DESIGN AND DEVELOPMENT OF A ROTARY STIRLING CYCLE ENGINE
THE DESIGN AND DEVELOPMENT
OF A ROTARY STIRLING CYCLE ENGINE

by

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ABSTRACT

A theory is presented for the prediction of the dynamic behaviour of a rotary Stirling engine. The system is broken up into a number of stationary and moving control volumes and the solution consists of a series of differential and partial differential equations that must then be solved for the four main dependent system variables: pressure, temperature, mass content, and mass flow. Since such a solution would require the knowledge of certain input parameters not obtainable elsewhere, a simplified model is proposed so that these parameters may be measured. The design and construction of this model is discussed and the short-lived operational experience is presented. In conclusion, several recommendations are made concerning the model, its configuration and future work in this field.
SOMMAIRE

Cette thèse décrit une méthode de calcul du comportement dynamique d'un moteur rotatif de type Stirling. Le système à l'étude est subdivisé en un certain nombre de volumes de contrôle, les uns fixes, les autres mobiles sur lesquels on se base pour obtenir une série d'équations différentielles ordinaires et partielles dont la solution donnerait les quatre principales variables dépendantes du système: la pression, la température, le débit massique et la masse enfermée. Toutefois, cette méthode de solution suppose une connaissance de certaines données pour lesquelles on ne dispose d'aucune valeur numérique. Pour contourner cette difficulté, on a donc conçu un modèle simplifié grâce auquel il est possible de mesurer les paramètres manquants. Une discussion de la réalisation de ce moteur d'essai suit donc, ainsi qu'un compte-rendu des quelques brèves expériences qu'il a permises. La conclusion présente plusieurs recommandations concernant le modèle, sa configuration et propose des orientations pour l'accomplissement d'autres travaux en ce domaine.
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LIST OF NOTATIONS

A Flow areas (Appendix A)

a Heat transfer surface area

A \psi, A \psi Cross-sectional area from reference line and cross-sectional area between two adjacent blades (Appendix B)

b Width of working sector (Appendix B)

C_p, C_v Constant pressure and constant volume specific heats

C Thermal capacitance of a component (Appendix A)

E Load Torque

e Eccentricity

F Static friction coefficient

G Dynamic friction coefficient

H, h Conductance (Appendix A)

h Heat transfer film coefficient (section 4.4)

I Mass moment of inertia

J Mechanical equivalent of heat

K Equivalent friction or discharge coefficient (Appendix A)

L Flow lengths (Appendix A)

l, l_0 Distance from pivot to C.G. of platform and part (section 4.2)
\( M_R \) Mass of regenerator matrix (Appendix A)

\( m \) Mass flow

\( P, P \) Pressure

\( Q \) Heat transfer

\( R \) Gas constant

\( R_C, R_R \) Housing and rotor radius

\( T, t \) Temperature

\( U \) Total internal energy

\( V, V \) Volume

\( W \) Work

\( w \) Mass

\( w, w_b \) Weight of part and pendulum support (section 4.2)

**GREEK LETTERS**

\( \gamma \) Ratio of specific heats

\( \eta \) Efficiency

\( \theta \) Time

\( \lambda \) Number of sectors in model of Appendix A

\( \tau \) Machine torque (Appendix A)

\( \tau, \tau_b \) Period of small oscillations of platform with part and that of platform alone (section 4.2)
\[ \phi \] Angular displacement of radial line from centre of housing with reference line (Appendix B)

\[ \psi \] Angular displacement of rotor from reference line

\[ \omega \] Angular velocity of rotor

**SUBSCRIPTS**

The following subscripts are found in Appendix A. In the text, upper case Latin subscripts refer to the boundary wall or exterior of one of the spaces, while lower case Latin subscripts refer to a property of the working fluid inside one of the spaces.

- \( c_{bi} \): Compression buffer inlet
- \( c_i \): \( i^{th} \) compression section
- \( c_1 \): First compression section
- \( c_2 \): Second compression section
- \( cbo \): Compression buffer outlet
- \( cd, CD \): Compression duct
- \( cr, CR \): Compression regenerator
- \( h, H \): Heater
- \( e_{bi} \): Expansion buffer inlet
- \( e_i \): \( i^{th} \) expansion section
- \( e_1 \): First expansion section
- \( e_2 \): Second expansion section
- \( ebo \): Expansion buffer outlet
- \( ed, ED \): Expansion duct
er, ER  Expansion regenerator
k, K  Cooler
b  Buffer
R  regenerator or rotor (clear from context)
W  Wall
Chapter 1

1. INTRODUCTION

The aim of this project is the realization of a rotary Stirling engine. It must be reported that, to date, this aim has not been fulfilled. The experience gained from the present investigation, which is presented in this report, can nevertheless be used as a basis for further work. A mathematical model of the engine is presented in Chapter 2 while details of the design and fabrication of the prototype are given in Chapter 3. The prototype has yet to run as it failed during preliminary tests due to severe wear of both the cylinder and blade seal and the end-plate and side-seal surfaces. It is suggested that the engine configuration could be simplified so as to minimize the length of the leakage path and also reduce the number of relative motions involved.

1.1 LITERATURE SURVEY

Robert Stirling had patented his first air engine in 1816. Theoretically, this cycle was thermodynamically as efficient as the Carnot cycle, its economy depending on the reiterated use of the same air alternately giving out and absorbing the same heat. Descriptions of this and other early air engines are given in references 7, 10, 29 and 30. It was not until 1871, however, that Gustav Schmidt published the first analysis of Stirling type engines in which he succeeded in deriving expressions for
the cyclic pressure variations assuming isothermal conditions (ref. 1). His is considered the classical solution in spite of its gross oversimplifications. Soon after this, interest in air engines was almost completely eliminated with the advent of internal combustion engines. It was not until the late 1930's that work on "hot air engines" was once again resumed, this time by Philips of Holland. Their work has continued to the present day and is well-documented in several papers (refs. 2-6, 10, 15, 16 and 27). Their earlier papers (refs. 2-6) all assume a constant mean temperature in the cylinders. The main limitation with this is that all the processes in the cylinders are treated as strictly isothermal (as in Schmidt's analysis) and this calls for infinite heat transfer coefficients across the walls. Finkelstein (ref. 8) was the first to present an analysis accounting for the heat transfer properties of the walls. The bulk of the remaining papers deal with analytical studies of Stirling engines (refs. 13, 14, 17, 18, 20, 21, 23, 25 and 26), efforts at optimizing certain design parameters (refs. 9, 11 and 12), and presentation of experimental results (refs. 17, 22, 25 and 26). Other works that have been cited (refs. 15, 16, 19, 24, 27 and 28) provide an insight into the modern developments in this field.

All this interest in the Stirling engine is due to the numerous advantages of this powerplant over conventional internal combustion engines (refs. 7, 16, 19, 22, 24, 27 and 31). The most prominent of these advantages are the superior emission and
noise characteristics due to the external combustion. Also significant are the excellent thermal efficiencies obtainable, these having been reported (ref. 31) as high as 40%.

1.2 CHRONOLOGICAL HISTORY

Work on this project started at McGill in 1970. The vane pump type of configuration was chosen at that time and a model with floating side pressure plates was constructed. This failed to yield the required compression ratio and so the side sealing mechanism was redesigned. The side pressure plates were fixed and seal slots machined into them. Side seals were manufactured from teflon, but still the compression results were very poor. Rotor blades at this time were of carbon graphite and slot springs were handmade. The non-uniformity of these springs caused considerable doubt as to whether or not the blades were seating properly against the casing periphery. At this point, it was decided to redesign the system for better sealing characteristics. Work on this second model was completed in February 1973. During the first two hours of preliminary testing, the engine suffered severe wear and it was impossible to complete the tests.
Chapter 2

2. THEORY

2.1 THE IDEAL STIRLING CYCLE

A Stirling engine is a heat engine operating according to the idealized cycle shown in Figure 2.1. This cycle consists of two isothermal and two isometric phases. To illustrate how such a machine would produce a net power output, consider for the moment two pistons in open communication with each other, as shown in Figure 2.2. During the changes II to III and IV to I, no mechanical work is done (assuming an ideal frictionless model) since the volumes remain constant. During the change I to II, the gas is compressed and thus, work is done on the system, while during the change III to IV, the gas expands and thus, does work on the system. This latter work exceeds the compression work because it takes place at a higher temperature level and, thus, at a higher pressure since the same volume changes are involved. This then yields a surplus of work per cycle.

The preceding suggests that there could be all types of engine configurations employing this cycle. In practice, however, due to thermodynamic losses and design constraints, their number is limited indeed. In all cases, the cycle is far from the ideal, with practical units approximating more closely adiabatic compression and expansion phases. It must be understood then that today's Stirling engines are so-called because of their constituent parts (two variable volumes, two compact heat exchangers
FIG. 2.1 IDEALIZED STIRLING CYCLE
FIG. 2.2  CONFIGURATION DEMONSTRATING STIRLING CYCLE
and one regenerator) rather than the actual cycle they undergo. These engines have complex drive systems (cf. Philips' rhombic drive, refs. 6 and 10) and incorporate large amounts of dead volume due to the regenerator and heat exchangers. In a typical machine, this dead volume is roughly 60% of the total volume (ref. 9) and has considerable effect in reducing the engine output (ref. 12).

2.2 THE ROTARY CONCEPT

Just as the Wankel engine seems to have solved some of the problems of the internal combustion engines, so it was thought that a rotary configuration would help to overcome some of the Stirling engine problems. The configuration chosen for study is an internal-axis planetary rotation machine (this classification is due to Wankel, ref. 32) resembling the present day vane pump, as shown in Figure 2.3. Such a system completely eliminates the complex drive system while retaining a desirable sinusoidal volume variation. Also, it was hoped to significantly reduce the effect of the dead volume.

Referring to Figure 2.3, one sees that in this model there is no external ducting, regenerator or heat exchanger, and this represents the ideal situation as regards the dead volume. It is equally clear, however, that such a model would yield very poor thermal efficiencies and thus, one should consider the use
FIG. 2.3  PRESENT PROJECT CONFIGURATION

FIG. 2.4  ROTARY CONFIGURATION INCLUDING HEATER, COOLER AND REGENERATOR
of a regenerator with its associated dead volume and find the appropriate design tradeoff. One such configuration is shown schematically in Figure 2.4.

2.3 THE GENERAL MODEL

The ability of a mathematical model to accurately and repeatedly predict the performance of an actual engine is the best justification for its use. Since work on the present engine has not been completed, corroboration of the validity of the theory with actual operating experience cannot be presented. It is felt, however, that the model used has taken into account all factors that could have a significant effect on the performance calculations.

The model, shown in Figures 2.4 and 2.5 has been subdivided into 9 distinct parts which are labeled in Figure 2.5. It includes a regenerator, heater, cooler, variable working volumes, a buffer space and the associated ducting required. To be presented first is a short discussion of each section.

(i) The Compression Volume:

This is the space between planes 8 and 1 in Figure 2.5. Ideally, the compression should be isothermal, but in practice, this is approximated more closely by a process somewhere in between an isothermal and an adiabatic process. This is because
FIG. 2.5 SCHEMATIC REPRESENTATION OF GENERAL MODEL SHOWING THE VARIOUS WORKING SECTIONS
the heat rejection required to maintain the ideal isothermal process cannot take place fast enough and because of the generally high heat capacity of the wall. Taking the compression process to be adiabatic has a pronounced effect on the efficiency (together with an adiabatic expansion process). This latter assumption would, however, represent a more realistic (though somewhat conservative) estimate than the ideal Carnot efficiency one obtains assuming isothermal conditions. In actual practice, the compression surface can be cooled somewhat by using incoming combustion air. Also, since the inside surface film coefficients are expected to be quite high due to the turbulent character of the gas in the sections, then it is reasonable to expect a good rate of heat transfer to or from the gas as long as the outside surface coefficients are sufficiently high.

(ii) Compression Duct:

This is the ducting required to take the compressed gas to the regenerator. This process should be adiabatic and preferably a constant volume one. Thus, the only losses encountered here would be flow losses. Also associated with the ducting is a certain amount of dead volume.

