HARMONIC EFFECTS IN ROTATING ELECTRICAL MACHINES

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EFFETS HARMONIQUES DANS LES MACHINES TOURNANTES ELECTRIQUES.

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RESUME

Cette thèse contient une étude expérimentale et analytique des effets de la réaction d'armature, tenant compte des différents harmoniques. Grâce à un développement en série des fonctions trigonometriques rencontrées, on a pu étudier l'effet de chaque harmonique d'espace sur le courant et la tension en régime permanent. Ce travail a été possible par la mécanisation des opérations algébriques permises par l'utilisation du système FORMAC d'IBM. Les résultats obtenus sont discutés et confirment les conclusions théoriques et expérimentales déjà connues. Cependant, la formulation mathématique présentée dans cette étude, et basée sur la matrice d'impédance harmonique, permet de terminer les calculs cent ou mille fois plus rapidement que les méthodes itératives utilisées jusqu'à présent. Une attention particulière a été apportée à l'étude des harmoniques de la FMM créés dans une machine polyphasée, connectée à un réseau équilibré, déséquilibré, ou monophasé. Une étude expérimentale, ainsi qu'une représentation graphique des résultats théoriques obtenus dans le cas où la machine polyphasée est connectée en monophasé, prouvent que les effets des harmoniques de la réaction d'armature ne sont pas négligeable dans les machines polyphasées déséquilibrées.
HARMONIC EFFECTS IN ROTATING ELECTRICAL MACHINES

by

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ABSTRACT

In this thesis the phenomena of multiple armature reaction effects is studied analytically and experimentally. The study of the effect of space harmonics on the steady state current and voltage waveforms is investigated analytically by the method of direct algebraic expansion of trigonometric series. The solution depends for its success on the mechanization of this process by the use of the IBM FORMAC system. The results obtained are discussed and compared with the existing theoretical and experimental data and found in good general agreement. Using the harmonic impedance matrix formulation introduced in this work, a reduction of computing time between 2 to 3 orders of magnitudes is achieved over earlier numerical iterative techniques. Special attention is given to the mmf harmonics produced in the windings of balanced polyphase machines and unbalanced or single phase connections. Experimental study and predicted waveforms for single phase connections show that multiple armature reaction effects are extremely severe in unbalanced machines.
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CHAPTER 1

INTRODUCTION

It is now more than forty years since Kron unveiled his generalized electric machine theory and while interest in it was only slowly aroused, it is now firmly established as an analytic tool. The original theoretical treatment, although fully explored by Kron was left by him in a somewhat indigestible state. By the end of the 1950's and beginning of the 1960's Gibbs [1], Lynn [2], White and Woodson [3] and many others have so clarified the basic theoretical issues that attention was fruitfully turned to the closer correlation of the structure of Kron's primitives with the nonideal structure of practical machines. Hamdi - Sepen [4] extended the method of two-axis considerations by ascribing direct axis and quadrature axis saturation factors as well as direct axis and quadrature axis saturation coupling factors. The theoretical premises of the 2-axis theory have been shown by Carter et al [5] and Dunfield and Barton [6] to be invalid (or questionable) to varying degrees in actual machines. This is due to armature mmf harmonics in addition to the fundamental, and airgap permeance harmonics higher than the second, these showing up in the form of inductance coefficients that do not conform to the Park-Kron definition.

Recently, interest in the steady state behaviour in terms of machine analysis has been on the increase. Various transformations are available to simplify the analysis, Dunfield and Barton [7, 8] introduced a three-phase to two phase slip-ring transformation. Willems [9] shows that, in some cases where space harmonics are taken into consideration, a linear transformation can be set up to transform the non-stationary
equations describing an electrical machine to stationary equations. The paper shows that an interesting relationship exists between unified machine theory and linear system theory. Chalmers [10] puts many of the problems of the analysis of non-ideal electric machines in practical perspective. He also shows how a.c. machine windings may be arranged to reduce harmonic content [11].

Most of the study of harmonic effects in a.c. machines have been concerned with balanced polyphase conditions and have not included unbalanced or single-phase operation. Buchanan [12] specifically treated an equivalent circuit for a single phase motor having space harmonics in its magnetic field. Davis and Novotny [13] have developed the equivalent circuits for single-phase squirrel-cage induction machine considering the effects of both odd and even mmf harmonics. More recently, they studied both analytically and experimentally the even order mmf harmonics in squirrel cage induction motors under transient state conditions [14].

In this thesis, a simple formalism for steady state harmonic analysis in electric machines is presented. The analytical strategy is to apply a transformation which reduces the differential equations describing the machine into a linear matrix equation which is readily solved. This formalism has long been known, but is seldom used because of the great complexity of the algebraic and trigonometric expressions encountered. FORMAC, standing for "Formula Manipulation Compiler" is demonstrated to be an efficient and powerful computer aid for solving such problems.

In Chapter II, the effect of multiple armature reaction is explained, the analytical approach is discussed and FORMAC is introduced. Chapter III is con-
cerned with developing a computer program for a general machine. In Chapters IV and V, two typical examples are worked out, namely a 3-phase synchronous machine and a 2-phase induction machine. Chapter VI carries the experimental and analytical results for a single phase synchronous machine and a single phase/single phase machine.

In summary, this thesis as a contribution to the theory of electrical machines, introduces a new computer aid which, to the author's knowledge, has not been used so far in solving such problems.
2.1 Introduction

Real electrical machines differ from ideal primitive machines as follows:

1. The primitive machines have idealized air gap and winding geometries which is to say that airgap permeance harmonics higher than the second and all harmonics of winding mmf’s are considered negligible. Robinson [15], proves that the airgap flux density for an a. c. machine cannot be expressed as a simple rotating wave function, sinusoidal in space and time, but contains in general a combination of such rotating waves, harmonically related in space or in time. The method of approach is to regard any airgap flux-density distribution as the superposition of components having this simple form.

2. Some of the effects of departures from the ideal machine are related to magnetic saturation. The technique of using adjustable parameters in place of the constants in ideal machine equations can be handled with reasonable accuracy, e.g. a saturated reactance may give reasonable results for the behaviour of a machine, in terms of average and fundamental frequency phenomena, but it intentionally ignores the harmonic phenomena.
taking place within the machine and observable in its external
currents and voltages.

Jones [16], Carter [5], and Dunfield and Barton [6] have shown in both
analytical and experimental studies that real electrical machines have windings that
produce significant space harmonics of \( \text{mmf} \) and also have non-simple air gap geometry
which gives rise to air gap permeance harmonics higher than the second.

2.2 Multiple Armature Reaction Effects

When current flows in a normal machine winding, in general both positive
and negative rotating fields having pole numbers that are odd multiples of the funda-
mental number of poles are created. These cause multiple frequency currents to flow
in the secondary which in turn create multiple rotating fields reacting on the primary.
This phenomena has been called multiple armature reaction [17] a term which, while
perhaps not ideal, will be employed here.

2.2.1 The Basic Armature Reaction Cycle

If we consider a voltage of frequency \( f_p \) applied to the primary winding,
this will create a current of the same frequency, also it will create fields rotating at
a frequency of \( \pm f_p / m \) relative to the primary, where \( m \) is an odd integer. These
fields rotate at frequencies of \( \pm f_p / m - f_r \) relative to the secondary, \( f_r \) being
the frequency of rotation of the machine. The term frequency of rotation, synonymous
with speed of rotation in electrical revolutions per second, is used for convenience.

The primary fields induce voltages in the secondary windings at frequencies
\[(\pm \frac{f}{p} - \frac{m}{f_r})m = (\pm f - mf_r).\] Negative frequency components arising from this
expression need not cause concern, they merely signify a particular phase sequence.

The resulting secondary currents will produce fields which rotate at frequencies
\[\pm (\pm \frac{f}{p} - \frac{m}{f_r})/n,\] where \(n\) is an odd integer independent of \(m\). In their turn,
these fields rotate relative to the primary at \(\pm (\pm \frac{f}{p} - \frac{m}{f_r})/n + f_r\), inducing voltages
in the primary windings at frequencies \(\pm (\pm \frac{f}{p} - \frac{m}{f_r}) + nf_r = f \pm (m \pm n)f_r\), and
the cycle is complete, each voltage component producing its corresponding current as
before.

Thus associated with the primary excitation frequency \(f_p\), is a range of
primary frequencies \(f_p \pm lf_r\) where \(l = m \pm n\) is an even integer, and a range of
secondary frequencies \(f_p \pm mf_r\) where \(m\) is an odd integer. In the following, we will
use the term harmonic series to define any such series of sinusoidally varying components
no matter whether the frequencies are integral multiples of a fundamental or not.

2.3 General Equations of a Machine

It is well known that, when a balanced polyphase winding is excited from
a polyphase voltage source of sinusoidal waveform the general equation of the machine
can be written in a matrix form as:
\[ V = (R + DL) I \quad \text{(2-1)} \]

where \( D \) is the differential operator \( \frac{d}{dt} \).

Since we are here concerned with steady state response we may assume currents in the primary and secondary sides of the form:

\[ I_p = \sum_l l_{pl} \cos [(\omega_p + l \omega_r) t - \alpha_l], \quad \text{(2-2a)} \]

and

\[ I_s = \sum_m l_{sm} \cos [(\omega_p + m \omega_r) t - \alpha_m], \quad \text{(2-2b)} \]

where

\[ f_p \] is the supply frequency,

\[ f_r \] is the rotation frequency,

\[ l \] is any even integer positive or negative,

and \( m \) is any odd integer positive or negative.

Equation (2-1) can be written as:

\[
\begin{pmatrix}
V_p \\
V_s
\end{pmatrix} = 
\begin{pmatrix}
Z_{pp} & Z_{sp} \\
Z_{ps} & Z_{ss}
\end{pmatrix}
\begin{pmatrix}
I_p \\
I_s
\end{pmatrix}
\quad \text{(2-3)}
\]

The primary and secondary self inductances as well as the mutual inductance between two primary windings or two secondary windings of a real electrical machine can be expressed as an even harmonic series of the form...
\[ L = \sum_l l_L \cos \left( \frac{L \pi}{2} \right) \cos \left( L \omega_r t - \delta_L \right), \]

while the mutual inductances between primary and secondary windings can be expressed as odd harmonic series

\[ M = \sum_m \sum_l M_{lm} \cos \left( \frac{m \pi}{2} \right) \cos \left( m \omega_r t - \delta_{lm} \right), \]

where \( \delta_L \) and \( \delta_{lm} \) are angles depending on the relative geometrical configuration of the windings. In general, the magnitude factors \( L_L \) and \( M_{lm} \) depend on the permeance and winding constants of the machine.

Equation (2-3) can now be rewritten

\[
\begin{pmatrix}
V_p \\
V_s
\end{pmatrix}
= 
\begin{pmatrix}
Z^e_{pp} & Z^o_{sp} \\
Z^o_{ps} & Z^e_{ss}
\end{pmatrix}
\begin{pmatrix}
l^e_p \\
l^o_s
\end{pmatrix}
\]

(2-4)

where the superscripts \( e \) and \( o \) stand for even and odd harmonic series in \( \omega_r \) respectively. This matrix equation will lead to \( V_p \) being an even harmonic series, and \( V_s \) an odd harmonic series in \( \omega_r \).

2.3.1 Steady State Solution of the Machine Equations

Dunfield and Barton [18] obtained the steady state solution for a synchronous machine by the direct application of the Runge-Kutta numerical integration
technique. They found that the integration step size must be small enough to give at least ten steps per period of the highest significant frequency. Thus if we have a basically 60 Hz problem in which we feel that the fifth harmonic may be significant, the step size must be less than \( \frac{1}{10 \times 5 \times 60} \) sec., i.e., 0.33 msec.

For the steady state problems there is no way of knowing the correct initial values of the state variables. Thus every problem becomes a transient problem and a number of transient cycles must be followed before the desired steady state is reached. Even under the best conditions much useless information is generated. This method although easy to use is inefficient when the harmonic series representing the winding inductance include a large number of terms.

Since only the steady state solution is desired, another method of solution was sought with a trade-off between solution time and accuracy being made [19]. The form of the stator currents and rotor currents was assumed known:

\[
\mathbf{i} = \sum_{i} I_i \cos (\omega_i t - \alpha_i) . \tag{2-5}
\]

A set of equalities were established by substituting these currents in the matrix equation (2-1) and performing the necessary multiplications and differentiations and finally separating terms of the same frequency and phase on the L.H.S. and R.H.S. The expanded form yielded an equation of the form:

\[
V = Q (\sin \alpha_i, \cos \alpha_i) \cdot \mathbf{i}
\]

where \( V \) is the column vector \( [V_i] \) whose elements are the in-phase and quadrature
harmonic components, \( I \) is the column vector \([I_i]\) whose elements are the magnitudes of the harmonic currents, and \( Q \) is a rectangular matrix of coefficients depending on the impedance elements as well as the phases of the currents. A closed form solution of this equation is not possible since transcendental relationships are involved, but a solution may be obtained using numerical methods such as the Newton-Raphson technique [20, 21].

While the Runge-Kutta method is easily programmed and has a reasonable efficiency, solving the same system using Newton-Raphson method gives more insight into significant harmonic interactions and yields computed results in less time. A disadvantage of this method is that the elements of the matrix \( Q \) are functions of the phase angles \( \alpha_i \), which necessitates the repetition of the Newton-Raphson Search at every load angle desired. This inefficient utilization of computer time provided the incentive to search for an alternative method of solution.

