K) = 0.0
DO 100 J = 1, M
100 C(I; K) = C(I; K) + A(I; J)*B(J; K)
RETURN
END