A Methodology for Finite Element Analysis of Curvilinear Fiber Laminates with Defects, Fabricated by Automated Fiber Placement Technique.

By

Suhas Prabhakar

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Structures and Composite Materials Laboratory
Department of Mechanical Engineering
McGill University, Montreal

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Abstract

Composite materials have immense capabilities in terms of tailorability and studying in-depth the Automated Fiber Placement (AFP) technique can go a long way in achieving the above goal. Composite laminates fabricated by AFP can also induce defects such as gaps and overlaps. These defects mainly arise in laminates with curvilinear fiber paths. Gaps usually get filled with resin post-curing while the overlaps are thickness build-ups. This makes it important to analyze the effect of these defects and to incorporate these effects while designing and modeling composite laminates. The present work introduces a new finite element methodology that incorporates a ‘defect element’ which has material strength properties that vary with defect area percentage. The salient points of this methodology include ease of application and applicability of a single model to one side tow drop as well both side tow drop strategies. This methodology will equip designers with a computationally faster approach towards analysing curvilinear fiber laminates with defects and to model them as close to reality as possible.
Résumé

Les matériaux composites ont des capacités immenses vis-à-vis la flexibilité de leur design. Étudier en profondeur la technique de placement automatisé de fibres (AFP) est nécessaire pour atteindre l'objectif ci-haut mentionné. La méthode AFP, utilisée pour des composites stratifiés, peut induire des défauts tels que des écarts et chevauchements de plis. Ces défauts se manifestent principalement dans des composites dont les fibres suivent des trajectoires curvilinéaires. Les écarts de plis sont généralement remplis de résine post-cuisson, alors que les chevauchements contribuent à une augmentation d'épaisseur. Il est donc important d'analyser l'effet de ces défauts et d'incorporer ces modifications lors de la conception et de la modélisation de composites stratifiés. Le présent travail introduit une nouvelle méthodologie d'éléments finis qui intègre un « élément défectueux ». Ce dernier possède des propriétés de résistance de matériau qui varient avec le pourcentage de surface des défauts. Les points saillants de cette méthodologie sont la facilité d'application et l'applicabilité d'un modèle unique peu importe la configuration des fibres. Cette méthode permettra aux concepteurs d'approcher plus rapidement l'étude et l'analyse des composites à fibres curvilinéaires comportant des défauts et de les modéliser de façon réaliste.
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CHAPTER 1: Introduction

Fiber reinforced composite materials have gained a widespread popularity over conventional materials in fields such as aerospace, construction, consumer products, transportation and sporting goods. The most important of the properties that provide an edge over conventional monolithic materials are design tailorability, strength-per-unit-weight, good fatigue life and high stiffness-per-unit-weight. The quality of the finished products is comparable to the ones produced by aluminum in terms of both the surface finish and part integrity. Despite the apparent advantages of composite materials, their main disadvantage lies in the fabrication and processing. Processing of composite structures has traditionally been cost and labour intensive and this has prevented a far greater application of composites. This anomaly has been the propelling force behind many of the works done by various researchers to optimize the composite structures thus produced with the primary goals of reduced labour and material costs, reduced processing times and higher part quality. One important area of research is to improve and optimize manufacturing processes.

1.1 Conventional fabrication techniques

Composite materials consist of fibers and the matrix or resin material as two distinct constituents. These two components are macroscopically identifiable. Fidelity of any composite structure depends on the effectiveness of the matrix or
resin to bind the fibers compactly. The component performance is thus intimately linked with the manufacturing method. The manufacturing method varies according to the phase of the raw materials, i.e. different for pre-pregs (fibers that are pre-impregnated with resin) and dry fiber forms. Hand or manual lay-up method is preferred for simple structures or for those where weight does not need to be optimized. Advanced fiber placement techniques for placing the pre-pregs on to the desired geometry are utilised for highly specialised structures for which strength/weight ratio is important. These pre-pregs provide more precise fiber orientations than the hand lay-up. Some of the manufacturing techniques that are utilised are summarized here.

Hand lay-up or contact moulding technique is one of the historically earliest and simplest manufacturing techniques for composite structures. In this process, the components are produced on a mould coated with releasing agents. Structures with desired thicknesses are built by stacking layers of the composite material (dry fiber forms or pre-pregs) on the mould. In the case of dry fibers, liquid resin is applied that leads to wetting of the fibers. Layers are stacked by applying resin to the surface followed by reinforcements (fibers). Compaction is usually carried out by draining out the excess resin using hand rollers or ‘squeegees’. The advantage of this method lies in low capital cost due to simple tooling and mould requirements. However, major disadvantages of this method include limitation to
simple geometries, labour intensive and the part quality depends on the skill of the labour employed. Also, as fiber volume fractions cannot be monitored accurately, mechanical properties can be variable. An improvement to this process is to use pre-impregnated composite materials instead of dry fibers and liquid resins. This gives much better control over part quality and resin content. Nevertheless, using pre-preg on a mold still requires a great deal of manual labour. To overcome these problems, industries have started to employ automated cutters and laser guided lay-up markers. These reduce the lay-up time and help in reducing waste material.

Another fabrication technique is the Resin Transfer moulding (RTM) method. In this method, a fiber perform is inserted into the cavity of a closed mold tool and resin is injected into the tool. Compaction is performed by applying resin pressure and maintaining external pressure on the tool faces until the mold is full and excess resin bleeds out of the exit vent. The compaction caused by the advancing resin front expels air and the resin-impregnated reinforcement cures before removing the part. The major advantage of this method is the relatively high level of automation which removes manual labour as a variable in achieving desired part quality. The disadvantage of this method is the application of pressure which might offset one of the tooling surfaces with respect to other, fiber movement due to the pressure of the advancing resin front, and relatively low
fiber volume fractions that can be achieved. Also, as in many composite manufacturing processes, the part quality depends on the surface finish of the tooling surface.

Filament winding produces structures by winding bands of continuous fibers impregnated with resin onto the structure directly or onto a mandrel that can be removed once it is done. This is performed by using an automated technique such as a 5-axes filament winding machine. The mandrel has the shape of the part to be fabricated and it is rotated between supports while fibers are drawn out from large spools on carriages. The carriages move in a path parallel to the mandrel rotation axis. Fibers get wrapped onto the geometry in helical fashion where the helix angle is determined by the relative speeds of the carriage and the angular velocity of the mandrel rotation.

A schematic of a filament winding system is shown in Figure 1.1. After the desired lay-up has been achieved, the part is then cured and the mandrel may/may not be removed. This method is more efficient than hand lay-up in reducing the labour time and cost. However, the laminate manufactured by filament winding can have high void content that can degrade the part quality and lower the mechanical properties. This process is also limited to some specific part geometries due to the problem of fiber bridging [2] as shown in Figure 1.2, i.e. even if we increase the applied pressure via the tension of the drawn fibers; it
might be that the fibers do not follow the mandrel geometry. Filament winding is thus restricted to convex geometries in order to avoid fiber bridging.

Another disadvantage is the additional manufacturing of collapsible or removable mandrels. These problems associated with filament winding have given rise to automated techniques such as Tape Winding and Automated Fiber Placement (AFP).

In order to automate the composite manufacturing process, some methods can be used that will reduce some of the problems encountered by the methods explained in section 1.1. The goal of these automated processes is to reduce manual intervention and to achieve accurate placement of fibers. Two such techniques are described here: the Tape Winding process and the Automated Fiber Placement (AFP) process.
Tape winding comprises of placing of pre-preg tape onto a tool surface that has been coated with a release agent. Tape widths are usually available in dimensions of 3 in., 6 in. and 12 in. The pre-preg from the spool is placed onto the tool surface by the lay-up roller. The pre-preg is then heated and the pressure is applied for compaction. This compaction removes any trapped air and thus provides lower void content than the filament winding and heating decreases resin viscosity and increases tackiness. It is cut to size and shape by the cutters provided. This process is a high output process but with limited applications due to the high cost of tape laying machines and the pre-preg materials. Another major advantage of this method is the ability to cut and restart the material and as a result, geometries with convex or concave shapes can be efficiently fabricated using tape lay-up. Also, larger range of ply fiber orientations can be applied as compared to the filament winding technique. The disadvantage of this process is the limited deformation of the tape used in this process due to a wide bandwidth. As a result, constraints of minimum curvatures are placed on the manufactured part. To overcome this defect, Automated Fiber Placement (AFP) has been introduced that utilises a smaller band width. AFP provides flexibility and better part quality. Automated Fiber Placement is explained in detail in the next chapter.
CHAPTER 2: Automated Fiber Placement technique

2.1 AFP machine

Automated fiber placement (AFP) is a relatively new composite structure fabrication technique that has been developed to incorporate the advantages of tape placement and filament winding techniques and to alleviate their limitations such as fiber bridging and restriction on the fiber bandwidth. AFP provides the designer great accuracy and flexibility and is widely used in the aerospace industry for fabricating cost-effective high quality and large parts. AFP can also aid in manufacturing parts with complex geometries or severe curvatures.

Figure 2.1: Automated fiber placement machine¹.
A typical AFP machine, as shown in Figure 2.1, consists of a fiber placement head (shown in Figure 2.2) mounted on a robotic arm and has six or seven degrees of freedom (DOF) depending on the type of AFP machine. The material is fed to the arm in the form of a pre-preg slit tape consisting of bundles of fibers that are pre-impregnated with resin. A bundle of pre-preg fibers, called a tow is passed through a guiding system onto the fiber laying head and the head allows the alignment of the multiple tows to form a wide band. Each tow can vary from 3.18 mm to 6.35 mm (0.125" to 0.25") wide. During the formation of tows, the payout from spools is controlled individually and these tows can be cut and
restarted to provide for a continuous supply of the material. Each of the wide bands used to cover a particular dimension is called a course. Course widths vary according to the type of AFP machine. Usually, AFP machines can place up to 32 tows per course. The head guides the placement of the tape to the desired location and also performs the compaction of the tape using the compaction roller. It also heats the tape to increase the drapability and tackiness of the tape onto the surface. This combination of material tackiness and compaction aids the head to place the tows accurately onto concave surfaces without any peeling off effects and also guides the tows to follow the part geometry. The AFP head has the capability to cut and restart the tows individually and this provides it the ability to vary the fiber orientation and to drape geometries with high curvatures.

AFP technique provides the ability to fabricate laminates that have variable fiber orientations in the plane of the laminate that only helps in realizing the true potential of composite structures but also aids in weight reduction. This is achieved by better load distribution in the composite structure and thus aids in optimizing the composite structure. Despite all these advantages, AFP suffers from some limitations that are preventing industries from completely realizing the potential of AFP. During the fabrication of laminates using AFP, defects emerge that not only alter the properties but also reduce the quality of the product. Although, studies have been performed but there is still a need to understand
effect of these defects. This has prevented the designer to fully exploit AFP’s true potential.

2.2 Curvilinear fiber concept.

This section explains in detail the curvilinear fiber concept that aids in designing variable stiffness laminates.