2.3.2 The Analytical Approach

The current harmonic series of equation (2-5) can be analyzed into the in-phase and quadrature current components and rewritten

\[
i = \sum_i I_i \cos \Theta_i + I_i \sin \Theta_i
\]

(2-6)

where

\[
\Theta_i = \omega_i t + \beta \quad , \quad \beta \text{ being a constant}
\]

of integration depending on the initial rotor position.
From equation (2-4) the product of a row of the inductance matrix with the current column vector involves the products of elements having inductance harmonics and current harmonics. This in general yields a large number of elements of the type

$$E = A \cos (\Theta_1 - \gamma_1) \cos (\Theta_2 - \gamma_2)$$ (2-7)

This cosine product is analyzed into a cosine sum

$$E = A / 2 \left[ \cos \left( \Theta_1 + \Theta_2 - (\gamma_1 + \gamma_2) \right) + \cos \left( \Theta_1 - \Theta_2 - (\gamma_1 - \gamma_2) \right) \right]$$ (2-8)

which can be further analyzed into in-phase and quadrature elements,

$$E = A / 2 \left[ \cos (\gamma_1 + \gamma_2) \cos (\Theta_1 + \Theta_2) \\
+ \sin (\gamma_1 + \gamma_2) \sin (\Theta_1 + \Theta_2) \\
+ \cos (\gamma_1 - \gamma_2) \cos (\Theta_1 - \Theta_2) \\
+ \sin (\gamma_1 - \gamma_2) \sin (\Theta_1 - \Theta_2) \right]$$ (2-9)

which if differentiated with respect to time yield

$$\frac{dE}{dt} = A / 2 \left[ - (\omega_1 + \omega_2) \cos (\gamma_1 + \gamma_2) \sin (\Theta_1 + \Theta_2) \\
+ (\omega_1 + \omega_2) \sin (\gamma_1 + \gamma_2) \cos (\Theta_1 + \Theta_2) \\
- (\omega_1 - \omega_2) \cos (\gamma_1 - \gamma_2) \sin (\Theta_1 - \Theta_2) \\
+ (\omega_1 - \omega_2) \sin (\gamma_1 - \gamma_2) \cos (\Theta_1 - \Theta_2) \right]$$ (2-10)
The very large number of terms thus generated (4 times the original number), is scanned to group together terms of the same frequency and phase which are then separately equated to the corresponding voltage terms on the L.H.S. yielding,

\[ V = AI \]  \hspace{1cm} (2-11)

where \( A \) is a square matrix whose elements are only dependent on the machine constants. This matrix relates the harmonic components in the voltage waveforms to the corresponding harmonic components in the current waveforms, hence the name harmonic impedance matrix.

Performing such analysis requires in general weeks or months of hand manipulation of algebraic and trigonometric expressions. As in the case of numerical computations, the human is error-prone when manipulating long complicated formulae like these. FORMAC, standing for 'FORMULA MANIPULATION COMPILER' is a powerful computer aid for solving such problems.

2.4 Introduction to FORMAC

'FORMAC' is an experimental programming system which was designed to permit the engineer to use, in a practical way, analytic as well as numeric techniques on a digital computer [22, 23]. The advantages of using a digital computer for numeric computation apply in almost equally large measure to the use of a computer for non-numeric work.
The basic concepts of 'FORMAC' were developed by Jean E. Sammet [24, 25] (assisted by Robert G. Tobey) at IBM's Boston Advanced Programming Department in July 1962. At first FORMAC was developed as an extension of FORTRAN IV on the IBM 7090/94 Computer. Consideration of language and implementation for the IBM System 360 started in the fall of 1964, with the intent to provide a better capability and associate it with PL/I rather than FORTRAN. The PL/1 - FORMAC system was released in November 1967.

FORMAC expressions can contain variables, user defined functions, rational constants with up to 2295 digits, and symbolic constants representing $\pi$, $i$, and $e$, as well as trigonometric, logarithmic and exponential functions.

To demonstrate how FORMAC applies to the analysis of an expression term by term in every detail we shall discuss some of its powerful features by considering the application of the FORMAC operators NARGS, ARG and LOP to the simple expression of equation (2-7)

$$E = A \cos (\theta_1 - \gamma_1) \cos (\theta_2 - \gamma_2).$$

The operator NARGS (E) evaluates the number of arguments in the expression E. In our example, since in the FORMAC language, the expression E is regarded as the product of three arguments A, $\cos (\theta_1 - \gamma_1)$ and $\cos (\theta_2 - \gamma_2)$, therefore NARGS (E) returns the integer 3 to the main program.

The operator ARG (1, E) is used to separate the arguments contained in an expression. Thus the statement
\[ G = \text{ARG} \left( 2, E \right) \]

yields

\[ G = \cos \left( \theta_1 - \gamma_1 \right) . \]

In general \( \text{ARG} \left( 1, E \right) \) returns the 1st argument in the expression \( E \).

In order that they may be manipulated, the arguments are identified by code numbers by means of the operator \( \text{LOP} \). The function \( \text{LOP} \left( G \right) \) returns an integer code for the lead operator in the expression \( G \). The code for the subtraction sign \(( - )\) for instance is 25, while that for an exponential is 31. \( ' \cos ' \), \( ' \sin ' \) and constants are also recognized by \( \text{LOP} \) and given the codes 5, 4 and 37 respectively. Applying the above functions to the subexpressions in \( E \):

\[
\begin{align*}
G \left( 1 \right) & = \text{ARG} \left( 1, E \right) \\
& \rightarrow \\
G \left( 1 \right) & = A \\
G \left( 2 \right) & = \cos \left( \theta_1 - \gamma_1 \right) \\
G \left( 3 \right) & = \cos \left( \theta_2 - \gamma_2 \right)
\end{align*}
\]

and

\[
\begin{align*}
L \left( 1 \right) & = \text{LOP} \left( G \left( 1 \right) \right) \\
& \rightarrow \\
L \left( 1 \right) & = 37 \\
L \left( 2 \right) & = 5 \\
L \left( 3 \right) & = 5
\end{align*}
\]

To extract the argument of the cosine term of \( G \left( 2 \right) \) we reapply the function \( \text{ARG} \):  

\[
\text{ARG} \left[ 1, G \left( 2 \right) \right] = \text{ARG} \left[ 1, \text{ARG} \left( 2, E \right) \right] = \theta_1 - \gamma_1 .
\]
Now that the expression $E$ is dissected almost completely, the various components within it, in our case harmonics, can be identified and the expression can be reformed as desired. In our case, it is desirable to convert the cosine product into a sum. This process is described in detail in Chapter III.

Many other functions and routines are available in FORMAC and can easily be combined to treat almost any expression. For a more complete understanding of FORMAC the reader is referred to Reference [23].
CHAPTER III

COMPUTER PROGRAM FOR A GENERAL MACHINE

3.1 Number of Variables and Equations in the Analytical Solution

In the previous chapter, we found that the machine equations can be written according to equation (2-4). $I_p$ and $I_s$ as expressed by equations (2-2a) and (2-2b) can be rewritten in a form more suitable for machine computations using the in-phase and quadrature current components and putting

$$\theta_0 = \omega_p t$$

and

$$\theta = \omega_r t + \beta$$

where $\beta$ is a constant of integration depending on the initial rotor position,

$$I_p = I_{pl} \cos \left( (\omega_p + \omega_r) t - a_l \right)$$

$$= I_{pl} \cos (\theta_0 + l\theta) + I_{pQl} \sin (\theta_0 + l\theta)$$

where $I_{pl} = I_{pl} \cos (a_l + l\beta)$ is the in-phase current component and $I_{pQl} = I_{pl} \sin (a_l + l\beta)$ is the quadrature current component. Similarly

$$I_s = I_{slm} \cos (\theta_0 + m\theta) + I_{sQm} \sin (\theta_0 + m\theta)$$

where $l$ is any even integer and $m$ any odd integer, positive or negative.
In most practical cases convergence is so rapid that it is only necessary to consider a relatively small number of terms in the harmonic series. Restricting our problem to the $N$th harmonic, we proceed to estimate the number of unknowns and the required equations for a complete solution.

In the primary side the number of unknowns in the range $- N \leq l \leq N$ is

$$N_p = 4 \left(\frac{N}{2}\right) + 2$$

per primary winding.

Here $\left(\cdot\right)$ indicates an integer division with no round off. Thus if $N$ is 9 then the quotient $N / 2$ is returned as 4.

On the secondary side the number of unknowns in the range $- N \leq m \leq N$ is

$$N_s = 4 \left[\left(\frac{N+1}{2}\right)\right]$$

per secondary winding.

Thus, for a machine having $W_p$ primary windings and $W_s$ secondary windings, the solution up to the $N$th harmonic requires the evaluation of a total of

$$W_p \left[4 \left(\frac{N}{2}\right) + 2\right] + 4 W_s \left[\left(\frac{N+1}{2}\right)\right]$$

unknowns.

To solve for this large number of currents the harmonic series of the voltages evaluated on the right hand side of the machine equation should be equated term by term to the left hand side up to the $N$th harmonic providing just enough equations for the complete solution.

Many cases of great practical interest have symmetries which will drastically reduce the number of independent unknowns. Thus the number of unknowns and equations may be reduced in some specific cases by three factors:
1. Many components will have the same frequency and thus can be combined.

2. Many components are zero and can be eliminated in the initial machine equations.

3. Many components form balanced spatial systems, thus implying specific phase differences in both voltages and currents (e.g. 2-phase or 3-phase balanced systems). A more complete discussion of this effect is given in Appendix 1.

As an application of the above-mentioned principles we consider a balanced, 3-phase synchronous machine having 2 damper windings such as that depicted in Figure [3-1].

If the harmonic series of the primary and secondary currents are terminated at the 7th harmonic, this machine having 3 primary windings and 3 secondary windings will be represented by 42 current components on the primary side and 48 current components on the secondary side, a total of 90 unknowns.

However, in this special case, we can take advantage of the balanced three secondary windings and make the justifiable assumption that:

\[
\begin{align*}
I_a &= I_s(\omega t), \\
I_b &= I_s(\omega t - 2\pi/3), \\
I_c &= I_s(\omega t + 2\pi/3),
\end{align*}
\]

thus reducing the book-keeping labour on the secondary side by a factor of 3.
FIGURE 3-1. 3-PHASE SYNCHRONOUS MACHINE WITH 2 DAMPER WINDINGS.
On the primary side, the two damper windings being isolated from the field winding enable us to make the assumption that the harmonic of order zero (d.c.) in these windings is identical to zero, reducing the primary unknowns to 38.

Further, in a synchronous machine having the field winding connected to a d.c. supply, $\omega_p = 0$ while $\omega_f = \omega_s$, the synchronous frequency. $\omega_p$ and $\omega_s$ will therefore have terms of frequencies $\pm l\omega_s$ and $\pm m\omega_s$ respectively where $0 \leq l \leq N$ and $1 \leq m \leq N$. Terms having the same frequency and different signs can be combined with proper adjustment for the phases thus reducing the total burden by a factor of 2. The total number of unknowns will therefore be reduced from 90 to 27 out of which only 8 belong to the secondary side.

3.2 General Structure of the Computer Program

In this section, an outline of the computer program used in this study is given. In Appendices II and III computer listings of two typical versions of the program are shown.

The general structure of the program is depicted in the flow graph of Figure [3-2]. The program starts by reading the input data. This includes the number of windings $NW$, number of stator windings $NSW$, number of unknown current components $NU$, the highest harmonic order considered $NHAR$, and the number of equations to be solved $N$. 
FIGURE 3-2. GENERAL STRUCTURE OF THE ANALYSIS PROGRAM.
Next we read as alphanemic data the reactance matrix $W_L$, the resistance matrix $R$, the current vector $\text{CU}$ as harmonic series, and the current components vector $\text{CUDQ}$ containing the in-phase and quadrature current components arranged in a suitable order. A typical data set is shown with each program listing.

After initializing the control variables, the program proceeds by multiplying one row of the reactance matrix by the current vector resulting in the variable named $Q$. In general $Q$ has the form:

$$Q = \sum X \cdot \begin{bmatrix} \sin \\ \cos \end{bmatrix} (a \theta + a) \cdot \begin{bmatrix} \sin \\ \cos \end{bmatrix} (b \theta + \beta)$$

The analytical expression $Q$ is broken into sub-expressions $E$ which are analyzed in the trigonometric analysis routine explained in the following section. The output of this routine consists of the coefficients of the $1^{\text{st}}$ harmonic component assigned to the variables $C(1)$ in case they are associated with a cosine function or to $S(1)$ in case they are associated with a sine function. The differentiation of the expressions will be equivalent to an exchange between $C(1)$ and $S(1)$, with the proper adjustment for the sign and coefficient of the independent variable.

The coefficients thus obtained are stored and we proceed by forming the product $R1$ which is then analyzed using the same procedure. The resulting coefficients are added to those obtained previously forming the complete analytical expressions for the voltage components. Each of these expressions is further broken into a row vector of coefficients multiplied by the column current vector $\text{CUDQ}$, thus forming the harmonic impedance matrix $A$. 
3.3 **Trigonometric Analysis Routine**

The trigonometric analysis routine is depicted in Figure [3-3]. In the most general case every subexpression \( E \) consists of the product of several terms not more than two of which are the trigonometric functions sine and cosine,

\[
E = \prod_r \sin \{ \cos \} (a_q + a) \cdot \sin \{ \cos \} (b \theta + \beta).
\]

The problem is then to recognize these terms, and replace them with the proper expansion or alternatively rebuild the coefficients of the expanded form and label them correctly for further manipulations.

This is done using the following procedure:

First, \( E \) is broken into terms \( G_i; i = 1, \ldots, NE \). Since an exponent might be screened by the leading product operator, we first check whether \( G \) is operated upon by an exponent, and if so, what is the argument of the SIN or COS function. An index is associated with each case to facilitate future labeling of the different coefficients. In case the term is free from an exponential element, it is tested for SIN or COS and accordingly an index is associated. Then the remaining terms are tested for another SIN or COS in which case the index is changed again.