(iii) Compressed Gas Regenerator Section:

For this application, a rotary regenerator such as is used for gas turbine preheaters could be used, or else some other
compact design. The difference between the regenerator of the present model and those of other investigators is that in the present model the fluids must be kept unmixed. This has the effect of lowering the overall effectiveness of the regenerator thus necessitating more heat transfer in the heater and cooler. Temperature fluctuations occurring in the two variable volumes have little effect on the temperature in the regenerator (ref. 20) and thus, the performance of the regenerator can be calculated on the basis of constant end temperatures. Longitudinal and transverse heat conduction is usually negligible and so can be neglected.

(iv) Heater Section:

Sufficient heat must be added to the working gas while in the heater to raise its temperature to the required working level. This is a constant volume process, the effectiveness of a typical heater or cooler being from 90 to 95%.

(v) The Expansion Volume:

This is similar to the compression volume except that in this case, since the average pressure is higher than during compression, leakage will be out of the expansion space into the buffer space.
(vi) **Expansion Duct:**

Like the compression duct, this serves the purpose of transferring the working gas adiabatically and at constant volume to the regenerator.

(vii) **Expansion Gas Regenerator Section:**

This section can be treated the same as the compression side regenerator.

(viii) **Cooler Section:**

Cooling takes place at constant volume and the cooling air is used for the combustion process.

(ix) **Buffer Space:**

This is a constant volume space which is filled with working gas at the minimum cycle pressure. This reduces the leakage from the expansion space by reducing the pressure difference across the sealing surfaces. An average temperature and pressure are used for this section.

Each section shall be considered separately in the analysis. The various equations derived for each section must then be matched at the appropriate interfaces. With the aid of these boundary conditions and the following assumptions, the entire
analysis can then be carried out.

Assumptions:

1. The perfect gas law applies (i.e., $pv = wRT$). For hydrogen and helium, the two most commonly used working fluids, this holds very well, even at the considerable pressure levels involved, introducing only small errors.

2. The mass of working fluid is constant, there not being any leakage outside the system.

3. The volume variations in the compression and expansion phases are sinusoidal, as shown in Appendix B.

4. The temperature of the walls of the heater and cooler are constant and equal to the upper and lower temperature limits respectively.

5. Perfect mixing of the fluid in the compression and expansion cylinders is assumed. This leads to temperatures and pressures in these sectors that are only time-dependent (i.e., no fluctuations inside a sector). This also applies to the buffer space.
2.4 ANALYTICAL WORK

A detailed analysis, including three tables summarizing all pertinent information, is given in Appendix A. The analysis is presented in the form of a case study of a machine having 8 rotating sectors. The resulting set of differential equations (93 for the case of Appendix A) must now be solved in order to obtain a complete solution for the machine's dynamic behaviour. As noted in Table A-2, there are numerous input parameters that must be specified before any such solution could possibly be attempted. Some of these, such as the variable volume functions, can be calculated directly from basic geometric data (Appendix B). Operating temperature limits can be specified so as to attain a high Carnot efficiency, thereby assuring the highest possible realistic operating efficiency, while still remaining within metallurgically-safe operating limits. Even such factors as equivalent flow friction and discharge coefficients can be estimated with some degree of accuracy.

However, the variable conductances and the mechanical input parameters are difficult, if not impossible, to determine with accuracy. A survey of heat transfer literature has failed to reveal any work that might provide some information on film coefficients applicable in this case. For forced convection heat transfer involving gases, the heat transfer film coefficient $h$ can typically vary from 2 to 20 BTU/hr.ft$^2$°F (refs. 33 and 34).
This represents a tenfold difference between maximum and minimum values and thus would have considerable effect on the performance of the compression and expansion phases.

The dynamic and static friction coefficients are likewise very uncertain quantities. Even the mass moment of inertia becomes difficult to calculate accurately due to the complicated rotor geometry. An experimental program is therefore proposed that will enable the determination of these and other useful parameters so that a system solution may be obtained. The following section outlines the model chosen for experimental study, presents a "simplified" analysis in order to obtain some starting points for the design, and outlines the experimental program to be followed.

2.5 **PRESENT MODEL**

The model considered for the experimental investigation is as shown in Figure 2.3. It does not contain a regenerator, separate heater and cooler or any of the associated ducting as indicated in Figure 2.4. Information concerning heat transfer and pressure drops for these components are readily available in the literature (see for example, refs. 20, 25, 26 and 35). However, the model does contain all the essential features necessary for determining the various input parameters mentioned in the previous section. The methods whereby these values are to be
obtained will be detailed later in Chapter 4. An ideal air standard analysis will first be presented in order to determine the governing parameters for the design of the engine.

The following equations can be derived with the aid of Figure 2.3 and the assumptions of section 2.3:

Phase 1→2 Isothermal compression

\[ \delta T = \delta U = 0 \]

and

\[ |w_c| = |Q_c| = \int p\,dV = wR T_c \ln \frac{p_2}{p_1} \]

Phase 3→4 Isothermal expansion

\[ |w_e| = |Q_e| = wR T_H \ln \frac{p_3}{p_e} \]

Phase 2→3 Constant volume heat addition

\[ w = 0 \quad \text{and} \quad Q_a = \delta U \]

\[ |Q_a| = wC_v(T_H - T_C) \]

Phase 4→1 Constant volume heat rejection

\[ |Q_r| = wC_v(T_H - T_C) \]
Since, for an ideal gas one has $p_2/p_1 = p_3/p_4$, then

$$W_{\text{net}} = wR(T_H - T_C) \ln p_2/p_1 \quad \text{(2.1)}$$

and

$$Q_{\text{tot}} = wC_v(T_H - T_C) + wR T_H \ln p_2/p_1 \quad \text{(2.2)}$$

Using equations (2.1) and (2.2) one obtains for the thermal efficiency:

$$\eta_{\text{Thermal}} = \frac{(T_H - T_C) \ln p_2/p_1}{\frac{C_v}{R}(T_H - T_C) + T_H \ln p_2/p_1} \quad \text{(2.3)}$$

Note here that if perfect regeneration were provided, then $Q_a$, the constant volume heat addition, would be derived totally from $Q_f$ and the only remaining heat addition would be $Q_e$. This would lead to an expression for the efficiency in equation (2.3) equal to the Carnot efficiency.

From these three equations one can obtain all the information necessary to proceed with the model design. Equation (2.1) shows that in order to obtain maximum work output,

a) the product $wR$,

b) the temperature difference $(T_H - T_C)$, and

c) the pressure ratio $p_2/p_1$ (or alternatively, the compression ratio $V_1/V_2$),

should all be maximized. A means for controlling such an engine
is also revealed since the net work can be changed by simply varying the mass \( w \) of fluid taking part in the process. The ratio \( C_v/R \) is dependent on the working fluid and minimization of this factor, besides being consistent with maximization of the product \( wR \), has a pronounced effect on the thermal efficiency. Considering equation (2.3), one sees that for the numerator to be a maximum for a given pressure ratio the temperature difference should be a maximum. Then, for that maximum temperature difference, \( C_v/R \) should be a minimum for the denominator to be a minimum. This would then yield a maximum efficiency for that specific pressure ratio.

The gases in use in present Stirling engines are hydrogen, helium and nitrogen. Even though hydrogen exhibits the best performance characteristics, helium is preferred for the experimental application due to safety considerations. The use of helium has the added advantage that the fluid friction or flow losses will be less than with a denser gas. Typical values of \( C_v, R \) and \( \frac{C_v}{R} \) \( J \) for helium are as follows (ref. 36):

\[
\begin{align*}
C_v &= 0.753 \text{ BTU/lbm } \degree R \\
R &= 386.0 \text{ ft lbf/lbm } \degree R \\
\frac{C_v}{R} \ J &= 1.518
\end{align*}
\]

Combining these values with an assumed lower temperature limit, \( T_c \), results in a graph (Figure 2.6) showing variation of
FIG. 2.6 VARIATION OF THERMAL EFFICIENCY WITH PRESSURE RATIO AND TEMPERATURE DIFFERENCE

$T_c = 100 \degree F$

- $p_2/p_1 = 8$.
- $p_2/p_1 = 6$.
- $p_2/p_1 = 4$.
- $p_2/p_1 = 2$.
thermal efficiency with pressure ratio and temperature difference. This graph clearly indicates that for optimum thermal efficiency both the pressure ratio and the temperature difference should be maximums. However, present Stirling engines usually operate at a high mean pressure level while keeping the pressure ratio well below maximum (refs. 6 and 12). This lower pressure ratio helps considerably in reducing the leakage losses, while the higher pressure level increases considerably the net torque one can expect as indicated in equation (A103). Also shown in equation (A103) is the dependence of the net torque on the speed of the machine. This is explained if one considers the gas contained in any one of the operating sections moving past the casing wall. The amount of heat this gas can absorb or reject is directly proportional to the mass flow past a section of casing area and the specific heat \( C_p \) of the working fluid.

The following (Table 2-1) is a list of the major design parameters together with a summary of their effects on the system.
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>EFFECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Ratio</td>
<td>Increases efficiency, work output and leakage losses. Limited by design considerations.</td>
</tr>
<tr>
<td>Temperature Difference</td>
<td>Increases work efficiency and work output. Limited metallurgically.</td>
</tr>
<tr>
<td>Mass Content</td>
<td>Increases work output. Regulating parameter.</td>
</tr>
<tr>
<td>R</td>
<td>Increases work output and thermal efficiency. Fluid property.</td>
</tr>
<tr>
<td>C&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Increases work output. Fluid property.</td>
</tr>
<tr>
<td>C&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Decreases thermal efficiency. Fluid property.</td>
</tr>
</tbody>
</table>
Chapter 3

3. DESIGN

3.1 GENERAL

This project has undergone three distinct construction phases. The model shown in Figure 3.1 was developed during the first phase. End plates (see Figure C.1 in Appendix C where all drawings are presented) were mounted on either side of a cylindrical tube (Figure C.2) which served as a housing. The rotor (Figure C.3) was located in bearings mounted eccentrically in the end plates, with the eccentricity being such that the volume ratio was approximately 4:1 (giving an adiabatic pressure ratio of 7:1).

Sealing between rotor and end plate was to be provided by a bearing ring (Figure C.4) forced against the rotor by six set screws. Rotor vanes were of carbon graphite and were supported underneath by two springs as illustrated in Figure 3.2. Efforts at making this machine work proved unsuccessful. The main reasons for this failure were that leakage inside the engine (from section to section) was excessive as well as the leakage to the outside. Of these, it was decided that the inside leakage was the most important, and modifications aimed at remedying this problem resulted in the second model. Basic changes made during this phase of construction were as follows:

a) The bearing rings were fixed to the end plates, thereby
**FIG. 3.1** FIRST MODEL CONSTRUCTED IN FALL 1970

**FIG. 3.2** SPRINGS USED TO KEEP VANES IN CONTACT WITH HOUSING SURFACE
**FIG. 3.3** END PLATE WITH BEARING RING GROOVED AND MOUNTED IN PLACE

**FIG. 3.4** ROTOR SHOWING SIDE SEALS INSTALLED IN GROOVES
eliminating leakage behind these plates to the outside.

b) A groove was machined into each of the bearing rings and into each side of the rotor (Figures 3.3 and 3.4).

c) Seals of teflon were made so that they bridged the gap between rotor and end plates. Gas pressure inside the sections was to force the seals into continuous contact with the edge of the bearing plate groove thereby effecting the sealing action.

d) New springs (illustrated in Figure 3.5) were also made in an effort to produce more uniform springs and eliminate the cracking of the blades caused by the ends of the previous springs around the holes where they were seated.

![Figure 3.5 Springs Used in Second Model](image)

Again these efforts went unrewarded as only a very small pressure difference (less than 1/2 inch of mercury) was obtainable. At this stage, it was decided to rebuild the unit, saving as much as possible of the major components. Modifications
fall into the following categories: (i) sealing, (ii) rotor, and (iii) housing. Each of these shall be discussed separately in the following sections, with sealing being the first and major one as it bears so heavily on the other two considerations. Combined, they form the third, and to date final, model in this program.