2.2.1 Straight fiber laminates

Composite materials have been favoured over the conventional monolithic materials due to their high stiffness and high strength-to-weight ratio. Composite materials also provide the ability to customize mechanical properties and structural response. Conventional composite structures were limited to constant fiber orientations in the plane of the lamina. This means that for a particular layer, the fiber orientation is chosen and does not vary over the entire area of that layer. Although this approach provided better strength properties with respect to the metallic counterparts, a full composite tailoring capability was still not applied. Traditional laminates consist of multiple plies that are stacked on top of each other to a desired thickness. The plies have specific thicknesses and material properties and they have the same fiber orientation in the plane of the lamina. Due to constant fiber orientation for each ply, their structural response remains constant throughout each ply. In other words, the plies cannot be optimized for a particular loading condition. The tailoring capabilities in terms of design variables
are limited for constant stiffness laminates compared to variable stiffness laminates as will be discussed in the following section. Methods of tailoring composite laminates with constant fiber orientations include changing thickness, material properties and/or fiber orientation through the thickness. The major disadvantage of such an approach is the increase in costs and/or weight that might not be acceptable in the aerospace industry.

2.2.2 Variable stiffness laminates using AFP

As mentioned before, composite laminates can have optimized loading response but initially, researchers restricted tailoring of composite laminates to variable volume fraction [3] and/or varying thickness by dropping and adding plies and varying orientation angle through the entire ply. Leissa et al. [4] investigated the effect of varying the fiber volume fraction of a single layer composite laminate for vibration and buckling responses. The response of the laminate was shown to be x-direction dependant as the fiber volume fraction was also a function of the x-direction. They also showed that following such an approach and having a higher fiber density around the center of single layer laminates, the buckling load and the fundamental frequency was increased by 38% and 21%, respectively, compared to single layer laminates with uniform fiber densities. DiNardo et al. [5] investigated the effect of changing the thickness of the laminate for buckling and post-buckling responses. They employed the ply dropping technique to fabricate
laminates with variable thickness. Plies were terminated before they filled up the entire laminate area. They found that dropping of the plies led to complex buckling response and the change in the bending stiffness (from the drop-off to the non-drop-off region) had a greater effect on the buckling response than the unsymmetrical lay-up. These 2 approaches for generating variable stiffness laminates resulted in laminates that could not be optimized for both weight and cost considerations.

The other approach towards generating variable stiffness was by changing fiber angles throughout a lamina. Hyer et al. [6], presented one of the first studies performed on the curvilinear fiber laminates. Before their work, curvilinear fiber format was not analyzed completely as the implementation was still a problem. They based their design problem on a square plate with a central hole. The main objective of their work was to look at the possibility of increasing the load carrying capacity of the plate by varying the fiber orientations around the hole. They were also interested in looking at the dependence of this load carrying capacity on the width-to-hole diameter ratio. The finite element method employed by them provided a constant orientation within each element. An iterative process was used to arrive at fiber orientations oriented along the principle stress directions. The baseline laminate was a quasi-isotropic 16 ply laminate, [(±45/0/90)]₂s while the curvilinear laminate was denoted [C₈]ₙ. The major limitation of their work was
the coarseness of the mesh which meant that the fiber orientations changed by a large amount from one element to another. This posed problems for manufacturability of these laminates.

Hyer et al. [7], as a continuation of their previous work, increased the buckling resistance of a composite square plate with central circular hole as stress concentration (width-to-diameter ratio, \( W/D = 3.0 \)). The composite plate was loaded in compression and fiber orientation was determined using the gradient-search technique. In the beginning, the quarter plate was divided into 18 regions that were radially aligned as shown in Figure 2.3.

The element used was an 8-noded iso-parametric element and the material used was a typically available polymer matrix. The laminate was composed of 12
plies with varying fiber orientations that were sandwiched between \( \pm 45^\circ \) laminae. Fiber orientations for the 12 curvilinear plies were determined by gradient searches and sensitivity analyses were performed to determine which elements were affected more by a particular response. Displacement loading was applied to closely resemble the experimental set-up. Each element had a constant fiber orientation and the orientation changed from one element to another. The major problem with this approach was the large number of variables for sensitivity analysis. Also, results from the analysis showed that the orientation varied by large values between neighbouring elements. The optimized fiber orientations varied up to 75\(^\circ\) between the adjacent elements and this proved to be the most important limitation of their work. Current fiber placement machines cannot deal with such large variations in the fiber orientation angles. Also, a large discontinuity in the fiber angles would lead to a drastically changed stress states. This made the panel non-feasible to be manufactured by AFP as it would cause large kinks in the tows even though it increased the buckling load by 2.96 times that of the baseline. It was also found that buckling load was not sensitive to fiber orientations in some regions. They thus modified the design (the optimized design is shown in Figure 2.4) which provided improvements in terms of manufacturability but at the cost of buckling load. The increase was 1.85 times that of the baseline as compared to 2.96 times for the previous case. They also
demonstrated an increase in the tensile strength and buckling load, for the final specimen, of 35% and 30%, respectively. Their approach showed that alignment of the majority of the fibers along the loading directions can go a long way in improving the structural response of the laminates. They were efficient in removing the non-manufacturability aspects of their earlier design [6]. A limitation of their work was that they did not look into the free edge stresses that might have played a significant role in determining the effective fiber orientations. Another shortcoming of their approach was a large design space (fiber orientations in each element) which made the entire process computationally expensive.

Due to the apparent disadvantages of the above methods, researchers started working on methods that provided an alternative methodology to design variable stiffness laminates that resulted in not only optimizing them for loading responses but also for weight and cost requirements. As mentioned before, Automated Fiber Placement (AFP) machine fabricates laminates with curvilinear fiber paths suited to a particular loading requirement. The following section will explain in detail, the theoretical aspects behind generating a curvilinear fiber path.

2.3 Curvilinear Fiber path generation

Variable stiffness laminates, as mentioned before, have the ability to tailor the laminate according to the structural requirements. This tailorability induces
additional possibilities on the design formulation than the traditional constant stiffness laminates. In composite laminates, stress distribution is a function of the fiber orientation, stacking sequence and laminate thickness. While this stress distribution makes straight fiber composites perform better than isotropic materials in some cases, their reduced tailorability limits the possibilities for optimization of such laminates. Variable stiffness laminates alleviate this problem and offer more flexibility by varying the stiffness as a function of location and can thus distribute stresses throughout the laminate, thereby reducing the local stress concentration effects. However, the design space results in many variables which again is a hindrance in the optimization routines. This can be removed by applying certain restrictions on the available tow paths as explained below.

The fabrication methodology followed by typical AFP machines involves laying down number of plies over the surface of a mandrel. The machine software also facilitates the conversion of a three-dimensional geometry to ‘surface coordinates’ for thin plates and shells [8]. The AFP machine lays down curvilinear fiber tow paths through its fiber placement head that passes along the centerline of each course. These paths are laid side by side to cover the entire surface of the ply. There are a number of relations that can be formulated for specifying the change of fiber orientation but one of the simplest ones is the linearly varying fiber orientation. The path equation for the tow center line is a continuous linear
function such as the one defined by equation (1). This function takes into account the manufacturing constraints that are discussed later. These curvilinear fiber paths follow constant curvature arcs in order to simplify the numerical analyses and also to provide a relation to the minimum curvature constraint of any AFP machine (which is unique to every machine).

Curvilinear fiber path can be defined by adequately defining fiber orientation for any coordinate point of the lamina. This is achieved by first defining a ‘reference path’ [8]. The reference path is a curve that can accommodate various manufacturing constraints. Entire path function can be defined by six parameters, \(x_o, y_o, T_o, T_1, \Phi\) and \(d\). The above parameters are defined in Figure 2.5. The reference path commences from \(x_o, y_o\) and the initial orientation \(T_o\). Distance ‘\(d\)’ is a characteristic length which for a rectangular or a square plate is ½ of the side dimensions. \(T_1\) is the fiber orientation angle at a distance equal to \(d/2\). The path follows a constant curvature arc between points \{-d/2,-d/2\} to \{d/2, d/2\}. Finally, ‘\(\Phi\)’ defines the axis-rotation angle for fiber orientation. After rotation of the fiber orientation axes (\(x', y'\)) by ‘\(\Phi\)’, the fiber orientation changes along this direction. The fabrication axes are \(x\) and \(y\). In summary, any curvilinear fiber path can be defined as:

\[ \pm \Phi < T_0/T_1 > \] (1)
The ‘±’ denotes 2 adjacent layers with equal and opposite $T_0$ and $T_1$ angles. Depending on the requirements, the AFP machine can lay down tows so as to prevent big gaps and/or overlaps by following a ‘differential tow pay-out scheme’ where the tows can be cut and restarted whenever the desired thickness is achieved. Also, for covering the entire lamina, the reference path is offset by a set distance along the desired axis of shift. This shift can be ‘parallel’ or ‘shifted’. Description of the above 2 strategies is explained in the following section.
2.3.1 Parallel path configuration

The ‘parallel’ path strategy creates fiber paths such that every point on the path is at a constant distance from the reference curve. Points along a horizontal line will not have same fiber orientation i.e. the fiber orientation and thus the stiffness variation is along both, x’ and y’ axes. The solid lines in Figure 2.6 [9] represent the course centerlines while the dashed lines indicate the course boundaries. The parallel shift strategy requires the edges of the neighbouring tows in a course to be flush with each other and this causes changes in fiber orientation depending on the coordinates of the point under consideration. This might cause the radius of curvature to be less than the minimum radius of curvature that the AFP machine can handle. This will lead to out-of-plane buckling of the tows and lead to structural imperfections. The restriction on the minimum radius of curvature is related to the difference in $T_0$ and $T_1$: larger the difference, smaller will be the radius of curvature.

2.3.2 Shifted ply configuration

According to Waldhart [3], once the reference fiber path has been defined by equation [1], ‘shifted’ fiber paths are formed by shifting the reference path in the $y'$ direction. The remaining courses are formed by shifting the reference path by different amounts along the $y'$-direction as shown in Figure 2.7. In other words,
this strategy assumes that each subsequent path follows the same shape as the baseline case but shifted in a direction perpendicular to the stiffness variation.

The stiffness variation is perpendicular to the direction of the shift. Unlike the parallel shift, this strategy does not have tows inside a course flush with each other. This can lead to formation of defects such as gaps and/or overlaps. To reduce the size of gaps/overlaps, the AFP machines can cut/restart tows to fit required number of tows within a course. Post-curing, gaps are composed of resin and thus reduce the effective laminate material properties. The overlaps are local thickness changes which can improve stiffness locally and can thus aid in

Figure 2.6: Parallel fiber path. Figure 2.7: Shifted fiber paths.

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distributing stresses better than other laminates. However, overlaps can create locally unbalanced laminates and thus cause instabilities.