Table [3-1] shows in detail the indices and arguments in all possible cases.
FIGURE 3-3. TRIGONOMETRIC ANALYSIS ROUTINE.
To complete the analysis, we need to reconstruct the product of the non-trigonometric terms and also to find out the order of the harmonic elements associated with the expressions. This is done by forming the sum and difference of the arguments.

\[
X = A + B \quad \text{and} \quad Z = A - B, 
\]

and extracting the coefficients of \( \theta \) in both \( X \) and \( Z \), i.e., \( X = a + b \) and

<table>
<thead>
<tr>
<th>Index</th>
<th>Case</th>
<th>Argument A</th>
<th>Argument B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No sin or cos</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>( \sin^2(A) )</td>
<td>( a \theta + a )</td>
<td>( B = A )</td>
</tr>
<tr>
<td></td>
<td>( \sin(A) \sin(B) )</td>
<td>( a \theta + a )</td>
<td>( b \theta + \beta )</td>
</tr>
<tr>
<td>2</td>
<td>( \sin(A) )</td>
<td>( a \theta + a )</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( \sin(A) \cos(B) )</td>
<td>( a \theta + a )</td>
<td>( b \theta + \beta )</td>
</tr>
<tr>
<td>3</td>
<td>( \cos(A) \sin(B) )</td>
<td>( a \theta + a )</td>
<td>( b \theta + \beta )</td>
</tr>
<tr>
<td>4</td>
<td>( \cos^2(A) )</td>
<td>( a \theta + a )</td>
<td>( B = A )</td>
</tr>
<tr>
<td></td>
<td>( \cos(A) \cos(B) )</td>
<td>( a \theta + a )</td>
<td>( b \theta + \beta )</td>
</tr>
<tr>
<td></td>
<td>( \cos(A) )</td>
<td>( a \theta + a )</td>
<td>0</td>
</tr>
</tbody>
</table>
$Z = a - b$ which are the harmonic orders of concern. Finally, we allocate the reconstructed expression, preceded by the proper coefficient as implied by the indexing system, to the SIN and COS terms having the same harmonic order as computed.

Table [3-2] summarizes the coefficients attributed to each case.

<table>
<thead>
<tr>
<th>Index</th>
<th>$\sin (a + b) \theta$</th>
<th>$\cos (a + b) \theta$</th>
<th>$\sin (a - b) \theta$</th>
<th>$\cos (a - b) \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0.5 \sin (\alpha + \beta)$</td>
<td>$-0.5 \cos (\alpha + \beta)$</td>
<td>$-0.5 \sin (\alpha - \beta)$</td>
<td>$0.5 \cos (\alpha - \beta)$</td>
</tr>
<tr>
<td>2</td>
<td>$0.5 \cos (\alpha + \beta)$</td>
<td>$0.5 \sin (\alpha + \beta)$</td>
<td>$0.5 \cos (\alpha - \beta)$</td>
<td>$0.5 \sin (\alpha - \beta)$</td>
</tr>
<tr>
<td>3</td>
<td>$0.5 \cos (\alpha + \beta)$</td>
<td>$0.5 \sin (\alpha + \beta)$</td>
<td>$-0.5 \cos (\alpha - \beta)$</td>
<td>$-0.5 \sin (\alpha - \beta)$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.5 \sin (\alpha + \beta)$</td>
<td>$0.5 \cos (\alpha + \beta)$</td>
<td>$-0.5 \sin (\alpha - \beta)$</td>
<td>$0.5 \cos (\alpha - \beta)$</td>
</tr>
</tbody>
</table>

The output of this procedure is as mentioned before the coefficients $C(1)$ and $S(1)$ associated with the 1th order cosine and sine harmonics respectively, where 1 extends from $-\text{NHAR}$ to $\text{NHAR}$. (Note: Negative subscripts are allowed in PL/I - FORMAC, an advantage which we make use of).
CHAPTER IV

ANALYTICAL SOLUTION OF THE SYNCHRONOUS MACHINE

STEADY STATE PERFORMANCE

4.1 Introduction

In this chapter we study a special model of a salient pole rotating field synchronous machine. The configuration of the machine is defined in Figure [4-1], where a, b and c represent the three stator windings, 1 and 2 the direct and quadrature axis damper windings respectively and f the field winding.

This same example has been treated by Dunfield [19] using the Runge-Kutta numerical integration method and a modified Newton-Raphson method. Although both techniques give the accuracy required, the computation time involved is considerable. In the following the results obtained using the analytical solution are compared to those obtained numerically for both the 4-wire and 3-wire connections.

4.2 Steady State Performance of a 4-Wire Star Connected Synchronous Machine

Under steady state condition the relationship between the voltages and the currents in the different windings of the machine introduced in Figure [4-1], can be written in a matrix form.
FIGURE 4-1. 3-PHASE SYNCHRONOUS MACHINE WITH 2 DAMPER WINDINGS.
To determine the solution of equation (4-1) in the state-space form we rewrite

\[ V = (R + DL) I \]  

(4-2)

where \( D \) is the differential operator.

The matrices \( R \) and \( L \) are defined by equations (4-3) and (4-4)

\[
R = \begin{pmatrix}
R_f & \cdots \\
& R_1 & \cdots \\
& & R_2 & \cdots \\
& & & R_\alpha & \cdots \\
& & & & R_\alpha \\
& & & & & R_\alpha \\
& & & & & & R_\alpha \\
\end{pmatrix}
\]  

(4-3)
\[
\begin{array}{cccccc}
L_f & M_{f1} & M_{af} \cos \Theta & M_{af} \cos (\Theta - 2\pi/3) & M_{af} \cos (\Theta + 2\pi/3) \\
M_{f1} & L_1 & M_{a1} \cos \Theta & M_{a1} \cos (\Theta - 2\pi/3) & M_{a1} \cos (\Theta + 2\pi/3) \\
 & L_2 & -M_{a2} \sin \Theta & -M_{a2} \sin (\Theta - 2\pi/3) & -M_{a2} \sin (\Theta + 2\pi/3) \\
M_{af} \cos \Theta & M_{a1} \cos (\Theta - 2\pi/3) & -M_{a2} \sin (\Theta - 2\pi/3) & L_{aa} & -M_{ab} & -M_{ab} \\
+M_{af3} \cos 3 \Theta & M_{a1} \cos (\Theta - 2\pi/3) & -M_{a2} \sin (\Theta - 2\pi/3) & +L_{aa2} \cos 2 \Theta & +L_{aa4} \cos 4 \Theta & +M_{ab2} \cos 2 (\Theta - \pi/3) & +M_{ab2} \cos 2 (\Theta + \pi/3) \\
M_{af} \cos (\Theta - 2\pi/3) & M_{a1} \cos (\Theta - 2\pi/3) & -M_{a2} \sin (\Theta - 2\pi/3) & -M_{ab} & L_{aa} & -M_{ab} \\
+M_{af3} \cos 3 \Theta & M_{a1} \cos (\Theta - 2\pi/3) & -M_{a2} \sin (\Theta - 2\pi/3) & +L_{aa2} \cos 2 (\Theta - \pi/3) & +L_{aa4} \cos 4 (\Theta - \pi/3) & +M_{ab2} \cos 2 (\Theta + \pi/3) & +M_{ab2} \cos 2 \Theta \\
M_{af} \cos (\Theta + 2\pi/3) & M_{a1} \cos (\Theta + 2\pi/3) & -M_{a2} \sin (\Theta + 2\pi/3) & -M_{ab} & -M_{ab} & L_{aa} \\
+M_{af3} \cos 3 \Theta & M_{a1} \cos (\Theta + 2\pi/3) & -M_{a2} \sin (\Theta + 2\pi/3) & +L_{aa2} \cos 2 (\Theta + \pi/3) & +L_{aa4} \cos 4 (\Theta + \pi/3) & +M_{ab2} \cos 2 (\Theta + \pi/3) & +L_{aa4} \cos 4 (\Theta - \pi/3) \\
\end{array}
\]
During normal operation the field voltage is constant while the stator voltages form a balanced three phase set.

Thus

\[ v_f = V_f, \]
\[ v_a = V \cos (\omega t), \]
\[ v_b = V \cos (\omega t - 2\pi / 3), \]
\[ v_c = V \cos (\omega t + 2\pi / 3). \]

and

The instantaneous angular position \( \theta \) of the rotor is the time integral of the speed of rotation,

\[ \theta = \omega t + \beta \]

where \( \omega \) is the angular frequency of the supply and \( \beta \) is a constant of integration chosen so that

\[ \beta = 3\pi / 2 - \delta \]

\( \delta \) being the load angle defined as the angle between the direct axis and an axis based upon the three phase terminal voltages.

The currents in the field winding and the two damper windings are assumed

\[ i_f = i_0 + \sum_l (i_{fDL} \cos \theta + i_{fQL} \sin \theta) \]
\[ i_1 = \sum_l \left( l_{1DL} \cos l\theta + l_{1QL} \sin l\theta \right), \quad (4-7b) \]
\[ i_2 = \sum_l \left( l_{2DL} \cos l\theta + l_{2QL} \sin l\theta \right), \quad (4-7c) \]

while the stator currents are
\[ i_a = \sum_m \left( I_{Dm} \cos m\theta + I_{Qm} \sin m\theta \right), \quad (4-7d) \]
\[ i_b = i_a \left( \theta - \frac{2\pi}{3} \right), \quad (4-7e) \]
\[ i_c = i_a \left( \theta + \frac{2\pi}{3} \right), \quad (4-7f) \]

where
\[ l \quad \text{is any positive even integer}, \]
and
\[ m \quad \text{is any positive odd integer}. \]

4.2.1 Computer Program

In Appendix II the computer program used to obtain the analytical solution is listed together with a typical data set. We make use of the knowledge that the various harmonic series quoted in equations (4-7) converge very rapidly, to terminate the current and inductance harmonic series at the seventh harmonic. More or less terms can be used depending on the accuracy required and the time and cost of the computer program.
The first four equations of (4-1) are then expanded in detail, the fifth and sixth being unnecessary since they repeat the information contained in the fourth equation due to the assumption made in equations (4-5) and (4-7 e and f) about the balanced three phase set of voltages and currents. The output is in the form

\[ V = A I \]  

(4-8)

where \( V \) is the voltage column vector consisting of 27 elements which are the result of analyzing the harmonic components of each voltage in the direct and quadrature axes. \( A \) is a 27 x 27 harmonic impedance matrix and \( I \) is a current column vector of 27 elements constructed in a similar way to the voltage vector. Equation (4-8) is depicted more explicitly in the fold-out in Appendix II.

A numerical solution is now straightforward since it only involves the solution of a set of linear equations. However, inspection of the matrix equation suggests some results. The first row of this matrix equation yields

\[ I_{f0} = \frac{V_f}{R_f} \]
Rows 2, 3, 8 and 9 read:

\[
\begin{pmatrix}
R_f & 2X_f & 2X_{f_1} \\
-2X_f & R_f & -2X_{f_1} \\
2X_{f_1} & R_1 & 2X_1 \\
-2X_{f_1} & -2X_1 & R_1
\end{pmatrix}
\begin{pmatrix}
I_{fD2} \\
I_{fQ2} \\
I_{1D2} \\
I_{1Q2}
\end{pmatrix} = 0
\]

(4-9)

which, since the square matrix is not singular, imply that

\[I_{fD2} = I_{fQ2} = I_{1D2} = I_{1Q2} = 0.\]

Similarly, rows 4, 5, 10 and 11 give

\[I_{fD4} = I_{fQ4} = I_{1D4} = I_{1Q4} = 0.\]

The four rows 14, 15, 16 and 17 also yield

\[I_{2D2} = I_{2Q2} = I_{2D4} = I_{2Q4} = 0.\]

Actually, these results are expected since in a balanced three phase synchronous machine the induced stator mmf waves that couple to the rotor are the fifth, seventh, eleventh .... harmonics only [15]. The fifth harmonic mmf wave induced in the stator rotates at a speed \(\omega/5\) in a forward direction, thus the relative
The velocity with respect to the rotor is \((\omega + \omega/5) = 6 \omega/5\), and since it has five times the basic number of poles, the induced field in the rotor side has a frequency of \(6 \omega\). Similarly, the seventh harmonic stator mmf wave rotates backward at a relative velocity \((\omega - \omega/7) = 6 \omega/7\) and has seven times the basic number of poles thus contributing to the six harmonic rotor mmf waves only.

To solve for the remaining current components we use the measured parameters shown in Tables [4-1] and [4-2] quoted from Reference [19] for the synchronous machine under consideration. Substituting these values and solving the matrix equation (4-8) using a standard gaussian elimination routine (GELG, IBM scientific subroutine package), the elements of the current vector are readily obtained.

<table>
<thead>
<tr>
<th>TABLE [4-1]</th>
<th>WINDING RESISTANCE IN OHMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_f)</td>
<td>(R_1)</td>
</tr>
<tr>
<td>38.0</td>
<td>3.65</td>
</tr>
</tbody>
</table>
### TABLE [4-2]

**WINDING INDUCTANCE**

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_f$ (H)</td>
<td>10.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_1$ (mH)</td>
<td>100.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_2$ (mH)</td>
<td>141.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{aa}$ (mH)</td>
<td>60.1*</td>
<td>10.7</td>
<td></td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>$M_{f1}$ (mH)</td>
<td>770.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{a1}$ (mH)</td>
<td></td>
<td>65.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{a2}$ (mH)</td>
<td></td>
<td>49.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{af}$ (mH)</td>
<td></td>
<td>812.0</td>
<td></td>
<td></td>
<td>**</td>
</tr>
<tr>
<td>$M_{ab}$ (mH)</td>
<td>26.3</td>
<td>19.8</td>
<td></td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

* This value include the leakage inductance $L_\sigma$, which is equal to 4.35 mH.