3.2 SEALING

The truth of Dipl. Ing. Wolf Dieter Bensinger's words when referring to the sealing grid of a Wankel RC engine, he pointed out that "a sealing grid of this nature can only perform its allocated task if all primary and secondary sealing areas are in uninterrupted contact" (ref. 37), was discovered through bitter experience. Reviewing some of the literature on the Wankel engine (refs. 37-40) led to the realization that a sealing grid had to be developed if there was to be any chance of success. Compounding the problem over that encountered in the Wankel engine was the fact that the vanes moved relative to three surfaces at quite high speeds. Having decided that the rotor configuration would remain basically unchanged, the problem was broken down into two parts: (i) design of a sealing grid separating each rotor section from the others, and (ii) choice of proper mating materials.

The final design for the sealing grid is illustrated in
Figure 3.6 while Figures 3.7, 3.8 and 4.14 show the actual components. A continuous sealing path was thus provided between the following surfaces:

- a) blade and rotor
- b) blade and housing bore
- c) blade and end plates
- d) rotor and end plates.

Initial attempts at designing a sealing grid had in mind the use of TFE fluorocarbon resins. Unfilled teflon proved to be impractical due to the difficulties encountered trying to machine parts to proper dimension and also because of its high thermal expansion coefficient. Filled TFE (ref. 41) seemed to be the answer to these problems but was not available. Another duPont product, Vespel (ref. 42), is a polyimide material which seems to have all the required characteristics but again, this was not readily available. Also, it was extremely expensive. The choice of materials was thus narrowed down to those readily available which had shown good operating characteristics under similar operating conditions in the past. The materials finally decided upon were chosen with the aid of references 37 to 40, relying on Wankel RC engine experience, and are listed in Table 3.1.
FIG. 3.6 ILLUSTRATION OF (a) SEALING GRID AND (b) GRID AS INSTALLED IN A ROTOR SECTION
FIG. 3.7 Steel vanes with sealing elements

FIG. 3.8 Slot seals with backup bar and retaining bracket
<table>
<thead>
<tr>
<th>BORE SURFACE</th>
<th>BLADE SEALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard chrome</td>
<td>Piston ring cast iron</td>
</tr>
<tr>
<td>END. PLATES</td>
<td>SIDE SEALS</td>
</tr>
<tr>
<td>Hard chrome</td>
<td>Bronze</td>
</tr>
<tr>
<td>BLADES</td>
<td>SLOT SEALS</td>
</tr>
<tr>
<td>Mild steel</td>
<td>Piston ring cast iron</td>
</tr>
</tbody>
</table>

**Table 3-1  Material Combinations for Mating Surfaces**

All sealing elements were manufactured at McGill. Difficulties were encountered in producing a good surface finish on the contact faces of the blade seals, and this is believed to have been the cause of the eventual failure. Two small compression springs were used behind each of the rotor side seals and rotor slot seals to maintain these surfaces in contact with the end plates and blades respectively. Four of these springs were also used in each blade (two per side) to insure proper seating of the blade seal segments against the end plate surfaces. Here too efforts had initially been focused in another direction. Linear wave springs were considered so as to eliminate the mounting holes required for the compression springs but this idea proved unfeasible due to production difficulties. Such springs would have simplified the blade design considerably as well as the rotor design (see Figure C.5 for blade details).

Mounted vertically in each rotor slot were two additional
compression springs to supply uniform pressure to each blade. This was to assure continuous contact between the housing bore and the blade seal segments which had not been the case with the previous springs. These segments were overlapped at the centre so that the gas pressure would force the lips against each other producing a sealed joint while still allowing for motion of the segments to accommodate thermal expansion and wear. All sealing elements discussed in this section are detailed in Figures C.6 to C.10.

3.3 ROTOR

To accommodate the sealing grid described in the previous section, the rotor had to be redesigned. Details of the present design are shown in Figures C.11, 3.9, 3.10 and 3.11. One of the major problems of the original rotor was the difficulty in keeping the slot thickness within the tolerance limits. This was because the slots were milled with a slitting saw of large diameter (5 inches) and small width (3/16 inch) allowing considerable vibration of the cutting edge. Initially, two rows of slot seals together with a blade centering component along the bottom edge of each blade were envisioned. Their purpose would have been to prevent tilting of the blades in the oversized slots. This is still the case at present since these parts could not be incorporated because of space limitations.
FIG. 3.9 VIEW OF ROTOR WITH SLOT SEALING ELEMENTS REMOVED

FIG. 3.10 VIEW OF ROTOR INCORPORATING SLOT SEALING ELEMENTS
The modifications posed some manufacturing difficulties. Cutting of the surfaces (Figure 3.9) where the slot seals were to be mounted required great care and a special cutter had to be made to mill the through slots. Such problems could be easily eliminated by making the sectors separately and then fitting them together on a hub as is presently done with turbine blades. This would provide easier access to the surfaces being machined and also allow for more secure mounting (on one of the flat faces of a sector) than is possible with the entire unit mounted on a dividing head. Additional work done on the rotor involved the drilling of two holes in each sector for mounting and retaining the compression springs behind the side seals (Figure C.11).
3.4 HOUSING

In order that leakage to outside the system could be eliminated, both the end plates and housing were redesigned. With the end plates, this involved completely new parts. Details of the end plate with the pressure taps is given in Figures C.12 and 3.12. The major changes to the end plates were:

a) A flanged joint between housing and end plate was designed, allowing for easy and sure application of gasket material and sufficient surface area over which to apply the gasket. This is in contrast to the cantilever type of joint employed previously where the end plates had been rigidly secured to each other by draw bars with the cylindrical housing mounted between them (see Figures 3.1, 3.3).

b) The closely toleranced draw bars were replaced by standard bolts.

c) The end plate is of unit construction with hard chrome plating applied to the inside surface (see Figure 4.12).

Two flanges were welded onto the housing of Figure C.2 and the bore was plated with hard chrome to .010 inches and ground to dimension. The plating and grinding mentioned here were the only operations not done at McGill. Details of the housing are given in Figure C.13 while Figure 3.13 shows the
flange as welded to the cylinder. Finally, Figure 3.14 shows the bearing mounting bracket and sealing cover. There are two of these sets, one for each side of the engine, the only difference being that the one set not illustrated has a through hole allowing for the output shaft. Sealing of the shaft was provided for by installing two O-rings in this latter sealing cover.

FIG. 3.12 END VIEW OF ASSEMBLED ENGINE SHOWING END PLATES AND PRESSURE TAPS
FIG. 3.13 SIDE VIEW SHOWING END PLATES BOLTED TO FLANGED HOUSING

FIG. 3.14 BEARING, MOUNTING BRACKET AND SEALING COVER
Chapter 4

4. EXPERIMENTAL PROCEDURE

4.1 GENERAL

This chapter deals with the procedures and apparatuses to be used in measuring the input parameters listed here:

a) mass moment of inertia of assembled rotor;

b) static and dynamic friction torques;

c) heat transfer film coefficients;

d) pressure.

Also presented are details of the initial test during which the housing and end plates failed due to severe wear.

4.2 MASS MOMENT OF INERTIA

Procedures for the determination of the moment of inertia of complex forms such as the assembled rotor are available and one such method will be described here (ref. 43). It consists of mounting the part on a pendulum support as illustrated in Figure 4.1. The part is suspended in such a way that the centroidal axis c-c is directly below and parallel to the suspension axis o-o. Measuring the period of small oscillation, one can then determine I by application of the following equation:
FIG. 4.1 APPARATUS FOR USE IN INERTIA MEASUREMENT
\[ I = w \left( \frac{\tau^2}{4\pi^2} - \frac{\ell_0}{g} \right) + \frac{w_b \ell}{4\pi^2} (\tau^2 - \tau_b^2) \]  

(4.1)

where

- \( w \) = weight of part
- \( w_b \) = weight of pendulum support
- \( \ell_0 \) = distance from 0 to centre of gravity of part
- \( \ell \) = distance from 0 to centre of gravity of platform
- \( \tau \) = period of small oscillations of pendulum support and part
- \( \tau_b \) = period of pendulum support.

Best accuracy is obtained when \( \ell_0 \) is kept as small as possible though it must be accurately measurable. The experiment must then be carried out twice, once with just the casing and once with both casing and rotor. Subtracting the result for the casing from the latter will then yield the inertia of the assembled rotor.

### 4.3 Static and Dynamic Friction Torques

The procedure used for determining these two parameters is derived from references 44 and 45. An electric dynamometer was constructed according to the circuit diagram shown in Figure 4.2. A DC car generator is used for this application (Figure 4.3), its
FIG.4.2 DYNAMOMETER CIRCUIT

FIG.4.3 GENERATOR MOUNTED IN TRUNION BEARINGS
field structure cradled in trunion bearings so that the only restraint is that of the torque-weighing arm and the connecting wires. Since a source of DC power was readily available, this was also a relatively inexpensive procedure to adopt, the only additional requirements being for a variable transformer and a rectifier (Figure 4.4). The physical circuitry is shown in Figure 4.5.

With this setup, the engine can be motored at no load through the complete speed range to yield the dynamic friction torque. The static friction torque would correspond to that force measured just prior to the onset of motion. Speed measurements are taken manually through the centre of the rear trunion bearing and bearing adapter (Figure 4.6) using a Basler tachometer.

FIG. 4.4 THREE MAIN CONTROL COMPONENTS: VARIABLE TRANSFORMER, RECTIFIER, AND CIRCUIT BOARD
FIG. 4.5  PHYSICAL REALIZATION OF DYNAMOMETER CIRCUIT

FIG. 4.6  GENERATOR WITH REAR BEARING ADAPTOR SHOWING ACCESS TO GENERATOR SHAFT
4.4 HEAT TRANSFER FILM COEFFICIENTS

Considering the cold side first, the following equations can be written for the heat transfer through the cylinder wall (see Figure 4.7):

\[ Q_{w\rightarrow o} = h_o a_o (t_w - t_o) \]  
\[ Q_{i\rightarrow w} = h_i a_i (t_g - t_w) \]

Here, \( h \) = film coefficient, \( a \) = area, \( t \) = temperature, and subscripts:
- \( o \) refer to outside
- \( w \) = wall
- \( i \) = inside
- \( r \) = rotor

FIG. 4.7 SCHEMATIC IDENTIFYING HEAT TRANSFER TERMINOLOGY FOR A SECTOR

Neglecting heat loss by conduction along the wall gives:

\[ Q_{i\rightarrow w} = Q_{w\rightarrow o} \]  

If one adds to this the assumption that the heat loss of the cooling fluid to the atmosphere can be neglected, then:
\[ Q_{w+o} = m \cdot C_p \cdot \Delta T \]  
\[ (4.4) \]

where \( m \) = mass flow of coolant  
\( \Delta T \) = temperature between inlet and outlet.

This assumes steady-state operating conditions and no time variations as would be caused by a load change. Provision for measuring coolant inlet and outlet temperatures using partial immersion thermometers has been made (see Figure 3.13) and the coolant temperature \( t_o \) can be taken as the average of these two or be measured separately. Water was chosen as coolant because film coefficients in forced convection for water vary from 100 to 2000 BTU/hr.ft\(^2\)°F (ref. 34) and this assures that the inside resistance will be controlling. Since \( m \) can be measured (weighing coolant outflow over a certain time period) and \( a_o \) is known, then \( h_o \) can be calculated from equation \( (4.2a) \) once \( t_w \) is known. Similarly, \( h_i \) can be calculated once \( t_g \) is known. Both \( t_w \) and \( t_g \) must be measured experimentally. As of this writing, provision for these measurements has not been made. The techniques for making such measurements are described in the literature (eg., ref. 46). Measurement of the gas temperature is difficult since a thermocouple protruding into the working section and mounted in the rotor will be required, posing considerable problems in bringing the thermocouple leads out of the rotor housing.
An expression similar to equation (4.2) can be written for the rotor:

\[ Q_{1-r} = h_{a r} (t_r - t_g) \]  

(4.5)

Measurement of the rotor temperature presents the same problem as noted above. A good approximation would be an average between the hot and cold gas temperatures. In all these measurements, the connection problem appears to be the only difficulty, probes being readily available with time constants as small as $10^{-5}$ seconds and in materials usable to 5000°F and 10,000 psi pressure (ref. 46). Such thermocouples are commercially available from: (a) Nammac Corp., Indian Head, Md., USA, and (b) MO-RE', Inc., Bonners Springs, Kans., USA.