2.4 Generation of defects:

As mentioned before, ‘shifted’ strategy generates gaps and/or overlaps due to a constant tow width. Overlaps can be eliminated to some extent by cutting and restarting the tows by the AFP machine. Tatting [8], introduced a ‘coverage percentage’ parameter that determines if the curvilinear fiber laminate will have gaps and/or overlaps. Coverage percentage is shown in Figure 2.8. As shown in the figure, 0% coverage percentage is depicted by cutting of the tows as soon as the first edge reaches the course boundary. This leads to the formation of triangular gap regions (a, yellow) that will be filled with resin post-curing. A coverage percentage of 100% implies cutting of the tow when the second edge reaches the course boundary. This produces regions of overlaps (c, green) that are thickness build-up regions. A coverage percentage somewhere between 0 and 100% determines the distribution of gaps and overlaps (b). This cutting of the tows or ‘tow-dropping’ can occur from one side of the course or both the sides. As mentioned by Gürdal [8], defects such as gaps can be diminished by defining a suitable coverage percentage. Similarly overlaps produced can be
reduced by using the interweaving technique or shifting the fiber paths by a certain amount for every ply after the 1\textsuperscript{st} ply.

![Diagram showing coverage percentage](image)

**Figure 2.8:** Coverage percentage\textsuperscript{21}.

This shifting of the fiber paths helps to even out the thickness variations due to overlaps. The AFP machine head has some limitations which necessitated the development of more feasible curvilinear fiber path definitions than the one initially proposed by Hyer et al. [6,7]. These constraints include the minimum radius of curvature, fiber orientation range, speed of the AFP machine, compaction pressure, path tolerance and path definition [12].
Curvatures smaller than the minimum prescribed can cause fiber kinking, out-of-plane waviness and can thus drastically affect the structural response and dimensional stability of the composite structure. According to Waldhart [3], the effect of $R_{\text{min}}$ depends upon the strategy (shifted or parallel) applied to form the laminate. For shifted fiber paths, as all the paths follow the reference path for all the points, applying $R_{\text{min}}$ constraint on it can take care of this problem. For parallel fiber paths, application of this constraint can be a source of error as the paths consist of points that are equidistant from the reference path. This forces the center of curvature to be effectively placed outside the panel. Other issues
that might arise due to the spatially varying stiffness are balancing and symmetry.

Unbalanced laminates might introduce bending-extension and extension-shear couplings that are usually unacceptable. Symmetry is an important issue as non-symmetry about the middle surface of the laminate causes unintentional curvature post-curing. Balancing of variable stiffness laminates can be achieved by using the mirroring technique as defined by Gürdal [8] and is shown in Figure 2.11. In general, a minimum of 4 plies are required to obtain a balanced laminate on a structural level as ply level balancing is not possible for tow steered plies. Symmetry for variable stiffness laminates is achieved as long as fiber orientations are ‘identical’ in location as well as for parameters \{x_0, y_0\} and d as
defined in Figure 2.5. Again, for laminates that have local thickness variations due to overlaps, there can be extension/bending coupling due to this thickness change even if the laminate is symmetric about the mid plane. Generally for AFP machines, a symmetry counter is used that provides a 180° rotation to ensure the symmetry according to Figure 2.12. This will lead to symmetrical laminates in both overlap and non-overlap regions.

2.6 Thesis organization

The research presented in this thesis is a part of the Consortium for research and innovation in aerospace in Quebec (CRIAQ) COMP 413 project. As mentioned by Croft [12], the ultimate objective is to fabricate optimized composite structures using tow steered design via characterization of defects generated by Automated Fiber Placement (AFP). The present research is a continuation of the work done by Croft [12] and extends the experimental characterization of defects performed by him to finite element modeling of defects such as gaps and overlaps. The present research proceeds with the finite element model description of the various test specimens. Initially, the test specimens will be constant stiffness laminates with constant fiber orientation. The models will be validated using experimental testing done by another participating university (École Polytechnique de Montreal). After the initial validation, the model will be extended to cover a wider range of defect areas, an area of research that has not been
dealt with previously. Quasi-isotropic laminates will be modelled and validated in a similar fashion.

Finally, the model will be extended to curvilinear fiber path laminates and case studies performed to demonstrate the applicability of the methodology developed. As mentioned before, there are a number of defects that can be introduced in a laminate with curvilinear fiber paths such as gaps, overlaps, twisted tows and half gap-overlap. The present research mainly deals with gap, overlap and half gap-overlap configurations. The work performed in the present study will form a basis on which industry can evaluate their defect models and effectively predict the properties and structural response of curvilinear fiber laminates with manufacturing defects. Additionally, the results from the present work will be utilised for developing an optimization routine.
CHAPTER 3: Literature review

3.1 Research done on curvilinear fiber laminates and defects

We have seen from the previous section that curvilinear fiber path laminates have driven the researchers to look into optimizing the AFP process since it was started in 1990 [13]. Although, there are several studies available in literature that deal with the structural response of these laminates, yet only a handful of these studies deal with the defects introduced by AFP. This has been considered to be one of the major areas that have prevented the complete utilisation of the AFP process. The author of the present work believes that the defects will have an effect on the structural properties beyond a certain percentage and that these defects do not allow for the development of an optimum composite structure design. The analysis that has been performed in the studies done so far has been both experimental and numerical (including finite element analysis) and as the present work mainly deals with the finite element modeling of variable stiffness composites, review of these analyses assumes utmost importance.

Although the literature review is brief, yet for a detailed understanding of the work done, the reader is encouraged to explore suitable references that are cited in this section. The conditions starting from the material used, the lay-up definition to the boundary conditions applied are unique to the present course of study, so direct conclusions from already established studies cannot be used to compare
results but the results can help a designer to form an informative decision about the effect of the defect on the structural response for such laminates.

Croft [12] performed the characterization of defects produced by Automated Fiber Placement (AFP) in composite structures. The work performed is one of the few studies done to characterize defects like gaps, overlaps, twisted tows and half gap/overlap. Croft utilised CYCOM 5276-1 (unidirectional carbon/epoxy system) for his research which will also be the material under consideration in the present work. The mechanical tests performed were simple tensile, simple compression, in-plane shear, open-hole tensile (OHT) and open-hole compression (OHC) test. The first 3 tests were performed to ascertain lamina level properties, moduli and strength values while the last 2 tests provided the laminate level response on tensile and compressive loading with stress concentrations. All the tests followed ASTM specifications. The defects were introduced in the middle of the layers to keep the entire laminate symmetric. It was also assumed that as the tow width of the material laid by AFP is small, 1 defect will not affect the properties so 2 defects were symmetrically placed about the central axis. Micrographic analyses were performed and it was found that the effect of defects on tensile tests were to reduce the thickness variations by the movement of the tows. It was also revealed that the gap regions were not entirely
resin rich but also had neighbouring tows impinging into the gap areas. Some of
the micrographic pictures are shown in Figures 3.1-3.3.

![Micrographs of gaps](image1.png)  ![Micrographs of overlaps](image2.png)  ![Micrographs of half gap/overlap](image3.png)

**Figure 3.1:** Micrographs of gaps\(^{12}\).  **Figure 3.2:** Micrographs of overlaps\(^{12}\).  **Figure 3.3:** Micrographs of half gap/overlap\(^{12}\).

The overlaps produced thickness variations of the same order of magnitude as
the gaps. All the above observations applied for the compression testing as well.
All tensile samples failed catastrophically in the middle of the gage section while
the compressive samples failed according to the transverse shear or through-the-thickness mode of failure. For in-plane shear tests, orientation of the unidirectional tows played a big role in determining the compaction achieved while curing, making defect size a critical parameter. In this configuration, gap regions were composed of resin rich areas and thus gaps lowered the effective strengths of the cured laminates. Overlaps provided better rigidity to the structure and the fiber waviness was more constant than the gap case. Half-gap overlap scenario affects the properties on a smaller scale. Defects along the width of the specimen affected the strengths more than the defects aligned along the length. Open-Hole Tension (OHT) and Open-Hole compression (OHC) samples had similar defect geometries and it was found that random defects do not produce any major thickness changes as well as large amounts of fiber waviness. In OHC samples, the defect orientation affected the ultimate strength by causing rapid delaminations. The overall effect was seen more on the laminate level than the lamina level due to high levels of ply consolidation obtained by AFP. A brief summary of the results is shown in Figure 3.4. A limitation of his work was introduction of only 1 defect per test coupon. This can be a source of error when the defect density per coupon becomes more or if the defect covers a larger area percentage.
Gürdal et al. [14] studied the in-plane response of variable stiffness laminates. The fiber orientations were varied along the length of the composite laminate, resulting in varying fiber orientations with location. The in-plane elasticity problem was evaluated by solving the governing in-plane equilibrium equations of a plate under uniform end-shortening. The fiber orientation function as defined by them was,

$$\theta(x) = \frac{2(T_1 - T_0)}{a} x + T_0$$ , where ‘a’ is a characteristic length \( (2) \)
The fiber angle starts from the angle $T_0$ at the panel mid-length, $x=0$ and reaches a value of $T_1$ at a length $x=a/2$ (plate edge). The fiber path was assumed to be symmetric about $x=0$. The benefit of such an approach was the simplicity of the fiber path definition as well as the resulting fiber path was feasible to be manufactured. Due to the dependence of the fiber path on the location ($x$-coordinate) the modulus in $x$-direction, $E_x$ and poisson’s ratio $\nu_{xy}$ also were functions of $x$-coordinate. The coupled differential equations were solved using ELLPACK elliptic differential equation solver. Three different in-plane boundary conditions were considered,

Case 1: At transverse edges, $N_y(x,y)=0$, $N_{xy}(x,y)=0$ and $N_x=\text{constant}$ (transverse plate edges are free to move)

Case 2: Displacement in $y$-direction, $\nu$, was fixed at $y=\pm b/2$, where $b$ is the width of the panel as well as at the transverse edges $N_x=N_o$, $N_{xy}=0$ and $\epsilon_y=0$.

Case 3: $N_x=N_o$, $N_{xy}=0$, $\nu=\epsilon_o y$ and $\epsilon_y$.

Closed-form solutions to the above differential equations were computed by applying the above conditions. These solutions were then validated by a numerical model that was a $10\times10$ grid which was based on the convergence model based on displacements. They considered 3 cases which were free transverse edges, fixed but straight transverse edges and free but straight transverse edges. The first case provided similarities between the numerical and
closed-form solution for axial stress resultant for \( T_1-T_0 = 15^\circ \) while it was comparatively more for higher differences in the values of \( T_0 \) and \( T_1 \). As the value of \( y \) become \( y/b \), the closed form solution over-estimates the value of \( N_x \) and this error reaches the value of 8% at the panel end. For the remaining 2 cases, the results are nearly exact, with an error of around 0.04%. They also showed that the changes in the elastic properties will generate transverse stresses even when the transverse edges are not loaded as well as shear stresses when there is no shear-extension coupling present. They have also suggested that this can be a useful tool for the designers to further exploit the abilities of curvilinear fiber laminates.