** The value of $M_{af3}$ is negligibly small in the experimental machine.
4.2.2 Results

The results obtained from solving (4-8) at a stator phase voltage of 120 volts, and an average field voltage of 19 volts are compared with the results obtained by Dunfield in Table [4-3]. The agreement between the different methods is good. The results obtained using the analytical solution are close to those obtained using the Newton–Raphson technique, which is an indication that the analytical solution accuracy with the harmonic expansion limited to the seventh harmonic is acceptable.

Figure [4-2] shows the waveform of the stator line current at two load angles $\delta = -10^0$ and $\delta = -20^0$. We notice that the third harmonic component in the stator current is as large as the fundamental while the effects of the fifth and seventh harmonics are hardly noticed. The variations of the fundamental, third and fifth harmonic stator current with the load angle are shown in Figure [4-3]. It is clear that the amplitude of the third harmonic is always comparable to the fundamental while the amplitude of the fifth harmonic is within about 21%. The highest value of the seventh harmonic is 0.09 ampere reached at a load angle of $90^0$, which represents about 1.65% of the fundamental. Typical values of the ratios $I_3/I_1$, $I_5/I_1$ and $I_7/I_1$ are shown in Table [4-4].

The variation of the sixth harmonic current in the field winding and the two damper windings is shown in Figure [4-4]. The sixth harmonic current in the quadrature axis damper winding is higher than that in the direct axis damper winding especially at large load angles.
FIGURE 4-2. STATOR LINE CURRENT FOR A LOAD ANGLE $\delta = -20^\circ$ AND $\delta = -10^\circ$. 
FIGURE 4-3. FUNDAMENTAL, 3rd AND 5th HARMONIC LINE CURRENTS, 4-WIRE CONNECTION.

FIGURE 4-4. 6th HARMONIC FIELD AND DAMPER WINDINGS CURRENTS, 4-WIRE CONNECTION.
## TABLE [4-3]

**COMPARISON OF RESULTS FOR FORMAC, RUNGE-KUTTA AND NEWTON-RAPHSON METHODS**

**AT $s = -10^\circ$, 4-WIRE CONNECTION**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Harmonic Order</th>
<th>Magnitude (RMS)</th>
<th>Phase (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FORMAC</td>
<td>Runge-Kutta</td>
<td>Newton-Raphson</td>
</tr>
<tr>
<td>Stator Current (A)</td>
<td>1</td>
<td>1.046</td>
<td>1.047</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.086</td>
<td>1.096</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.223</td>
<td>0.229</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.017</td>
<td>0.037</td>
</tr>
<tr>
<td>Field Current (A)</td>
<td>6</td>
<td>0.022</td>
<td>0.024</td>
</tr>
<tr>
<td>Direct Axis</td>
<td>6</td>
<td>0.033</td>
<td>0.035</td>
</tr>
<tr>
<td>Damper Current (A)</td>
<td>6</td>
<td>0.126</td>
<td>0.105</td>
</tr>
</tbody>
</table>
TABLE [4 - 4]

THE RATIOS \( \frac{l_3}{l_1}, \frac{l_5}{l_1} \) AND \( \frac{l_7}{l_1} \) FOR THE 4 - WIRE CONNECTION

<table>
<thead>
<tr>
<th>( \frac{l_3}{l_1} ) %</th>
<th>( \frac{l_5}{l_1} ) %</th>
<th>( \frac{l_7}{l_1} ) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>107.2</td>
<td>21.8</td>
<td>1.84</td>
</tr>
</tbody>
</table>

The second and fourth harmonics in the field current and the two damper winding currents are identically zero as was discussed previously.

4.3 Steady State Solution of a Three-Wire Star Connected Synchronous Machine

Since for the three-wire star connection the sum of the stator currents is zero, \( i_a + i_b + i_c = 0 \). Equation (4-1) must be transformed so that a solution takes cognizance of this fact. The relation between the voltages and currents is therefore

\[
\begin{align*}
\begin{bmatrix}
\nu_f \\

\nu_a - v_c \\

\nu_b - v_c
\end{bmatrix}
&=
\begin{bmatrix}
Z_f \\

Z_{f1} \\

Z_{fa} - Z_{fc} \\

Z_{fb} - Z_{fc}
\end{bmatrix}
\begin{bmatrix}
i_f \\

i_1 \\

i_a \\

i_b
\end{bmatrix}
\end{align*}
\]

\[\text{(4-10)}\]
The resistances and inductances are determined from equations (4-3) and (4-4) with the proper re-arrangement for the solution.

4.3.1 Computer Program

A similar computer program to that used for the four wire connection is used to solve this problem. Again the first four rows of (4-10) are expanded to yield the complete analytical solution

\[ V = A I, \]  

(4-11)

which is shown explicitly in the fold-out in Appendix II. A similar analysis to that outlined in the previous case follows.

4.3.2 Results

Since for the three-wire star connection the sum of the stator currents is zero, the third harmonic in the stator current does not exist. In Figure [4-5] the fundamental and fifth harmonic components of the stator current for various load angles are shown, the seventh harmonic being very small, with a maximum value of 0.088 ampere at a load angle of 90°.

The sixth harmonic components in the two damper windings, and the field winding currents are depicted in Figure [4-6]. It is clear that the harmonic current
FIGURE 4-5. FUNDAMENTAL AND 5th HARMONIC LINE CURRENTS, 3-WIRE CONNECTION.

FIGURE 4-6. 6th HARMONIC FIELD AND DAMPER WINDINGS CURRENTS, 3-WIRE CONNECTION.
in the quadrature axis damper winding is greater than that in the direct axis damper winding at the same load angle, and both reduce to zero at zero load angle. The a.c. component of field current is very small, typically 0.031 ampere at a load angle of 90°.

In addition we notice that this connection of the machine causes the generation of identical third harmonic neutral voltages in each phase of the stator windings. These voltages appear when the voltage is measured between line terminal and stator neutral or between stator neutral and source neutral. The neutral voltage may be readily calculated as the variation between source neutral and machine neutral. The relationships are those of equation (4-8) with \( I_{D3} = I_{Q3} = 0 \). Therefore, the neutral voltage \( V'_{n'n} \), where \( n \) denotes the source neutral and \( n' \) the stator neutral is

\[
V'_{n'n} = V_{D3} \cos 3\theta + V_{Q3} \sin 3\theta
\]

where

\[
V_{D3} = 1.5 X_{af3} I_f Q6
\]

\[
+ 1.5 (X_{a2} - X_{aa4} - X_{ab2} - X_{ab4}) I_{Q1}
\]

\[
+ 1.5 (X_{aa2} - X_{ab2}) I_{Q5}
\]

\[
+ 1.5 (X_{aa4} + X_{ab4}) I_{Q7}
\]
\[ V_{Q3} = -3 X_{af3} l_0 - 1.5 X_{af3} l_{D6} \]
\[-1.5 (X_{aa2} + X_{aa4} - X_{ab2} + X_{ab4}) l_{D1} \]
\[-1.5 (X_{aa2} - X_{ab2}) l_{D5} \]
\[-1.5 (X_{aa4} + X_{ab4}) l_{D7} \]

Thus, substituting in equation (4-12) the values of the currents obtained from the solution of equation (4-11), \( V_{n'n} \) is obtained. The variation of the neutral voltage with respect to the load angle is depicted in Figure [4-7] from which we notice that the neutral voltage increases with the load angle reading \( V_{n'n} = 46 \) volts at a load angle \( \delta = 90^\circ \).

Table [4-5] compares the magnitudes and phases of the different harmonic currents and neutral voltage obtained using different computational techniques. As in Table [4-3] the agreement between the different results indicates that the accuracy obtained using the series expansion limited to the seventh harmonic is acceptable.

4.4 The 3-Wire Versus the 4-Wire Connection

Comparing the results obtained for the 3-wire connection with those previously discussed for the 4-wire connection, we easily notice that the former is to be preferred, as with this connection the harmonic interactions are greatly reduced
FIGURE 4-7. THIRD HARMONIC NEUTRAL VOLTAGE $V_{nn}$. 
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Harmonic Order</th>
<th>FORMAC</th>
<th>Runge-Kutta</th>
<th>Newton-Raphson</th>
<th>FORMAC</th>
<th>Runge-Kutta</th>
<th>Newton-Raphson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Current (A)</td>
<td>1</td>
<td>0.925</td>
<td>0.926</td>
<td>0.925</td>
<td>153.4</td>
<td>153.1</td>
<td>153.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>109.7</td>
<td>109.6</td>
<td>109.7</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.014</td>
<td>0.015</td>
<td>0.015</td>
<td>269.3</td>
<td>269.0</td>
<td>268.8</td>
</tr>
<tr>
<td>Field Current (A)</td>
<td>6</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>9.6</td>
<td>9.4</td>
<td>9.4</td>
</tr>
<tr>
<td>Direct Axis Damper Current (A)</td>
<td>6</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>8.5</td>
<td>8.2</td>
<td>8.4</td>
</tr>
<tr>
<td>Quadrature Axis Damper Current (A)</td>
<td>6</td>
<td>0.010</td>
<td>0.007</td>
<td>0.010</td>
<td>277.9</td>
<td>277.8</td>
<td>277.9</td>
</tr>
<tr>
<td>Stator Neutral Voltage (V)</td>
<td>3</td>
<td>7.400</td>
<td>7.313</td>
<td>7.311</td>
<td>235.0</td>
<td>236.1</td>
<td>235.0</td>
</tr>
</tbody>
</table>
and the third harmonic of the stator current is eliminated. Table [4-6] compares between both connections at a load angle of $20^\circ$.

![Table 4-6](image)

A drawback of the 3-wire connection is the neutral voltage which appears between the line terminal and stator neutral. Although this voltage is small it can be troublesome because it gives rise to a winding voltage gradient higher than that expected on the basis of the applied stator voltage and thereby reduces the stator insulation safety factor.

4.5 Conclusion

In this chapter, the machine equations for a 3-phase synchronous machine are expanded to yield the harmonic impedance matrix relating the voltage components to the current components. The matrix equation thus obtained gives a closed form solu-
tion for the current components. The results are in good agreement with those obtained experimentally and analytically in Reference [19]. This indicates that limiting the current and impedance harmonic series to the seventh harmonic in the analytical solution leads to a reasonable accuracy. Considering computation time, typically a run of 3 minutes might be required for a poor choice of initial conditions for the Runge-Kutta program to evaluate the currents at a typical load angle. The same results are obtained in less than 0.3 seconds using the harmonic impedance matrix, an improvement of about 2 to 3 orders of magnitude.
CHAPTER V

ANALYTICAL SOLUTION FOR INDUCTION MOTORS

WITH PHASE-WOUND ROTORS

5.1 Introduction

As shown in Chapter II due to the multiple armature reaction effects, when current flows in a machine winding it will create fields in the primary which will cause multiple frequency currents to flow in the secondary, in turn these currents create multiple fields that will react back in the primary.

Oberretl [17] and Robinson [15] indicated that the assumption of sinusoidal mmf's is invalid for induction machines in which multiple armature reaction can give rise to appreciable harmonic effects. Dunfield and Barton [26] discussed the theoretical background and reported the experimental results for such phenomena in a 2-phase induction machine. In this chapter we will consider the same simple example, namely a machine with two primary and two secondary windings as shown in Figure [5-1], for which the relationship between the voltages and currents can be written in a matrix form as:

$$
\begin{bmatrix}
V_s \\
0
\end{bmatrix} =
\begin{bmatrix}
Z_{ss} & Z_{sr} \\
Z_{rs} & Z_{rr}
\end{bmatrix}
\begin{bmatrix}
I_s \\
I_r
\end{bmatrix}
$$

(5-1)
FIGURE 5-1. WINDING CONFIGURATION OF A 2-PHASE INDUCTION MACHINE.
The impedance matrix of such a machine is

\[
Z = \begin{bmatrix}
R_s + D L_s & D \Sigma M_h \cos h \omega t & - D \Sigma M_h \\
R_s + D L_s & D \Sigma M_h \sin h \pi/2 \sin h \omega t & D \Sigma M_h \cos h \omega t \\
D \Sigma M_h \cos h \omega t & D \Sigma M_h \sin h \pi/2 \sin h \omega t & R_r + D L_r \\
-D \Sigma M_h \sin h \pi/2 \sin h \omega t & D \Sigma M_h \cos h \omega t & R_r + D L_r
\end{bmatrix}
\] (5-2)

where \( h \) is any positive odd number and \( D \) the differential operator.

### 5.2 Harmonic Interactions in Induction Machines

If the rotor rotates at an angular frequency \( \omega_r \) electrical radians per second, and the stator is excited from a 2-phase supply of frequency \( f_o \), producing a fundamental synchronous speed \( \omega_o \) radians/sec., the fundamental component of the stator mmf induces voltages of frequency \( (\omega_o - \omega_r) \) in the rotor windings. The
resulting rotor currents produce a fundamental field which rotates in a forward direction relative to the rotor at \((\omega_o - \omega_r)\) and in synchronism with the originating field. These rotor currents also produce harmonic fields which move, relative to the rotor, with the corresponding fractional speed, and which then induce voltages in the stator which are, in general, not at the supply frequency. As an example consider the effect of the third harmonic component of rotor mmf. This rotates backwards with respect to the rotor at an angular frequency \((\omega_o - \omega_r)/3\), and induces voltages in the stator windings of angular frequency \((\omega_o - 4\omega_r)\).