Turning next to the hot side, one can again use the two equations (4.2a) and (4.2b). In this case however, the heat transfer rate is not easily measurable. This is seen if one considers the method of heating that is proposed: two burners (Figure 4.8) mounted in an arc over the hot side utilizing direct flame impingement. Even though facilities were provided for metering the fuel and combustion air, thereby allowing calculation of the heat of combustion, heat losses to the surroundings cannot be determined accurately using the measuring techniques already described. Heat flux sensors would be required to do this. Operating principles of these devices are
given in reference 46. These sensors are available commercially from Heat Technology Laboratory, Inc., Huntsville, Ala., USA. These sensors measure $Q$ directly, and along with measurements of gas and wall temperatures, allow determination of the hot side film coefficients.

4.5 PRESSURE

Cyclic pressure variations can be measured by locating around the periphery of the engine (either in the cylinder or side walls) a sufficient number of static pressure taps. The readings obtained would enable one to plot the variation of pressure with rotor angle, yielding a sinusoidal variation.
This is to be expected because of the sinusoidal volume variation. So far in this project, capability exists for measuring only the compression ratio, this being chiefly to test the effectiveness of the sealing grid. Four holes drilled into one end plate (Figure C.12) were used for this purpose. The one at the start of the compression cycle was used for injecting working fluid into the engine, while the other three were connected to pressure gages to register readings at their respective locations. Under operating conditions, this would then give the pressure ratio (comparing the pressure at the end of the expansion phase to that at the end of the compression phase) and show the effect of heat addition (comparing pressure at end of compression phase to that at the start of the expansion phase).

In Figure 4.1 the engine is shown during initial tests with three pressure gages connected to the above mentioned taps. Bourdon-type gages can be employed in this case because the pressure at any one location around the periphery should be essentially constant. Small fluctuations in pressure due to the volume variation that occurs while one sector passes over the opening have little effect on the gage reading and can be damped out almost entirely by crimping the tubing connection to the gage.
4.6 AUXILIARY EQUIPMENT

Discussed in this section are the burners (Figure 4.8) and the flow metering apparatus (Figure 4.9). The burners were designed and made at McGill. Air and gas are supplied to the burner as shown in Figure 4.10. This results in excellent mixing of the fuel with the air because the two gases are injected in such a manner that two counter-rotating flows are established.

Two screens have been used, one as the outside bottom surface and the other just above this on the inside. These serve two purposes: (i) a pre-mixed flame will not propagate through a fine mesh screen, thus assuring safety in operation; (ii) excellent flame stability is achieved with this configuration.

The mixture is controlled by the apparatus shown in Figure 4.9. Missing at the right of this illustration is the flowmeter (rotameter type). Each stream is controlled by an on-off valve, a regulator and a needle valve. Both streams are metered through the same rotameter, by selector valves, and once the flow rates have been set can be bypassed completely.

4.7 PRELIMINARY TESTS

Following completion of the construction phase, the next step consisted of mounting the engine on a lathe and motoring it at various speeds. The purpose of this operation was twofold:
FIG. 4.9 Mixture control apparatus

FIG. 4.10 Schematic showing pattern of air-fuel mixing in burners
(i) to allow the parts to wear in and seat themselves properly, and (ii) to check the effectiveness of the sealing grid. With the engine set-up on the lathe (Figure 4.11), compressed air from the cylinder shown in the background was introduced through the tap located at the start of the compression phase while the remaining three taps were connected to the three pressure gages. The rotor was mounted in a collet, with the other side being supported by a centre mounted in the tailstock. To keep the housing stationary, a tool bit was locked behind one of the end plate bolts.

**FIG.4.11** SETUP FOR MOTORYING ENGINE ON LATHE
Prior to any test a liberal amount of oil was injected into the engine through one of the pressure tap openings. This tap would then be reconnected to the appropriate gage and the entire system pressurized. In this limited series of tests, these minimum pressure levels were used: 50 psig, 100 psig and 150 psig. These were all operated at 60, 80, 120 and 160 rpm while a speed of 224 rpm was also used for the 100 psig and 150 psig runs. One additional run (before the others) was made without pressurizing the engine and lasted for 1 1/2 hours. Here all four taps were left disconnected and sufficient oil was added to assure smooth operation. This test ran at 120 rpm. Since the assembly of the entire engine was very complicated, it was assumed after this test that everything was all right. Had the engine been disassembled and inspected at this point, some indication of the future cause of failure might have been evident. This was not done, however, and all the tests were carried out without an inspection. The results showed that there was a marked increase in pressure difference as both speed and pressure level were increased. A maximum pressure difference of 15 psia was recorded during the 150 psig test at 224 rpm. Although this did not come close to the desired compression level, it was a considerable improvement over the previous results.

It was during the above mentioned maximum run that unusual noises were suddenly heard. The test was halted at this point.
and it was decided to disassemble the unit to see if any damage had been done. Shown in Figures 4.12, 4.13 and 4.14 are the inside surfaces of the end plate, housing bore and side seals, respectively. Both the end plates and housing bore had been hard chrome plated and ground to a surface finish of 8μin. As seen in the figures, these surfaces were severely damaged. The two side seals shown in Figure 4.14 were also damaged and are typical of the condition of the other side seals. Barely visible in the bottom seal is an inclusion of cast iron. Examination of the blade seal segments revealed considerable chipping and wear of these surfaces especially on the forward side (i.e., in the direction of rotation). It was not possible to take a picture of these segments that properly showed these results.
FIG. 4.12 END PLATE SHOWING SEVERE WEAR ALONG OUTER CIRCUMFERENCE OF PLATED SURFACE

FIG. 4.13 HOUSING BORE SHOWING SURFACE WEAR
5. CONCLUSIONS AND RECOMMENDATIONS

The model as described in this paper failed during preliminary testing. Results necessary for solving the analytical model could therefore not be obtained. Failure of the present design resulted from material wear as described in section 4.7. New sealing elements should be constructed utilizing different materials. Recommended are either Vespel (ref. 42) or a filled teflon (ref. 41) as these materials combine excellent wear characteristics with high operating temperature limits and low friction coefficients. If such materials are used, care must be taken in the design to allow for thermal expansion which is several times that of ordinary materials (steel, cast iron, etc.).

Some improvement in the sealing was noted with the present design. A second sealing grid mounted directly below and identical to the one presently in use should further help in this respect. This would be similar to a set of piston rings on a piston, each ring taking a certain amount of the entire pressure drop. Combined with new low friction seal materials this promises to be a good tradeoff between the degree of sealing and the friction power required to effect this sealing.

Provision for measuring the gas temperature must still be provided as outlined in section 4.4. The apparatus for measuring the mass moment of inertia also remains to be constructed. Once this work has been done a new series of tests can be carried out.
to determine the parameters necessary for the solution of the analytical work. With this information, a complete design can then be contemplated incorporating all the equipment (heater, cooler, regenerator) necessary to produce a working model of the engine.

Thought should also be given to how the overall configuration could be simplified to reduce the number of relative motions involved as well as shortening the length of the sealing path. One such modification suggested for further study would have the end plates attached directly to the rotor. Sealing would be effected by piston ring-type seals mounted directly in the housing flanges and bearing on the rotating end plates (ref. 5.). This would allow for blade support and easier slot sealing.
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<thead>
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APPENDIX A

CASE STUDY FOR $\lambda = 8$

The work presented below follows closely Finkelstein's work (refs. 8, 13 and 21) on the generalized thermodynamic analysis of Stirling engines. The procedures will be the same, the differences being in the model used. Expressions for mass distribution, mass flow, pressure drop, expansion and compression work, heat transfer, etc., are presented. The ensuing differential equations, when combined with the appropriate input parameters, will then allow the complete determination of all major machine characteristics. To start the analysis, one must assume certain directions for mass flow, these being shown in Figure A.1 along with other pertinent terminology.

(i) Mass Distribution:

The model chosen for this work has 8 rotating chambers, this number not necessarily being the optimum, but serving well to demonstrate the concepts involved. The mass of gas contained in each of the $15$ sections of the engine is given by the following equations:

$$w_{cbi} = \frac{P_{cbi} v_{cbi}}{R t_{cbi}}$$  \hspace{1cm} (A1)

$$w_{ci} = \frac{P_{ci} v_{ci}}{R t_{ci}}$$

where $i=1,\ldots, \lambda/2 - 2$ where $\lambda =$ number of sections.
FIG.A.1 SCHEMATIC FOR CASE STUDY SHOWING SECTIONS AND FLOW DIRECTIONS
Thus, for the present case, this gives:

\[ w_{cl} = \frac{P_{cl} v_{cl}}{R_{t_{cl}}} \quad (A2) \]

and

\[ w_{c2} = \frac{P_{c2} v_{c2}}{R_{t_{c2}}} \quad (A3) \]

\[ w_{cbo} = \frac{P_{cbo} v_{cbo}}{R_{t_{cbo}}} \quad (A4) \]

\[ w_{cd} = \frac{P_{cd} v_{CD}}{R_{t_{cd}}} \quad (A5) \]

\[ w_{cr} = \frac{P_{cr} v_{CR}}{R_{t_{cr}}} \quad (A6) \]

As before, one has for the expansion side:

\[ w_{ei} = \frac{P_{ei} v_{ei}}{R_{t_{ei}}} \quad (A8) \]

where \( i = 1, 2, \ldots, \lambda/2 - \lambda \), and therefore:

\[ w_{el} = \frac{P_{el} v_{el}}{R_{t_{el}}} \quad (A9) \]
The temperatures \( t_{c_{bi}}, t_{c_{1}}, t_{c_{2}}, t_{c_{bo}}, t_{e_{bi}}, t_{e_{1}}, t_{e_{2}} \) and \( t_{e_{bo}} \) are all taken as being the instantaneous bulk temperatures in these spaces. In the other sections, ducting, heater, cooler and regenerator, the temperature can vary considerably with distance and thus, one can define these temperatures as follows:

\[
\bar{t}_{cd} = \frac{1}{L_{CD}} \int_{0}^{L_{CD}} t \, dx \quad (A16)
\]

\[
\bar{t}_{cr} = \frac{1}{L_{R}} \int_{0}^{L_{R}} t \, dx \quad (A17)
\]
The positive flow directions are as indicated in Figure A.1.
These will be useful for determining the time rate of change of mass content in each space. The general expression for this rate of change is:

\[
\begin{align*}
\text{Rate of change of mass content} &= \text{Sum of mass flow} - \text{Sum of mass flow} \\
\frac{\partial w}{\partial t} &= \sum m_{\text{in}} - \sum m_{\text{out}}
\end{align*}
\]

Writing this expression for each section in turn gives the following:
\[ \frac{dw_{cbi}}{d\theta} = m_8 + m_{10} + m_{11} - m_9 \quad (A22) \]

\[ \frac{dw_{c1}}{d\theta} = m_{12} - m_{18} - m_{11} \quad (A23) \]

\[ \frac{dw_{c2}}{d\theta} = m_{13} - m_{12} - m_{19} \quad (A24) \]

\[ \frac{dw_{cbo}}{d\theta} = m_{14} - m_1 - m_{13} - m_{20} \quad (A25) \]

\[ \frac{dw_{cd}}{d\theta} = m_1 - m_2 \quad (A26) \]

\[ \frac{dw_{cr}}{d\theta} = m_2 - m_3 \quad (A27) \]

\[ \frac{dw_{h}}{d\theta} = m_3 - m_4 \quad (A28) \]

\[ \frac{dw_{ebi}}{d\theta} = m_4 - m_{21} - m_{14} - m_{15} \quad (A29) \]
\[ \frac{d\omega_{el}}{d\theta} = m_{15} - m_{16} - m_{22} \]  
(A30)

\[ \frac{d\omega_{e2}}{d\theta} = m_{16} - m_{17} - m_{23} \]  
(A31)

\[ \frac{d\omega_{ebo}}{d\theta} = m_{17} - m_{10} - m_{5} - m_{24} \]  
(A32)

\[ \frac{d\omega_{ed}}{d\theta} = m_{5} - m_{6} \]  
(A33)

\[ \frac{d\omega_{er}}{d\theta} = m_{6} - m_{7} \]  
(A34)

\[ \frac{d\omega_{k}}{d\theta} = m_{7} - m_{8} \]  
(A35)

and

\[ \frac{d\omega_{b}}{d\theta} = m_{9} + m_{19} + m_{19} + m_{20} + m_{21} + m_{22} + m_{23} + m_{24} \]  
(A36)
(iii) Pressure Drop:

Following Finkelstein (ref. 13), a leakage factor $K$ will be used. This is taken to be a combined flow resistance acting at the planes separating the different sections. Therefore:

\[ m_1 = K_1 (p_{cbo} - p_{cd}) \]  \hspace{1cm} (A37)

\[ m_2 = K_2 (p_{cd} - p_{cr}) \]  \hspace{1cm} (A38)

\[ m_3 = K_3 (p_{cr} - p_h) \]  \hspace{1cm} (A39)

\[ m_4 = K_4 (p_h - p_{ebi}) \]  \hspace{1cm} (A40)

\[ m_5 = K_5 (p_{ebi} - p_{ed}) \]  \hspace{1cm} (A41)

\[ m_6 = K_6 (p_{ed} - p_{er}) \]  \hspace{1cm} (A42)

\[ m_7 = K_7 (p_{er} - p_k) \]  \hspace{1cm} (A43)

\[ m_8 = K_8 (p_k - p_{cbi}) \]  \hspace{1cm} (A44)

\[ m_9 = K_9 (p_{cbi} - p_b) \]  \hspace{1cm} (A45)

\[ m_{10} = K_{10} (p_{ebi} - p_{cbi}) \]  \hspace{1cm} (A46)

\[ m_{11} = K_{11} (p_{c1} - p_{cbi}) \]  \hspace{1cm} (A47)

\[ m_{12} = K_{12} (p_{c2} - p_{c1}) \]  \hspace{1cm} (A48)

\[ m_{13} = K_{13} (p_{cbo} - p_{c2}) \]  \hspace{1cm} (A49)

\[ m_{14} = K_{14} (p_{ebi} - p_{cbo}) \]  \hspace{1cm} (A50)
By using equations (A37) to (A44), one tacitly assumes a linear pressure variation along the flow sections. The coefficients $K_9$ through $K_{24}$ can be thought of as equivalent discharge coefficients for flow through an orifice equivalent to the respective leakage area. These flows (eqns. A45 to A60) have been represented as being directly proportional to the pressure differences but could be equally well represented by some other power expression.
(iv) **Temperature:**

The relationships for the temperature distributions are, by far, the most involved in this series. Each section will be considered separately with appropriate matchup equations included as boundary conditions. All equations are presented with constant or variable conductances. The procedure is to write an energy balance over a time interval $\delta \theta$ for the gas temperature as follows:

$$\begin{align*}
\text{Heat transferred to gas in } \delta \theta + & \text{Enthalpy addition to gas in } \delta \theta \\
= & \text{Work flow done by gas in } \delta \theta + \text{Internal energy increase in gas during } \delta \theta \\
\end{align*}$$

For the wall and rotor temperatures, the following balance is used:

$$\begin{align*}
\text{Heat transfer to wall during } \delta \theta - & \text{Heat transfer to gas during } \delta \theta = \text{Heat stored in wall during } \delta \theta \\
\end{align*}$$
(iva) **Compression Buffer Inlet:**

For the gas temperature, one has:

\[
\left[ h_w(t_w - t_{cbl}) + h_r(t_R - t_{cbl}) \right] \delta \theta + C_p \left[ m_{10ebo} + m_{11c1} + m_{s} t_{k} - m_{10ebo} t_{bi} \right] \delta \theta = p_{cbl} \frac{dv_{cbl}}{d\theta} + C_v \left[ t_{cbl} \frac{dw_{cbl}}{d\theta} + w_{cbl} \frac{dt_{cbl}}{d\theta} \right] \delta \theta
\]

Dividing by \( \delta \theta \) and rearranging:

\[
\frac{dt_{cbl}}{d\theta} = \frac{1}{w_{cbl}} \left\{ \frac{h_w}{C_v} t_{w} + \frac{h_r}{C_v} t_{R} - \frac{h_w h_r}{C_v} + \gamma_{s} \right\} t_{cbl}
\]

\[
+ \frac{\gamma_{s}}{C_v} m_{10ebo} + m_{11c1} + m_{s} t_{k} - p_{cbl} \frac{dv_{cbl}}{d\theta} - t_{cbl} \frac{dt_{cbl}}{d\theta} \right\}
\]

(A61)

For the wall and rotor we have:

\[
H_w(T_w - t_{cbl}) \delta \theta - h_w(t_w - t_{cbl}) \delta \theta = C_w \frac{dt_w}{d\theta} \delta \theta
\]

and

\[
h_r(t_R - t_{cbl}) \delta \theta = C_r \frac{dt_R}{d\theta} \delta \theta
\]

These yield:

\[
\frac{dt_w}{d\theta} = \frac{1}{C_w} \left[ H_w(T_w - t_{cbl}) - h_w(t_w - t_{cbl}) \right]
\]

(A62)

and

\[
\frac{dt_R}{d\theta} = \frac{1}{C_r} \left[ h_r(t_R - t_{cbl}) \right]
\]

(A63)
The remaining compression spaces and the expansion spaces will yield similar expressions.

\begin{align*}
\text{(ivb) Compression 1:} \\
\frac{dt_{c1}}{d\theta} &= \frac{1}{w_c} \left\{ \frac{h_w}{c_v} t_w + \frac{h_r}{c_v} t_r - \left( \frac{h_w + h_r}{c_v} \right) t_{c1} \\
&+ \gamma [m_{12} t_{c2} - (m_{11} + m_{18}) t_{c1}] - \frac{p_{c1}}{c_v} \frac{dv_{c1}}{d\theta} - t_{c1} \frac{dw_{c1}}{d\theta} \right\} \\
&= \frac{1}{c_w} \left[ v_w (T_C - t_w) - h_w (t_w - t_{c1}) \right] \tag{A64}
\end{align*}

\begin{align*}
\text{(ivc) Compression 2:} \\
\frac{dt_{c2}}{d\theta} &= \frac{1}{w_c} \left\{ \frac{h_w}{c_v} t_w + \frac{h_r}{c_v} t_r - \left( \frac{h_w + h_r}{c_v} \right) t_{c2} \\
&+ \gamma [m_{ij3} t_{cbo} - (m_{12} + m_{19}) t_{c2}] - \frac{p_{c2}}{c_v} \frac{dv_{c2}}{d\theta} - t_{c2} \frac{dw_{c2}}{d\theta} \right\} \tag{A67}
\end{align*}
\[
\frac{dt_w}{d\theta} = \frac{1}{C_w} [H_w(t_c - t_w) - h_w(t_w - t_{c2})]
\]  
(A68)

\[
\frac{dt_R}{d\theta} = \frac{1}{C_R} [h_R(t_R - t_{c2})]
\]  
(A69)

(ivd) **Compression Buffer Outlet:**

\[
\frac{dt_{cbo}}{d\theta} = \frac{1}{C_{v cbo}} \left\{ \frac{h_w}{C_v} t_w + \frac{h_R}{C_v} t_R - \left( \frac{h_w + h_R}{C_v} \right) t_{cbo} \\
+ \gamma [m_{i1 + t cbo} - (m_{i1} + m_{i2} + m_{i20}) t_{cbo}] \\
- \frac{P_{cbo}}{C_v} \frac{dv_{cbo}}{d\theta} - \frac{dw_{cbo}}{d\theta} \cdot t_{cbo} \right\}
\]  
(A70)

\[
\frac{dt_w}{d\theta} = \frac{1}{C_w} [H_w(t_c - t_w) - h_w(t_w - t_{cbo})]
\]  
(A71)

\[
\frac{dt_R}{d\theta} = \frac{1}{C_R} [h_R(t_R - t_{cbo})]
\]  
(A72)
Compression and Expansion Duct:

It is considered that these two sections of the engine would operate under essentially constant volume and adiabatic conditions. The heat capacity of the duct's wall will nevertheless be accounted for.

Now, since the gas temperature will be a function of both time and position along the duct, one obtains the following expressions from an energy balance:

\[
\frac{h_{CD}}{L_{CD}} (t_{\text{CD}} - t_{\text{cd}}) = C_P \left( m_{\text{cd}} \frac{\partial t_{\text{cd}}}{\partial x} + t_{\text{cd}} \frac{\partial m_{\text{cd}}}{\partial x} \right) + \frac{A}{J(\gamma-1)} \frac{\partial p_{\text{cd}}}{\partial \theta} \quad (A73)
\]

Here, the left-hand side represents the heat transferred to the gas from the wall, while the right-hand side represents the net enthalpy flux out of the control volume, and the increase in internal energy respectively. The term \( \frac{\partial p_{\text{cd}}}{\partial \theta} \) can be eliminated by performing a mass balance on an elemental volume of the duct. Thus:

\[
- \frac{\partial m}{\partial x} \delta x \delta \theta = \frac{\partial}{\partial \theta} \left( \frac{p_{\text{cd}} A \delta x}{R t_{\text{cd}}} \right) \delta \theta
\]

Differentiating and rearranging, one obtains:

\[
\frac{\partial p_{\text{cd}}}{\partial \theta} = \frac{p_{\text{cd}}}{t_{\text{cd}}} \frac{\partial t_{\text{cd}}}{\partial \theta} - \frac{R t_{\text{cd}}}{A_{\text{CD}}} \frac{\partial m_{\text{cd}}}{\partial x} \quad (A74)
\]
Substituting equation (A74) into equation (A73), one has:

\[
\frac{A_{CD} \rho_{cd}}{J(\gamma-1) t_{cd}} \frac{\partial t_{cd}}{\partial \theta} + C_{Pm_{cd}} \frac{\partial t_{cd}}{\partial x} = \frac{h_{CD}}{\rho_{cd} L_{CD}} (t_{cd} - t_{cd}) - \frac{R}{J} t_{cd} \frac{\partial m_{ed}}{\partial x} \tag{A75}
\]

A similar expression can be written for the expansion duct:

\[
\frac{A_{ED} \rho_{ed}}{J(\gamma-1) t_{ed}} \frac{\partial t_{ed}}{\partial \theta} + C_{Pm_{ed}} \frac{\partial t_{ed}}{\partial x} = \frac{h_{ED}}{\rho_{ed} L_{ED}} (t_{ed} - t_{ed}) - \frac{R}{J} t_{ed} \frac{\partial m_{ed}}{\partial x} \tag{A76}
\]

The terms \( \frac{\partial m_{cd}}{\partial x} \) and \( \frac{\partial m_{ed}}{\partial x} \) can be written as follows:

\[
\frac{\partial m_{cd}}{\partial x} = \frac{l}{L_{CD}} (m_2 - m_1) = \frac{1}{L_{CD}} \frac{dW_{cd}}{d\theta}
\]

and

\[
\frac{\partial m_{ed}}{\partial x} = \frac{l}{L_{ED}} (m_4 - m_5) = \frac{1}{L_{ED}} \frac{dW_{ed}}{d\theta}
\]

Substituting these two expressions into equations (A75) and (A76), one obtains:

\[
\frac{A_{CD} \rho_{cd}}{J(\gamma-1) t_{cd}} \frac{\partial t_{cd}}{\partial \theta} + C_{Pm_{cd}} \frac{\partial t_{cd}}{\partial x} = \frac{h_{CD}}{\rho_{cd} L_{CD}} (t_{cd} - t_{cd}) - \frac{R}{J} t_{cd} \frac{dW_{cd}}{d\theta} + \frac{R}{J L_{CD}} t_{cd} \frac{dW_{ed}}{d\theta} \tag{A77}
\]
In addition to these latter two equations, one has two more equations relating the heat content in the walls of the ducts to the heat transfer. It is considered that the wall temperatures vary with time only as longitudinal conduction is high. Therefore:

\[ h_{CD} (t_{CD} - \bar{t}_{cd}) = C_{CD} \frac{dt_{CD}}{d\theta} \]  \hspace{1cm} (A79)

and

\[ h_{ED} (t_{ED} - \bar{t}_{ed}) = C_{ED} \frac{dt_{ED}}{d\theta} \]  \hspace{1cm} (A80)

where \( \bar{t}_{cd} \) and \( \bar{t}_{ed} \) are as defined in equations (A16) and (A19). These ducting sections were included here for completeness, as it is apparent that any reasonable prototype would involve some such ducting.