Nagendra et al. [15] demonstrated that the fiber orientation definition for curvilinear fiber paths can be optimized and tailored for various loading conditions. The main focus of their work was to generate optimized fiber paths that could be easily passed on to the AFP machines. Their optimization routine was based on NURBS (Non-Uniform rational B-splines). The AFP machine used in their research was developed Cincinnati Milacron Inc. and consisted a fiber placement head mounted on a 7-axis CNC machine. According to them, the tow fiber path optimization problem was that of a shape optimization formulation. A basis shape function was used to obtain optimized fiber path corresponding to a given layer. The final fiber path was determined as a function of scalar multipliers
of the basis fiber path. Each basis fiber path was a NURBS curve generated by the interpolation of control points. They utilized CSTEM® code for finite element analysis and for the optimization based on approximation based concepts. TAGUS® software was used as geometry engine and it worked in tandem with CSTEM®. Ply courses were generated as fitting curves ('lofted curves') which passed through a series of semi-circular arcs formed by sets of three points that were defined as “based on the offset distance, 3 points are defined along the negative surface normals and the along the cross-product of the normal and the curve tangent”. These ‘lofted curves’ were then repeated to cover the entire plate or geometry. The next step was to compute fiber orientation for each element in the finite element model. This was obtained by the dot product, $t_e . b$, where $t_e$ was a tangent vector along the ply course and $b$ is the vector parallel to material 1 axis as shown in Figure 3.5.

![Diagram of fiber orientation angle calculation](image)

Figure 3.5: Calculation of the fiber orientation angle.
They assume the fiber orientation to be a piecewise linear function as Gürdal [14]. Failure analysis was done on the basis of Tsai-Hill and Modified Hashin criteria. The case studies performed by them comprised of a flat plate with a central hole (maximized for critical buckling load) and a composite engine fan blade (minimize twist-bend coupling parameter in the portion of the blade excluding the dovetail). The FEM model for the first case consisted of radially located elements as Hyer [6,7] and the results were compared to the baseline laminate \([45/-45/0_6]_s\). The optimized lay-up was designated \([45/-45/C_6]_s\). The mesh consisted of 18, 8-noded brick element (1 element through the thickness). Improvements in the buckling load were 1.85 times that of the baseline. The overall effect of varying fiber orientation was to reduce the stress concentration factor \((K_T, \text{ratio of the stresses near the hole to the far-field values})\) by 20%. The buckling load factor was increased from 0.77 for the baseline to 1.30 for the optimized lay-up. For case 2, only 1 element through the thickness was used and the resulting twist-bend coupling was around 0.403% of the baseline.

Waldhart [3], investigated variable stiffness laminates subjected to uni-axial compression and in-plane shear deformation. The fiber path defined was again a piecewise linear function that assumed one value at the center of a square plate to another value at a characteristic distance, as in equation 1. It was found that the curvilinear fiber path aided in redistributing stresses away from the stress
concentration regions and/or created beneficial stress distributions at the stress concentration regions. ‘Shifted’ and ‘parallel’ fiber paths were compared to the equivalent straight fiber baseline. For the laminate \([90\pm<30/75>]_{9s}\), the critical buckling load and the axial stiffness were increased by 44% and 124% respectively compared to \([\pm45]_{9s}\). For in-plane shear loading there was an increase of 46% for \([\pm (45<15/60>)]_{9s}\) for the critical shear buckling load over \([\pm 45]_{9s}\). The results also showed that the ‘parallel’ fiber paths did not show much improvement compared to the ‘shifted’ laminates as the stiffness did not change as dramatically as the shifted fiber path. The limitations of this work include the non-involvement of the course boundaries i.e. the course boundaries were assumed to be smooth between neighboring courses. As the work was an analytical approach towards tailoring variable stiffness laminates, experimental validation was not done. He assumed that the tow paths could be defined by individual fiber paths and this made the orientation at any point to be path dependent. In actuality, the fiber orientation is only a function of the location. Also, another assumption was that there is no thickness change anywhere in the laminate. Indirectly, manufacturing defects such as gaps/overlaps were not considered for their analysis.

Sawicki et al. [16] were among the first researchers that performed the analysis of effect of intraply gaps and overlaps on the compression strength of composite
laminates. According to them, there is higher likelihood for the laminate; fabricated using the AFP machine, to have more gaps/overlaps due to greater number of tows being laid compared to hand lay-up. As a result, more defects cause higher levels of out-of-plane fiber waviness in adjacent plies. The defects ranged from 0.03" to 0.12" wide and the laminates were made of fiber placed IM6/3501-6. 3 different quasi-isotropic laminates were considered for their study. Defects were introduced as a strip of required width between the plies. Tests were performed according to ASTM standard and an IITRI anti buckling fixture was utilized.

Figure 3.6(a): Sawicki defect specimen\(^\text{16}\). Figure 3.6(b): Test coupon for Sawicki\(^\text{16}\).

The specimen and the test coupon are shown in Figures 3.6(a) and 3.6(b), respectively. Finite element model was made in MSC/PATRAN\(^\text{®}\) and 2944, 4-noded quad elements with 2-D solid properties. The defects were modeled by
varying the thickness of the 90° plies locally. They found that reductions varied from 5% to 27% for defects greater than 0.03" wide and no further reductions were observed for wider defects. Finite element analyses demonstrated the main cause of failure to be a combination of in-plane compression and interlaminar shear stresses caused to the out-of-plane waviness. The shear stress levels were shown to be proportional to the slope of the out-of-plane waviness. Compression strength was also found to be affected by the presence of high humidity and temperature. Tension strengths were affected on a smaller scale as compared to compression strength reductions. Another conclusion of their work was that after the degradation of strengths due to the first defect, rest of the defects if present, did not cause any more significant reductions. They did not consider curvilinear fiber paths with defects as well as other strength parameters.

Langley [2], performed finite element modeling of tow placed variable stiffness laminates and was one of the first studies done to look into the analytical modeling of the variable stiffness laminates. Golden section search was used to find the distance of the center point of every element from the reference path and hence the orientation. The reference path was then shifted to over the entire plate. FEM analysis was done using GENESIS software and 2 cases were considered for the analysis. Case 1 was a square plate with uniform end shortening and free upper and lower edges. Case 2 was again a square plate
with uniform end shortening but with constrained upper and lower edges. A quarter plate was utilized to reduce the computational time and the mesh size was checked for convergence defined by proper characterization of the static response and enough elements to model the thickness variation throughout the plate. For laminate $[\pm<0/45>]_s$, it was found that the thicker regions took more load compared to less thick regions. This was true for both tensile and compressive stress resultants. This led to proper stress distribution throughout the plate. There was a drop in stiffness near the displaced edge due to high level of transverse strain. For shear stress resultant, it is zero along the free edge due to zero value of the shear force. The overlap regions are shown to have near zero values due to the unsymmetrical nature of the local laminate definition (approximately $[-\theta/\theta/\theta]_s$ or $[-\theta/\theta/\theta]_s$). The results from his work showed improvements over the work done by Waldhart [3] and tried to remove the limitations of his work as mentioned before. $N_x$ distribution for $[90\pm<45/75>]_s$ showed by him was more variable in nature than Waldhart. The reason for the change was presence of overlaps that were not considered by Waldhart. Results for $[0\pm<45/75>]_s$ follow the same trend for $N_x$ as before and are approximately twice the values predicted by the closed-form solution of Waldhart due to the thickness changes caused by overlaps. $N_y$ and $N_{xy}$ results showed that the trends...
of finite element results as well as the closed-from solution were consistent with each other.

Turoski [17] investigated the effect of manufacturing defects on the performance of carbon/epoxy composite pre-pregs. The material used was Torayca T800H/3900-2 and the layup was a quasi-isotropic with 31 plies but with varying resin gap configurations. The gaps were generated by removing different number of tows, one, two, three and four gaps. Gaps formed by four ply orientations are shown in Figure 3.7.

![4-Ply gap configuration for Turoski](image)

Figure 3.7: 4-Ply gap configuration for Turoski\textsuperscript{17}.

Each sample had a different stagger arrangement, details of which can be found in the literature. 3 main cases were considered as shown in Figure 3.8(a-c). Case 1 was an un-notched specimen (a) with no stress concentrations, case 2 (b)
consisted of an open hole with a centrally placed defect and finally case 3 (c) comprised of open hole with offset defect.

![Defect configuration models for Turoska](image)

**Figure 3.8: Defect configuration models for Turoska**

For compression tests, both notched and unnotched specimens showed reductions of 3-11% in failure stress with increase in the number of gaps. Additionally, offsetting the defect reduced the failure stresses 12 % more than the non-offset defect. One possible reason mentioned for the above reduction was the unbalanced fiber orientation around the hole due to the offset. Comparison of the results between the numerical and experimental results showed that the 4-gap case did not differ much from the 3-gap scenario. In plates with holes and for all gap cases the failure was predicted in the 0° plies. His work also demonstrated that gaps caused out-of-plane waviness and thickness variations. The small difference in the results for the experimental and numerical analyses
showed that the interaction between waviness and thickness variations was minimal.

Huang et al. [18] optimized fiber orientations around a hole for increased load carrying capacity. The laminate for their study was a symmetric six layer 
\[(C-0)/45/-45\]s laminate with a central hole. The C-0 layer was further divided into 2 distinct regions namely; far field regions with fiber orientations equal to 0° and the near-field regions around the hole that had varying fiber orientations defined by continuous bilinear interpolation functions. Tsai-wu failure criterion was used for failure analysis and \[(C-0)/45/-45\]s and \[(C-0)\]6 laminates were optimized using gradient based search method and genetic algorithm. For analysis purposes, they divided the near field region of the quarter-plate into 25 quadrilateral regions. The bilinear fiber orientation functions were obtained from the interpolation of fiber orientations at the four nodes of each of the 25 quadrilateral regions. The finite element model was built in NASTRAN/PATRAN and consisted of 4800 bilinear quadrilateral elements for a quarter-plate as shown in Figure 3.9. Similar ‘symmetry’ boundary condition was imposed on all the three layers. They found that the C-0 fiber orientation distribution pattern followed closely the principle stress distribution pattern.
Mesh sensitivity to failure load was checked for 3200 elements and 4800 elements and it was found that models with 4800 elements were able to accurately predict stresses and fiber angle gradients. It was found that on varying the fiber orientation around the hole region, the load carrying capacity of $[(C-0)]_6$ was increased by 100% and the load carrying capacity of $[(C-0)/45/-45]_6$ was increased by 88%. Another observation was that the near field fibers, formed concentric circles and they merged into far-field fiber orientations away from the hole.

Blom et al. [19] demonstrated the methodology behind fabricating composite cylinder with circumferentially varying fiber orientations and hence stiffness. Their
main aim was to optimize the cylinder for maximum buckling load and utilised ABAQUS for analysing the structural response of such shells. They applied the minimum turning radius constraint on the fiber paths. A reference path was defined and it was shifted perpendicularly to cover the entire surface. In their approach, they allowed overlaps while prevented gaps and assumed that the course boundaries are smooth. The fiber orientation was defined as a cosine function of the circumferential coordinate varied per stage and is shown in Figure 3.10 for an expanded cylinder.

![Figure 3.10: Blom cylinder model](image)

The finite element model was built in ABAQUS and composed of S4 shell elements with 4 nodes and 4 integration points. A FORTRAN routine, UGENS was used to pass shell stiffness for each integration point to ABAQUS. The
information about the local stacking sequence, material properties are passed to ABAQUS for calculation of stiffness matrices using classical laminate theory. In reality, the actual fiber orientation varies from the ideal fiber orientation due to finite width of the courses. Manufacturing constraints that were looked at were minimum radius of curvature, tow drop interfaces, minimum cut length and compaction pressure. The choice of constraint to be applied was dependant on the geometry and the final fiber angle configuration. Although improvements of up to 17% were obtained for buckling loads with straight fiber panels of the same weight, further improvements were limited by the manufacturing constraints. The panels analyzed by them were assumed to have a constant thickness and any defects were not considered.

Blom et al. [20], investigated the effect of the tow drop areas on the strength and stiffness of variable stiffness laminates. Previous studies performed on the variable stiffness laminates considered the course boundaries to be smooth while in reality, the course boundaries are jagged due to cutting and restarting of the tows. This leads to the generation of gaps and/or overlaps and may also reduce the structural properties. In their work, they took the studies done so far a step further by including these defects. They built a finite element model in ABAQUS and passed the local fiber orientation, material properties for the calculation of stiffness matrices. They chose a constant turning radius of 635 mm for their
study. They also defined the centerline of a course and then shifted the centerline by a specified amount to cover the entire ply. Their finite element model consisted of a 300 × 300 mm plate which was loaded under compression until failure. Out-of-plane displacements were prevented to prevent buckling of the test samples. The test panels were symmetric, [±<T₁/T₀/T₁>ₙ]ₘ laminates and the fibers followed a constant curvature paths. The mesh consisted of 10,000 S4 elements, 1.5 ×1.5 mm in size. The FEM boundary conditions are shown in Figures 3.11 and 3.12. Parameters that were investigated were tow width, number of plies and laminate thickness. It was noted that with larger width of the tows, the tow drop areas became larger and hence decreased the strength of the panels. Damage was seen to be initiated at the tow drop regions and it was proposed that in order to incorporate the tow drop regions, the values of material allowable can be reduced. The staggering technique was found to increase the strength of both, baseline as well as tow drop panels.
Fayazbakhsh et al. [21], investigated the effect of the manufacturing parameters on the tow drop regions of a variable stiffness laminate. The work presented by them can be applied to 3D and other complex geometries. They assumed straight fiber paths for non-planar geometries and a MATLAB® subroutine was developed to locate gaps and overlaps for such geometries. The design parameters investigated in their study were fiber orientation, number of tows and tow width and their effect on the gap area percentage for a truncated cone. The generation of fiber paths for a flat plate was similar to the one used by Blom [20]. The reference path was defined for the fiber paths and the machine head was offset horizontally to cover the entire plate. For location of the gaps and overlaps, tows from both sides were dropped. Also, any value for the coverage percentage
could be used in the MATLAB® routine developed. These were the additional things incorporated to the work already done by Blom [20]. For the truncated cone geometry, they incorporated the effect of gaps and overlaps and extended the work done by Blom [20]. They maintained a constant course width and thus captured the AFP process closely. The gap area percentages decreased with increase in tow number as well as an increase in the tow width. The effect of fiber orientation on the gap area percentage was not significant.

3.2 Motivation of this thesis

The main limitations of the works performed until now on variable stiffness or tow-placed laminates are the lack of data about the effect of manufacturing defects such as gaps and overlaps. This lack of knowledge about the knockdowns has forced the aerospace industry to conservatively reduce the properties of the laminates manufactured by AFP. There is a need to efficiently characterize these defects and aid the designer in manufacturing composite structures with automated fiber placement technique. The studies that have been performed until now as mentioned in the literature review, are either incomplete in their characterization or they assume conditions such as smooth course boundaries and/or one side tow drops.
The research objectives include characterizing the defects using finite element modeling for straight fiber and curvilinear fiber path laminates and to provide the designer a tool to gauge the actual knockdowns to be incorporated. Finite element analysis that takes into account the defect locations, defect area percentage and the type of defect have not been completed to date in the literature and is thus the motivation for this thesis. The main objective of the present work is to present a new ‘composite element’ that can be directly incorporated into any finite element software with composite capability. As we have seen before, curvilinear fiber paths can improve the structural response as compared to a straight fiber laminate; better methods to characterize the defects will go a long way to fully exploit the capabilities of the AFP process.
CHAPTER 4: Finite element modeling of composite laminates

4.1 Introduction

Since the early 1990s, researchers have demonstrated the tremendous possibilities of steering the tows in the plane of the laminate and thus tailor the composite properties to follow the load paths. The present section will begin by giving an introduction about finite element modeling with composites and then proceed towards development of the finite element model along with its initial validation using experimental results. These results will then form a basis for the development of the “composite defect element” as mentioned in the previous section. FEA softwares such as ANSYS® have capabilities to generate elemental stiffness matrices for shell elements and also allow for user fed input command lines. ANSYS® shell elements follow a first-order deformation which assumes that a straight line drawn through the thickness of the shell may rotate but remains straight post deformation. The thickness of the shell also remains the same even as the shell deforms. The reader should refer the reference [24] for the detailed theory.

Fabrication of tow steered laminates produce unwanted and often damaging defects such as gaps and overlaps and it becomes necessary to develop a defect model to accurately predict their effects. The model developed in this
thesis is a simple one that incorporates the presence of these defects in terms of the following parameters:

- Defect area percentage within the element.
- Defect area location
  - Laminate level.
  - Lamina level.
- Defect element’s orientation.

The above points will become clear in the following sections.

4.2 Case study-I: FEM analysis of defects in straight fiber laminates ([45/0/-45/90]3s)

The present case study involves the analysis of the effect of gap and overlap area percentages on the effective longitudinal compression strengths ($X_C$) of the straight fiber quasi-isotropic laminate with lay-up [45/0/-45/90]3s. The results presented for this case study are from the study performed by the author of this thesis and his colleague as mentioned in reference [21].

4.2.1 Problem definition:

The present problem deals with the study of the effect of gaps and overlaps in a composite laminate manufactured by the automated fiber placement technique. The geometry used is a rectangular plate with dimensions of 80.8 mm × 12.7 mm
× 3.81 mm as shown in Figure 4.1 below. The straight fiber laminates were fabricated using a VIPER 4000 fiber placement machine at the EADS, Composite Atlantic facility in Mirabel, Quebec. The laminates were cut from panels 0.914 m (3 ft.) × 0.914 m (3 ft.). The defect was 3.175 mm (1/8 inch, 1 tow width) wide and was in the form of a tow removed along the complete length, from each of the defectives plies. For generating overlaps, a tow was added in the defective plies (tow width= 1/8 inch) at desired locations.

![Figure 4.1: Straight fiber laminate test coupon](image)

The above geometry is standard for compression testing as specified by the Boeing-Modified ASTM D695 compression test [26]. The actual gauge length is
3mm while the glass-epoxy tabs provide the clamping supports. Compression loading is predicted to be the critical loading condition for laminates manufactured by AFP as a combination of defect and non-defect areas might bring about failure via the fiber kinking mode. As a result, the work presented in this case study primarily deals with results obtained under compression loading of the laminate. The anti-buckling loading fixture used in the test set-up restricts the out-of-plane displacements of the test coupon. This case study assumes importance as researchers found knock-downs of strengths due to these gaps. Based on previous results from the literature review about the influence of the defect area percentage on the strength knockdowns, it is necessary to undertake a deeper study on the influence of the bending dominated local stresses based on buckling.

The Experimental tests were performed and results were gathered by project partners at École Polytechnique de Montréal and mentioned in [22] and in Table 2. Defects were introduced in 0° and 90° plies as mentioned before.

### 4.2.2 Finite element model description

#### 4.2.2.1 Geometry

The finite element model was made using ANSYS® and consisted of SHELL 181 elements. These elements are 4-node finite strain shell elements that can be
used to model thin to moderately thick shell structures and have six degrees-of-freedom (DOFs) [23]. These elements were used due to their simplicity in application and ability to use both full and reduced integration schemes. They also have a smaller stiffness matrix generation times compared to the other composite capability elements such as SHELL43 or SHELL99 [24]. The lay-up was same as the experimental coupons, [45/0/-45/90]_{3s} which is a 24 ply balanced and symmetric laminate. The defect geometries in ANSYS® are shown in Figures 4.2 and 4.3 below (the figures are only for understanding and are not to scale). In both the figures, areas A1 and A3 represent the 'non-defective' regions of the composite laminate while area A2 represents the 'defect' area with one tow removed to produce a gap.

Figure 4.2: Defect geometry for defects in 0° plies.²¹
Alternatively, one tow is added for producing overlaps in this same A2 region. 0° plies are oriented along X-axis while the 90° plies are along the Y-axis. Consequently, the laminate thickness is along the Z-axis. The following cases are considered for both experimental testing as well as finite element modeling.

Cases:

1. Only 0° gap (6 defective plies in total) as shown in Figure 4.2.
2. Only 90° gap (6 defective plies in total) as shown in Figure 4.3.
3. 0° and 90° gaps (12 defective plies in total) as shown in Figure 4.4.
4. 0° overlap (6 defective plies in total) as shown in Figure 4.2.

For case 3, Figure 4.4 shows 9 different areas. Areas A1, A3, A7, A9 are ‘non-defective’ regions of the composite laminate. Areas A2 and A8 represent the ‘defect’ regions with defect in all 90° plies (6 regions through the thickness) while
Areas A4 and A6 represent the ‘defect’ regions with defect in all 0° plies (6 regions through the thickness).

The common area, A5 represents region with defect in both 0° and 90° plies (12 regions through the thickness).

Figure 4.4: Defect geometry for defects in 0 and 90 plies\textsuperscript{21}.

Figure 4.5: Top view of the defect in 0° plies\textsuperscript{21}.

Figure 4.6: Top view of the defect in 90° plies\textsuperscript{21}.
4.2.2.2 Meshing and boundary conditions

The finite element mesh consisted of fully integrated SHELL 181 elements. The element number varied from 3,600 to 14,400 while the element size varied from $7.9375 \times 10^{-2}$ mm to $3.175 \times 10^{-1}$ mm. The loading was a constant displacement ($= 1 \mu m$) at the right edge while the left edge was simply supported as shown in Figure 4.8. Constant displacement loading was applied to make sure that all the regions get equal amounts of strain and thus deform equally such that the stiffer regions (such as Areas A1 and A3 in Figures 4.2 and 4.3) took greater loads than the less stiff regions (such as Area A2 in the same figures). This approach brought our model closer to the actual experimental testing conditions. The reaction forces for the baseline (without defects) and a defect laminate are shown in Figures 4.10 and 4.11, respectively. Gaps reduced the reaction forces in the defect regions while the overlaps increased the reaction forces in the defect regions which, stated in other terms, overlaps increased the stiffness locally in the defect regions.

Figure 4.7: Top view of the defect in 0° and 90° plies\textsuperscript{21}. 

![Image of a defect in 0° and 90° plies]
Figure 4.8: ANSYS® geometry with defect areas and boundary conditions.

Figure 4.9: Deformed shape under uniform compression at the right edge.
The number of elements and sizes required was governed by the number of element divisions at the right edge. The number of element divisions along the right edge was calculated so as to provide a correct value of the reaction force for the baseline samples in defect and non-defect region along the right edge. This number changed to keep a constant element size in the defect and non-defect areas. For example, the ratio of the number of elements along the right edge of area A1 to the number of elements along the right edge of area A2 in Figure 4.2 was equal to the ratio of the corresponding right edge lengths. The above modification was necessitated by the observation that incorrect element divisions generated incorrect value of the line pressure for the baseline samples (defect-free samples). This led to errors in modeling the baseline samples. For defects in 0° plies only, the above procedure is implemented and the element sizes remain constant in the defect and the non-defect areas. The defect configurations with gaps or overlaps in 90° plies only, do not require equal size of the elements. This is due to the fact that the 0° defect area is along the direction of loading while the 90° defect area is perpendicular to it. Furthermore, the number of elements for each scenario is such that they capture the complete defect area along with maintaining a reasonable computation time.
Figure 4.10: Constant reaction forces at the right edge for the baseline.

Figure 4.11: Lower reaction forces in the gap regions along the right edge (center portion).
4.2.3 Material properties

Straight fiber laminate coupons were made of CYTEC-5276-1, material properties of which are mentioned in the Table 1 on page 72. Also, as shown by Croft [12], the gaps are filled by resin post curing, so the gaps were modelled as resin rich areas the properties of which are mentioned in Table 1 while the overlaps were modelled as additional thickness build-ups.

4.2.4 Calculation of strength values

The strength values for the baseline as well as defective laminates were calculated as follows:

\[
\text{Effective strength} = \frac{P_L}{(FI*H_o)}
\] (3)
where,

\[ P_L = \text{Effective line pressure} = \frac{R_F}{L}, \]

\( R_F \) is the summed reaction forces on the right edge and \( L \) is the length of the right edge of the region in consideration.

\( F_I \) = Failure index and 

\( H_0 \) is the layer thickness for the laminate, 0.15833 mm.

The values of \( F_I \) and \( R_F \) were calculated separately for each ‘region’ (For example, Areas A1, A2, A3 in Figure 4.2). Failure index (FI) was averaged throughout the region. This is done as the presence of the defect layer facilitated redistribution of the load so that stiffer regions took more of the load while less stiff regions took less load. The above methodology assumed that the entire laminate fails at the same time or the first ply failure was catastrophic. The reaction force (\( R_F \)) taken into account was the effective cumulative force in X-direction for any one of the vertical edges. Finally, the weighted strength of the laminate was calculated based on the areas of the defect and non-defect regions.
Table 4.1: Material properties [13, 27].

<table>
<thead>
<tr>
<th>Property</th>
<th>CYTEC 5276-1</th>
<th>Epoxy resin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_X$</td>
<td>143 GPa$^1$</td>
<td>3.72 GPa$^1$</td>
</tr>
<tr>
<td>$E_Y$</td>
<td>9.10 GPa$^1$</td>
<td>3.72 GPa$^{11}$</td>
</tr>
<tr>
<td>$E_Z$</td>
<td>9.10 GPa$^2$</td>
<td>3.72 GPa$^{12}$</td>
</tr>
<tr>
<td>$\nu_{XY}$</td>
<td>0.3$^1$</td>
<td>0.3$^{13}$</td>
</tr>
<tr>
<td>$\nu_{YZ}$</td>
<td>0.3$^3$</td>
<td>0.3$^{13}$</td>
</tr>
<tr>
<td>$\nu_{XZ}$</td>
<td>0.3$^4$</td>
<td>0.3$^{13}$</td>
</tr>
<tr>
<td>$G_{XY}$</td>
<td>4.82 GPa$^1$</td>
<td>1.43 GPa$^{14}$</td>
</tr>
<tr>
<td>$G_{YZ}$</td>
<td>3.50 GPa$^5$</td>
<td>1.43 GPa$^{15}$</td>
</tr>
<tr>
<td>$G_{XZ}$</td>
<td>4.82 GPa$^6$</td>
<td>1.43 GPa$^{15}$</td>
</tr>
<tr>
<td>$X_T$</td>
<td>3010 MPa$^1$</td>
<td>58.3 MPa$^1$</td>
</tr>
<tr>
<td>$X_C$</td>
<td>1740 MPa$^1$</td>
<td>58.3 MPa$^{16}$</td>
</tr>
<tr>
<td>$Y_T$</td>
<td>90.3 MPa$^1$</td>
<td>58.3 MPa$^{16}$</td>
</tr>
<tr>
<td>$Y_C$</td>
<td>200 MPa$^1$</td>
<td>58.3 MPa$^{16}$</td>
</tr>
<tr>
<td>$Z_T$</td>
<td>90.3 MPa$^7$</td>
<td>58.3 MPa$^{16}$</td>
</tr>
<tr>
<td>$Z_C$</td>
<td>200 MPa$^6$</td>
<td>58.3 MPa$^{16}$</td>
</tr>
<tr>
<td>$S_{XY}$</td>
<td>139 MPa$^1$</td>
<td>29.2 MPa$^{17}$</td>
</tr>
<tr>
<td>$S_{YZ}$</td>
<td>29.2 MPa$^9$</td>
<td>29.2 MPa$^{17}$</td>
</tr>
<tr>
<td>$S_{XZ}$</td>
<td>139 MPa$^{10}$</td>
<td>29.2 MPa$^{17}$</td>
</tr>
</tbody>
</table>

1. References [13,27].
2. $E_Z=E_Y$.
3. Equal to $\nu_{XY}$(resin).
4. $\nu_{XZ}=\nu_{XY}$.
5. Equal to $E/(2(1+\nu_{YZ}))$.
6. $G_{XZ}=G_{XY}$.
8. $Z_C=Y_C$.
9. Equal to $S_{XY}$.
10. Equal to $G_{XY}$.
11. $E_X=E_Y$.
12. $E_X=E_Y$.
14. Equal to $E/(2(1+\nu))$.
15. $G_{XZ}=G_{XY}$.
16. Equal to $X_T$.
17. Equal to $X_T/2.0$
4.2.5 Model validation

The developed finite element model developed above was validated by comparing theoretical values to the values obtained by experimental testing. Finite element and experimental results are tabulated in Table 2 below. The longitudinal compressive strength for the baseline can be calculated by classical laminate theory [25]. The theoretical longitudinal compressive strength ($X_C$) of the defect free laminate was 666.7 MPa. The finite element model provided the same value. The baseline value for the experimental procedure was 9% lower than the theoretical. Similarly, the error percentages in the first three cases, i.e. defect-free baseline, 0° gap, and 0° and 90° gap are in the range of 9%-13%.

Table 4.2: Comparison of Finite element simulation and experimental results.

<table>
<thead>
<tr>
<th>Test type</th>
<th>Longitudinal compression strength (MPa)</th>
<th>Change w.r.t. baseline (%)</th>
<th>Experimental (MPa)</th>
<th>Change w.r.t. baseline (%)</th>
<th>Error between FE and experimental (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>667</td>
<td>---</td>
<td>611</td>
<td>---</td>
<td>-9%</td>
</tr>
<tr>
<td>0° gap</td>
<td>579</td>
<td>-13%</td>
<td>529</td>
<td>-13%</td>
<td>9.5%</td>
</tr>
<tr>
<td>0° and 90° gap</td>
<td>552.4</td>
<td>-17%</td>
<td>504</td>
<td>-16%</td>
<td>9.6%</td>
</tr>
<tr>
<td>0° overlap</td>
<td>721.5</td>
<td>8%</td>
<td>708</td>
<td>16%</td>
<td>2%</td>
</tr>
</tbody>
</table>
Some of the error in the compression strength values can be attributed to the fixture used in the experimental setup. It may be recalled that the ASTM D695 compression test fixture is an “end-loaded” method that tends to cause premature failure at the loaded end, otherwise known as “end-brooming failure”. This mode of failure gives lower values as compared to the expected strength. The actual error percentages or the deviation in the values of the FEM model and experiments should thus be lower than shown in the fifth column of Table 2. However, if one calculates deviation of the results with respect to the baseline result (columns 2 and 4 of Table 2), then one sees that the FEA provides reasonable knockdown factors for the different defect cases.

4.2.6 Results and discussions

The geometry with defect is again shown in Figure 4.13. Graphs for the theoretical variation of the effective (normalized) longitudinal compression strength with increase in gap/overlap areas are plotted. Defect area is increased by removing more tows in the specified ply. In other words, as defect area is a function of the width of the defect area $A_2$, ‘$W$’ in the Figure 4.13 below, we increase ‘$W$’ to provide a greater defect area (For example, a defect area of 25% is formed by removing 1 tow ( tow width =1/8 inch) and greater areas are formed by increasing ‘$W$’). Values are thus obtained from the model and are presented next.
4.2.6.1 Variation for 0° gap

Figure 4.14 shows the variation of $X_C$ with increasing gap area percentage. Effective value of $X_C$ is normalized with respect to $X_C$ for baseline (defect-free) laminate which for CYTEC 5276-1 is 1740 MPa. The variation between the parameters ($X_C$ and gap area percentage in 0° plies) is linear (regression coefficient, $R^2=1$) as shown by the curve equation of the curve fitting equation ($y = -0.5311x + 1.001$). This implies that the longitudinal compression, $X_C$ decreases with increase in gap area percentage and with this relation, we can interpolate effective $X_C$ value for any gap area percentage. The value of normalized $X_C$ varies from 1 for the baseline (1740 MPa) to 0.47 (817.8 MPa) or 53% reduction in $X_C$ for 100% gap area. It should be noted that a 100% gap signifies presence of a resin layer only instead of the pre-preg. The above results were quite intuitive as the defect was along the loading direction and hence the in-plane strengths decreased and also led to a redistribution of the load.
Figure 4.14: Variation of $X_c$ with increase in $0^\circ$ gap area.

The relationship can be described by the equation:

$y = -0.5311x + 1.001$

with $R^2 = 1$.
4.2.6.2 Variation for 90° gap

Figure 4.15: Variation of $X_C$ with increase in 90° gap area.

The results for knock-downs for gaps in 90° plies (Figure 4.15) were not quite as straight forward as the ones obtained for 0° plies. The variation was not linear as was with the previous case. One reason for it can be due to the fact that the defect is perpendicular to the loading direction so the effect is not as severe as the previous case. But still there is a decrease in $X_C$ with increase in gap area percentage (from 1 to 0.905 or about 10% reduction). The high normalized value of 0.9 is observed for compressive strength even for 60% gap area percentage.
(equal to 15 times tow width) in 90° plies. This drop is even smaller than the knockdown for 0° gap for 25% gap area case (equal to 1 times tow width). The large departure between these two sets of values (0° gaps and 90° gaps) is due to the large difference in the longitudinal values of pre-preg fiber direction strength and the resin strength (1740 MPa and 58.3 MPa, respectively) compared to the much smaller difference in the transverse values (90.3 MPa and 58.3 MPa).