Tables [5-1a] and [5-1b] show respectively the non-supply stator and rotor frequencies of interest which we can express as

\[
\omega_S = (-1)^{k+1} \omega_o + [m + (-1)^k] \omega_r \quad (5-3)
\]

and

\[
\omega_R = (-1)^{k+1} \omega_o - m \omega_r \quad (5-4)
\]

where

\[
m = 2k - 1 , \; k \text{ being any positive integer.}
\]

We conclude that the currents in the stator and rotor windings can be expressed as harmonic series of the above frequencies only, while other components are expected to be zero.
### TABLE [5-1a]

**STATOR FREQUENCIES OF INTEREST**

<table>
<thead>
<tr>
<th>m</th>
<th>Induced Rotor mmf Frequency</th>
<th>Rotation Frequency</th>
<th>Stator Voltage Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>(- (\omega_o - \omega_r)/3)</td>
<td>(\omega_r)</td>
<td>(- (\omega_o - 4 \omega_r))</td>
</tr>
<tr>
<td>5</td>
<td>((\omega_o - \omega_r)/5)</td>
<td>(\omega_r)</td>
<td>((\omega_o + 4 \omega_r))</td>
</tr>
<tr>
<td>7</td>
<td>(- (\omega_o - \omega_r)/7)</td>
<td>(\omega_r)</td>
<td>(- (\omega_o - 8 \omega_r))</td>
</tr>
<tr>
<td>9</td>
<td>((\omega_o - \omega_r)/9)</td>
<td>(\omega_r)</td>
<td>((\omega_o + 8 \omega_r))</td>
</tr>
</tbody>
</table>

### TABLE [5-1b]

**ROTOR FREQUENCIES OF INTEREST**

<table>
<thead>
<tr>
<th>m</th>
<th>Induced Stator mmf Frequency</th>
<th>Rotation Frequency</th>
<th>Rotor Voltage Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\omega_o)</td>
<td>(\omega_r)</td>
<td>((\omega_o - \omega_r))</td>
</tr>
<tr>
<td>3</td>
<td>(- \omega_o / 3)</td>
<td>(\omega_r)</td>
<td>(- (\omega_o + 3 \omega_r))</td>
</tr>
<tr>
<td>5</td>
<td>(\omega_o / 5)</td>
<td>(\omega_r)</td>
<td>((\omega_o - 5 \omega_r))</td>
</tr>
<tr>
<td>7</td>
<td>(- \omega_o / 7)</td>
<td>(\omega_r)</td>
<td>(- (\omega_o + 7 \omega_r))</td>
</tr>
<tr>
<td>9</td>
<td>(\omega_o / 9)</td>
<td>(\omega_r)</td>
<td>((\omega_o - 9 \omega_r))</td>
</tr>
</tbody>
</table>
5.2.1 Harmonic Effects in a Symmetrical Two Phase Machine

Under steady state conditions, we can assume the current in the stator windings

$$I_s = \sum_l l_{sl} \cos \left[ (\omega_o \pm l \omega_r) t - \alpha_l \right]$$

or using the quadrature and direct axis representation

$$I_s = \sum_l l_{sl} \cos (\omega_o + l \omega_r) t + l_{ql} \sin (\omega_o + l \omega_r) t \quad (5-5a)$$

and rotor currents are

$$I_r = \sum_m I_{rm} \cos (\omega_o + m \omega_r) t + I_{qm} \sin (\omega_o + m \omega_r) t \quad (5-5b)$$

where

- $\omega_r$ is the rotation frequency rad/sec,
- $l$ is an even integer positive or negative,
- $m$ is an odd integer positive or negative.

In general, solving equation (5-1) analytically will lead to a large number of linear equations as explained before. If we terminate the harmonic current series at the ninth harmonic and using the same procedure explained in Chapter III, the number of equations to be solved will be 76 equations in 76 unknowns. This large number of equations will be difficult to handle from a computational point of view since the array area core storage needed for the FORMAC exit routines will be very large.
To reduce the number of equations and make them in a suitable form for computation one can take advantage of the following:

1. The interaction between the stator and rotor mmf's will be such that some harmonics will be zero as shown in the previous section.

2. The symmetry of the supply voltage and the configuration of the machine windings help us to solve the problem for one stator and one rotor windings only, since the currents in the two other windings will be of the same amplitude but shifted in phase by $\pi/2$.

5.3 Computer Program

A typical data set is listed in Appendix III with the FORMAC computer program. In the data, both currents in the stator and rotor are expressed as in equation (5-5) in complete harmonic series to get the complete solution and a better insight into the results. The solution is only for one stator winding and one rotor winding thus involving only 38 unknowns and is of the form:

$$V = A I$$  \hspace{1cm} (5-6)

where $V$ is the voltage vector of 38 elements, $A$ is the harmonic impedance matrix $(38 \times 38)$ and $I$ is the current vector. This matrix equation is shown explicitly in
the fold-out in Appendix III*. The first 18 rows of $A$ are related to the stator winding and the remaining 20 rows to the rotor winding.

5.4 Results

For the machine we are considering the values of the stator and rotor reactances at 60 Hz are

\[
X_s = 179.24 \text{ ohm} \\
X_r = 179.47 \text{ ohm}
\]

and the 60 Hz magnetizing reactances are

\[
X_1 = 174.0 \text{ ohm} \\
X_3 = 0.695 \text{ ohm} \\
X_5 = 0.365 \text{ ohm} \\
X_7 = 0.043 \text{ ohm} \\
X_9 = 0.03 \text{ ohm}
\]

while the stator and rotor resistances are

\[
R_s = 5.74 \text{ ohm} \quad \text{and} \quad R_r = 5.89 \text{ ohm}.
\]

* In the computer print-out, the current components are indexed with the letter $P$ for $m$ or $l$ positive and with the letter $N$ for $m$ or $l$ negative.
The numerical solution of equation (5-6) confirmed the expectation that the stator and rotor current series contain the frequencies obtained in Tables [5-1 a] and [5-1 b] only. Using the computed results we plot the waveforms of the stator current at various frequencies of rotation. These are shown in Figure [5-2]. The upper curve of each pair is the current waveform, while the lower curve is the same waveform with the supply frequency component removed and with the vertical current scale increased five times. The magnitude of the harmonic content is surprisingly large in view of the very low harmonic content of the inductances. The excellent agreement between predicted and measured [26] waveforms gives confidence in the theory.

The waveforms of the rotor current at various frequencies of rotation are shown in Figure [5-3]. These waveforms are distorted from the sinusoidal shape due to the harmonic effects.

The amplitudes and phases of the stator harmonic currents are drawn in Figure [5-4] as a function of the ratio $f_r/f_o$. From this figure we notice the trivial result that the amplitude of the $l$th harmonic current vanishes when

$$ f_o + l f_r = 0 $$

e.g. the current component of frequency $f_o + 4 f_r$ is zero at a relative frequency $f_r/f_o = -0.25$, while that of frequency $f_o - 8 f_r$ reduces to zero at $f_r/f_o = +0.125$. In general, the harmonic contents represent a small percentage of the supply current. At a rotational frequency of $0.6 f_o$ typical values for the fourth and eighth harmonics are 9 percent and 1.6 percent of the supply current respectively, a result which agrees with the experimental data [26].
FIGURE 5-2. STATOR CURRENT WAVEFORMS FOR VARIOUS VALUES OF RELATIVE ROTATIONAL FREQUENCY $f_r/f_o$, (VERTICAL SCALE 2 AMP./INCH). THE LOWER TRACE OF EACH PAIR IS THE SAME WAVEFORM WITH THE FUNDAMENTAL REMOVED AND THE SCALE INCREASED 5 TIMES.
FIGURE 5-3. ROTOR CURRENT WAVEFORM FOR VARIOUS VALUES OF RELATIVE ROTATIONAL FREQUENCY $f_r / f_o$, (SUPPLY CURRENT 2 AMP.).
FIGURE 5-4. MAGNITUDE AND PHASE OF THE STATOR HARMONIC CURRENTS AT FREQUENCIES $f_0 \pm 4f_r$ AND $f_0 \pm 8f_r$ AS FUNCTION OF THE RELATIVE ROTATIONAL FREQUENCY $f_r/f_0$. 
Figure 5-5. Magnitude and phase of the rotor harmonic currents at frequencies $f_o + 3f_r$, $f_o - 5f_r$, $f_o + 7f_r$ and $f_o - 9f_r$ as function of the relative rotational frequency $f_r/f_o$. 
In Figure [5-5] the magnitudes and phases of the rotor currents are drawn with respect to the ratio $f_r/f_0$. Comparing Figures [5-4] and [5-5], we clearly see the similarity and the dependence of the interacting stator and rotor currents. We also notice a shift of approximately 180 degrees in the phase of the rotor current with respect to the phase of the corresponding interacting stator current, a result which implies the transformer-like nature of the induction machine.

5.5. Conclusion

In this chapter, the machine equations for an induction machine are solved analytically. The analysis shows that not all the harmonic orders expected in the stator and rotor of a general machine are excited in the induction machine. Further, in the example considered, the symmetry of the 2 phase machine reduces the initial problem to a simpler one where only the interaction between one stator and one rotor windings are considered.

The analytical solution yields the harmonics impedance matrix of the machine. Numerical results are in excellent agreement with those obtained experimentally. This merely shows the reasonable accuracy that can be achieved by truncating the current and impedance harmonic series at a not-so-high order harmonic, say the ninth.
CHAPTER VI

SINGLE PHASE MACHINE, EXPERIMENTAL STUDY

6.1 Introduction

The purpose of the study in the previous two chapters was to check the validity of the analytical solution and to evaluate the accuracy of the predicted results by comparing them to those obtained experimentally.

In this chapter a more searching test of the validity of the proposed technique than those balanced three phase situations is provided by some single-phase machines. The single phase synchronous generator is a device of some practical significance in which extreme precautions must be taken to combat the harmonic generating effect of the negative sequence field [15]. Accordingly the synchronous machine described earlier has also been tested in a single-phase configuration in which, since it is only equipped with a modest damper winding, multiple armature reaction effects are severe.

In addition a single-phase, single-phase induction machine was tested. Such a machine, while of no practical significance, shows in the most extreme degree the multiple armature reaction effects under investigation.

For both machines analytical results are also obtained and compared to the experimental results enhancing our confidence in the analytical method.
FIGURE 6-1. SINGLE PHASE CONNECTION FOR THE EXPERIMENTAL STUDY OF THE SYNCHRONOUS MACHINE.
6.2 Single Phase Synchronous Machine

The three phase machine used as an example in the analytical study in Chapter IV is used here in a special connection to simulate a single phase machine. This is a salient pole alternator rated at 220 volt, 3 kW, 0.8 PF lagging. The field is on the rotor and there are four salient poles each having a 600 turn exciting winding, a 2 turn search coil at each end of the main field winding and a damping cage in the pole face. The 48 slots stator carries a balanced, three phase, double layer winding of 5/6 pitch and 52 turns per pole and phase.

The test was carried by connecting the phases a and b of the stator windings together to a single phase supply of 120 volt, while phase c was left un-connected. The field winding was connected to a d.c. source adjusted to supply an average field current of 0.5 ampere. The configuration of the machine is shown in Figure [6-1].

6.2.1 Analysis

The machine equations are derived from the 3 phase synchronous machine matrix equation (4-1) by putting \( i_a = -i_b, i_c = 0 \) and \( v = v_a - v_b \). The resulting matrix equation is:
The harmonic impedance matrix is obtained using a slightly modified version of the program used for the 3 phase machine. The data used as well as the harmonics impedance matrix are shown in Appendix IV.

6.2.2 Results

Photographs of line current, field current and stator voltage waveforms were taken with the synchronous machine operating both as a motor and as a generator at various load angles. The load angle $\delta$ was measured using a line frequency synchronized stroboscope, by observing the shaft position at zero load angle, i.e. when the output current is virtually zero (or minimum) and noticing the change in angular position as the load is varied. Due to a slight hunting the accuracy in measuring the load angle is considered to be $\pm 2$ degrees.

<table>
<thead>
<tr>
<th>$v_f$</th>
<th>$Z_f$</th>
<th>$Z_{f1}$</th>
<th>$Z_{fa} - Z_{fb}$</th>
<th>$i_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{f1}$</td>
<td>$Z_{11}$</td>
<td>$Z_{1a} - Z_{1b}$</td>
<td>$i_1$</td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>$Z_{fa} - Z_{fb}$</td>
<td>$Z_{1a} - Z_{1b}$</td>
<td>$Z_{2a} - Z_{2b}$</td>
<td>$Z_{aa} + Z_{bb}$</td>
</tr>
</tbody>
</table>

(6-1)
FIGURE 6-2. PREDICTED AND EXPERIMENTAL WAVEFORMS OF THE STATOR CURRENT (UPPER TRACE) AND FIELD CURRENT (LOWER TRACE) AT LOAD ANGLES (a) $\delta = 40^\circ$ AND (b) $\delta = 20^\circ$. Continued......
FIGURE 6-2 (CONTINUED) - STATOR AND FIELD CURRENT WAVEFORMS AT (c) $\delta = -20^\circ$ AND (d) $\delta = 0$. 

60 Hz VOLTAGE REFERENCE 

********

(c)

********

(d)
Figure [6-2] shows oscillograms of actual waveforms of the supply voltage, the stator induced current and the field current compared with the predicted waveforms at different load angles. These waveforms show that the a.c. component of the field current contains all the even harmonics, thus confirming the results discussed in Reference [15].

Measured and computed waveforms of line current at a load angle of 20 degrees are shown superimposed in Figure [6-3]. This figure shows the reasonable agreement between the predicted waveform and the experimental one. However, a small discrepancy points out a source of error either in truncating the harmonic series at the seventh harmonic or in the measured machine parameters. At the same load angle the frequency spectrum of the stator current waveform with a logarithmic vertical scale is shown in Figure [6-4]. The ratio of the third harmonic to the fundamental is 37% while that of the fifth is 15%. The seventh harmonic is only 2.8% of the fundamental which suggests a reasonable accuracy by terminating the harmonic series after the seventh order.

6.3 Single-Phase / Single-Phase Induction Machine

The harmonic interactions in the case of a simple model of a single phase induction machine is studied experimentally and analytically. The stator of the machine used in this study has a single layer 2 pole a.c. winding in 24 slots. The rotor winding is of the double layer lap type, wound in 36 slots, with a coil pitch of
FIGURE 6-3. PREDICTED AND EXPERIMENTAL STATOR CURRENT WAVEFORMS AT A LOAD ANGLE $\delta = 20^\circ$.

FIGURE 6-4. FREQUENCY SPECTRUM OF THE STATOR CURRENT AT A LOAD ANGLE $\delta = 20^\circ$. THE VERTICAL SCALE IS LOGARITHMIC.
1 - 19 slots. In the single phase configuration the machine is connected to a single phase supply of 120 volts. A d.c. dynamometer is used to drive the machine at the desired speed.

6.3.1 Analysis

The relationship between the currents and the voltages in the single phase machine is

\[
\begin{bmatrix}
    v_s \\
    0
\end{bmatrix}
= \begin{bmatrix}
    R_s + DL_s & D \sum M_h \cos \omega t \\
    D \sum M_h \cos \omega t & R_r + DL_r
\end{bmatrix}
\begin{bmatrix}
    i_s \\
    i_r
\end{bmatrix}
\]

(6-2)

which is derived from the 2-phase case, equation (5-1), by forcing \( i_{s2} \) and \( i_{r2} \) to be identically zero. The data used and the output matrix are listed in Appendix V.

6.3.2 Results

To obtain the machine constants, an open-circuit and a short-circuit tests were performed. Ignoring the effect of harmonics, the equivalent circuit shown in Figure [6-5] is obtained. The total stator and rotor winding reactances at the supply frequency are \( X_s = 154.3 \) ohm and \( X_r = 158.6 \) ohm respectively. The
FIGURE 6-5. SIMPLIFIED EQUIVALENT CIRCUIT OF EXPERIMENTAL INDUCTION MACHINE. RESISTANCES AND REACTANCES ARE IN OHMS AT 60 Hz.
The mutual reactance between the stator and rotor is \( X_1 = 149 \) ohm while the stator and rotor resistances are \( R_r = 1.8 \) ohm and \( R_s = 1.24 \) ohm respectively. The 60 Hz – harmonic magnetizing reactances are calculated from equation (6) of Reference [6]:

\[
\begin{align*}
X_3 &= 0.596 \text{ ohm} \\
X_5 &= 0.310 \text{ ohm} \\
X_7 &= 0.037 \text{ ohm} \\
X_9 &= 0.025 \text{ ohm}
\end{align*}
\]

The excellence of the predictions of the rotor current waveforms is demonstrated in Figure [6-6] where oscillograms are compared to calculated plots at different speeds of rotation. At a slip \( s = 1 - (\omega_r/\omega_o) \), the significant frequencies are obtained by putting \( m = 1 \) in the set \( [\omega_o \pm m \omega_r] = [1 \pm m (1 - s)] \omega_o \), which is the frequency spectrum of the rotor current. We focus attention on one of the waveforms, say at a slip of 0.1. Compared to the 60 Hz reference waveform, the frequencies of interest are the \( 1.9 \omega_o \) spiky waveform and the \( 0.1 \omega_o \) envelope. Figure [6-7] shows the complete frequency spectrum of the rotor and stator current waveforms at 0.1 slip. The magnitudes of the current components are listed in Tables [6-1] and [6-2], from which we notice that in the analytical solution truncating the harmonic series at the ninth order is a justifiable approximation. A similar analysis and deduction can be performed on any of the waveforms shown in Figure [6-6].
FIGURE 6-6. PREDICTED AND EXPERIMENTAL ROTOR CURRENT WAVEFORMS AT VARIOUS VALUES OF SLIP.
FIGURE 6-7. FREQUENCY SPECTRUM OF: (a) THE ROTOR CURRENT, (b) THE STATOR CURRENT AT A SLIP OF 0.1.
### TABLE [6-1]

**ROTOR HARMONIC CURRENTS AT A SLIP OF 0.1**

<table>
<thead>
<tr>
<th>Harmonic Component Frequency</th>
<th>Current Component (A)</th>
<th>Percent from Supply Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_o \pm \omega_r$</td>
<td>3.5</td>
<td>75</td>
</tr>
<tr>
<td>$\omega_o \pm 3\omega_r$</td>
<td>2.2</td>
<td>47</td>
</tr>
<tr>
<td>$\omega_o \pm 5\omega_r$</td>
<td>1.35</td>
<td>29</td>
</tr>
<tr>
<td>$\omega_o \pm 7\omega_r$</td>
<td>0.73</td>
<td>15</td>
</tr>
<tr>
<td>$\omega_o \pm 9\omega_r$</td>
<td>0.23</td>
<td>4.9</td>
</tr>
</tbody>
</table>

### TABLE [6-2]

**STATOR HARMONIC CURRENTS AT A SLIP OF 0.1**

<table>
<thead>
<tr>
<th>Harmonic Component Frequency</th>
<th>Current Component (A)</th>
<th>Percent from Supply Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_o \pm 2\omega_r$</td>
<td>2.9</td>
<td>60</td>
</tr>
<tr>
<td>$\omega_o \pm 4\omega_r$</td>
<td>1.8</td>
<td>38</td>
</tr>
<tr>
<td>$\omega_o \pm 6\omega_r$</td>
<td>1.04</td>
<td>22</td>
</tr>
<tr>
<td>$\omega_o \pm 8\omega_r$</td>
<td>0.48</td>
<td>10</td>
</tr>
</tbody>
</table>
6.4 Conclusion

Experimental verification and confirmation of the analytical formalism are done in this chapter using as an example a single phase synchronous machine and a single phase induction machine. It is demonstrated that in single phase machines, a wide frequency spectrum of harmonics exists.

Using the measured parameters for the induction machine, the predicted results are found to be in very good agreement with the experimental results. For the synchronous machine the parameters were obtained from Reference [19], analytical and experimental results agree within an error margin which is thought to be due to the inaccuracy of measurement of some of the machine parameters.
CHAPTER VII

SUMMARY AND CONCLUSIONS

The main concern of this thesis is the development of a new analytical approach to the study of the effects of space harmonics on the steady state current and voltage waveforms in electrical rotating machines. The analytical approach involves the manipulation of trigonometric and algebraic functions of moderate complexity. The completion of this work was only made possible by the use of the facilities of the IBM - FORMAC language available at McGill University Computing Centre. Thus, computer programs have been written in FORMAC and used to obtain a matrix formulation relating the harmonic current components to the applied voltages for some typical examples such as a 3-phase synchronous machine, a 2-phase induction machine and single phase synchronous and induction machines. The harmonic impedance matrix obtained using any one of these programs is then used in a conventional FORTRAN program to obtain numerical values for the current components under prescribed conditions.

Using the analytically computed matrices, a reduction of computing time of between 2 to 3 orders of magnitudes is achieved over earlier numerical iterative techniques. The results obtained using different methods are compared and found to be in very good agreement.

Experimental results for the single phase synchronous and induction machines show the excellence of the predictions of the analytical solution. The experimental and analytical studies for single phase machines show also that if single phase windings, or
their equivalent exist on both sides of the airgap, an infinite series of harmonic components field is set up. A similar situation is expected to arise when unbalanced loading or unbalanced faults occur in a 3-phase alternator without a complete damping winding on the rotor.

The areas of the future investigation based on this thesis are:

1. To develop a general computer program in FORMAC, or any similar language, to be available for expanding the equations of any machine in the generalized harmonic impedance matrix form.

2. To make use of such programs in predicting harmonics induced in machine windings due to different design considerations, and,

3. To develop programs able to optimize a certain machine design or connection in view of minimizing the harmonic effects.

A saturated machine, because of its large magnetizing current, has a low power factor and a slightly lower efficiency. A study of space harmonics as the result of magnetic saturation in the machine is also another field of application of the techniques described in this thesis. These space harmonics are of great practical significance especially at high saturation levels and their effect must be included to obtain greater accuracy in performance calculations.
APPENDIX 1

VOLTAGES INDUCED IN SYMMETRICAL WINDINGS

If we consider a wave of \( m \) pole pairs and \( \omega \) as the basic electrical frequency in rad / sec, the general form of the flux density component at an angle \( \varphi \) in the airgap is

\[
B_{\varphi} = B \sin (m \varphi - \omega t)
\]

where \( B \) is the peak flux density (webers / m\(^2\)).

The flux linkage with a winding whose axis is at an angle \( \beta \) is

\[
\lambda = \int_{\beta - \pi/2}^{\beta + \pi/2} B \sin (m \varphi - \omega t) \, d\varphi
\]

\[
= \frac{2B}{m} \sin \left( \frac{m \pi}{2} \right) \sin \left( m \beta - \omega t \right)
\]

The voltage induced in the winding is given by

\[
v = - \frac{d\lambda}{dt}
\]

\[
= \frac{2B}{m} \omega \sin \left( \frac{m \pi}{2} \right) \cos \left( m \beta - \omega t \right)
\]

This can be rewritten as

\[
V(m, \beta) = V_m \cos (\omega t - m \beta)
\]

This general formula is applied to spatially symmetrical windings to determine the relations between their induced voltages and currents.
For example in symmetrical 3-phase windings, the voltages induced can be expressed as:

\[ V_a = \sum_m V_m(m, \beta) = \sum_m V_m \cos(\omega t - m \beta) \]

\[ V_b = \sum_m V_m(m, \beta + 2\pi/3) = \sum_m V_m \cos(\omega t - m (\beta + 2\pi/3)) \]

and

\[ V_c = \sum_m V_m(m, \beta - 2\pi/3) = \sum_m V_m \cos(\omega t - m (\beta - 2\pi/3)) \]

while in symmetrical 2-phase windings

\[ V_a = \sum_m V_m(m, \beta) = \sum_m V_m \cos(\omega t - m \beta) \]

and

\[ V_b = \sum_m V_m(m, \beta + \pi/2) = \sum_m V_m \cos(\omega t - m (\beta + \pi/2)) \]

It is also deduced that the currents induced in the windings will have the same phase relations as the voltages.
APPENDIX II

3 - PHASE SYNCHRONOUS MACHINE

In the following sections, a conventional notation is used for the magnitudes of the different components of the voltages, currents, resistances and reactances. These are denoted by \( V \), \( I \), \( R \) and \( X \) followed by an alphanemic string indicating the particular component. \( F \) denotes the field winding, \( 1 \) and \( 2 \) stand for the direct and quadrature axis damper windings, while \( A \), \( B \) and \( C \) denote the different stator windings. The letters \( D \) and \( Q \) point to the direct and quadrature axes respectively. A stator related quantity is also denoted by \( S \) while \( R \) denotes the rotor related quantities. The letter \( T \) is used in the computer print-outs for \( Q \) and the FORMAC symbol \( \# P \) for \( \pi \). Some other abbreviations are used to contract the notations, and these are pointed out wherever felt necessary.
3-Phase Synchronous Machine,

4-Wire Star-Connection

1. Program Listing.

2. A Typical Data Set.

3. The Harmonic Impedance Matrix.

Note:

$V_D$ and $V_Q$ are derived from:

$v = v_a = V \cos \omega t$

$= V_D \cos \theta + V_Q \sin \theta$

where

$\theta = \omega t + \beta$

and

$\beta = 3\pi / 2 - \delta$

$\delta$ being the load angle.
/* THIS PROGRAM ANALYZES THE M.M.F. HARMONIC EFFECTS IN 3 PHASE SYNCHRONOUS MACHINES. */

THE FOLLOWING IS A LIST OF INPUT VARIABLES

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEANING</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW</td>
<td>NUMBER OF WINDINGS IN THE MACHINE</td>
</tr>
<tr>
<td>N</td>
<td>NUMBER OF EQUATIONS TO BE SOLVED</td>
</tr>
<tr>
<td>NSW</td>
<td>NUMBER OF STATOR WINDINGS</td>
</tr>
<tr>
<td>NU</td>
<td>NUMBER OF UNKNOWNS (CURRENT COMPONENTS)</td>
</tr>
<tr>
<td>NHAR</td>
<td>HIGHEST HARMONIC ORDER</td>
</tr>
<tr>
<td>WL</td>
<td>THE REACTANCE MATRIX</td>
</tr>
<tr>
<td>R</td>
<td>THE RESISTANCE DIAGONAL MATRIX</td>
</tr>
<tr>
<td>CU</td>
<td>THE CURRENT VECTOR</td>
</tr>
</tbody>
</table>
| CUDQ     | THE VARIABLE ARRANGEMENT VECTOR */

/* PROCEDURE OPTIONS (MAIN) */
FORMAC_OPTIONS;
DECLARE PUNCH FILE;
DCL STRING CHAR (66) VARYING;
DCL WL(6,6)CHAR(80);    
DCL CU(6) CHAR(240);    
DCL R (6) CHAR(10);    
DCL CUDQ (27) CHAR(10);    

/* INPUT DATA */
GET LIST (NW,N,NSW,NU,NHAR);
GET LIST (WL,R,CU,CUDQ);
/* INITIALIZATION */
IU=0;
IEI=0;
MNHAR =NHAR ;

/* MAIN DO LOOP FOR N EQUATIONS */
DO II = 1 TO N;
/* INITIALIZATION */
LET (II="II";Q=0);
NT=1;
DO I=MNHAR TO NHAR ;
    LET (I="I"; S(I)=0; C(I)=0 );
END ;
/* FOR NT=1 ANALYZE Q=D(I*1)/D(T) */
/* NT=2 ANALYZE Q=R*I */

START: IF NT = 2 THEN DO ;
    LET Q="(R(II)-""C(I)(II)"" )
END ;
ELSE DO ;
    DO JJ=1 TO NW ;
    LET ( JJ="JJ";
LET (Q=EXPAND (Q) );
NQ =NARGS (Q);  
//*******************************/  // ANALYSE EACH SUBEXPRESSION (E) OUT OF (NQ)  */  
//*******************************/
DO K =1 TO NQ ;
    LET (K"K"; E=ARG (K~Q); E1D =1);  
    //*******************************/  // IDENTIFY NEGATIVE (E)  */  
    /*******************************/
    ID =LOP (E);  
    IF ID =25 THEN DO ;
        LET (E=ARG(1,E) 1EID=-1) J
    END ;
    //*******************************/  // BREAK (E) INTO NE TERMS (G)*/
    /*******************************/
    NE =NARGS (E);  
    DO J=1 TO NE ;
        LET (J="J"; G(J)= ARG(J~E ) J
    END ;
    /*******************************/  // TEST EVERY G INDEX WILL TAKF THE FOLLOWING VALUES */
    /*******************************/
    IF LI =4 THEN INDEX =1  */
    /*******************************/
    IF LI =4 THEN INDEX =2  */
    /*******************************/
    IF LI =4 THEN INDEX =3  */
    /*******************************/
    IF LI =4 THEN INDEX =4  */
    /*******************************/

INDEX =0 ;
DO I=1 TO NE ; LET (I="I") ;
    /*******************************/  // TEST FOR SIN**2 OR COS**2 */
    /*******************************/
    LI =LOP (G(I) ) ;
    IF LI =31 THEN DO ;
        LET (G(I)=ARG(1,G(I)) I  
    END ;
    IF LI =4 THEN INDEX =1 ;
    ELSE INDEX =4 ;
    IF LI =4 THEN INDEX =2 ;
    ELSE INDEX =4 ;
    LET (A=ARG (1,G(I) ) ) B=A ) ;
    GO TO LBL1 ;
END ;

SYNC0590
SYNC0600
SYNC0610
SYNC0620
SYNC0630
SYNC0640
SYNC0650
SYNC0660
SYNC0670
SYNC0680
SYNC0690
SYNC0700
SYNC0710
SYNC0720
SYNC0730
SYNC0740
SYNC0750
SYNC0760
SYNC0770
SYNC0780
SYNC0790
SYNC0800
SYNC0810
SYNC0820
SYNC0830
SYNC0840
SYNC0850
SYNC0860
SYNC0870
SYNC0880
SYNC0890
SYNC0900
SYNC0910
SYNC0920
SYNC0930
SYNC0940
SYNC0950
SYNC0960
SYNC0970
SYNC0980
SYNC0990
SYNC1000
SYNC1010
SYNC1020
SYNC1030
SYNC1040
SYNC1050
SYNC1060
SYNC1070
SYNC1080
SYNC1090
SYNC1100
SYNC1110
SYNC1120
SYNC1130
SYNC1140
SYNC1150
SYNC1160
IF I=NE THEN GO TO LBL1;
I= I+1;
******************************************************************************
/* TEST REMAINING TERMS FOR A:OTHER SIN OR COS */
******************************************************************************
DO J=1 TO NE J
LET (J= "J") J;
LJ= LN P (G(J));
IF LJ =4 L J =5
THEN DO J
  IF LJ=4 THEN INDEX=INDEX-1
  J=J-1;
  LET(B=ARG(1,G(J)));
  GO TO LBL1;
END J;
END J;
******************************************************************************
/* INDEX =0 RETURN E */
******************************************************************************
LBL1: IF INDEX =0 THEN DO J
  LET ( C(0)= C(0) +E*EID ) J
  GO TO CONT J
END J;
******************************************************************************
/* RECONSTRUCT * E* EXCEPT FOR SING COS */
******************************************************************************
LET (E=EID J);
DO J =1 TO NE J
  IF J =JO J =10 THEN GO TO SKIPl;
  LET(J= "J") E=E*G(J));
SKIPl: END J;
******************************************************************************
/* FORM THE ARGUMENTS OF SIN(A+B) & COS(A+B) */
******************************************************************************
LET (X=A+B J;
  Z=A-B J;
  IX=COEFF(X,T) J;
  IZ=COEFF(Z,T) J;
  X=X-IX*T J;
  Z=IZ-T J;
  M=0.5 J;
******************************************************************************
/* ACCORDING TO INDEX BREAK E INTO COEFFICIENTS */
******************************************************************************
IF INDEX=31 INDEX=4 THEN DO J
  LET (M=-M J;
IF INDEX=11 INDEX=4 THEN DO J
  LET(S(IX)= M*E*SIN(X)+S(IX) J;
  C(IX)= -M*E*COS(X)+C(IX) J;
  S(IZ)=0.5*E*SIN(Z)+S(IZ) J;
  C(IZ)= 0.5*E*COS(Z)+C(IZ) J;
  GO TO CONT J;
END J;
IF INDEX=21 INDEX=3 THEN DO J
  END J;

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7
LET(S(IX) = 0.5*E*COS(X) + S(IX)) SYNCl750
C(IX) = 0.5*E*F^2 + C(IX)) SYNCl760
S(IZ) = H*E*COS(Z) + S(IZ)) SYNCl770
C(IZ) = H*E*SIN(Z) + C(IZ)) SYNCl780
END ;

CONT: END ;

IF NT=2 THEN GO TO CLCT;

END ;

IF NT=1 THEN DIFFERENTIATE EXPRESSIONS ;

END ;

LET (C(0)=0);

DO I=1 TO NHAR ;

END ;

NT = NT+1 ;

GO TO START ;

END ;

IF NT=1 THEN GO TO CLCT;

END ;

IF NT=2 THEN GO TO CLCT ;

DO I=1 TO NHAR ;

END ;

IF NT=1 THEN DIFFERENTIATE EXPRESSIONS ;

END ;

IF NT=2 THEN GO TO CLCT ;

DO I=1 TO NHAR ;

END ;

IF NT=1 THEN DIFFERENTIATE EXPRESSIONS ;

END ;
/* TYPICAL DATA FOR A 4 WIRE, STAR CONNECTED, 3 PHASE */
/* SYNCHRONOUS MACHINE, WITH 2 DAMPER WINDINGS. */

6 4 3 27 7

`X1F`
`X2F`
`XAF*COS(T)+XAF3*COS(3*T)`
`XAF*COS(T-2*#P/3)+XAF3*COS(3*T)`
`XAF*COS(T+2*#P/3)+XAF3*COS(3*T)`
`XFI`
`X1I`
`XAI*COS(T)`
`XAI*COS(T-2*#P/3)`
`XAI*COS(T+2*#P/3)`
`X2I`
`X2I`

`X1F= X2F= XAF*COS(T)+XAF3*COS(3*T)`
`XAF=COS(T-2*#P/3)+XAF3*COS(3*T)`
`XAF=COS(T+2*#P/3)+XAF3*COS(3*T)`
`XFI= X1I= XAI*COS(T)`
`XAI=COS(T-2*#P/3)`
`XAI=COS(T+2*#P/3)`
`X2I= X2I= XAF*COS(T)+XAF3*COS(3*T)`

`RF` 'RA' 'R2' 'RA' 'RA' 'RA'

`IF0+IFD2*COS(2*T)+IFG2*SIN(2*T)+IF04*COS(4*T)+IFQ4*SIN(4*T)`
`+IFD6*COS(6*T)+IFG6*SIN(6*T)`
`+ID02*COS(2*T)+ID02*SIN(2*T)+ID04*COS(4*T)+IDQ4*SIN(4*T)`
`+ID06*COS(6*T)+IDQ6*SIN(6*T)`
`+IZ02*COS(2*T)+IZ02*SIN(2*T)+IZ04*COS(4*T)+IZQ4*SIN(4*T)`
`+IZ06*COS(6*T)+IZQ6*SIN(6*T)`
`+ID1*COS(T)+ID1*SIN(T)+ID3*COS(3*T)+IDQ3*SIN(3*T)`
`+ID5*COS(5*T)+IDQ5*SIN(5*T)`
`+IQ5*SIN(5*T)+IQ7*SIN(7*T)`
`+IQ7*SIN(7*T-2*#P/3)`

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\[101 \cdot \cos(T + 2\pi/3) + \text{IQ1} \cdot \sin(T + 2\pi/3) + \text{IQ3} \cdot \cos(3T) + \text{IQ3} \cdot \sin(3T) + \text{IQ5} \cdot \cos(5T - 2\pi/3) + \text{IQ7} \cdot \sin(5T - 2\pi/3) + \text{IQ7} \cdot \sin(7T + 2\pi/3)\]
<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>RF</td>
<td>R</td>
<td>X=02</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>RF</td>
<td>XF=6</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>RF</td>
<td>XF=6</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td>RF</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>X=02</td>
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</tr>
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</tr>
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<td>22</td>
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<td>XAF=1.5</td>
<td>XAF=1.5</td>
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<tr>
<td>24</td>
<td></td>
<td></td>
<td>XAF=2.5</td>
<td>XAF=2.5</td>
</tr>
<tr>
<td>26</td>
<td></td>
<td></td>
<td>XAF=3.5</td>
<td>XAF=3.5</td>
</tr>
</tbody>
</table>
B - 3-PHASE SYNCHRONOUS MACHINE,

3-WIRE STAR-CONNECTION

1. A Typical Data Set.

2. The Harmonic Impedance Matrix.

Notes:

1. The following abbreviations are used:

\[ XX = X_{AA} + X_{AB} \]
\[ XX2 = X_{AA}^2 + 2 \times X_{AB}^2 \]
\[ XX4 = X_{AA}^4 - 2 \times X_{AB}^4 \]

2. \( V_D \) and \( V_Q \) are derived from

\[ V = V_a - V_c \]
\[ = V \cos \omega t - V \cos (\omega t + 2\pi/3) \]
\[ = V_D \cos \theta + V_Q \sin \theta \]
TYPICAL DATA FOR 3 WIRE STAR CONNECTED, 3 PHASE SYNCHRONOUS MACHINE WITH 2 DAMPER WINDINGS

\[
\begin{align*}
&FX' \\
&XF' \\
&'0' \\
&'Z0*XAF*SIN(T+\#P/3)*SQR(3.0)/2.0' \\
&'Z0*XAF*SIN(T)*SQR(3.0)/2.0' \\
&'XF' \\
&'X1' \\
&'0' \\
&'Z0*XAL+SIN(T+\#P/3)*SQR(3.0)/2.0' \\
&'Z0*XAL+SIN(T)*SQR(3.0)/2.0' \\
&'X1' \\
&'X2' \\
&'2.0*X2*SQR(3.0)/2.0*COS(T+\#P/3)' \\
&'2.0*X2*SQR(3.0)/2.0*COS(T)' \\
&'2.0*X2+SQR(3.0)/2.0*COS(T+\#P/3)' \\
&'XX-XX2*SQR(2*P/3)-XX4*COS(4*P/3)' \\
&'XX-XX2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XX-XX2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XX-XX2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XX-XX2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
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&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
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&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
&'XY+XY2*COS(2*P/3)-XX4*COS(4*P/3)' \\
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APPENDIX III

2 - PHASE INDUCTION MOTOR

1. Program Listing.

2. A Typical Data Set.

3. The Harmonic Impedance Matrix.

Notes:

1. The abbreviation

\[ F = \frac{\text{Rotation Frequency}}{\text{Supply Frequency}} = \frac{f}{f_0} \]

is used.

2. The letters \( P \) and \( N \) are added to the harmonic currents to denote the positive and negative frequency components respectively.
/* THIS PROGRAM ANALYZES THE M, M, F, HARMONIC
   EFFECTS IN 2 PHASE INDUCTION MOTOR WITH PHASE
   WOUND ROTORS.

   THE FOLLOWING IS A LIST OF INPUT VARIABLES

   VARIABLE                     MEANING
   ------------------------------------------------------------
   NW  NUMBER OF WINDINGS IN THE MACHINE
   N   NUMBER OF EQUATIONS TO BE SOLVED
   NU  NUMBER OF UNKNOWNS (CURRENT COMPONENTS)
   NHAR HIGHEST HARMONIC ORDER
   WL  THE REACTANCE MATRIX
   R   THE RESISTANCE DIAGONAL MATRIX
   CU  THE CURRENT VECTOR
   CUOQ THE VARIABLE ARRANGEMENT VECTOR */

WVFRT: PROCEDURE OPTIONS (MAIN);
FORMAC_OPTIONS;
DECLARE PUNCH FILE;
DCL STRING CHAR(66) VARYING;
DCL WL(4,4)CHAR(80)J
DCL CU(4)  CHAR(400);
DCL R(4)  CHAR(10); CUDQ(38) CHAR(10);

GET LIST (NW,N,NU,NHAR);
GET LIST (WL,R,CU,CUDQ);

** INITIALIZATION */

IU=0;
IEI=0;
MNHR=-NHAR;

GET(ZERO=0);

** MAIN DO LOOP FOR N EQUATIONS */

DO II=1 TO N;
    /* INITIALIZATION */

LET (II="II";Q=0);
NT=1;
DO I=MNHR TO NHAR;
    LET (I="I"; S(I)=0; C(I)=0 );
END;

END;

** FOR NT=1  ANALYZE Q=D(L*I)/D(Y) */

** FOR NT=2  ANALYZE Q=R*I */

START: IF NT =2 THEN DO;
    LET (Q="R(II)"**C(I="II")");
END;
ELSE DO;
    DO JJ=1 TO NW;
LET (JJ="JJ");
Q="ML(II;JJ)*""CU(JJ)+Q");
END;
END;
LET (Q=EXPAND (Q));
NQ = NARGS (Q);
******************************************************************************
/* ANALYZE EACH SUBEXPRESSION (E) OUT OF (NQ) */
******************************************************************************
DO K = 1 TO NQ :
   LET(K="K"); E=ARG (K;Q); EID = 1);
******************************************************************************
/* IDENTIFY NEGATIVE (E) */
******************************************************************************
   ID = LOP (E);
   IF ID = 25 THEN DO ; LET(E=ARG(1;E);EID=1) J
END ;
******************************************************************************
/* BREAK (E) INTO NE TERMS (G) */
******************************************************************************
NE = NARGS (E);
DO J=1 TO NE ;
   LET (J="J"); G(J)= ARG(J;E));
END ;
ID=0;
******************************************************************************
/* TEST EVERY G INDEX WILL TAKE THE FOLLOWING VALUES */
******************************************************************************
/*
0 NO SIN OR COS
1 SIN
2 SIN*2 OR SIN*SIN
3 COS
4 COS**2 OR COS*COS OR COS
******************************************************************************
INDEX =0 ;
DO I=1 TO NE ; LET (I="I");
   JO=0 J
******************************************************************************
/* TEST FOR SIN**2 OR COS**2 */
******************************************************************************
   LI = LOP (G(I));
   IF LI=31 THEN DO ;
      LET(G(I)=ARG(1;G(I))) J
      LI=LOP(G(I));
      IF LI =4 THEN INDEX =1;
      ELSE INDEX =4 ;
      IO=I ;
      LET (A=ARG(1;G(I)); B=A) ;
      GO TO LBL1;
   END ;
******************************************************************************
/* TEST FOR SIN OR COS */
******************************************************************************
   IF LI=4 AND LI=5 THEN DO ;
      IF LI =4 THEN INDEX =2 ;
      ELSE INDEX =4 ;
      IO=I ;
LET (A=ARG (1, G(I))); B=0 ;
IF I=NE THEN GO TO LBL1;
I=I+1 ;
******************************************************************************
/* TEST REMAINING TERMS FOR ANOTHER SIN OR COS */
******************************************************************************
DO J=I1 TO NE ;
LET (J=J") ;
LJ=LOP (G(J));
IF LJ =4 | LJ =5 THEN DO J ;
    IF LJ=4 THEN INDEX=INDEX-1 ;
    J0=J ;
    LET(B=ARG(1,G(J)));
    GO TO LBL1 ;
END ;
END ;
******************************************************************************
LBL1 :IF INDEX =0 THEN DO ;
    LET ( C(0)= C(0) *E=EID ) ;
    GO TO CONT ;
END ;
******************************************************************************
/* RECONSTRUCT " E" EXCEPT FOR SIN & COS */
******************************************************************************
LET ( E=EID : );
DO J =1 TO NE ;
    IF J =JO| J=10 THEN GO TO SKIP1 ;
    LET(J="J"; E=E*G(J)) ;
SKIP1 : END ;
******************************************************************************
/* FORM THE ARGUMENTS OF SIN(A+B) & COS(A+B) */
******************************************************************************
LET ( X=A+B ;
    Z=A-B ;
    JX=COEFF (X; TO) ;
    JZ=COEFF (Z; TO) ;
    IZ=-1 ;
    M=0 .5 );
    IF IDENT(JZ;I1) THEN DO ;
        LET(JZ=-JZ ;
        Z=-Z);
        IF INDEX =2 | IND FX =3 THEN DO ;
            LET (M=M) ;
        END ;
        LET ( IX=COEFF (X; T) ;
        IZ=COEFF (Z; T) ;
        X=X-IX*T-JX*TO ;
        Z=IZ*T-JZ*TO ) ;
******************************************************************************
/* ACCORDING TO INDEX BREAK E INTO COEFFICIENTS */
/* FOR SIN & COS */
******************************************************************************
IF INDEX=3 I INDEX=4 THEN DO
    LET (M=-M);
END;

IF INDEX=1 I INDEX=4 THEN DO
    LET(S(I)= M*E*SIN(X)+S(I))
    C(I)= M*E*COS(X)+C(I))
    S(I)=0.5*E*SIN(Z)+S(I))
    C(I)= 0.5*E*COS(Z)+C(I))
GO TO CONT ;
END;

IF INDEX=1 I INDEX=3 THEN DO
    LET SCIX) =
    CCIX) = .H*E*COS(X)+C(IX))
    CCII) = 0.5*E*SIN(Z)+S(IZ))
    CCII) = 0.5*E*COS(Z)+S(IZ))
    GO TO START ;
END;

CONT: END :

IF NT=2 THEN GO TO OUTP ;

/***************************************************************************/
/* IF NT=1 ,DIFFERENTIATE EXPRESSIONS */
/***************************************************************************/
DO I=MHAR TO NHR :
    LET (I="I"
    E=S(I) ;
    S(I)=EXPAND((-1.0+I+F )*EI));
    C(I)=EXPAND((1.0+I+F )*E));
END ;
NT =NT+1 ;
GO TO START ;

/***************************************************************************/
/* PRINT & PUNCH OUTPUT IN A PROPERLY SELECTED FORM */
/***************************************************************************/
OUTP: IF II=2 THEN IU=1 :
DO J =1 TO NU :
    LET ( J="J" : W="CUDQ(J)" ) ;
    IE =IEI :
    DO I = IU TO NHR BY 2 :
        IE = IE + 1 :
        LET ( IE ="IE" ; I="I"
        A(IE,J) = COEFF ( C(-I),W ));
        IF IDENT(A(IE,J),ZERO) THEN GO TO NEXT1 ;
        PRINT_OUT (A(IE,J) ;
        CHAREX (STRING = A(IE,J)) ;
        PUT FILE (PUNCH) EDIT(STRING) (SKIP(1),X(6),A(66)) ;
        IF IE=IE+1 ;
        LET ( IE ="IE" ;
        A(IE,J)=COEFF(S(-I),W ));
        IF IDENT(A(IE,J),ZERO) THEN GO TO NEXT2 ;
        PRINT_OUT (A(IE,J)) ;
        NEXT1:

        CHAREX (STRING = A(IE,J)) ;
        PUT FILE (PUNCH) EDIT(STRING) (SKIP(1),X(6),A(66)) ;
        IF I=0 THEN DO ; IE =IE+1 ;
        LET ( IE = "IE" ;
        A(IE,J)= COEFF ( C(1),W ));
        IF IDENT(A(IE,J),ZERO) THEN GO TO NEXT3 ;
        PRINT_OUT(A(IE,J)) ;
        NEXT2:

        CHAREX (STRING = A(IE,J)) ;
        PUT FILE (PUNCH) EDIT(STRING) (SKIP(1),X(6),A(66)) ;
        IF I=0 THEN DO ; IE =IE+1 ;
IE=IE+1;
LET (IE="IE";
A(IE,J)=COEFF(S(I),W ));
IF IDENT(A(IE,J); ZERO) THEN GO TO NEXT4;
PRINT_OUT ( A(IE,J));
NEXT4:
CHAREX (STRING = A(IE,J));
PUT FILE (PUNCH) EDIT(STRING) (SKIP(1),X(6),A(66));
END;
END;
IE=IE;
END;
END WVFRM ;
TYPICAL DATA FOR 2 PHASE INDUCTION MOTOR

\[
\begin{align*}
X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
0
\end{align*}
\]

\[
\begin{align*}
-X_1 \sin(T) + X_3 \sin(3T) - X_5 \sin(5T) + X_7 \sin(7T) - X_9 \sin(9T) \\
0
\end{align*}
\]

\[
\begin{align*}
X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
0
\end{align*}
\]

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\begin{align*}
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\begin{align*}
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\begin{align*}
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\end{align*}
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X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
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-X_1 \sin(T) + X_3 \sin(3T) - X_5 \sin(5T) + X_7 \sin(7T) - X_9 \sin(9T) \\
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X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
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\end{align*}
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\begin{align*}
-X_1 \sin(T) + X_3 \sin(3T) - X_5 \sin(5T) + X_7 \sin(7T) - X_9 \sin(9T) \\
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\begin{align*}
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\begin{align*}
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0
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\]

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\begin{align*}
-X_1 \sin(T) + X_3 \sin(3T) - X_5 \sin(5T) + X_7 \sin(7T) - X_9 \sin(9T) \\
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\end{align*}
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\begin{align*}
X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
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\begin{align*}
X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
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\end{align*}
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\begin{align*}
-X_1 \sin(T) + X_3 \sin(3T) - X_5 \sin(5T) + X_7 \sin(7T) - X_9 \sin(9T) \\
0
\end{align*}
\]

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\begin{align*}
X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
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X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
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\begin{align*}
X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
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0
\end{align*}
\]

\[
\begin{align*}
X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \\
0
\end{align*}
\]
APPENDIX IV

SINGLE PHASE SYNCHRONOUS MACHINE

1. A Typical Data Set.

2. The Harmonic Impedance Matrix.

Notes:

1. The following abbreviations are used:

\[
XX = XAA + XAB \\
XX2 = XAA2 + 2 \times XAB2 \\
XX4 = XAA4 - 2 \times XAB4
\]

2. \( V_D \) and \( V_Q \) are derived from

\[
V = V_a - V_b \\
= V \cos \omega t - V \cos (\omega t - \frac{2\pi}{3}) \\
= V_D \cos \theta + V_Q \sin \theta
\]

where

\[
\theta = \omega t + \beta ,
\]

and

\[
\beta = \frac{3\pi}{2} - \delta .
\]
/*******************************************************/
/* TYPICAL DATA FOR SINGLE PHASE SYNCHRONOUS MACHINE*/
/*******************************************************/

4 4 1 27 7
'XF1'
'XF1'
'0'
'\sqrt{3.0} \cdot X_2 \cdot \cos(T + \pi / 6.0)''
'X1'
'0'
'\sqrt{3.0} \cdot X_1 \cdot \cos(T + \pi / 6.0)''
'0'
'0'
'X2'
'\sqrt{3.0} \cdot X_2 \cdot \sin(T + \pi / 6.0)''
'\sqrt{3.0} \cdot X_1 \cdot \cos(T + \pi / 6.0)''
'\sqrt{3.0} \cdot X_2 \cdot \sin(T + \pi / 6.0)''
'2 \cdot X_2 \cdot \cos(2 \cdot T + \pi / 3) + X_4 \cdot \cos(4 \cdot T - \pi / 3)''
'RE1' 'R1' 'R2' '2*RA'
'\text{IF0} + \text{IFD2} \cdot \cos(2 \cdot T) + \text{IFQ2} \cdot \sin(2 \cdot T) + \text{IFD4} \cdot \cos(4 \cdot T) + \text{IFQ4} \cdot \sin(4 \cdot T)'
'\text{IF06} \cdot \cos(6 \cdot T) + \text{IF06} \cdot \sin(6 \cdot T)''
'\text{ID02} \cdot \cos(2 \cdot T) + \text{ID02} \cdot \sin(2 \cdot T) + \text{ID04} \cdot \cos(4 \cdot T) + \text{ID04} \cdot \sin(4 \cdot T)'
'\text{ID06} \cdot \cos(6 \cdot T) + \text{ID06} \cdot \sin(6 \cdot T)''
'\text{ID1} \cdot \cos(T) + \text{ID1} \cdot \sin(T) + \text{ID3} \cdot \cos(3 \cdot T) + \text{ID3} \cdot \sin(3 \cdot T) + \text{ID5} \cdot \cos(5 \cdot T)'
'\text{ID06} \cdot \sin(6 \cdot T) + \text{ID06} \cdot \sin(6 \cdot T)''
'\text{ID1} \cdot \cos(T) + \text{ID02} \cdot \sin(T) + \text{ID02} \cdot \cos(T) + \text{ID04} \cdot \sin(T) + \text{ID04} \cdot \cos(T)'
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APPENDIX V

SINGLE PHASE - SINGLE PHASE INDUCTION MACHINE

1. A Typical Data Set.

2. The Harmonic Impedance Matrix.

Note:

The abbreviation

\[ F = \frac{\text{Rotation Frequency}}{\text{Supply Frequency}} = \frac{f_r}{f_o} \]

is used.
# TYPICAL DATA FOR A SINGLE PHASE/SINGLE PHASE MACHINE

\[ X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \]

\[ X_1 \cos(T) + X_3 \cos(3T) + X_5 \cos(5T) + X_7 \cos(7T) + X_9 \cos(9T) \]

\[ X_{R1} \cos(T_0) + X_{R2} \cos(T_0 + 2T) + X_{R3} \cos(T_0 + 3T) + X_{R4} \cos(T_0 + 4T) + X_{R5} \cos(T_0 + 5T) + X_{R6} \cos(T_0 + 6T) + X_{R7} \cos(T_0 + 7T) + X_{R8} \cos(T_0 + 8T) + X_{R9} \cos(T_0 + 9T) \]

\[ X_{R1} \cos(T_0) + X_{R2} \cos(T_0 + 2T) + X_{R3} \cos(T_0 + 3T) + X_{R4} \cos(T_0 + 4T) + X_{R5} \cos(T_0 + 5T) + X_{R6} \cos(T_0 + 6T) + X_{R7} \cos(T_0 + 7T) + X_{R8} \cos(T_0 + 8T) + X_{R9} \cos(T_0 + 9T) \]
REFERENCES