(ivf) Compression and Expansion Regenerator:

A rotary regenerator would seem ideally suited to this problem and the equations to be presented are for such a machine. Several papers on the design theory and performance of rotary
regenerators are referenced (refs: 47, 48, 49, 50). The following equations for temperature distributions are obtained from reference 47.

Using the approximate relationships:

\[ m_{cr} = \frac{m_2 + m_3}{2} \quad \text{and} \quad m_{er} = \frac{m_5 + m_7}{2} \]

one obtains the following:

\[ m_{cr} \cdot C_p \frac{\partial t_{cr}}{\partial x} + \frac{A_R}{J(\gamma - 1)} t_{cr} \frac{\partial t_{cr}}{\partial \theta} = \frac{M_{CR}}{L_R} \frac{\partial t_R}{\partial \theta} \quad (A81) \]

\[ = \frac{h_{er} A_{ER}}{L_R} (t_{cr} - t_R) \quad (A82) \]

and

\[ m_{er} \cdot C_p \frac{\partial t_{er}}{\partial x} + \frac{A_R}{J(\gamma - 1)} t_{er} \frac{\partial t_{er}}{\partial \theta} = -\frac{M_{CR}}{L_R} \frac{\partial t_R}{\partial \theta} \quad (A83) \]

\[ = -\frac{h_{er} A_{ER}}{L_R} (t_{er} - t_R) \quad (A84) \]

with the following boundary conditions:

For interval of hot flow: \( t_{er(in)} = \text{constant at } x = 0 \);

For interval of cold flow: \( t_{cr(in)} = \text{constant at } x = L_R \).
(ivg) **Heater and Cooler:**

The heater and cooler are taken to be in communication with reservoirs at $T_H$ and $T_K$ respectively. The equations for these two sections are as follows:

$$\frac{dt_H}{d\theta} = \frac{1}{C_H} \left[ H_H (T_H - t_H) - h_H (t_H - \bar{T}_H) \right]$$  \hspace{1cm} (A85)

$$\frac{dt_K}{d\theta} = \frac{1}{C_K} \left[ H_K (T_K - t_K) - h_K (t_K - \bar{T}_K) \right]$$  \hspace{1cm} (A86)

$$\frac{A_H}{\gamma-1} \frac{1}{t_H} \frac{\partial x}{\partial \theta} + C_{pH} \frac{1}{x} \frac{\partial t_H}{\partial x} = \frac{R}{JL_H} \frac{d \omega_H}{d\theta} + \frac{h_H}{JH} (t_H - \bar{t}_H)$$  \hspace{1cm} (A87)

and

$$\frac{A_K}{\gamma-1} \frac{1}{t_K} \frac{\partial x}{\partial \theta} + C_{pK} \frac{1}{x} \frac{\partial t_K}{\partial x} = \frac{R}{JL_K} \frac{d \omega_K}{d\theta} + \frac{h_K}{JK} (t_K - \bar{t}_K)$$  \hspace{1cm} (A88)

The mass flows $m_h$ and $m_k$ can be approximated by

$$m_h = \frac{m_3 + m_4}{2} \quad \text{and} \quad m_k = \frac{m_7 + m_8}{2}$$

since the volumes in the heat exchanger are comparatively small.
(ivh) Expansion Buffer Inlet

The equations for this and the following three sections are similar to those of the compression sections and, therefore, will be presented without further development.

\[
\frac{dt_{ebi}}{d\theta} = \frac{1}{w_{ebi}} \left( \frac{h_w}{C_V} t_w + \frac{h_R}{C_V} t_R - \left[ \frac{h_w + h_R}{C_V} + \gamma (m_{14} + m_{15} + m_{21}) \right] t_{ebi} \right.
\]

\[
+ \gamma m_v t_h - \frac{P_{ebi}}{C_V} \frac{dv_{ebi}}{d\theta} - t_{ebi} \frac{dw_{ebi}}{d\theta} \right) \tag{A89}
\]

\[
\frac{dt_w}{d\theta} = \frac{1}{C_V} \left[ h_w(T_e - t_w) - h_w(t_w - t_{ebi}) \right] \tag{A90}
\]

\[
\frac{dt_R}{d\theta} = \frac{1}{C_R} \left[ h_R(t_R - t_{ebi}) \right] \tag{A91}
\]

(ivii) Expansion 1:

\[
\frac{dt_{el}}{d\theta} = \frac{1}{w_{el}} \left( \frac{h_w}{C_V} t_w + \frac{h_R}{C_V} t_R - \left[ \frac{h_w + h_R}{C_V} + \gamma (m_{14} + m_{21}) \right] t_{el} \right.
\]

\[
+ \gamma m_s t_{ebi} - \frac{P_{el}}{C_V} \frac{dv_{el}}{d\theta} - t_{el} \frac{dw_{el}}{d\theta} \right) \tag{A92} \]
\[
\frac{dt_w}{d\theta} = \frac{1}{C_w} \left[ H_w(T_E - t_w) - h_w(t_w - t_e) \right] \tag{A93}
\]

\[
\frac{dt_R}{d\theta} = \frac{1}{C_R} \left[ h_R(t_R - t_e) \right] \tag{A94}
\]

(ivj) Expansion 2:

\[
\frac{dt_{e2}}{d\theta} = \frac{1}{w_{e2}} \left\{ \frac{h_w}{C_V} t_w + \frac{h_R}{C_V} t_R - \left[ \frac{h_w + h_R}{C_V} + \gamma (m_{17} + m_{23}) \right] t_{e2}
\]
\[
+ \gamma m_{16} t_e - \frac{P_{e2}}{C_V} \frac{dv_{e2}}{d\theta} - t_{e2} \frac{dw_{e2}}{d\theta} \right\} \tag{A95}
\]

\[
\frac{dt_w}{d\theta} = \frac{1}{C_w} \left[ H_w(T_E - t_w) - h_w(t_w - t_{e2}) \right] \tag{A96}
\]

\[
\frac{dt_R}{d\theta} = \frac{1}{C_R} \left[ h_R(t_R - t_{e2}) \right] \tag{A97}
\]
(ivk) Expansion Buffer Outlet:

\[
\frac{d t_{ebo}}{d \theta} = \frac{1}{w_{ebo}} \left( \frac{h_w}{C_V} t_w + \frac{h_R}{C_V} t_R - \left[ \frac{h_w + h_R}{C_V} + \gamma (m_{10} + m_{28} + m_{5}) \right] t_{ebo} \right)
\]

\[+ \gamma m_{28} t_{c2} = \frac{P_{ebo} dV_{ebo}}{C_V} - t_{ebo} \frac{d w_{ebo}}{d \theta} \]  \hspace{1cm} (A98)

\[
\frac{d t_w}{d \theta} = \frac{1}{C_w} \left[ h_w (t_E - t_w) - h_w (t_w - t_{ebo}) \right] \]  \hspace{1cm} (A99)

\[
\frac{d t_R}{d \theta} = \frac{1}{C_R} \left[ h_R (t_R - t_{ebo}) \right] \]  \hspace{1cm} (A100)

(ivl) Buffer Section:

For this section, the following assumptions can be made:

1. The gas is well-mixed within this region and thus, a bulk temperature will be used.

2. Heat transfer from the wall and rotor to the gas will be neglected here as this would involve extremely complicated heat transfer relations depending on \( \psi \), \( \theta \) and radius and involving tortuous boundary conditions.
Performing an energy balance, one then has:

\[
\begin{bmatrix}
\text{Net enthalpy addition} \\
\text{increase in internal energy}
\end{bmatrix} = \gamma C_v (m t_{cbi} t_{cl} t_{c2} t_{cbo} t_{ebi} t_{el})
\]

\[
+ m_{23} t_{e2} t_{eb0} = C_v (t_b \frac{dw_b}{d\theta} + \omega_b \frac{dt_b}{d\theta})
\]

Rearranging, one obtains the final equation in this series:

\[
\frac{dt_b}{d\theta} = \frac{1}{\omega_b} \left[ \gamma (m t_{cbi} t_{cl} t_{c2} t_{cbo} t_{ebi} t_{el})
\right.
\]

\[
+ m_{23} t_{e2} t_{eb0} - t_b \frac{dw_b}{d\theta} \right]
\]

(v) **Machine Characteristics:**

All that remains now is to present the dynamical equation for the machine. This equates the driving torque to the sum of the accelerating torque, frictional resistance and load. These can be stated as follows:

\[
\tau = I \frac{d^2 \psi}{d\theta^2} + G \frac{d\psi}{d\theta} + E + F
\]

(A102)

and

\[
\tau = \sum \text{Expansion torque} - \sum \text{Compression torque}
\]
Typically, this would be:

\[
\tau = \sum (A_\psi \bar{r}_f - A_\psi \bar{r}_r) \rho \frac{d\psi}{d\theta} - \sum (A_\psi \bar{r}_f - A_\psi \bar{r}_r) \rho \frac{d\psi}{d\theta}
\]  \hspace{1cm} (A03)

Here \( \bar{r} \) represents an average moment arm given by:

\[
\bar{r}^2 - e \bar{r} \cos(\psi - \pi) + \frac{e^2 - (R_R + R_C - e)^2}{4} = 0
\]  \hspace{1cm} (A04)

and the variable vane surface area is given by:

\[
\left\{ e \cos(\psi - \pi) \pm \sqrt{e^2 \cos^2(\psi - \pi) - 1} + R_C^2 \right\} b = A_\psi
\]  \hspace{1cm} (A05)

(vi) **Summary**

Presented in the following three tables is a summary of all information required to obtain a solution for the present system (\( \lambda = 8 \)). This is not to say that one could now blindly proceed to solve this problem, for as indicated in Table A-2, there are a host of input parameters that require definition and, in some cases, optimization. To this end, an analysis of the volume variation is presented in Appendix B.

There exist, in addition to the relations presented in Table A-3, three other important relationships for the mass, pressure and temperature. These are as follows:
1. \[ \int \text{Mass contained in each section} = \text{constant.} \]
   \[ \text{Total # of sections} \]

2. This second point is best explained if one considers a specific section. Basically, it is a statement of continuity of mass content. If one chooses the compression buffer inlet, one can take this to be a moving control volume that changes its identity as it revolves (i.e., from cbi to c1 to c2 to...to ebo to cbi). Considering the expansion buffer outlet just immediately before it reaches \( \psi = \alpha \) (Figure A.1), it contains a certain amount of mass \( w_{ebo} \). An instant later, the forward face of this sector has crossed \( \psi = \alpha \) and now this sector is the compression buffer inlet. At this instant, one can say that:

\[ \begin{align*}
   w_{ebo} \quad \text{just before} & \quad w_{cbi} \quad \text{just after} \\
   \text{crossing } \alpha & \quad \text{crossing } \alpha
\end{align*} \]

Similar mass boundary conditions apply to all other rotating sectors.

3. Analogous to the case of mass continuity, there must exist a temperature continuity in the gas. Thus, as an example:

\[ \begin{align*}
   t_{ebo} \quad \text{just before} & \quad t_{cbi} \quad \text{just after} \\
   \text{crossing } \alpha & \quad \text{crossing } \alpha
\end{align*} \]

This too holds for all the other rotating sectors.