**4.2.6.3 Variation for 0° and 90° gaps**

The graph (Figure 4.16) was obtained by plotting the normalized compressive strength, $X_C$ against increasing 90° gap area percentage for a constant 0° gap area percentage. Four cases considered are 25% 0° gap area (equal to 1 tow width), 37.5% 0° gap area (equal to 1.5 times tow width), 50% 0° gap area (equal to 2 times tow width) and the baseline (no-defect). The dashed line represents baseline while the solid colored lines represent the above mentioned variation. The results for the variation of $X_C$ with increase in gap area percentage provided some interesting insights. For all the 3 defect trend lines (solid) we see that the slopes of the curves are large until the second point(s) (A, B, C) while the trend somewhat shows a smaller slope in the rest of the plot. This can be interpreted as a greater effect of gaps in 0° for lower 90° gap area percentages. This effect then starts to wear away as the percentage of 90° gap increases or in other
words greater 90° gap area compared to 0° gap area prevents severe degradation of \( X_C \). The results from this set of simulations can be postulated to be a cumulative effect of 0° and 90° defects.

![Variation of \( X_C \) with increase in 0° and 90° gap area](image)

Figure 4.16: Variation of \( X_C \) with increase in 0° and 90° gap areas.

### 4.2.6.4 Variation of 0° overlap

Figure 4.17 shows the variation of \( X_C \) with increase in the overlap area. As mentioned before, overlaps are modeled as thickness build-ups and thus provide
a local stiffening mechanism. From classical laminate theory [25], this causes the stiffness to increase locally and thus more load carrying and distributing capacity.

The same effect is seen from plot in Figure 4.17 where there is a linear \((R^2 = 1)\) increase in the normalized \(X_C\) value with increase in the defect (overlap) area. The value increases from 1 (baseline) to 1.32 or 32% increase for 100% overlap area percentage. A value of 100% overlap area, signifies a layer with 2 times the thickness \((H_T=2\times H_O, \ H_O\ being\ the\ ply\ thickness)\).
4.2.7 Conclusions for case study-I

As mentioned before the present case study was performed to determine the effect of parameters such as:

- Defect area percentage within the element.
- Defect area location
  - Laminate level.
  - Lamina level.

It was shown that the defect information regarding its location with respect to the laminate sequence degraded the properties more or less. For example, the properties degraded more for gaps in $0^\circ$ plies than in $90^\circ$ plies. Finally, the area percentage also affected the normalized $X_C$ value. Gaps were shown to decrease the effective longitudinal compression strengths while the overlaps which were thickness build-ups increased the longitudinal compression strengths. Also, the decrease in $X_C$ due to gaps was more severe than increase in $X_C$ due to overlaps.
CHAPTER 5: Finite element model for curvilinear fiber laminates

The results from the previous section demonstrated the need to understand the effect of defects such as gaps and overlaps on the mechanical properties of laminates fabricated by AFP. The FEM methodology developed previously will now be modified to develop a ‘defect element’ and finally apply the ‘defect element’ to variable stiffness or curvilinear fiber laminates. It is essential that the approach must not be geometry or fiber path dependent. This will enable the element to be used efficiently for optimizing VS laminates. Finally, this model will be employed to another case study to demonstrate its ease of use and application for the tow steered laminates.

5.1 Case study II: FEM analysis of defects in straight fiber laminates ([0]τ)

This case study presents the effect of gap and overlap on the effective material properties ((strengths, $X_T$, $X_C$, $Y_T$, $Y_C$, $S$) of the straight fiber unidirectional laminate with lay-up [0]τ) as a function of defect area percentages. The selection of the above lay-up was determined by ease of application and analysis. Also, presently all the Finite element analysis software (ANSYS® as well) use shell elements to model thin composite laminates by passing the layer definition in terms of ply thickness ($H_0$) and ply orientation ($\theta$). The material properties for any
layer having ply orientation angle different from 0° are calculated by using the composite transformation matrices [25] as shown in Equations 4 and 5 below.

\[
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{bmatrix} = [T_\sigma]^{-1} \begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
Q_{12} & Q_{22} & Q_{26} \\
Q_{16} & Q_{26} & Q_{66}
\end{bmatrix} [T_\varepsilon]
\]

where \([T_\varepsilon]\) and \([T_\sigma]^{-1}\) represent the transformation matrices and \([\bar{Q}_{ij}]\) and \([Q_{ij}]\) represent the stiffness matrices of the composite ply.

\[
[T_\sigma] = \begin{bmatrix}
c^2 & s^2 & 2cs \\
s^2 & c^2 & -2cs \\
-2cs & 2cs & c^2 - s^2
\end{bmatrix}
\]

and \([T_\varepsilon] = \begin{bmatrix}
c^2 & s^2 & cs \\
s^2 & c^2 & -cs \\
-2cs & 2cs & c^2 - s^2
\end{bmatrix}\)

\[c = \cos \theta \text{ and } s = \sin \theta\]

So, we see that although the material properties are passed in terms of properties of unidirectional fibers but the stiffness matrices are formulated by using the above transformations. In the following sections, it will be shown that the above methodology is suitable for modeling curvilinear fiber laminates with defects.

### 5.1 Problem definition

The present problem deals with the study of the effect of gaps and overlaps on a single layer \([0]_T\) laminate. The geometry used is a square plate with dimensions of 3mm × 3mm as shown in Figure 5.1 below. The defect introduction is similar to previous case study where the defect area is increased by increasing the width ‘W’ in Figure 4.12. The choice of square geometry and the dimensions were a
result of the calculations performed for the actual gap sizes in MATLAB® and this will be dealt with in the subsequent sections.

5.1.2 Finite element model description

The finite element model was made in ANSYS® and consisted of SHELL 181 elements. The lay-up was [0]_T and the material properties used was that of CYTEC 5276-1 and high grade epoxy resin as before.
In the Figure 5.2 above, areas A1 and A3 represent the ‘pure’ or ‘non-defective’ regions of the composite laminate while area A2 represents the ‘defect’ area with one tow removed to produce a gap while the one tow is added for producing overlaps. $0^\circ$ plies are oriented along X-axis and the laminate thickness is along Z-axis. The following cases are considered for the finite element modeling.

Cases:

1. $0^\circ$ gap.
2. 0° overlap.

The properties that were investigated were:

1. Longitudinal tensile strength, $X_T$.
2. Longitudinal compressive strength, $X_C$.
3. Transverse tensile strength, $Y_T$.
4. Transverse compression strength, $Y_C$.
5. In-plane shear strength, $S$.

The finite element mesh consisted of fully integrated SHELL 181 elements. The element number varied from 3200 to 16800 while the element size varied from $2.81\times10^{-3}$ mm to $5.35\times10^{-4}$ mm. Again as before in the previous case study, the number of the elements is such that the reaction forces at the right edge are constant for the baseline configuration. The loading was a constant displacement ($=1\mu$m) at the right edge while the left edge was simply supported as shown in Figure 5.1.

5.1.3 Calculation of strength values

The calculation of the strength values was performed in exactly the same manner as for the strength of the $[45/0/-45/90]_{3s}$ laminate performed in section 4.2.4.

5.1.4 Results and discussions

Graphs for the variation of the effective (normalized) properties (mentioned above) as a function of gap/overlap area percentages were plotted. Defect area
is increased by removing more tows symmetrically from the ply. Values are obtained from the model and are presented next. The results show trends similar to the case study-I of the previous chapter.

5.1.4.1 Gaps

The results show that the gaps decrease the effective strengths while the overlaps result in increasing the effective strengths. The above observation is due to the difference in implementation of the defects. As mentioned before, gaps are modeled as resin pockets while overlaps are thickness build-ups. Also, the trends for gaps are more drastic compared to the quasi-isotropic case of the previous section as the effect of gaps was constrained due to other plies, while in the present case, there is only one ply. Therefore, increasing gap area has a direct correlation with the respective properties of CYTEC 5276-1 and the high grade epoxy resin. Normalisation of the strength is done with respect to the corresponding value of the baseline (defect free) laminate.

a.) Variation for $X_T$:

Figure 5.3 shows the variation of effective longitudinal tensile strength, $X_T$ with increase in the gap area percentage. The decrease is linear ($R^2=1$) with a normalized value of 1 for 0% gap area while it decreases to a minimum of
0.01932 \( (=X_{T(\text{resin})} / X_{T(\text{CYTEC})}) \) for a 100% gap area. Again, 100% gap area represents a complete layer with only resin properties.

b.) Variation for \( X_C \):

The relationship is again linear \( (R^2=1) \) (Figure 5.4). Normalized \( X_C \) value also varied from a value of 1 for the baseline to a decrease of 0.035 \( (=X_{C(\text{resin})} / X_{C(\text{CYTEC})}) \).

c.) Variations of \( Y_T \) and \( Y_C \):

The variations of transverse strength properties, \( Y_T \) and \( Y_C \) were not linear (Figures 5.5 and 5.6). This is due to the perpendicular orientation of the defect with respect to the loading. The normalise \( Y_T \) value varies from 1 for the baseline to a value of 0.65 \( (=Y_{T(\text{resin})} / Y_{T(\text{CYTEC})}) \). The corresponding values for \( Y_C \) are 1 and 0.29 \( (=Y_{C(\text{resin})} / Y_{C(\text{CYTEC})}) \), respectively.

d.) Variation of in-plane shear strength, \( S \):

The graph in Figure 5.7 shows that the variation of the shear strength with increase in gap area is a linear one \( (R^2=1) \). The value varies from 1 for the baseline to a value of 0.21 \( (= S_{xy(\text{resin})} / S_{xy(\text{CYTEC})}) \), for 100% gap area percentage.
Figure 5.3: Variation of $X_T$ with increase in $0^\circ$ gap area.

Figure 5.4: Variation of $X_C$ with increase in $0^\circ$ gap area.
Figure 5.5: Variation of $Y_T$ with increase in 0° gap area.

Figure 5.6: Variation of $Y_C$ with increase in 0° gap area.
5.1.4.2 Overlaps

The results for overlaps, modeled as thickness variations were straightforward. As the overlaps were modeled as thickness build-ups for a single layer 0°, the results for the strengths showed linear variations. For longitudinal tensile, longitudinal compression and in-plane shear strength values, the trend was linear. Transverse tensile and compression strength again showed a non-linear variation as was evident with gaps. 100% overlap area meant that the total thickness of the laminate was doubled (H₁=2 x Hₒ).

Figure 5.7: Variation of in-plane shear, S with increase in 0° gap area.

\[
y = -0.7904x + 1
\]

\[R^2 = 1\]
Figure 5.8: Variation of $X_T$ with increase in 0° overlap area.

Figure 5.9: Variation of $X_C$ with increase in 0° overlap area.
Figure 5.10: Variation of $Y_T$ with increase in $0^\circ$ overlap area.

Figure 5.11: Variation of $Y_C$ with increase in $0^\circ$ overlap area.
Figure 5.12: Variation of in-plane shear, S with increase in 0° overlap area.
5.2 Curvilinear FEM model

Discussions performed in the previous sections demonstrated that defects such as gaps and overlaps can affect strength properties of composite laminates. The present section will utilise the results from case study-II ([0]T) to formulate a methodology for incorporating these results to analyze VS laminates. The first step to achieve this goal is to look into the designing of the curvilinear fiber paths using MATLAB® and passing the geometry information to ANSYS®.

5.2.1 Geometry information from MATLAB®

A MATLAB® subroutine was initially devised by Fayazbakhsh et al. [11] and was extended by the author of the present work to incorporate the ability to transfer the mathematical code from MATLAB® to a FEA code in ANSYS®. A brief description of the MATLAB® code is provided here. The inputs to MATLAB® subroutine were the geometry (dimensions), starting point for the reference course (along the horizontal to provide for the distance for the first center line, \( x_0 \)), manufacturing radius (\( \rho \)), number of tows in one course (\( N \), usually 32 tows per course), tow width (\( W_t \)), start angle (\( T_0 \)), and coverage percentage (0-100%) as inputs. For detailed mathematical considerations the reader should refer to the above mentioned reference. It must be mentioned here that although in this study only one case study with only left side tow drop will be presented to facilitate the applicability of the approach, this approach can be easily applied to
VS laminates with both side tow drops as well. Due to constraints of time, the present study will only present one case study for applying the methodology. The first centerline is created from the point, \((x_0, \theta_0, T_0)\). Centerline angle \((\varphi)\) changes continuously from \(T_0\) at the start of the centerline to \(T_1\) at the end of the plate and is a function of the \(y\)-coordinate of the point under consideration. The parameters described above can be understood by referring to Figure 5.13.

The left course and the right course boundaries are calculated by keeping \(W_e\) (course width along the horizontal as shown in Figure 5.14) constant. This makes the perpendicular width of the course \((\tilde{W})\) (course width along the perpendicular drawn at the point under consideration), variable as the tows are dropped from the left side to keep \(W_e\) constant.

![Figure 5.13: Reference course parameters](image)

Figure 5.13: Reference course parameters\textsuperscript{11}.  

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The points along the left boundary and the right boundary are calculated analytically by offsetting the points on the centerline along the horizontal. Now, for considering the tow drop areas, the intersection points of the tows with the left course boundary are considered. The tows are generated by shifting the centerline by a distance of $D = \frac{W_e}{N}$, $N$ is the total number of tows in a course) to the left and to the right as shown in Figure 5.14. After generating the tows, the intersection points of tows (green lines in Figures 5.15 and 5.16) with the left course boundary (blue line in Figures 5.15 and 5.16) are determined. The points $A', B', C', D', E', F'$ (shown in Figure 5.16) are generated as intersection points of the tows with the perpendicular drawn at points A,B,C,D,E,F as the AFP machine.
head drops tows perpendicular to the course centerline. This approach is repeated to cover the entire plate. The point A lies in the tow drop region as the coverage percentage for this case study is 0%. This means that the tow is dropped as soon as the first edge of the tow (leading edge) touches the left course boundary. The above set of points (A,B,C,D,E,F and A’,B’,C’,D’,E’,F’ for each course some of which are shown in Figure 5.16) is passed to ANSYS® as tow drop areas and triangular defect areas are thus generated in ANSYS®. A point to be noted here is that above number of intersection points (A,B,C,D,E,F = 6 points) are only for understanding while actually it will vary according to the value of the parameters defined above.

The final geometry in Figure 5.15 consists of blue left course boundaries with tow drop areas while the white areas represent non-defect portions of the ply. The values of the parameters used are

\[ x_0 = 0.5, \]
\[ T_0 = 15^\circ, \]
\[ T_1 = 45^\circ, \]
\[ N=32, \]
\[ \text{Coverage percentage= 0\% (no overlaps)}, \]
\[ \text{Tow-width=1/8 in. or 3.125 mm,} \]
\[ \text{Minimum radius of curvature= 635 mm}. \]
Figure 5.15: Curvilinear fiber geometry in MATLAB®.

Figure 5.16: Magnified area selected above.
5.2.2 Meshing of the defect geometry

The geometry is meshed as SHELL181 square elements and the size is determined so as to keep the computation time as small as possible as well as capturing the entire defect geometry. For the present approach, element size will not be dependent on the geometry dimensions so as to be small enough to capture the defect and be computationally inexpensive at the same time. Previous research by Blom [20] incorporated 10,000 elements for geometry 300 × 300 mm and they could only incorporate one side tow drop but with the present approach, one side as well as both side tow drops can be applied. Their approach will require higher number of elements for a bigger rectangular plate or in other words, their approach is based on the aspect ratio of the geometry under consideration. This will provide the present model a definite advantage over its counterparts.

5.2.2.1 Element orientation

Element axis orientations are determined at the center of the element by passing the coordinate information of the 4 nodes. As mentioned before, the orientation at any point in the geometry is a function of the y-coordinate as mentioned in Equation 6 [11].
Another point to be stressed here is that the element size must be small enough so as to approximate the curvilinear fiber path to be almost straight within every element. This means that the fibers inside every element, act like straight fiber laminates as shown in Figures 5.17(a), 5.17 (b), 5.18(a) and 5.18(b) below. So every element has a different orientation determined by the above equation at the centroid. Calculation for the centroid is quite straightforward on account of the square shape of the element (weighted average of the coordinates of the 4 nodes at the vertices of the element).

\[
\sin \phi = \frac{1}{\rho} (y + \rho \sin T_0) = \frac{y}{\rho} + \sin T_0
\]  

(6)

Figure 5.17(a): Meshed curvilinear fiber geometry.

Figure 5.17(b): Magnified view of the selected area.
5.2.2.2 Material (strength) property:

In the previous case study with \([0]_T\), we emphasized the fact that the results from the case study will be used to devise a “defect element” routine. We define the “defect element” to be a composite element having a constant fiber orientation and material properties as functions of the defect area percentages. The equations for the variations obtained in the strength properties of the ‘defect-element(s)’ are obtained from the curve-fitting equations of Figures 5.3-5.12. For example, consider an element which has say 20% of gap area and has an element orientation of ‘\(\theta\)’, the strength properties will be computed as follows:
$X_T = a_1 x + a_2$

$X_C = a_3 x + a_4$

$Y_T = a_5 x^4 + a_6 x^3 + a_7 x^2 + a_8 x + a_9$

$Y_C = a_{10} x^4 + a_{11} x^3 + a_{12} x^2 + a_{13} x + a_{14}$

$S = a_{15} x + a_{16}$

The constants $a_i (i=1:16)$ are presented in table 3 below.

**Table 5.1: Value of the constants.**

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>-0.9805</td>
<td>$a_9$</td>
<td>1.0001</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.9988</td>
<td>$a_{10}$</td>
<td>1.5602</td>
</tr>
<tr>
<td>$a_3$</td>
<td>-0.9653</td>
<td>$a_{11}$</td>
<td>-4.2016</td>
</tr>
<tr>
<td>$a_4$</td>
<td>1.0002</td>
<td>$a_{12}$</td>
<td>4.3598</td>
</tr>
<tr>
<td>$a_5$</td>
<td>0.1543</td>
<td>$a_{13}$</td>
<td>-2.4229</td>
</tr>
<tr>
<td>$a_6$</td>
<td>-0.4366</td>
<td>$a_{14}$</td>
<td>0.9979</td>
</tr>
<tr>
<td>$a_7$</td>
<td>0.5489</td>
<td>$a_{15}$</td>
<td>-0.7904</td>
</tr>
<tr>
<td>$a_8$</td>
<td>-0.6209</td>
<td>$a_{16}$</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
For an area of 20%, substitute \( x=0.20 \) and calculate the above properties. As the element has an orientation of \( \theta \), the above properties will be passed on to ANSYS®, where the material value transformations will take place according to equations 4 and 5. This approach removes the dependence of the elements on the shape of the geometry as well as the tow-width in determining the element size. The following section will now summarize the salient points from chapters 4 and 5 and will thus form a foundation for concluding remarks and future works.

### 5.3 Summary of chapters 4 and 5

The work presented in chapters 4 and 5 provided an alternate methodology to analyze curvilinear fiber laminates with tow drops that led to defects such as gaps and/or overlaps. Chapter 4 dealt specifically with the reason for introducing the present approach by performing finite element modeling on a quasi-isotropic laminate. This analysis provided us an in-depth view of the effect of gaps (resin rich areas) and the results can now be visualised in terms of other defects such as overlaps (thickness build-up). The results showed a significant effect on mechanical properties which in turn led us to analyze a single layer laminate in the beginning of chapter 5. Following the analysis, the new methodology was demonstrated by defining a new composite ‘defect element’ that was square in shape. This shape also facilitated ease of calculation of fiber orientation as a
weighted mean of the 4 nodes at the vertices. The material properties were passed in the form of curve fitting equations that were a function of the defect area percentage. This approach was very efficient in modeling the actual defect areas and due to this aspect of the modeling; it was closer to the reality than the previous researches performed. This method of calculating the material properties as a function of the defect area percentage of the element is different from the approach by Blom et al. [20] where they considered the area in Figure 5.18(b) to be completely filled by the resin despite the fact that the actual defect area is lesser than the total element area. They are thus more inaccurate in determining the properties of a particular element by calculating only if a particular point was lying in a tow drop region or not. If the answer to the above question was affirmative, they passed resin properties irrespective of the defect area percentage. The present work removes this anomaly and tries to simulate the actual tow drop areas and therefore, the present methodology can go a long way in reducing the computational times and be relatively accurate at the same time.
CHAPTER 6: Conclusions and future work

6.1 Conclusions

The present work presents a new methodology for analyzing curvilinear fiber laminates using any FEA software such as ANSYS®. The methodology was based on the observation, like that of most of the previous researches performed on variable stiffness laminates, which assumes that the boundaries were smooth and that there were defects such as gaps and/or overlaps. Although, some recent research works did take these defects into consideration but still the FE modeling was more conservative i.e. the defects were assumed to completely fill the elements. This paved the way for further investigation into the effect of the defect area percentages on the strength reduction and increment in the case of gaps and overlaps, respectively.

A quasi-isotropic laminate ([45/0/-45/90]_{3s}) was investigated for the above knock-downs and it was found that the variations of $X_C$ with gaps was dependant on the material used as well as the defect orientation. Defects aligned along the loading direction degraded properties more than the defects aligned perpendicular to the loading direction. Overlaps increased the effective strength of the laminate due to an increase in the local stiffness in the region around overlaps.

Finally, above findings led to the development of a ‘defect element’, the properties of which were computed via analysis of $[0]_T$ laminate. The ‘effective’
material strength properties were passed to the elements as a function of defect area percentages. This approach facilitates lower computational time owing to lesser number of elements which do not need to be of the same order as the tow width. Another observation that warrants mentioning is the fact that the above approach of ‘defect element’ is general in the sense that the applicability is limited to symmetric laminates (both variable stiffness and constant stiffness laminates) as mentioned before in section 2.4. Additionally, the entire approach of determining constants in Table 5.1 is strictly dependent upon the material properties of the composite material used.
6.2 Future work

In this thesis, a new methodology was formulated for analysing curvilinear fiber laminates using finite element software such as ANSYS®. Future work can include validation of the ‘defect element’ methodology to variable stiffness laminates via experimentation using multiple samples for each set. Single layer and multi-layer laminates should be fabricated using the AFP machine so as to quantify the effectiveness of the approach.

Another aspect of analysis of variable stiffness laminates can be to investigate the applicability of the approach to alternate and more complex 3D geometries to look at the computational times and feasibility of the approach.
References

Cone Made Out Of Automated Fiber Placement. In 42nd ISTC, SAMPE, 2010. Salt Lake City, UT; USA.