4. The pressure too can be considered continuous as outlined above.
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<thead>
<tr>
<th>SECTION</th>
<th>NUM. OF VARIABLES</th>
<th>PARAMETER</th>
<th>SYMBOLS</th>
<th>DIMEN.</th>
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TABLE A-1 (CONT'D)
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TABLE A-1 (CONT'D)
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<td>CHANGE OF MASS CONTENT</td>
<td>22</td>
<td>[ \frac{dw_{eb1}}{dt} = m_o + m_{1e} + m_{e1} - m_0 ]</td>
<td>[ b_y = b_y(t) ]</td>
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<td>MASS FLOW</td>
<td>44</td>
<td>[ m = -T_s(-p_h - p_{eb1}) ]</td>
<td>[ b_y = b_y(t) ]</td>
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<td>GAS TEMPERATURE</td>
<td>46</td>
<td>[ m_{1e} = \frac{p_{eb1}}{R} ]</td>
<td>[ b_y = b_y(t) ]</td>
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<td>WALL TEMPERATURE</td>
<td>47</td>
<td>[ m_{e1} = \frac{p_{eb1}}{R} ]</td>
<td>[ b_y = b_y(t) ]</td>
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<td></td>
<td>ROTOR TEMPERATURE</td>
<td>61</td>
<td>[ \frac{dT_{eb1}}{dt} = \frac{1}{v_{eb1}} \left[ \frac{\partial^2 \phi}{\partial x^2} \left( \frac{h_x}{c_p} + k_x - h_{xy} \frac{\partial \phi}{\partial y} \right) \right] + \frac{T_o \cdot m_{e1} \cdot \left( n_s \cdot \eta_{e1} - n_{e1} \cdot \eta_{eb1} \right)}{c_p} - \frac{\partial \tau_{eb1}}{\partial \phi} ]</td>
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<td></td>
<td>PRESSURE</td>
<td>2</td>
<td>[ P_{c1} = \frac{v_{c1} \cdot R \cdot T_{c1}}{v_{c1}} ]</td>
<td>[ v_{c1} = v_{c1}(t), h_y = b_y(t) ]</td>
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<td>CHANGE OF MASS CONTENT</td>
<td>23</td>
<td>[ \frac{dw_{c1}}{dt} = m_{1e} - m_{e1} - m_{e1} ]</td>
<td>[ b_y = b_y(t) ]</td>
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<td>MASS FLOW</td>
<td>48</td>
<td>[ m_{1e} = \frac{p_{c1}}{R} ]</td>
<td>[ b_y = b_y(t) ]</td>
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<td>GAS TEMPERATURE</td>
<td>49</td>
<td>[ m_{e1} = \frac{p_{c1}}{R} ]</td>
<td>[ b_y = b_y(t) ]</td>
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<td>WALL TEMPERATURE</td>
<td>50</td>
<td>[ \frac{dT_{c1}}{dt} = \frac{1}{c_p} \left[ h_x \left( c_x \cdot T_x - h_{xy} \cdot T_{c1} \right) \right] + \frac{T_o \cdot m_{c1} \cdot \left( n_s \cdot \eta_{c1} - n_{c1} \cdot \eta_{c1} \right)}{c_p} - \frac{\partial \tau_{c1}}{\partial \phi} ]</td>
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<td>ROTOR TEMPERATURE</td>
<td>63</td>
<td>[ \frac{dT_{c1}}{dt} = \frac{1}{c_p} \left[ h_x \left( c_x \cdot T_x - h_{xy} \cdot T_{c1} \right) \right] ]</td>
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<td>COMPRESSOR</td>
<td>PRESSURE</td>
<td>3</td>
<td>( \frac{du_{c}}{dt} = \frac{u_{c2} \cdot u_{c2}}{u_{c2}} )</td>
<td>( v_{c3} = v_{c2}(t) ), ( h_{c} = h_{c}(t) )</td>
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<td>COMPRESSOR</td>
<td>CHANGE OF MASS CON- TENT</td>
<td>24</td>
<td>( \frac{dw_{c}}{dt} = w_{c2} - w_{c1} = -w_{c1} )</td>
<td>( h_{c} = h_{c}(t) )</td>
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<td>COMPRESSOR</td>
<td>MASS FLOW IN</td>
<td>49</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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<td>COMPRESSOR</td>
<td>GAS TEMPERATURE</td>
<td>67</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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<td>COMPRESSOR</td>
<td>WALL TEMPERATURE</td>
<td>68</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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<tr>
<td>COMPRESSOR</td>
<td>ROTOR TEMPERATURE</td>
<td>69</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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<td>COMPRESSOR</td>
<td>PRESSURE</td>
<td>5</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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<td>COMPRESSOR</td>
<td>CHANGE OF MASS CON- TENT</td>
<td>25</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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<tr>
<td>COMPRESSOR</td>
<td>MASS FLOW IN</td>
<td>50</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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<td>COMPRESSOR BUFFER OUTLET</td>
<td>GAS TEMPERATURE</td>
<td>70</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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<td>COMPRESSOR BUFFER OUTLET</td>
<td>WALL TEMPERATURE</td>
<td>71</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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<td>COMPRESSOR BUFFER OUTLET</td>
<td>ROTOR TEMPERATURE</td>
<td>72</td>
<td>( \frac{dsw_{c2}}{dt} = \frac{1}{\gamma_{c}} \left[ \frac{\gamma_{c}}{\gamma_{s}} \cdot \frac{\gamma_{s}}{\gamma_{c}} \right] - \frac{(h_{w} \cdot h_{c})}{\gamma_{c}} \cdot \frac{dw_{c}}{dt} + \frac{dw_{c}}{dt} )</td>
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**TABLE A-5 (cont'd)**
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<td>COMPRESSION</td>
<td>PRESSURE</td>
<td>$p_{cd} = \frac{u_{cd} R T_{cd}}{v_{cd}}$</td>
<td>$E_{cd} = \frac{1}{\frac{W}{v_{cd}}} \int E_{cd} dx$</td>
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<td>DUCT</td>
<td>CHANGE OF MASS</td>
<td>$\frac{d u_{cd}}{d x} = u_{1} - u_{2}$</td>
<td>$E_{cd} = E_{2cd}$ at $x=0$ and $u_{1} &gt; 0$</td>
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<td>CONTENT</td>
<td>$u_{1} = E_{1}(T_{ce} - p_{cd})$</td>
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<td></td>
<td>MASS FLOW IN</td>
<td>$f_{ce} = \int_{x_{2}}^{x_{1}} \frac{d u_{cd}}{d x} dx$</td>
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<td></td>
<td>GAS TEMPERATURE</td>
<td>$h_{ce}(T_{ce} - E_{cd}) = C_{cd} \frac{d E_{cd}}{d x}$</td>
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<tr>
<td>WALL TEMPERATURE</td>
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</tr>
<tr>
<td></td>
<td>PRESSURE</td>
<td>$p_{ex} = \frac{u_{ex} R T_{ex}}{v_{ex}}$</td>
<td>$E_{ex} = \frac{1}{\frac{W}{v_{ex}}} \int E_{ex} dx$</td>
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<tr>
<td>REGENERATOR</td>
<td>CHANGE OF MASS</td>
<td>$\frac{d u_{ex}}{d x} = u_{1} - u_{2}$</td>
<td>$E_{ex}(x_{2})$ - constant at $x_{ex} = L_x$</td>
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<tr>
<td>CONTENT</td>
<td>MASS FLOW IN</td>
<td>$u_{1} = E_{1}(p_{cd} - p_{ex})$</td>
<td>$E_{cd} = E_{ex}$ at $x_{ex} = L_x$</td>
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<td>GAS TEMPERATURE</td>
<td>$h_{ex} C_{p} = \int_{x_{2}}^{x_{1}} \frac{A_{ex} P_{ex} \frac{d x}{d x}}{x_{ex}} dx$</td>
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<td>MEASUREMENT</td>
<td>PRESSURE</td>
<td>$P_{h} = \frac{u_{h} E_{h}}{R}$</td>
<td>$E_{h} = \frac{1}{\frac{W}{v_{h}}} \int E_{h} dx$</td>
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<td>TEMPERATURE</td>
<td>CHANGE OF MASS</td>
<td>$\frac{d u_{h}}{d x} = u_{1} - u_{2}$</td>
<td>$E_{h}$ at $x_{2} = 0 = E_{ex}$ at $x_{ex} = 0$</td>
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<td>HEATER</td>
<td>MASS FLOW IN</td>
<td>$u_{1} = E_{1}(p_{ex} - p_{h})$</td>
<td>$E_{h} = \frac{W_{h}}{V_{h}}$</td>
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<td>WALL TEMPERATURE</td>
<td>$\frac{d t_{ex}}{d x} = \frac{1}{L_{h}} \left[ h_{h}(T_{h} - t_{ex}) - h_{h}(t_{ex} - t_{ex}) \right]$</td>
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<tr>
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<td>GAS TEMPERATURE</td>
<td>$h_{ex} C_{p} = \int_{x_{2}}^{x_{1}} \frac{A_{ex} P_{ex} \frac{d x}{d x}}{x_{ex}} dx$</td>
<td>$E_{ex} = \frac{1}{\frac{W}{v_{ex}}} \int E_{ex} dx$</td>
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**TABLE A-3 (cont'd)**
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<th>AUXILIARY EQUATIONS AND BOUNDARY CONDITIONS</th>
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<tbody>
<tr>
<td>PRESSURE</td>
<td>$p_{ab1} = \frac{u_{ab1} R T_{ab1}}{\varepsilon_{ab1}}$</td>
<td></td>
<td>$w_{ab1}{v}<em>{ab1} = \delta</em>{ab1} v_{ab1}$</td>
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<tr>
<td>CHANGE OF MASS CONTENT</td>
<td>$\frac{dw_{al}}{\delta} = m_a - m_{as} - m_{al} - m_{al}$</td>
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<tr>
<td>MASS FLOW IN</td>
<td>$n_e = \varepsilon_e (p_{al} - p_{as})$</td>
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<tr>
<td>GAS TEMPERATURE</td>
<td>$\frac{dT_{ab1}}{\delta} = \frac{1}{\varepsilon_{ab1}} \left( \frac{\delta}{\delta} T_{ab1} + \frac{h_a}{\varepsilon_{ab1}} T_{ab1} + \left( \frac{h_a}{\varepsilon_{ab1}} \right) T_{ab1} - \left( \frac{h_a}{\varepsilon_{ab1}} \right) T_{ab1} \right)$</td>
<td></td>
<td>$T_{p_{ab1}} = \frac{\delta}{\delta} + m_a T_{ab1}$</td>
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<tr>
<td>WALL TEMPERATURE</td>
<td>$\frac{dT_{al1}}{\delta} = \frac{1}{\varepsilon_{al1}} \left( h_a (c_s - c_{al1}) - h_a (c_s - c_{al1}) \right)$</td>
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<td>$T_{p_{al1}} = \frac{\delta}{\delta} + m_a T_{al1}$</td>
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<tr>
<td>ROTOR TEMPERATURE</td>
<td>$\frac{dT_{al1}}{\delta} = \frac{1}{\varepsilon_{al1}} \left( h_a (c_s - c_{al1}) - h_a (c_s - c_{al1}) \right)$</td>
<td></td>
<td>$T_{p_{al1}} = \frac{\delta}{\delta} + m_a T_{al1}$</td>
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<tr>
<td>PRESSURE</td>
<td>$p_{al} = \frac{u_{al} R T_{al}}{\varepsilon_{al}}$</td>
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<td>$w_{al}{v}<em>{al} = \delta</em>{al} v_{al}$</td>
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<tr>
<td>CHANGE OF MASS CONTENT</td>
<td>$\frac{dw_{al}}{\delta} = m_a - m_{as} - m_{al} - m_{al}$</td>
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<tr>
<td>MASS FLOW IN</td>
<td>$n_{al} = \varepsilon_{al} (p_{al} - p_{as})$</td>
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<td>GAS TEMPERATURE</td>
<td>$\frac{dT_{al1}}{\delta} = \frac{1}{\varepsilon_{al1}} \left( \frac{\delta}{\delta} T_{al1} + \frac{h_a}{\varepsilon_{al1}} T_{al1} + \left( \frac{h_a}{\varepsilon_{al1}} \right) T_{al1} - \left( \frac{h_a}{\varepsilon_{al1}} \right) T_{al1} \right)$</td>
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<td>$T_{p_{al1}} = \frac{\delta}{\delta} + m_a T_{al1}$</td>
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<td>WALL TEMPERATURE</td>
<td>$\frac{dT_{al1}}{\delta} = \frac{1}{\varepsilon_{al1}} \left( h_a (c_s - c_{al1}) - h_a (c_s - c_{al1}) \right)$</td>
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<td>$T_{p_{al1}} = \frac{\delta}{\delta} + m_a T_{al1}$</td>
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<td>$\frac{dT_{al1}}{\delta} = \frac{1}{\varepsilon_{al1}} \left( h_a (c_s - c_{al1}) - h_a (c_s - c_{al1}) \right)$</td>
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<td>$T_{p_{al1}} = \frac{\delta}{\delta} + m_a T_{al1}$</td>
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**TABLE A-3 (cont'd)**
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<th>AUXILIARY EQUATIONS AND BOUNDARY CONDITIONS</th>
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<td><strong>EXPANSION 1</strong></td>
<td><strong>PRESSURE</strong></td>
<td>10</td>
<td>( p_{a_2} = \frac{w_{21} R T_{a_2}}{V_{a_2}} )</td>
<td>( v_{a_2} = v_{a_2}(\theta), \ h_{a_2} = h_{a_2}(\theta), \ b_{a_2} = b_{a_2}(\theta) )</td>
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<td><strong>CHANGE OF MASS CONTENT</strong></td>
<td>31</td>
<td>( \frac{dw_{2}}{dt} = \theta_{1} - \theta_{2} - \theta_{3} )</td>
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<td><strong>MASS FLOW IN</strong></td>
<td>52</td>
<td>( \theta_{1} = \theta_{12} (p_{a_2} - p_{a_2}) )</td>
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<td><strong>GAS TEMPERATURE</strong></td>
<td>95</td>
<td>( \frac{dV_{a_2}}{dt} = \frac{1}{h_{a_2}} \left[ \frac{\rho_{a_2}}{\gamma_{a_2}} + \frac{h_{a_2}}{\gamma_{a_2}} + \frac{b_{a_2}}{\gamma_{a_2}} \right] )</td>
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<tr>
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<td><strong>WALL TEMPERATURE</strong></td>
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<td>( \frac{dV_{a_2}}{dt} = \frac{1}{h_{a_2}} \left[ \frac{\rho_{a_2}}{\gamma_{a_2}} \right] )</td>
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<td>( \frac{dV_{a_2}}{dt} = \frac{1}{h_{a_2}} \left[ \frac{\rho_{a_2}}{\gamma_{a_2}} \right] )</td>
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<td><strong>EXPANSION BUFFER OUTLET</strong></td>
<td><strong>PRESSURE</strong></td>
<td>11</td>
<td>( p_{a_2} = \frac{w_{21} R T_{a_2}}{V_{a_2}} )</td>
<td>( v_{a_2} = v_{a_2}(\theta), \ h_{a_2} = h_{a_2}(\theta), \ b_{a_2} = b_{a_2}(\theta) )</td>
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<td>( \frac{dw_{2}}{dt} = \theta_{1} - \theta_{2} - \theta_{3} - \theta_{4} )</td>
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<td><strong>MASS FLOW IN</strong></td>
<td>53</td>
<td>( \theta_{1} = \theta_{12} (p_{a_2} - p_{a_2}) )</td>
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<tr>
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<td><strong>GAS TEMPERATURE</strong></td>
<td>98</td>
<td>( \frac{dV_{a_2}}{dt} = \frac{1}{h_{a_2}} \left[ \frac{\rho_{a_2}}{\gamma_{a_2}} + \frac{h_{a_2}}{\gamma_{a_2}} + \frac{b_{a_2}}{\gamma_{a_2}} \right] )</td>
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<td>( \frac{dV_{a_2}}{dt} = \frac{1}{h_{a_2}} \left[ \frac{\rho_{a_2}}{\gamma_{a_2}} \right] )</td>
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<td>( \frac{dV_{a_2}}{dt} = \frac{1}{h_{a_2}} \left[ \frac{\rho_{a_2}}{\gamma_{a_2}} \right] )</td>
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**TABLE A-3 (cont’d)**
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<td>CHANGE OF MASS CONTENT</td>
<td>33</td>
<td>$\frac{dw_{ed}}{dx} = n_{i} - n_{a}$</td>
<td></td>
<td>$t_{ed} = \text{constant at } x = 0$ when $n_{a} &gt; 0$.</td>
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<td>MASS FLOW IN</td>
<td>41</td>
<td>$n_{i} = K_{a} (p_{ed} - p_{ed})$</td>
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<td>GAS TEMPERATURE</td>
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<td>$\frac{dV_{ed}}{dx} = \frac{R}{T_{ed}} - \frac{dw_{ed}}{dx}$</td>
<td>$t_{ed} = \text{constant at } x = 0$ when $n_{a} &gt; 0$.</td>
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<td>$\frac{dV_{ed}}{dx} = \frac{h_{ed}}{T_{ed}} (T_{ed} - T_{ed})$</td>
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<td>$p_{ed} = \frac{w_{ed} R T_{ed}}{V_{ed}}$</td>
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<td>$t_{ed} = \frac{1}{T_{ed}} \int t_{ed} , dt$</td>
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<td>$\frac{dw_{ed}}{dx} = n_{i} - n_{a}$</td>
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</tr>
<tr>
<td>MASS FLOW IN</td>
<td>42</td>
<td>$n_{i} = K_{a} (p_{ed} - p_{ed})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>REGENERATOR TEMPERATURE</td>
<td>63</td>
<td>$\frac{dV_{ed}}{dx} = \frac{R}{T_{ed}} - \frac{dw_{ed}}{dx}$</td>
<td>$t_{ed} = \text{constant at } x = 0$ when $n_{a} &gt; 0$.</td>
<td></td>
</tr>
<tr>
<td>GAS TEMPERATURE</td>
<td>83</td>
<td>$\frac{dV_{ed}}{dx} = \frac{R}{T_{ed}} - \frac{dw_{ed}}{dx}$</td>
<td>$t_{ed} = \text{constant at } x = 0$ when $n_{a} &gt; 0$.</td>
<td></td>
</tr>
<tr>
<td>PRESSURE</td>
<td>14</td>
<td>$p_{ed} = \frac{w_{ed} R T_{ed}}{V_{ed}}$</td>
<td></td>
<td>$t_{ed} = \frac{1}{T_{ed}} \int t_{ed} , dt$</td>
</tr>
<tr>
<td>CHANGE OF MASS CONTENT</td>
<td>35</td>
<td>$\frac{dw_{ed}}{dx} = n_{i} - n_{a}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MASS FLOW IN</td>
<td>43</td>
<td>$n_{i} = K_{a} (p_{ed} - p_{ed})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WALL TEMPERATURE</td>
<td>86</td>
<td>$\frac{dV_{ed}}{dx} = \frac{R}{T_{ed}} - \frac{dw_{ed}}{dx}$</td>
<td>$t_{ed} = \text{constant at } x = 0$ when $n_{a} &gt; 0$.</td>
<td></td>
</tr>
<tr>
<td>GAS TEMPERATURE</td>
<td>88</td>
<td>$\frac{dV_{ed}}{dx} = \frac{R}{T_{ed}} - \frac{dw_{ed}}{dx}$</td>
<td>$t_{ed} = \text{constant at } x = 0$ when $n_{a} &gt; 0$.</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE A.3 (cont'd)**
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>APPENDIX EQU. NO.</th>
<th>EQUATION</th>
<th>AUXILIARY EQUATIONS AND BOUNDARY CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE</td>
<td>15</td>
<td>( p_b = \frac{u_b \cdot R \cdot T_b}{b} )</td>
<td></td>
</tr>
<tr>
<td>CHARGE OF MASS CONTENT</td>
<td>36</td>
<td>( \frac{dW_a}{dt} = m_a + m_{a1} + m_{a2} + m_{a3} + m_{a4} + m_{a5} + m_{a6} )</td>
<td></td>
</tr>
<tr>
<td>MASS FLOW IN</td>
<td>45, 54</td>
<td>( n_a = E_a \cdot (p_{a3} - p_b) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>( n_{a5} = E_{a5} \cdot (p_{a3} - p_b) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>56</td>
<td>( n_{a6} = E_{a6} \cdot (p_{a3} - p_b) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>57</td>
<td>( n_{a1} = E_{a1} \cdot (p_{a1} - p_b) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>58</td>
<td>( n_{a2} = E_{a2} \cdot (p_{a2} - p_b) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>59</td>
<td>( n_{a3} = E_{a3} \cdot (p_{a3} - p_b) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>( n_{a4} = E_{a4} \cdot (p_{a4} - p_b) )</td>
<td></td>
</tr>
<tr>
<td>GAS TEMPERATURE</td>
<td>101</td>
<td>( \frac{dV_b}{dt} = \frac{1}{b_b} \left[ \tau \left( \frac{m_a \cdot c_{a1} \cdot m_{a1} \cdot c_{a1}^m \cdot m_{a2} \cdot c_{a2} \cdot m_{a3} \cdot c_{a3} \cdot m_{a4} \cdot c_{a4} \cdot m_{a5} \cdot c_{a5} \cdot m_{a6} \cdot c_{a6} \right) \cdot T_b \right] )</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE A-3 (cont'd)**
To obtain the volume variation as a function of the crank angle, one can use the model illustrated in Figure B1. One proceeds as follows:

\[ \text{Area (ABEF)} = \text{Area (ADF)} - \text{Area (BCE)} - \text{Area (DCF)} \]

Letting \( A_\psi \) = Area (ABEF) as a function of \( \psi \) gives:

\[ A_\psi = \frac{\psi \pi R_c^2}{360} - \frac{\psi \pi R_R^2}{360} - \frac{1}{2} R_c \cos \phi \]

**APPENDIX B**

**VOLUME VARIATION**

**FIG. B1** MODEL USED FOR DETERMINING VOLUME VARIATIONS
From \( \Delta DCF \), we get a relation between \( \phi \) and \( \psi \) as follows:

\[
\sin(\psi - \phi) = \frac{e}{R_C} \sin \psi
\]

Therefore:

\[
\phi = \psi - \sin^{-1}\left[\frac{e}{R_C} \sin \psi\right]
\]

Therefore:

\[
A_\psi = \left\{\psi - \sin^{-1}\left[\frac{e}{R_C} \sin \psi\right]\right\} \frac{\pi R_C^2}{360} - \frac{\psi \pi R^2}{360} - \frac{1}{2} R_C \sin\{\psi - \sin^{-1}\left[\frac{e}{R_C} \sin \psi\right]\}
\]

Considering a system with \( \lambda \) blades yields \( 360/\lambda \) degrees between 2 adjacent blades. Thus, when one blade is at an angle \( \psi \), the next blade will be at an angle \( (\psi - 360/\lambda) \). Thus, the area between 2 adjacent blades is given as:

\[
A_\psi = A_{\psi} - A_{(\psi-360/\lambda)}
\]

This gives as a volume for each chamber:

\[
v_\psi = a_\psi^b
\]
Appendix C

COMPONENT DRAWINGS
Note: 4 holes "3/4" bottom square

Note: 2 holes "3/32" drill
Fig. C.8

2 holes #37 drill ±0.000
0.155 ±0.002

0.250 ±0.003

2.170 ±0.003

0.500

0.125

SLOT SEAL BACKUP PLATE

MATERIAL: BRASS

QTY: 16

DATE 24/05/72

McGILL UNIVERSITY — MONTREAL

DRAWN L.W. Rosseger

CHECKED

REVISIONS

SCALE 2:1

DRAWING NO.
Fig. C.10

- ROTOR SIDE SEAL  
  QTY: 16  
- MATERIAL: BRONZE

DRAWN
L.W. Rosenegger

CHECKED

McGILL UNIVERSITY — MONTREAL

DATE 24/05/72

SCALE 2:1

DRAWING NO: